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A continuous-time approach to networked control of nonlinear systems

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Abstract—**A nonlinear networked control system in which the system information is transmitted over the network through multiple communication channels is considered. Each channel is modeled by a time-varying delay element. Stability of the control system is studied using the Lyapunov-Krasovskii method for continuous-time delayed systems. The sufficient condition for stability of the networked control system is presented in the form of a compact LMI. The results are applied to a robot arm networked control system to show the capabilities of the proposed method for nonlinear networked control applications.**

I. Introduction

As a new way of implementing control engineering solutions, the field of networked control has attracted a considerable attention from the researchers [1, 2, 3]. Implementing control systems over communication networks is associated with several problems of different nature ranging from the communication system analysis [4, 5] to control and communication scheduling co-design [6, 7] and control in presence of the network effects such as sampling issues [8, 9, 10], delay [11, 12, 19] and packet loss [13, 14].

Nonlinear networked control is a much more difficult problem and the attempts toward improved results are in progress. For example, a small gain Theorem is derived in [15] for a nonlinear networked control system (NCS) with multiple delayed channels. However, the results are not directly applicable for numerical analysis. In [16, 10], the analysis is based on a hybrid system modeling of the NCS. In [16], the NCS with multiple delayed channels is studied. Again it is difficult to use the results for the nonlinear case and the authors have applied the results to a linear NCS.

An important category of the works on the NCS problems are based on the Lyapunov-Krasovskii method for time delay systems [17]. This method is a natural extension of the Lyapunov method when time delays are present. In the case of linear systems, this method can result in LMI sufficient conditions for stability and performance. This method has been applied to linear NCS problems in several works including [12, 18, 19] where both communication delay and packet loss are handled at the same time.

In this work, a nonlinear NCS with multiple time-varying bounded delays is considered and a simple stability analysis is presented based on the Lyapunov-Krasovskii method. The controller can be designed using the ordinary Lyapunov based methods in continuous-time without considering the effects of communication. Then, the tolerance of the NCS to the communication delays can be studied using the analysis method proposed in this work which is a compact LMI condition for stability. The results are applied to a robot arm control problem where the sensors and actuators at each joint are independently connected to a serial communication control bus. In the remaining the problem is stated in Section two. The main results are presented in Section three. The robot control problem is studied in Section four and conclusions are made at the end.

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II. Problem Statement

A nonlinear closed loop control system with delayed communication channels can be illustrated as the block diagram in Figure 1. The augmented system in Figure 1 may be composed of several subsystems including interacting processes and controllers. The signal that passes through the communication channel experiences a time delay which varies with time in general. It is mentioned that a data packet loss in communication networks can be also considered as a delay equal to one sampling interval until receiving the next data sample (this idea is applied for example in [19]).

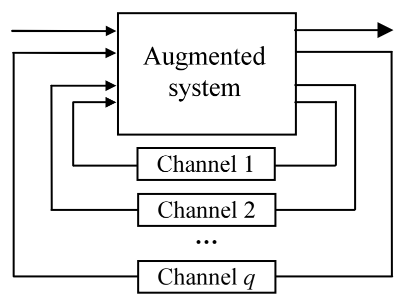

Figure 1. A control system with multiple communication channels on the signal paths.

In this work, the system in Figure 1, is modeled by the nonlinear state space model with time varying delays (1) in which $t\in\mathbb{R}$ is the time ($\mathbb{R}$ is the set of real numbers), $x(t)\in\mathbb{R}^{n_x}$ is the state of the augmented system and $d_i(t)\in\mathbb{R}$ is the time-varying delay in the ith communication channel for $1\le i\le q$.

$$\dot{x}(t) = f\big(t, x(t), x(t-d_1(t)),\ldots, x(t-d_q(t))\big) \tag{1}$$

In the remaining, $x(t)$ and $x(t-d_i(t))$ are written as x and x_{d_i} for brevity (the dependency on the current time t is omitted). The ideal communication is identified as the case in which $d_i(t) = 0$ for every $1 \le i \le q$. In this case, the set of system equations in (1) is simplified to $\dot{x} = f(t, x, ..., x)$. It is assumed that the control system is designed for the ideal communication case and there exist a bounded Lyapunov function $V_1(t,x)$ that satisfies (2).

$$V_1(t,x) \ge c_u \|x\|^2 \tag{2.1}$$

$$\frac{\partial V_1}{\partial t} + \frac{\partial V_1}{\partial x} f(t,x,...,x) \le -c_d \|x\|^2 \tag{2.2}$$

III. Main Results

In this Section stability of the system (1) is studied under the following assumptions.

Assumption A1: For every $1 \le i \le q$, there exist a positive constants r_i such that the delay value $d_i(t)$ in (1) is bounded as $d_i(t) \le r_i$.

Assumption A2: There exist positive constants c_v, c_f and c_{g_i} such that the relations (3), (4) and (5) are satisfied for every $1 \le i \le q$.

$$\left\| \frac{\partial V_1}{\partial x} \right\| \le 2 c_v \|x\| \tag{3}$$

$$\| f(t,x,...,x) \| \le c_f \|x\| \tag{4}$$

$$\left\| \frac{\partial f}{\partial x_{d_i}} \right\| \le c_{g_i} \tag{5}$$

Stability of the system (1) can be analyzed by the means of Theorem 1 in the following.

Theorem 1: If assumptions A1 and A2 hold, then the system (1) is asymptotically stable if there exist real numbers y_1, y_2 and positive real numbers $z_i > 0$ for $1 \le i \le q$ such that the matrix Ψ defined in (6) is negative definite.

$$\Psi = \begin{bmatrix} -c_d + 2y_1 c_f & y_1 + y_2 c_f & c_v c_{g_1} r_1 + y_1 c_{g_1} r_1 & c_v c_{g_2} r_2 + y_1 c_{g_2} r_2 & \cdots & c_v c_{g_q} r_q + y_1 c_{g_q} r_q \\ y_1 + y_2 c_f & -2y_2 + \sum_{i=1}^{q} z_i & y_2 c_{g_1} r_1 & y_2 c_{g_2} r_2 & \cdots & y_2 c_{g_q} r_q \\ c_v c_{g_1} r_1 + y_1 c_{g_1} r_1 & y_2 c_{g_1} r_1 & -z_1 & 0 & \cdots & 0 \\ c_v c_{g_2} r_2 + y_1 c_{g_2} r_2 & y_2 c_{g_2} r_2 & 0 & -z_2 & & \vdots \\ \vdots & \vdots & \vdots & & \ddots & 0 \\ c_v c_{g_q} r_q + y_1 c_{g_q} r_q & y_2 c_{g_q} r_q & 0 & \cdots & 0 & -z_q \end{bmatrix} \tag{6}$$

Proof: The following Lyapunov-Krasovskii functional is used to prove the Theorem.

$$V = V_1(t, x(t)) + \sum_{i=1}^{q} z_i \frac{1}{r_i} \int_{-r_i} (\theta - t + r_i)\, \dot{x}(\theta)^T \dot{x}(\theta) d\theta \tag{7}$$

Time derivative of V is calculated as

$$\dot{V} = \frac{\partial V_1}{\partial t} + \frac{\partial V_1}{\partial x} f(t, x, x_{d_1}, ..., x_{d_q}) + \sum_{i=1}^{q} z_i\, \dot{x}^T \dot{x} - \sum_{i=1}^{q} z_i \frac{1}{r_i} \int_{-r_i} \dot{x}(\theta)^T \dot{x}(\theta) d\theta \tag{8}$$

The following equation can be verified by eliminating the opposite terms from the right hand side.

$$f(t, x, x_{d_1}, ..., x_{d_q}) = \sum_{i=1}^{q} [f(t, x, ..., x, x_{d_i}, x_{d_{i+1}} ..., x_{d_q}) - f(t, x, ..., x, x_{d_{i+1}}, x_{d_{i+2}}, ..., x_{d_q})] + f(t, x, x, ..., x)$$

The above equation can be written as the following.

$$f(t, x, x_{d_1}, ..., x_{d_q}) = -\sum_{i=1}^{q} \int_{t-d_i(t)}^{t} \overline{G}_i(\theta) \dot{x}(\theta) d\theta + f(t, x, x, ..., x) \tag{9.1}$$

$$\overline{G}_i(\theta) = G_i(t, x, ..., x, x(\theta), x_{d_{i+1}}, ..., x_{d_q}) \tag{9.2}$$

$$G_i(t, x, x_{d_1}, ..., x_{d_q}) = \frac{\partial}{\partial x_{d_i}} f(t, x, x_{d_1}, ..., x_{d_q}) \tag{9.3}$$

Using (9), the system equations (1) can be written as below.

$$\dot{x} = f(t, x, ..., x) - \sum_{i=1}^{q} \int_{-d_i(t)} \overline{G}_i(\theta) \dot{x}(\theta) d\theta \tag{10}$$

Replacing (9.1) in (8) and adding a vanishing term according to (10) multiplied by $(y_1 x + y_2 \dot{x})$ for arbitrary values y_1 and y_2 we have

$$\begin{aligned} \dot{V} = & \left(\frac{\partial V_1}{\partial t} + \frac{\partial V_1}{\partial x} f(t, x, ..., x) \right) - \frac{\partial V_1}{\partial x} \left(\sum_{i=1}^{q} \int_{t-d_i(t)}^{t} \overline{G}_i(\theta) \dot{x}(\theta) d\theta \right) \\ & + \sum_{i=1}^{q} z_i \dot{x}^T \dot{x} - \sum_{i=1}^{q} z_i \frac{1}{r_i} \int_{-r_i} \dot{x}(\theta)^T \dot{x}(\theta) d\theta \\ & + 2 (y_1 x + y_2 \dot{x})^T \Big(-\dot{x} + f(t, x, ..., x) \\ & - \sum_{i=1}^{q} \int_{-d_i(t)} \overline{G}_i(\theta) \dot{x}(\theta) d\theta \Big) \end{aligned} \tag{11}$$

An upper bound can be obtained for $\dot{V}$ in (11) by using (2.2), (3), (4), (5) as below

$$
\begin{aligned}
\dot{V} < & -c_d\|x\|^2 + 2c_v\|x\|\left(\sum_{i=1}^{q}\int_{-d_i(t)}^{t} c_{g_i}\|\dot{x}(\theta)\|d\theta\right) \\
& + \sum_{i=1}^{q} z_i \dot{x}^T\dot{x} - \sum_{i=1}^{q} z_i \frac{1}{r_i}\int_{-r_i}^{t}\dot{x}(\theta)^T\dot{x}(\theta)d\theta \\
& + 2\Bigg[y_1\|x\|\|\dot{x}\| + y_1 c_f\|x\|^2 + y_1\|x\|\sum_{i=1}^{q}\int_{-d_i(t)}^{t} c_{g_i}\|\dot{x}(\theta)\|d\theta \\
& - y_2\dot{x}^T\dot{x} + y_2 c_f\|x\|\|\dot{x}\| + y_2\|\dot{x}\|\sum_{i=1}^{q}\int_{-d_i(t)}^{t} c_{g_i}\|\dot{x}(\theta)\|d\theta\Bigg]
\end{aligned} \tag{12}
$$

The following inequalities hold because r_i, z_i and the integrands (the squared expressions) are non-negative.

$$
0 \le \sum_{i=1}^{q} z_i \frac{1}{r_i}\int_{t-d_i(t)}^{t}\left[\frac{r_i c_{g_i}}{z_i}\left(c_v\|x\| + y_1\|x\| + y_2\|\dot{x}\|\right) - \|\dot{x}(\theta)\|\right]^2 d\theta \tag{13.1}
$$

$$
0 \le \sum_{i=1}^{q} z_i \frac{1}{r_i}\int_{t-r_i}^{t-d_i(t)}\|\dot{x}(\theta)\|^2 d\theta \tag{13.2}
$$

Adding (13.1) and (13.2) to (12) and simplifying the result we obtain

$$
\begin{aligned}
\dot{V} < & -c_d\|x\|^2 + 2\left[y_1\|x\|\|\dot{x}\| + y_1 c_f\|x\|^2 - y_2\|\dot{x}\|^2 + y_2 c_f\|x\|\|\dot{x}\|\right] \\
& + \sum_{i=1}^{q} z_i\dot{x}^T\dot{x} + \sum_{i=1}^{q}\frac{r_i d_i(t) c_{g_i}^2}{z_i}\left(c_v\|x\| + y_1\|x\| + y_2\|\dot{x}\|\right)^2
\end{aligned}
$$

Substituting $d_i(t)$ by its upper-bound r_i for $1 \le i \le q$, the above inequality can be written as below

$$
\dot{V} < \begin{bmatrix}\|x\| \\ \|\dot{x}\|\end{bmatrix}^T \Psi_c \begin{bmatrix}\|x\| \\ \|\dot{x}\|\end{bmatrix} \tag{14.1}
$$

$$
\begin{aligned}
\Psi_c = & \left(\begin{bmatrix} -c_d + 2y_1 c_f & y_1 + y_2 c_f \\ y_1 + y_2 c_f & -2y_2 + \sum_{i=1}^{q} z_i \end{bmatrix} + \right. \\
& \left. \sum_{i=1}^{q}\begin{bmatrix} c_v + y_1 \\ y_2 \end{bmatrix} c_{g_i} r_i \frac{1}{z_i} c_{g_i} r_i \begin{bmatrix} c_v + y_1 \\ y_2 \end{bmatrix}^T \right)
\end{aligned} \tag{14.2}
$$

If the matrix Ψ in (6) is negative definite, then it can be concluded that Ψ_c in (14.2) is also negative definite using the Schur complements technique. Hence, according to (14.1) there exist a constant $c_e > 0$ such that $\dot{V} < -c_e\,\|x\|^2$. On the other hand (2.1) and (7) imply that $V > c_u\,\|x\|^2$. Therefore according to Lyapunov-Krasovskii Theorem the system (1) is asymptotically stable (proposition 5.2 in [17] with $\varepsilon = \min\{c_u, c_e\}$). □

IV. CASE STUDY

In this Section control of a robot arm with two degrees of freedom is considered. The control system is implemented over a control network as shown in Figure 2. The control bus is passed through the robot links and connected to the various devices installed on the robot including the sensors and actuators for each joint of the robot. The sensor S1 measures $q_1, \dot{q}_1$ and the sensor S2 measures $q_2, \dot{q}_2$. The actuators A1 and A2 apply the torque control commands τ_1, τ_2 to the first and second joints respectively.

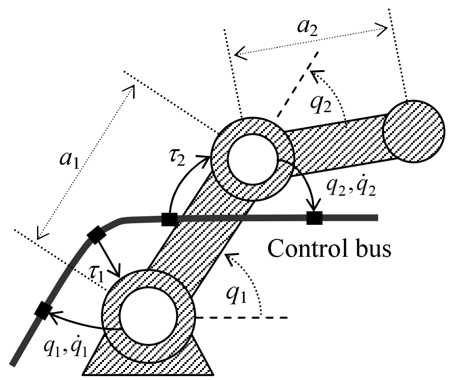

Figure 2. A robot arm equipped with a control bus for serial communication of the control data.

The equations of the motion of the robot arm can be written as in (15) where $m_1 = 1.5$ Kg, $m_2 = 0.8$ Kg, $a_1 = 0.5$ m, $a_2 = 0.4$ m and $g = 9.8$ m/sec^2 are the weight of link 1, weight of link 2, length of link 1, length of link 2 and acceleration of gravity respectively [20]. For simplicity, it is assumed that the mass of each link is concentrated at its end.

$$
M_{11}(q_2)\ddot{q}_1 + M_{12}(q_2)\ddot{q}_2 - N_1(q_1,\dot{q}_1,q_2,\dot{q}_2) = \tau_1 \tag{15.1}
$$

$$
M_{12}(q_2)\ddot{q}_1 + M_{22}\ddot{q}_2 - N_2(q_1,\dot{q}_1,q_2,\dot{q}_2) = \tau_2 \tag{15.2}
$$

$$
M_{11}(q_2) = (m_1 + m_2)a_1^2 + m_2 a_2^2 + 2m_2 a_1 a_2\cos(q_2) \tag{15.3}
$$

$$
M_{12}(q_2) = m_2 a_2[a_2 + a_1\cos(q_2)] \tag{15.4}
$$

$$
M_{22} = m_2 a_2^2 \tag{15.5}
$$

$$
\begin{aligned}
N_1(q_1,\dot{q}_1,q_2,\dot{q}_2) = & -m_2 a_1 a_2(2\dot{q}_1\dot{q}_2 + \dot{q}_2^2)\sin(q_2) \\
& + (m_1 + m_2)g a_1\cos(q_1) + m_2 g a_2\cos(q_1 + q_2)
\end{aligned} \tag{15.6}
$$

$$
N_2(q_1,\dot{q}_1,q_2,\dot{q}_2) = m_2 a_1 a_2\dot{q}_1^2\sin(q_2) + m_2 g a_2\cos(q_1 + q_2) \tag{15.7}
$$

By neglecting the control network delays, the robot controller can be designed such that q_1 and q_2 track the desired trajectories q_{d_1} and q_{d_2} respectively with the error dynamics in (16) using the computed torque method [20]. The positive constants α_i and β_i are the design parameters that determine the error convergence behavior.

$$
\ddot{e}_i + \alpha_i\dot{e}_i + \beta_i e_i = 0\ , \quad i \in \{1,2\} \tag{16.1}
$$

$$
e_i = q_i - q_{d_i}\ , \quad i \in \{1,2\} \tag{16.2}
$$

The required control torques τ_1, τ_2 are calculated in terms of $q_1, \dot{q}_1, q_2, \dot{q}_2$ by obtaining $\ddot{q}_1, \ddot{q}_2$ from (16) and substituting them in (15.1) and (15.2). If q_{d_1}, q_{d_2} are constant with respect to the time, then the resulting control law is expressed as (17).

$$\tau_1 = M_{11}(q_2)[-\alpha_1 \dot{e}_1 - \beta_1 e_1] + M_{12}(q_2)[-\alpha_2 \dot{e}_2 - \beta_2 e_2] - N_1(q_1, \dot{q}_1, q_2, \dot{q}_2) \tag{17.1}$$

$$\tau_2 = M_{12}(q_2)[-\alpha_1 \dot{e}_1 - \beta_1 e_1] + M_{22}[-\alpha_2 \dot{e}_2 - \beta_2 e_2] - N_2(q_1, \dot{q}_1, q_2, \dot{q}_2) \tag{17.2}$$

A set of controller coefficients for stable error dynamics can be selected as $\alpha_1 = \alpha_2 = 3.8$, $\beta_1 = \beta_2 = 7.1$ by designing LQR state feedback $u_i = -\alpha_i \dot{e}_i - \beta_i e_i$ for $\ddot{e}_i = u_i$ that minimizes $\int_0^\infty (e_i^2 + 0.01\,\dot{e}_i^2 + 0.02\,u_i^2)dt$. Since the closed loop dynamics (16.1) is linear, a quadratic Lyapunov function can be found for the closed loop system as below.

$$V = x^T \begin{bmatrix} P & 0 \\ 0 & P \end{bmatrix} x, \quad P = \begin{bmatrix} 0.54 & 0.14 \\ 0.14 & 0.077 \end{bmatrix} \tag{18.1}$$

$$\dot{V} = -x^T \begin{bmatrix} Q & 0 \\ 0 & Q \end{bmatrix} x, \quad Q = \begin{bmatrix} 2 & 0.54 \\ 0.54 & 0.3 \end{bmatrix} \tag{18.2}$$

$$x(t) = [e_1(t)\ \dot{e}_1(t)\ e_2(t)\ \dot{e}_2(t)]^T \tag{18.3}$$

If the control network delays are taken into account, then the delay from sensor i to controller is denoted by $d_{s_i}(t)$ and the delay from the controller to actuator i is denoted by $d_{a_i}(t)$ for $i \in \{1,2\}$. In this case, the closed loop dynamics is determined by the nonlinear set of equations in (19) instead of (16.1). The time dependencies of the delays are not shown in the following for brevity.

$$M_{11}(q_{d_2} + e_2)\ddot{e}_1 + M_{12}(q_{d_2} + e_2)\ddot{e}_2 - N_1(q_{d_1} + e_1, \dot{e}_1, q_{d_2} + e_2, \dot{e}_2) = \tau_1(t - d_{a_1}) \tag{19.1}$$

$$M_{12}(q_{d_2} + e_2)\ddot{e}_1 + M_{22}\ddot{e}_2 - N_2(q_{d_1} + e_1, \dot{e}_1, q_{d_2} + e_2, \dot{e}_2) = \tau_2(t - d_{a_2}) \tag{19.2}$$

$$\begin{aligned}\tau_1(t) = {} & M_{11}(q_{d_2} + e_2(t - d_{s_2}))[-\alpha_1 \dot{e}_1(t - d_{s_1}) - \beta_1 e_1(t - d_{s_1})] \\ & + M_{12}(q_{d_2} + e_2(t - d_{s_2}))[-\alpha_2 \dot{e}_2(t - d_{s_2}) - \beta_2 e_2(t - d_{s_2})] \\ & - N_1(q_{d_1} + e_1(t - d_{s_1}), \dot{e}_1(t - d_{s_1}), q_{d_2} + e_2(t - d_{s_2}), \dot{e}_2(t - d_{s_2}))\end{aligned} \tag{19.3}$$

$$\begin{aligned}\tau_2(t) = {} & M_{12}(q_{d_2} + e_2(t - d_{s_2}))[-\alpha_1 \dot{e}_1(t - d_{s_1}) - \beta_1 e_1(t - d_{s_1})] \\ & + M_{22}[-\alpha_2 \dot{e}_2(t - d_{s_2}) - \beta_2 e_2(t - d_{s_2})] \\ & - N_2(q_{d_1} + e_1(t - d_{s_1}), \dot{e}_1(t - d_{s_1}), q_{d_2} + e_2(t - d_{s_2}), \dot{e}_2(t - d_{s_2}))\end{aligned} \tag{19.4}$$

Since the controller is static, the delays from sensors to controller can be added to the delays from controller to the actuators. The total delays in the above equations include four time-dependent delay values $d_1 = d_{s_1} + d_{a_1}$, $d_2 = d_{s_1} + d_{a_2}$, $d_3 = d_{s_2} + d_{a_1}$, $d_4 = d_{s_2} + d_{a_2}$. Each delay value contains two parts where the first part appears in (19.1), (19.2) and the second one appears in (19.3), (19.4). The set of equations in (19) is simplified to (16.1) when the delays $d_{s_1}, d_{s_2}, d_{a_1}, d_{a_2}$ are zero.

The timing diagram of the networked control is shown in Figure 3. In the kth control cycle, the controller polls the sensors S1, S2 at $t_{k,0}$, $t_{k,1}$ respectively to collect the feedback information, then it starts to compute the control torques at $t_{k,2}$ and sends them to the actuators A1, A2 where they are received at $t_{k,3}$, $t_{k,4}$ respectively.

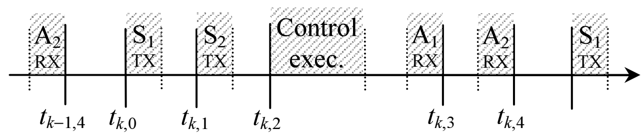

Figure 3. The timing diagram of the kth control cycle.

It can be assumed that the length of control cycle is limited such that for every k and i we have $t_{k,i} - t_{k-1,i} < T$. According to Figure 3, for the kth control cycle one can write $d_{s_1} = t_{k,2} - t_{k,0}$ and $d_{s_2} = t_{k,2} - t_{k,1}$. For the controller to actuator delays, the maximum delay is experienced just before receiving the control data by an actuator at which the delay becomes $d_{a_1} = t_{k,3} - t_{k-1,2}$ for A1 and $d_{a_2} = t_{k,4} - t_{k-1,2}$ for A2. Therefore we can write

$$d_1 < t_{k,3} - t_{k-1,0} < t_{k+1,0} - t_{k-1,0} < 2T \tag{20.1}$$

$$d_2 < t_{k,4} - t_{k-1,0} < t_{k+1,0} - t_{k-1,0} < 2T \tag{20.2}$$

$$d_3 < t_{k,3} - t_{k-1,1} < t_{k+1,0} - t_{k-1,0} < 2T \tag{20.3}$$

$$d_4 < t_{k,4} - t_{k-1,1} < t_{k+1,0} - t_{k-1,0} < 2T \tag{20.4}$$

If more information is available about the time offsets of the events within a control cycle, then tighter bounds than (20) can be obtained.

In the case of digital control without delays there exists a sufficiently small sampling period that can stabilize the control system which is designed in the continuous time frameworks [20, 10]. But this result is not applicable to the robot control problem in this Section because of the time delays. To use Theorem 1 in the previous Section, the closed loop dynamics (19) can be transformed into the form of (1) with x defined in (18.3). Based on (16), (18) and (20), the coefficients in (6) are calculated as $c_f = 8.05$, $c_d = \underline{\sigma}(Q) = 0.145$, $c_v = 2\,\overline{\sigma}(P) = 1.16$ and $r_i = 2T$ for $1 \le i \le 4$ ($\underline{\sigma}$ and $\overline{\sigma}$ denote the smallest and the largest singular values of a matrix respectively). To determine c_{g_i} in (5), it is assumed that $\|\dot{q}_i\| < 10$ rad/sec according to the limited producible torque and physical limitations. Then $\partial f / \partial x_{d_i}$ ($1 \le i \le 4$) are calculated symbolically and their upper bounds are obtained as $c_{g_1} = 249$, $c_{g_2} = 870$, $c_{g_3} = 259$, $c_{g_4} = 922$. By applying Theorem 1, the maximum value of T that guarantees the stability of the robot control system is 0.21 msec.

The response of the robot control system to $q_{d_1} = 1.2$ rad and $q_{d_2} = -0.8$ rad is plotted in Figure 4. The sampling time is $t_{k+1,0} - t_{k,0} = 1$ msec and $t_{k,4} - t_{k,3} = t_{k,3} - t_{k,2} = t_{k,2} - t_{k,1} = t_{k,1} - t_{k,0} = 0.2$ msec for every k. It can be seen that despite of violating the obtained limit $T = 0.21$ msec the tracking performance is satisfactory which shows that the sufficient stability condition in Theorem 1 is conservative. However, according to the existing results for analysis of nonlinear control systems this conservativeness is almost unavoidable. For example, if we assume that $t_{k,4} = t_{k,3} = t_{k,2} = t_{k,1} = t_{k,0}$ then the results of [10] are applicable to obtain a bound on T which is equal to 0.49 msec. This bound is also smaller than the sampling time that can deteriorate the performance. However, these bounds are still useful as design criteria.

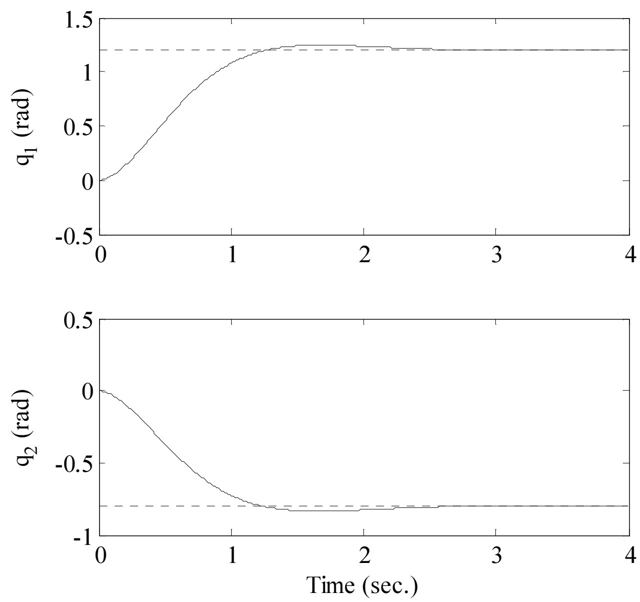

Figure 4. Control performance for 1 msec sampling intervals. Top: response of q_1 (solid) to the command of 1.2 rad (dotted). Bottom: response of q_2 (solid) to the command of −0.8 rad (dotted).

V. Conclusions

A relatively simple stability analysis Theorem was presented for nonlinear systems with multiple time-varying delayed channels. The analysis was based on the Lyapunov-Krasovskii method that has been applied to linear NCS problems in previous works. Application to a robotic arm control system showed that the results are useful especially for studying stability of networked control systems that are designed by neglecting the effects of communication.

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