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Multivariable State Feedback for Output Tracking MRAC for Piecewise Linear Systems

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***Abstract*—This paper studies the adaptive control problem of multi-input, multi-output (MIMO) piecewise linear systems, a class of linear systems with switched parameters. A direct state feedback model reference adaptive control (MRAC) scheme is developed based on the LDS decomposition of high frequency gain matrices of such systems to achieve closed-loop signal boundedness and asymptotic output tracking performance. Simulation results on linearized NASA GTM models are presented to demonstrate the effectiveness of the proposed scheme.**

I. Introduction

Adaptive control of plants that are modeled or approximated by linear time-invariant (LTI) systems has been studied extensively in the literature [7], [15]. However, in many practical applications an LTI system model may be insufficient to describe an actual plant. As an attempt to meet increasing performance requirements over a wide range of operating conditions, we consider the adaptive control problem of piecewise linear systems in this paper.

By a piecewise system (also called a "switched system" in the literature by many researchers), we mean a dynamical system whose dynamics switches among a set of continuous-time subsystems according to certain switching criteria. The motivation to study piecewise linear systems is two-fold. On the one hand, as shown in [10], [12], a nonlinear system may be modeled as a piecewise linear system for control design, which is expected to be capable of expanding the system operating range. On the other hand, many practical systems are of a hybrid nature and require several dynamical subsystems to describe their behavior [8], e.g., the motion of an automobile subject to a manual or an automatic transmission [1] and power electronics [14]. Such systems also arise in aircraft flight control applications, and a typical example is the linearized dynamics of an aircraft at some chosen operating points over its flight envelope, each corresponding to a set of constant parameter matrices. With a sufficient number of operating points chosen, transitions among them can be modeled as parameter switches.

Despite the tremendous growth of research interest in stability analysis and control design of such systems over the past two decades (see [8], [13] and the references therein), an adaptive control approach to piecewise systems is largely unexploited. An adaptive control scheme was presented in [3] for bimodal piecewise linear systems, but the assumption of canonical forms limits its applicability to system with more general structures. The proposed research focuses on the development of model reference adaptive control (MRAC) designs for piecewise linear systems to achieve closed-loop stability (signal boundedness) and asymptotic tracking performance, in the presence of structural and parametric uncertainties and repetitive system mode switches. Preliminary investigation by the authors has shown that under a slow system mode switch condition, the state feedback for state tracking design [10] can achieve closed-loop stability and a small state tracking error in the mean-square sense (asymptotic tracking for arbitrary system mode switches under certain matching conditions [2]). Asymptotic tracking performance is restored under the persistency of excitation condition. For the output tracking design [11], asymptotic tracking is ensured in addition to closed-loop stability. For both designs, stability and asymptotic tracking are accomplished for arbitrary system mode switches if a common Lyapunov function exists. This paper is an extension of the adaptive control design in [11] for SISO piecewise linear systems to the MIMO case. It is shown that with the proposed MRAC schemes, closed-loop stability and asymptotic tracking performance are achieved for such systems, if the occurrence frequency of parameter discontinuities is sufficiently low.

The paper is organized as follows. The formulation of the adaptive control problem for piecewise linear systems is presented in section II. In section III, the non-adaptive model reference control problem is considered, and an MRAC design is proposed, with the stability results established in Section IV. For demonstration of the effectiveness of the proposed adaptive control schemes, an illustrative example is given in Section V, and some concluding remarks and future work are given in Section VI.

II. Problem Statement

The adaptive state feedback control problem is formulated for a piecewise linear system to make its output track a desired trajectory generated from a reference model system. Indicator functions are introduced to characterize system parameter discontinuities, based on which an MRAC approach to such a control problem is proposed in Section III.

A. *Piecewise Linear Systems*

Consider an M-input, M-output piecewise linear system

$$\begin{aligned}\dot{\boldsymbol{x}}(t) &= \boldsymbol{A}(t)\boldsymbol{x}(t) + \boldsymbol{B}(t)\boldsymbol{u}(t),\\ \boldsymbol{y}(t) &= \boldsymbol{C}^{\mathsf{T}}\boldsymbol{x}(t), \;\; \boldsymbol{x}(0) = \boldsymbol{x}_0,\end{aligned} \tag{1}$$

where $\boldsymbol{x}(t) \in \mathbb{R}^n$ is the state vector and is available for measurement, $\boldsymbol{u}(t) \in \mathbb{R}^M$ is the control input to be specified by an adaptive control law, $\boldsymbol{y}(t) \in \mathbb{R}^M$ is the controlled output, $\boldsymbol{A}(t) \in \mathbb{R}^{n\times n}$ and $\boldsymbol{B}(t) \in \mathbb{R}^{n\times M}$ are unknown time-varying system parameter matrices, and $\boldsymbol{C} \in \mathbb{R}^{n\times M}$ is an unknown constant parameter matrix. The unknown time-varying matrices $\boldsymbol{A}(t)$ and $\boldsymbol{B}(t)$ can be expressed as

$$\boldsymbol{A}(t) = \sum_{i=1}^{l} \boldsymbol{A}_i \chi_i(t), \quad \boldsymbol{B}(t) = \sum_{i=1}^{l} \boldsymbol{B}_i \chi_i(t), \tag{2}$$

where the parameter matrix set $(\boldsymbol{A}_i, \boldsymbol{B}_i)$, $i \in \mathcal{I} \triangleq \{1, 2, \ldots, l\}$, is called the ith system mode, and l is the total number of system modes. Here, to characterize the system mode switches and for a simple notation, the *indicator functions* are introduced as follows:

$$\chi_i(t) = \begin{cases} 1, & \text{if } (\boldsymbol{A}(t), \boldsymbol{B}(t)) = (\boldsymbol{A}_i, \boldsymbol{B}_i), \\ 0, & \text{otherwise.} \end{cases}$$

Since at each specific time instant t, the piecewise linear system (1) can operate in one and only one system mode, the indicator functions have the following properties $\sum_{i=1}^{l} \chi_i(t) = 1$, $\chi_j(t)\chi_k(t) = 0$, $j \neq k$, $j, k \in \mathcal{I}$.

The indicator functions contain knowledge of the durations of time the system resides in each mode and the time instants at which mode switches occur, which is useful for adaptive control design. It is assumed that system mode switches can be detected instantaneously; that is, although $\chi_i(t)$ may *not* be known *a priori*, they are available during system operation.

B. *Control Objective*

The control objective is to develop a state feedback control law $\boldsymbol{u}(t)$ for the piecewise linear system (1) with parameter variations characterized in (2) such that all signals in the closed-loop system are bounded, and the plant output $\boldsymbol{y}(t)$ asymptotically tracks a reference signal $\boldsymbol{y}_m(t)$, i.e., $\lim_{t\to\infty}(\boldsymbol{y}(t) - \boldsymbol{y}_m(t)) = 0$, with $\boldsymbol{y}_m(t)$ generated from a reference model system

$$\boldsymbol{y}_m(t) = \boldsymbol{W}_m(s)[\boldsymbol{r}](t), \ \boldsymbol{W}_m(s) = \boldsymbol{\xi}_m^{-1}(s), \tag{3}$$

where $\boldsymbol{\xi}_m(s)$ is a common modified left interactor matrix for the transfer matrix of each system mode, i.e.,

$$\boldsymbol{G}_i(s) = \boldsymbol{C}^\top (s\boldsymbol{I} - \boldsymbol{A}_i)^{-1} \boldsymbol{B}_i, \ \ i \in \mathcal{I}, \tag{4}$$

and $\boldsymbol{r}(t)$ is a bounded, piecewise continuous external reference input signal.

III. Adaptive Control Design

A new state feedback controller structure is proposed in this section for the piecewise linear system (1) to achieve closed-loop stability (signal boundedness) and asymptotic output tracking performance. The non-adaptive model reference control problem is considered first, and a gradient design is then presented to solve the adaptive control problem.

Assumptions. Suppose for the ith system mode the transfer matrix of the system is as in (4). To design an adaptive state feedback control law for output tracking, the following assumptions are made for $i \in \mathcal{I}$:

(A1) $(\boldsymbol{A}_i, \boldsymbol{B}_i)$ are stabilizable and $(\boldsymbol{C}, \boldsymbol{A}_i)$ are observable.

(A2) The zeros of $\boldsymbol{G}_i(s)$ are stable.

(A3) $\boldsymbol{G}_i(s)$ are strictly proper, have full rank, and a common modified left interactor matrix $\boldsymbol{\xi}_m(s)$ is known.

(A4) All leading principle minors Δ_{ij}, $j = 1, 2, \ldots, M$, of the high frequency gain matrices, defined as $\boldsymbol{K}_{pi} = \lim_{s\to\infty} \boldsymbol{\xi}_m(s)\boldsymbol{G}_i(s)$, are nonzero with known signs.

(A5) The LDS decompositions of $\boldsymbol{K}_{pi}$ [6] are such that

$$\boldsymbol{K}_{pi} = \boldsymbol{L}_s \boldsymbol{D}_{si} \boldsymbol{S}_i, \tag{5}$$

where $\boldsymbol{L}_s \in \mathbb{R}^{M\times M}$ is unity lower triangular, $\boldsymbol{S}_i \in \mathbb{R}^{M\times M}$ are symmetric, positive definite, and

$$\boldsymbol{D}_{si} = \text{diag}\{\text{sign}[\Delta_{i1}]\gamma_{i1}, \ldots, \text{sign}[\frac{\Delta_{iM}}{\Delta_{iM-1}}]\gamma_{iM}\}$$

with $\gamma_{ij} > 0$, $j = 1, 2, \ldots, M$, being arbitrary.

Under Assumption (A4), a gain matrix $\boldsymbol{K}_p \in \mathbb{R}^{M\times M}$ with nonzero leading principle minors Δ_j, $j = 1, 2, \ldots, M$, has a unique LDU decomposition [6], [15], i.e., $\boldsymbol{K}_p = \boldsymbol{L}\boldsymbol{D}^*\boldsymbol{U}$, where $\boldsymbol{L}$ is unity lower triangular, $\boldsymbol{U}$ is unity upper triangular, and $\boldsymbol{D}^* = \text{diag}\{\Delta_1, \frac{\Delta_2}{\Delta_1}, \ldots, \frac{\Delta_M}{\Delta_{M-1}}\}$. Note that its LDS decomposition, $\boldsymbol{K}_p = \boldsymbol{L}_s\boldsymbol{D}_s\boldsymbol{S}$, is *not* unique in that it follows from the unique LDU decomposition with $\boldsymbol{L}_s = \boldsymbol{L}\boldsymbol{D}_s\boldsymbol{U}^{-\top}\boldsymbol{D}_s^{-1}$, $\boldsymbol{S} = \boldsymbol{U}^\top\boldsymbol{D}_s^{-1}\boldsymbol{D}^*\boldsymbol{U}$, and $\boldsymbol{D}_s = \text{diag}\{\text{sign}[\Delta_1]\gamma_1, \ldots, \text{sign}[\frac{\Delta_M}{\Delta_{M-1}}]\gamma_M\}$, whose diagonal elements $\gamma_j > 0$ may be chosen arbitrarily.

Assumption (A5), that is, the high frequency gain matrix $\boldsymbol{K}_{pi}$ of each system mode is assumed to have a common $\boldsymbol{L}_s$ matrix in their LDS decompositions (5), is essential in the derivation of the error model for adaptive control design to avoid differentiation operations on the output tracking error signal $\boldsymbol{e}(t)$, which is undesirable in practical applications with the presence of noises in signals. This assumption is automatically satisfied for the case when the system is single-input, single-output [11]. For a set of high frequency gain matrices $\boldsymbol{K}_{pi} \in \mathbb{R}^{M\times M}$ with $M > 1$, $\boldsymbol{D}_s$ may be carefully chosen such that a common $\boldsymbol{L}_s$ follows. This is illustrated with a simulation example in Section V.

A. *Nominal Control Scheme*

When system parameters are known, a model reference controller can be applied to achieve closed-loop signal boundedness and asymptotic (exponential) tracking performance.

Controller structure. If $\boldsymbol{A}_i$ and $\boldsymbol{B}_i$, $i \in \mathcal{I}$, are known, the following state feedback control law is applied

$$\boldsymbol{u}(t) = \boldsymbol{K}_x^{*\top}(t)\boldsymbol{x}(t) + \boldsymbol{K}_r^*(t)\boldsymbol{r}(t) \tag{6}$$

with the controller parameters $\boldsymbol{K}_x^*(t) = \sum_{i=1}^{l} \boldsymbol{K}_{xi}^*\chi_i(t)$ and $\boldsymbol{K}_r^*(t) = \sum_{i=1}^{l} \boldsymbol{K}_{ri}^*\chi_i(t)$, where $\boldsymbol{K}_{xi}^* \in \mathbb{R}^{n\times M}$ and $\boldsymbol{K}_{ri}^* \in \mathbb{R}^M$ are defined to satisfy the plant-model matching condition:

$$\boldsymbol{C}^\top(s\boldsymbol{I} - \boldsymbol{A}_i - \boldsymbol{B}_i\boldsymbol{K}_{xi}^{*\top})\boldsymbol{B}_i\boldsymbol{K}_{ri}^* = \boldsymbol{W}_m(s), \ \boldsymbol{K}_{ri}^{*-1} = \boldsymbol{K}_{pi}. \tag{7}$$

The existence of $\boldsymbol{K}_{xi}^*$ and $\boldsymbol{K}_{ri}^*$ is guaranteed under Assumptions (A1)–(A2) [4]. Furthermore, $\boldsymbol{A}_i + \boldsymbol{B}_i\boldsymbol{K}_{xi}^{*\top}$ are stable.

Substituting (6) in (1) leads to the closed-loop system

$$\begin{aligned} \dot{\boldsymbol{x}}(t) &= \sum_{i=1}^{l} \left((\boldsymbol{A}_i + \boldsymbol{B}_i\boldsymbol{K}_{xi}^{*\top})\chi_i\boldsymbol{x}(t) + \boldsymbol{B}_i\boldsymbol{K}_{ri}^*\chi_i\boldsymbol{r}(t)\right) \\ \boldsymbol{y}(t) &= \boldsymbol{C}^\top\boldsymbol{x}(t), \end{aligned} \tag{8}$$

Note that from (7)–(8), the output tracking error can be expressed as $\boldsymbol{e}(t) = \boldsymbol{y}(t) - \boldsymbol{y}_m(t) = \boldsymbol{\epsilon}_0(t)$ with $\boldsymbol{\epsilon}_0(t)$ an initial condition related term, $\boldsymbol{\epsilon}_0(t) = \boldsymbol{C}^\top\boldsymbol{\Phi}(t,t_0)\boldsymbol{x}(t_0)$, where $\boldsymbol{\Phi}(t,t_0)$ is the state transition matrix associated with the homogeneous part of the system (8):

$$\dot{\boldsymbol{z}}(t) = \boldsymbol{A}_m(t)\boldsymbol{z}(t), \tag{9}$$

where $\boldsymbol{A}_m(t) = \sum_{i=1}^{l}\boldsymbol{A}_{mi}\chi_i(t)$ with $\boldsymbol{A}_{mi} \triangleq \boldsymbol{A}_i + \boldsymbol{B}_i\boldsymbol{K}_{xi}^{*\top}$ being stable.

Exponential stability of (9) is sufficient for stability of (8), which has been studied in [5]. It is well known that for (9) to be exponentially stable, the time interval between two consecutive mode switches should be long enough. Let the increasing time sequence $\{t_k\}_{k=1}^{\infty}$ denote the time instants at which system mode switches occur, T_0 the minimum switching time interval, i.e., $T_0 = \min_{k\in\mathbb{Z}^+}\{t_k - t_{k-1}\}$, where $\mathbb{Z}^+$ stands for all positive integers, and $\boldsymbol{P}_{mi}, \boldsymbol{Q}_{mi} \in \mathbb{R}^{n\times n}$ be symmetric, positive definite satisfying

$$\boldsymbol{A}_{mi}^\top\boldsymbol{P}_{mi} + \boldsymbol{P}_{mi}\boldsymbol{A}_{mi} = -\boldsymbol{Q}_{mi}, \quad i \in \mathcal{I}. \tag{10}$$

Due to the stability of $\boldsymbol{A}_{mi}$, there exist $a_{mi}, \lambda_{mi} > 0$ such that $\|e^{\boldsymbol{A}_{mi}t}\| \leq a_{mi}e^{-\lambda_{mi}t}$. Define $a_m = \max_{i\in\mathcal{I}} a_{mi}$, $\lambda_m = \min_{i\in\mathcal{I}}\lambda_{mi}$, $\alpha = \max_{i\in\mathcal{I}}\lambda_{\max}[\boldsymbol{P}_{mi}]$, and $\beta = \min_{i\in\mathcal{I}}\lambda_{\min}[\boldsymbol{P}_{mi}]$, where $\lambda_{\min}[\cdot]$ and $\lambda_{\max}[\cdot]$ denote the minimum and maximum eigenvalues of a matrix. The following lemma gives a lower bound on T_0 that ensures exponential stability of (9):

Lemma 1 [11]. *The homogeneous system (9) is exponentially stable with decay rate $\sigma \in (0, 1/2\alpha)$ if the minimum switching time interval T_0 satisfies*

$$T_0 \geq \frac{\alpha}{1-2\sigma\alpha}\ln(1+\mu\Delta_{\boldsymbol{A}_m}), \quad \mu = \frac{a_m^2}{\lambda_m\beta}\max_{i\in\mathcal{I}}\|\boldsymbol{P}_{mi}\|, \tag{11}$$

where $\Delta_{\boldsymbol{A}_m}$ stands for the largest difference between any two modes of $\boldsymbol{A}_m(t)$, i.e., $\Delta_{\boldsymbol{A}_m} = \max_{i,j\in\mathcal{I}}\|\boldsymbol{A}_{mi} - \boldsymbol{A}_{mj}\|$.

Stability and tracking performance. With the model reference controller (6) applied to the piecewise linear system (1), we have closed-loop stability and exponential tracking performance as stated in the following lemma:

Lemma 2. *All signals in the closed-loop system (8) are bounded and the output tracking error $\boldsymbol{e}(t) = \boldsymbol{y}(t) - \boldsymbol{y}_m(t)$ with $\boldsymbol{y}_m(t)$ from (3) converges exponentially to zero if the minimum switching time interval T_0 satisfies (11).*

Proof: Closed-loop signal boundedness and the fact that $\boldsymbol{\Phi}(t,t_0) \leq ce^{-\sigma(t-t_0)}$ for some $c > 0$ under the condition (11) follow from Lemma 1. Exponential output tracking performance can be concluded since $\boldsymbol{e}(t) = \boldsymbol{y}(t) - \boldsymbol{y}_m(t) = \boldsymbol{C}^\top\boldsymbol{\Phi}(t,t_0)\boldsymbol{x}(t_0)$. ∇

Remark 1: In the state feedback state tracking design for piecewise linear systems [10], a plant-model matching condition in the form of $\boldsymbol{A}_i + \boldsymbol{B}_i\boldsymbol{K}_{xi}^{*\top} = \boldsymbol{A}_{mi}$, $\boldsymbol{B}_i\boldsymbol{K}_{ri}^* = \boldsymbol{B}_{mi}$ is crucial and certain structural information about the plant parameter matrices $\boldsymbol{A}_i$, $\boldsymbol{B}_i$ are needed for the specification of $\boldsymbol{A}_{mi}$, $\boldsymbol{B}_{mi}$ in the piecewise linear reference model system. In the output tracking case, such restrictive matching conditions are relaxed; in particular, the triple $(\boldsymbol{A}_i + \boldsymbol{B}_i\boldsymbol{K}_{xi}^{*\top}, \boldsymbol{B}_i\boldsymbol{K}_{ri}^*, \boldsymbol{C}^\top)$ here is only a state space realization of the transfer matrix $\boldsymbol{W}_m(s)$, to ensure input-output, piecewise plant-model matching, which can always be satisfied under the stated assumptions. In other words, the existence of the parameter matrices $\boldsymbol{K}_{xi}^*$, $\boldsymbol{K}_{ri}^*$ is guaranteed. $\square$

B. Adaptive Control Scheme

Since $\boldsymbol{A}_i$, $\boldsymbol{B}_i$, $i \in \mathcal{I}$, are unknown, the nominal controller parameters $\boldsymbol{K}_{xi}^*$ and $\boldsymbol{K}_{ri}^*$ are also unknown, and the model reference control law (6) cannot be implemented. An adaptive control law with its parameters updated from some adaptive laws is needed.

Controller structure. The adaptive version of (6) is proposed as follows:

$$\boldsymbol{u}(t) = \boldsymbol{K}_x^\top(t)\boldsymbol{x}(t) + \boldsymbol{K}_r(t)\boldsymbol{r}(t), \tag{12}$$

where $\boldsymbol{K}_x(t) = \sum_{i=1}^{l}\boldsymbol{K}_{xi}(t)\chi_i(t)$, $\boldsymbol{K}_r(t) = \sum_{i=1}^{l}\boldsymbol{K}_{ri}(t)\chi_i(t)$. The parameter matrices $\boldsymbol{K}_{xi}(t)$, $\boldsymbol{K}_{ri}(t)$ are the adaptive estimates of $\boldsymbol{K}_{xi}^*(t)$ and $\boldsymbol{K}_{ri}^*(t)$, respectively, and are updated from some adaptive laws to be developed.

By applying the adaptive control law (12) to the plant (1) and defining $\tilde{\boldsymbol{K}}_x(t) = \boldsymbol{K}_x(t) - \boldsymbol{K}_x^*(t)$, $\tilde{\boldsymbol{K}}_r(t) = \boldsymbol{K}_r(t) - \boldsymbol{K}_r^*(t)$, $\tilde{\boldsymbol{K}}_{xi}(t) = \boldsymbol{K}_{xi}(t) - \boldsymbol{K}_{xi}^*$, $\tilde{\boldsymbol{K}}_{ri}(t) = \boldsymbol{K}_{ri}(t) - \boldsymbol{K}_{ri}^*$, the closed-loop system follows:

$$\begin{aligned} \dot{\boldsymbol{x}}(t) &= \sum_{i=1}^{l}\left((\boldsymbol{A}_i + \boldsymbol{B}_i\boldsymbol{K}_{xi}^{*\top})\chi_i(t)\boldsymbol{x}(t) + \boldsymbol{B}_i\boldsymbol{K}_{ri}^*\chi_i(t)\boldsymbol{r}(t)\right) \\ &\quad + \sum_{i=1}^{l}\boldsymbol{B}_i\left(\tilde{\boldsymbol{K}}_{xi}^\top(t)\chi_i(t)\boldsymbol{x}(t) + \tilde{\boldsymbol{K}}_{ri}(t)\chi_i(t)\boldsymbol{r}(t)\right), \\ \boldsymbol{y}(t) &= \boldsymbol{C}^\top\boldsymbol{x}(t). \end{aligned} \tag{13}$$

In view of (3) and (7), the tracking error equation is

$$\boldsymbol{e}(t) = \boldsymbol{W}_m(s)\left[\sum_{i=1}^{l}\boldsymbol{K}_{pi}\tilde{\boldsymbol{\Theta}}_i^\top\chi_i\boldsymbol{\omega}\right](t) + \boldsymbol{\epsilon}_0(t) \tag{14}$$

where $\boldsymbol{\epsilon}_0(t)$ is an initial condition related term, and $\tilde{\boldsymbol{\Theta}}_i(t) = \boldsymbol{\Theta}_i(t) - \boldsymbol{\Theta}_i^*(t)$, $\boldsymbol{\Theta}_i(t) = [\boldsymbol{K}_{xi}^\top(t), \boldsymbol{K}_{ri}(t)]^\top$, $\boldsymbol{\Theta}_i^* = [\boldsymbol{K}_{xi}^{*\top}, \boldsymbol{K}_{ri}^*]^\top$, and $\boldsymbol{\omega}(t) = [\boldsymbol{x}^\top(t), \boldsymbol{r}(t)]^\top$.

Error model. With (3) and (14), it follows that

$$\boldsymbol{\xi}_m(s)[e](t) = \sum_{i=1}^{l}\boldsymbol{K}_{pi}\tilde{\boldsymbol{\Theta}}_i^\top(t)\chi_i(t)\boldsymbol{\omega}(t), \tag{15}$$

after ignoring the term $\boldsymbol{\epsilon}_0(t)$. To deal with the parametric uncertainties in $\boldsymbol{K}_{pi}$, their LDS decompositions are used under Assumptions (A4)–(A5). Substituting (5) into (15) leads to

$$\boldsymbol{L}_s^{-1}\boldsymbol{\xi}_m(s)[\boldsymbol{e}](t) = \sum_{i=1}^{l} \boldsymbol{D}_{si}\boldsymbol{S}_i\tilde{\boldsymbol{\Theta}}_i^{\mathsf{T}}(t)\chi_i(t)\boldsymbol{\omega}(t). \tag{16}$$

By introducing $\boldsymbol{\Theta}_0^* = \boldsymbol{L}_s^{-1} - \boldsymbol{I}$, which is lower triangular with zero diagonal elements, we obtain

$$\boldsymbol{\Theta}_0^*\boldsymbol{\xi}_m(s)[\boldsymbol{e}](t) + \boldsymbol{\xi}_m(s)[\boldsymbol{e}](t) = \sum_{i=1}^{l} \boldsymbol{D}_{si}\boldsymbol{S}_i\tilde{\boldsymbol{\Theta}}_i^{\mathsf{T}}(t)\chi_i(t)\boldsymbol{\omega}(t).$$

Operating both sides of the above equation by $h(s)\boldsymbol{I}$, where $h(s) = 1/f(s)$ with $f(s)$ a monic stable polynomial of degree equal to the maximum degree of $\boldsymbol{\xi}_m(s)$, leads to

$$\begin{aligned} &[0, \boldsymbol{\theta}_2^{*\mathsf{T}}\boldsymbol{\eta}_2(t), \boldsymbol{\theta}_3^{*\mathsf{T}}\boldsymbol{\eta}_3(t), \ldots, \boldsymbol{\theta}_M^{*\mathsf{T}}\boldsymbol{\eta}_M(t)]^{\mathsf{T}} + \bar{\boldsymbol{e}}(t) \\ &= \sum_{i=1}^{l} \boldsymbol{D}_{si}\boldsymbol{S}_i h(s)[\tilde{\boldsymbol{\Theta}}_i^{\mathsf{T}}\chi_i\boldsymbol{\omega}](t). \end{aligned}$$

where $\boldsymbol{\theta}_j^* \in \mathbb{R}^{j-1}$, $j = 2, \ldots, M$, denotes the jth row elements of $\boldsymbol{\Theta}_0^*$ under its diagonal, $\bar{\boldsymbol{e}}(t) = \boldsymbol{\xi}_m(s)h(s)[\boldsymbol{e}](t) = [\bar{e}_1(t), \bar{e}_2(t), \ldots, \bar{e}_M(t)]^{\mathsf{T}}$, $\boldsymbol{\eta}_j(t) = [\bar{e}_1(t), \bar{e}_2(t), \ldots, \bar{e}_{j-1}(t)]^{\mathsf{T}} \in \mathbb{R}^{j-1}$.

Define the estimation error signal

$$\begin{aligned} \boldsymbol{\epsilon}(t) =& [0, \boldsymbol{\theta}_2^{\mathsf{T}}(t)\boldsymbol{\eta}_2(t), \boldsymbol{\theta}_3^{\mathsf{T}}(t)\boldsymbol{\eta}_3(t), \ldots, \boldsymbol{\theta}_M^{\mathsf{T}}(t)\boldsymbol{\eta}_M(t)]^{\mathsf{T}} \\ &+ \bar{\boldsymbol{e}}(t) + \sum_{i=1}^{l} \boldsymbol{\Psi}_i(t)\boldsymbol{\xi}_i(t), \end{aligned} \tag{17}$$

where $\boldsymbol{\theta}_j(t)$, $j = 2, 3, \ldots, M$, are the estimates of $\boldsymbol{\theta}_j^*$, and $\boldsymbol{\Psi}_i(t)$ are the estimates of $\boldsymbol{\Psi}_i^* = \boldsymbol{D}_{si}\boldsymbol{S}_i$, and

$$\boldsymbol{\xi}_i(t) = \boldsymbol{\Theta}_i^{\mathsf{T}}(t)\boldsymbol{\zeta}_i(t) - h(s)[\boldsymbol{\Theta}_i^{\mathsf{T}}\chi_i\boldsymbol{\omega}](t),\ \boldsymbol{\zeta}_i(t) = h(s)[\chi_i\boldsymbol{\omega}](t).$$

We can obtain the error model

$$\begin{aligned} \boldsymbol{\epsilon}(t) =& [0, \tilde{\boldsymbol{\theta}}_2^{\mathsf{T}}(t)\boldsymbol{\eta}_2(t), \tilde{\boldsymbol{\theta}}_3^{\mathsf{T}}(t)\boldsymbol{\eta}_3(t), \ldots, \tilde{\boldsymbol{\theta}}_M^{\mathsf{T}}(t)\boldsymbol{\eta}_M(t)]^{\mathsf{T}} \\ &+ \sum_{i=1}^{l} \tilde{\boldsymbol{\Psi}}_i(t)\boldsymbol{\xi}_i(t) + \sum_{i=1}^{l} \boldsymbol{D}_{si}\boldsymbol{S}_i\tilde{\boldsymbol{\Theta}}_i^{\mathsf{T}}(t)\boldsymbol{\zeta}_i(t), \end{aligned} \tag{18}$$

where $\tilde{\boldsymbol{\theta}}_j(t) = \boldsymbol{\theta}_j(t) - \boldsymbol{\theta}_j^*$, $\tilde{\boldsymbol{\Psi}}_i(t) = \boldsymbol{\Psi}_i(t) - \boldsymbol{\Psi}_i^*(t)$ are the parameter errors.

Adaptive laws. Based on the error model (18), we propose the following gradient adaptive laws to update $\boldsymbol{\theta}_j(t)$, $\boldsymbol{\Theta}_i(t)$, and $\boldsymbol{\Psi}_i(t)$, $i \in \mathcal{I}$, $j = 2, 3, \ldots, M$:

$$\dot{\boldsymbol{\theta}}_j(t) = -\frac{\boldsymbol{\Gamma}_{\theta_j}\boldsymbol{\eta}_j(t)\epsilon_j(t)}{m^2(t)},\ \ \boldsymbol{\Gamma}_{\theta_j} = \boldsymbol{\Gamma}_{\theta_j}^{\mathsf{T}} > \boldsymbol{0} \tag{19}$$

$$\dot{\boldsymbol{\Theta}}_i^{\mathsf{T}}(t) = -\frac{\boldsymbol{D}_{si}\boldsymbol{\epsilon}(t)\boldsymbol{\zeta}_i^{\mathsf{T}}(t)}{m^2(t)}, \tag{20}$$

$$\dot{\boldsymbol{\Psi}}_i(t) = -\frac{\boldsymbol{\Gamma}_i\boldsymbol{\epsilon}(t)\boldsymbol{\xi}_i^{\mathsf{T}}(t)}{m^2(t)},\ \ \boldsymbol{\Gamma}_i = \boldsymbol{\Gamma}_i^{\mathsf{T}} > \boldsymbol{0} \tag{21}$$

where $\boldsymbol{\epsilon}(t) = [\epsilon_1(t), \epsilon_2(t), \ldots, \epsilon_M(t)]^{\mathsf{T}}$ is as defined in (17), $\boldsymbol{\Gamma}_{\theta_j}$ and $\boldsymbol{\Gamma}_i$ are the adaptation gain matrices, and $m^2(t) = 1 + \sum_{i=1}^{l}\left(\boldsymbol{\zeta}_i^{\mathsf{T}}(t)\boldsymbol{\zeta}_i(t) + \boldsymbol{\xi}_i^{\mathsf{T}}(t)\boldsymbol{\xi}_i(t)\right) + \sum_{j=2}^{M}\boldsymbol{\eta}_j^{\mathsf{T}}(t)\boldsymbol{\eta}_j(t)$ is the normalizing signal.

Along the line of derivation in this section, LDU and SDU decomposition based designs can be developed for adaptive state feedback control of piecewise linear systems for output tracking. Similar to Assumption (A5), the controlled system modes in these designs need to share certain common structural characteristics (a common $\boldsymbol{L}$ matrix in the LDU based deisign, or a common $\boldsymbol{S}$ matrix in the SDU based design). When these assumptions are not satisfied, however, the unparameterized uncertainties would lead to the loss of the desired signal properties (see Lemma 3 in the next section), and closed-loop system stability and asymptotic tracking performance may not be concluded.

IV. Stability Analysis

In this section, we analyze the stability and asymptotic tracking performance of the closed-loop system with the piecewise linear system (1), the reference model system (3), and the adaptive control law (12) updated from the adaptive laws (19)–(21). Some desired properties of the adaptive laws are presented first, which will then be used to establish the asymptotic tracking performance.

The adaptive laws (19)–(21) have the following desired properties for $i \in \mathcal{I}$, $j = 2, 3, \ldots, M$:

Lemma 3. *The adaptive laws (19)–(21) ensure that* $\boldsymbol{\theta}_j(t), \boldsymbol{\Theta}_i(t), \boldsymbol{\Psi}_i(t) \in \mathcal{L}^\infty$, *and* $\frac{\boldsymbol{\epsilon}(t)}{m(t)}, \dot{\boldsymbol{\theta}}_j(t), \dot{\boldsymbol{\Theta}}_i(t), \dot{\boldsymbol{\Psi}}_i(t) \in \mathcal{L}^2 \cap \mathcal{L}^\infty$.

Proof: Consider the Lyapunov function candidate

$$V = \frac{1}{2}\sum_{j=2}^{M}\tilde{\boldsymbol{\theta}}_j^{\mathsf{T}}\boldsymbol{\Gamma}_{\theta_j}^{-1}\tilde{\boldsymbol{\theta}}_j + \sum_{i=1}^{l}\left(\mathrm{tr}[\tilde{\boldsymbol{\Psi}}_i^{\mathsf{T}}\boldsymbol{\Gamma}_i^{-1}\tilde{\boldsymbol{\Psi}}_i] + \mathrm{tr}[\tilde{\boldsymbol{\Theta}}_i\boldsymbol{S}_i\tilde{\boldsymbol{\Theta}}_i^{\mathsf{T}}]\right).$$

Its time derivative along the trajectories of (19)–(21) is

$$\begin{aligned} \dot{V} &= -\sum_{j=2}^{M}\frac{\tilde{\boldsymbol{\theta}}_j^{\mathsf{T}}\boldsymbol{\eta}_j\epsilon_j}{m^2} - \sum_{i=1}^{l}\frac{\boldsymbol{\xi}_i^{\mathsf{T}}s\tilde{\boldsymbol{\Psi}}_i^{\mathsf{T}}\boldsymbol{\epsilon}}{m^2} - \sum_{i=1}^{l}\frac{\boldsymbol{\zeta}_i^{\mathsf{T}}\tilde{\boldsymbol{\Theta}}_i\boldsymbol{S}_i\boldsymbol{D}_{si}\boldsymbol{\epsilon}}{m^2} \\ &\leq \frac{\boldsymbol{\epsilon}^{\mathsf{T}}\boldsymbol{\epsilon}}{m^2} \leq 0 \end{aligned}$$

This implies that $\boldsymbol{\theta}_j(t), \boldsymbol{\Theta}_i(t), \boldsymbol{\Psi}_i(t) \in \mathcal{L}^\infty$, $\frac{\boldsymbol{\epsilon}(t)}{m(t)} \in \mathcal{L}^2 \cap \mathcal{L}^\infty$, and from (19)–(21) and the boundedness of $\frac{\boldsymbol{\eta}_j(t)}{m(t)}$, $\frac{\boldsymbol{\zeta}_i(t)}{m(t)}$, $\frac{\boldsymbol{\xi}_i(t)}{m(t)}$, we also have $\dot{\boldsymbol{\theta}}_j(t), \dot{\boldsymbol{\Theta}}_i(t), \dot{\boldsymbol{\Psi}}_i(t) \in \mathcal{L}^2 \cap \mathcal{L}^\infty$. ∇

With Lemma 3, the following results can be eastablished:

Theorem 1. *All signals in the closed-loop system with the piecewise linear system (1), the reference model system (3), and the control law (12) updated by the adaptive laws (19)–(21) are bounded, and the tracking error satisfies*

$$\lim_{t\to\infty}\boldsymbol{e}(t) = \lim_{t\to\infty}(\boldsymbol{y}(t) - \boldsymbol{y}_m(t)) = \boldsymbol{0},\ \ \boldsymbol{e}(t) \in \mathcal{L}^2,$$

for any initial condition, if the minimum switching time interval T_0 *satisfies (11).*

Although this is a state feedback control design, a direct Lyapunov stability analysis is not applicable because the state error is not available in this output tracking case. Closed-loop signal boundedness can be proved by first using a reduced-order state observer design of the piecewise linear system (1) to parameterize the state feedback controller structure in (6) into an output feedback form. A filtered system output $\boldsymbol{y}(t)$ is then expressed in a feedback framework that is suitable for the application of the small gain theorem and with the the Barbălat Lemma and the signal properties as above to conclude signal boundedness and asymptotic output tracking.

The minimum switching time interval requirement (11) is for ensuring internal stability in the presence of system mode switchings. It can be relaxed to arbitrarily fast mode switchings under additional conditions, such as these stated in the following corollaries:

Corollary 1. *If $\Delta_{\boldsymbol{A}_m} = 0$, i.e., $\boldsymbol{A}_i + \boldsymbol{B}_i \boldsymbol{K}_{xi}^{*\mathsf{T}} = \boldsymbol{A}_m$, $i \in \mathcal{I}$, for $\boldsymbol{K}_{xi}^*$ in (7), then closed-loop stability and asymptotic output tracking are achieved for arbitrary system mode switches.*

Proof: The condition of (11) reduces to $T_0 > 0$ with $\Delta_{\boldsymbol{A}_m} = 0$, and the stability and asymptotic tracking performance follow from Theorem 1. ∇

Corollary 2. *If the $\boldsymbol{A}_{mi}$ matrices in (10) share a common Lyapunov matrix $\boldsymbol{P}_m$, such that $\boldsymbol{A}_{mi}^{\mathsf{T}} \boldsymbol{P}_m + \boldsymbol{P}_m \boldsymbol{A}_{mi} = -\boldsymbol{Q}_{mi}$ for some symmetric, positive definite $\boldsymbol{Q}_{mi}$, then closed-loop stability and asymptotic output tracking are achieved for arbitrary system mode switches.*

Proof: Consider a continuous Lyapunov function candidate $V = \boldsymbol{z}^{\mathsf{T}}(t) \boldsymbol{P}_m \boldsymbol{z}(t)$ for the homogeneous system (9). It follows that $\dot{V} \le -(\gamma/\alpha)V$, for $\gamma = \min_{i \in \mathcal{I}} \lambda_{\min}[\boldsymbol{Q}_{mi}]$, $\alpha = \lambda_{\max}[\boldsymbol{P}_m]$. Therefore exponential stability of (9) is concluded for arbitrary mode switches, so is the closed-loop stability. Asymptotic output tracking performance follows from Theorem 1. ∇

Compared with the state tracking case [10], [12], where the reference state trajectory is specified by a piecewise linear system in state space form, the conditions in Corollary 2 are stronger, because in addition to the existence of a common Lyapunov matrix, the set of $\boldsymbol{A}_{mi}$ have to satisfy the plant model matching condition (7) due to the specification of the LTI reference model system (3) in input/output form. Systems in certain canonical forms, e.g., controllable canonical form, fit in this context, and an illustrative example is given in [11] for the SISO case.

In the output tracking design, the analysis method used is analogous to the conventional state feedback output tracking design for an LTI plant. An estimation error $\boldsymbol{\epsilon}(t)$ (along with some auxiliary signals) is defined as in (17) and the proposed gradient adaptive laws are such that the desired signal properties in Lemma 3, i.e., the boundedness of parameter estimates, and $\boldsymbol{\theta}_{ij}(t), \boldsymbol{\Theta}_i(t), \boldsymbol{\Psi}_i(t) \in \mathcal{L}^\infty$, and $\frac{\boldsymbol{\epsilon}(t)}{m(t)}, \dot{\boldsymbol{\theta}}_{ij}(t), \dot{\boldsymbol{\Theta}}_i(t), \dot{\boldsymbol{\Psi}}_i(t) \in \mathcal{L}^2 \cap \mathcal{L}^\infty$ remain in spite of the presence of mode switchings, and these properties help establish signal boundedness and the asymptotic tracking performance in a feedback framework for which small gain theorem can be applied. The only difference in the analysis with respect to the conventional output tracking design is the requirement on a minimum mode switching time interval T_0, which is needed to ensure the exponential decaying of the initial condition related term $\boldsymbol{\epsilon}_0(t)$ and internal stability of the closed-loop system.

On the other hand, in the state feedback state tracking design for the general case [10], an analogous Lyapunov analysis to the conventional state tracking design cannot be carried out due to the nonexistence of a common $\boldsymbol{P}_m$ matrix for $\boldsymbol{A}_{mi}$, in general. A piecewise continuous Lyapunov function was considered, instead. However, the minimum switching time interval requirement for ensuring a stable reference model system cannot ensure bounded parameter estimates, thus a parameter projection algorithm is needed. Additional switching time interval requirements are imposed for establishing the boundedness of $\boldsymbol{e}(t)$ with the boundedness of parameter estimates. Asymptotic state tracking performance cannot be concluded due to the loss of $\boldsymbol{e}(t) \in \mathcal{L}^2$ property.

V. An Illustrative Example

Simulations are performed to demonstrate the system stability and tracking performance with the proposed adaptive control schemes applied to a piecewise linear system model of the NASA GTM [9] at multiple operating points. It is to be noted that in the simulations, switches between the chosen linearized GTM models are time-dependent for illustration purposes, while transitions of operating points of the nonlinear GTM system are state-dependent. Extensions of the proposed adaptive control scheme in this paper for piecewise linear systems to be applicable to nonlinear systems are under investigation.

For simplicity of presentation, we choose $l = 2$, and trim the GTM at steady-state, straight, wings-level flight conditions at 80 knots and 90 knots at 800 ft., respectively, to obtain a piecewise linear lateral system model in the form of (1), where $\boldsymbol{x} = [v, p, r, \phi, \psi]^{\mathsf{T}}$ with the elements being the perturbed aircraft velocity component along the y-body-axis (fps), angular velocity along the x- and z-body-axis (crad/s), roll angle (crad), and yaw angle (crad), respectively. The outputs are chosen as $\boldsymbol{y} = [v, \psi]^{\mathsf{T}}$, while the control inputs are the perturbed aileron deflection δ_a and rudder deflection δ_r, i.e., $\boldsymbol{u} = [\delta_a, \delta_r]^{\mathsf{T}}$, and the nominal parameter matrices are

$$\boldsymbol{A}_1 = \begin{bmatrix} -0.6137 & 0.0959 & -1.3454 & 0.3210 & 0 \\ -66.3000 & -6.7565 & 1.8813 & 0 & 0 \\ 24.1200 & -0.3162 & -1.4992 & 0 & 0 \\ 0 & 1.0000 & 0.0691 & 0.0002 & 0 \\ 0 & 0 & 1.0000 & 0 & 0 \end{bmatrix},$$

$$\boldsymbol{A}_2 = \begin{bmatrix} -0.6870 & 0.0801 & -1.5153 & 0.3213 & 0 \\ -72.9200 & -7.5625 & 1.8623 & 0 & 0 \\ 27.3600 & -0.3078 & -1.6865 & 0 & 0 \\ 0 & 1.0000 & 0.0513 & 0.0001 & 0 \\ 0 & 0 & 1.0000 & 0 & 0 \end{bmatrix},$$

$$\boldsymbol{B}_1 = \begin{bmatrix} -0.0274 & 0.4892 \\ -90.0900 & 29.6300 \\ -2.5200 & -46.8300 \\ 0 & 0 \\ 0 & 0 \end{bmatrix},$$

$$\boldsymbol{B}_2 = \begin{bmatrix} -0.0396 & 0.6160 \\ -116.5300 & 38.2700 \\ -3.1300 & -59.5300 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad \boldsymbol{C} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}.$$

It is clear that $M = 2$, $n = 5$, and with the choice of the modified left interactor matrix $\boldsymbol{\xi}_m(s) = \text{diag}\{s+1, (s+1)^2\}$, the high frequency gain matrices can be computed, from which it follows that $\text{sign}[\Delta_{11}] = -1, \text{sign}[\Delta_{12}] = 1$, $\text{sign}[\Delta_{21}] = -1$, $\text{sign}[\Delta_{22}] = 1$, and with the choice of $\boldsymbol{D}_{s1} = -100\boldsymbol{I}$, $\boldsymbol{D}_{s2} = \text{diag}\{-100, -197.8998\}$, we have

$$\boldsymbol{L}_{s1} = \begin{bmatrix} 1 & 0 \\ 109.8248 & 1 \end{bmatrix}, \ \boldsymbol{S}_1 = \begin{bmatrix} 0.0003 & -0.0049 \\ -0.0049 & 1.0056 \end{bmatrix},$$

$$\boldsymbol{L}_{s2} = \begin{bmatrix} 1 & 0 \\ 109.8248 & 1 \end{bmatrix}, \ \boldsymbol{S}_2 = \begin{bmatrix} 0.0004 & -0.0062 \\ -0.0062 & 0.6427 \end{bmatrix}.$$

The reference model system is specified by (3) where $\boldsymbol{r}(t)$ is a bounded, piecewise continuous reference input vector signal. Furthermore, it can be verified that the plant model matching condition (7) for $i = 1, 2$, are satisfied.

The switching time interval is chosen to be $T = 50$s. The initial system state is $[2, 0, 0, 0, 10]^\mathsf{T}$ with zero reference model initial condition, and the initial parameter estimates are set at 70% of their nominal values. The adaptation gains are chosen as $\Gamma_{\theta_2} = 10^3$, $\boldsymbol{\Gamma}_1 = \boldsymbol{\Gamma}_2 = 10^5\boldsymbol{I}$. The parameters of the filter $h(s)$ are such that $f(s) = (s+6)^2$.

Fig. 1 shows the output tracking error $\boldsymbol{e}(t)$ with the reference input $\boldsymbol{r}(t) = [2\sin(0.014t), 10\sin(0.014t)]^\mathsf{T}$, corresponding to fluctuations of the lateral velocity in between ± 2 fps and of the yaw angle in between $\pm 5.73^\circ$ (± 10 crad). It can be seen that the desired closed-loop signal boundedness and asymptotic output tracking performance are achieved.

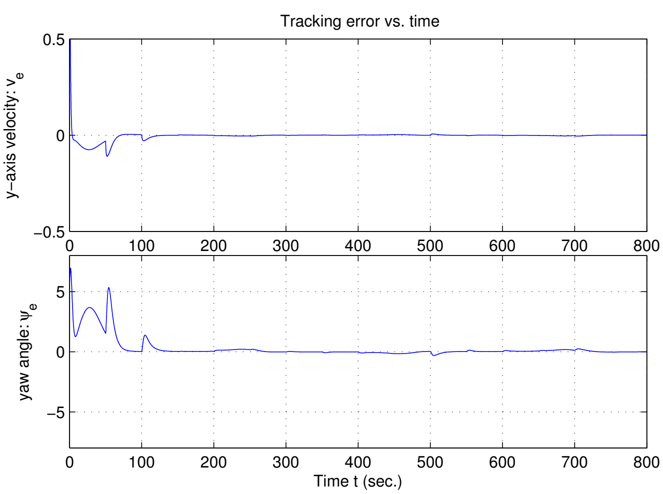

Fig. 1. Tracking error $\boldsymbol{e}(t)$ for $\boldsymbol{r}(t) = [2\sin(0.014t), 10\sin(0.014t)]^\mathsf{T}$.

VI. Conclusions

A direct state feedback model reference adaptive control (MRAC) scheme is developed in this paper for MIMO piecewise linear systems. The proposed control design employs the knowledge of the time instants of parameter discontinuities, which is characterized by the indicator functions. Closed-loop signal boundedness and asymptotic output tracking are achieved for sufficiently slow system mode switchings. Simulation results for a piecewise linear model of the GTM lateral dynamics demonstrate the effectiveness of the proposed adaptive control scheme. A current research topic under investigation is the extension of the adaptive control design proposed in this paper to be applicable to nonlinear systems over multiple operating conditions.

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