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Data Projection Method for Sensor Fault Detection and Isolation on Nonlinear Systems Based on Takagi Sugeno Model

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***Abstract*— Robust sensor Fault Detection and Isolation for nonlinear system using partial knowledge of the system is proposed in the paper. First, a suitable Takagi-Sugeno model for uncertain nonlinear system is determined using sector nonlinear method. Then the matrix input-output relation is given and the robust FDI residual is computed by projecting the relation onto the Kernel of the predetermined matrix.**

I. INTRODUCTION

Nowadays, Fault Detection and Isolation (FDI) methods have a growing interest ([1]-[3] and [25]).

The goal of FDI methods is to detect fault occurrence and to locate which component is subject to the fault. Model-based methods generally use mathematical model ([6]) and measurable input-output signals of the system to generate fault indicator signals named residuals ([5], [12]). These residuals indicate no fault situation when they are equal to zero mean and fault occurrence is detected when this mean differs significantly from zero. Consequently, an accurate mathematical model is required.

However, especially for nonlinear systems, its exact modeling is difficult to realize. Therefore, the system's model is subject to parameter uncertainties. These uncertainties disturb the residual and can lead to false alarms or no fault detection.

To cope with these uncertainties, robust fault detection methods are proposed ([4] and [25]). The task here is to find robust residuals against constant model uncertainties.

The goal of this paper is to provide a robust fault detection methods based on a partial knowledge of the system and using Takagi Sugeno (TS) approach. Only constant uncertainties are considered here. The TS approach can provide a mathematical model of different kind of systems using identification techniques ([16], [20] and [21]) or the nonlinear sector method ([9]-[11]). Sector nonlinear technique is used in this article to provide a particular form of TS model which suited for robust fault detection and isolation. All model uncertainties are set in local models and weighting functions are kept free from the uncertainties. Finally, robust residual is generated by projecting the system output matrix onto the kernel of a predetermined data matrix.

A simulation of a simplified quarter car model which is assumed as a mass-spring-damper model system is used to illustrate the method.

The paper is organized as follows: The TS model and its construction from the sector nonlinearity approach are presented in section 2. The proposed method based on data projection of the output matrix onto the weighted input matrix is provided in section 3. Section 4 is devoted to the description of mass-springer-damper model which is subject to uncertainties. Then, illustration of the proposed method is provided.

II. TAKAGI SUGENO MODEL

The target of this section is to provide an exact TS representation of nonlinear uncertain systems. The sector nonlinear approach is used in the following to reach this goal. The TS model is designed such as all uncertainties are in the local state and the activation functions contain no uncertainty. This representation is used in the next sections to yield a robust residual for fault detection and isolation.

Given the nonlinear system represented by:

$$\begin{aligned} x_{k+1} &= f\left(x_k, u_k, \Delta_f\right) x_k + g\left(x_k, u_k, \Delta_g\right) u_k \\ y_k &= h\left(x_k, u_k\right) + v\left(x_k, u_k\right) u_k + e_k \end{aligned} \tag{1}$$

where f, g, h and v are nonlinear matrix functions where terms are assumed to be bounded and Δ_f and Δ_g are uncertainties in functions f and g. The input, output and state vectors are respectively $u_k \in \mathcal{R}^m$, $y_k \in \mathcal{R}^\ell$ and $x_k \in \mathcal{R}^n$. The vector $e_k \in \mathcal{R}^\ell$ is a colored centered noise. The objective is to determine a TS uncertain model of the nonlinear system (1) such as no uncertainty is contained in the weighting functions.

Let us determine the TS uncertain model. Using a sector nonlinear approach ([5]), a TS representation of the nonlinear system is given by N ($N = 2^p$, where p is the number of nonlinearities in the nonlinear function (1)) rules. Rule R_γ $(1 < \gamma < N)$ is defined as:

$$\begin{aligned} &Model\ Rule\ \gamma: \\ &If\ z_1\ in\ \mathcal{O}_1^\gamma\ and\ \ldots\ and\ z_p\ in\ \mathcal{O}_p^\gamma\ Then \\ &\left\{ x_{k+1}^\gamma = \left(A_\gamma + \Delta A_\gamma\right) x_k + \left(B_\gamma + \Delta B_\gamma\right) u_k \right. \end{aligned} \tag{2}$$

where $A_\gamma \in \mathcal{R}^{n\times n}$, $B_\gamma \in \mathcal{R}^{n\times m}$ are subsystem matrices and $\Delta A_\gamma \in \mathcal{R}^{n\times n}$, $\Delta B_\gamma \in \mathcal{R}^{n\times m}$ are constant uncertain matrices with appropriate dimensions. The variable $z_j, 1 \leq j \leq p$ denotes chosen scheduled non-constant terms in f, g, h and v, and $\mathcal{O}_j^\gamma,\ 1 \leq \gamma \leq N,\ 1 \leq j \leq p$ are the fuzzy sets.

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We assume that these variables are measurable or can be computed from the input-output system.

Remark: x_{k+1}^{γ} denotes the value of the state variable obtained from the γ^{th} rule.

For a given premise vector $Z = (z_1 \; z_2 \; \ldots z_p)$, the resulting TS model-based dynamic system with constant-parametric uncertainties inferred from (2) is given by:

$$\begin{cases} x_{k+1} = \sum\limits_{\gamma=1}^{N} \omega_k^{\gamma}(Z)\left((A_\gamma + \Delta A_\gamma)x_k + (B_\gamma + \Delta B_\gamma)u_k\right) \\ y_k = \sum\limits_{\gamma=1}^{N} \omega_k^{\gamma}(Z)\left(C_\gamma x_k + D_\gamma u_k\right) + e_k \end{cases} \tag{3}$$

where the weighting function that corresponds to the fuzzy set in the rule γ is given as follows:

$$\omega_k^{\gamma}(Z) = \frac{\varpi_k^{\gamma}(Z)}{\sum\limits_{i=1}^{N} \varpi_k^{i}(Z)} \tag{4}$$

$$\omega_k^{\gamma}(Z) \geq 0, \quad \sum_{\gamma=1}^{N} \omega_k^{\gamma}(Z) = 1 \tag{5}$$

with $\varpi_k^{\gamma}(Z)$ is the degree of fulfillment of the rule γ and obtained by the product of the membership grade of the premise variables to their corresponding modalities $\mathcal{O}_j^{\gamma}$:

$$\varpi_k^{\gamma}(Z) = \prod_{j=1}^{p} \mathcal{O}_j^{\gamma} \tag{6}$$

III. Data-Projection method

This section describes the proposed residual generation method using only input-output data and weighting functions of the TS model expressed by equation (3). For this purpose, input-output matrix relation is established on a given time window of size L. Secondly, a projection matrix is computed on-line to eliminate both unknown state and the model parameters ([19]-[21]). Finally, sensitivity against constant model uncertainties is discussed.

A. Input-Output relation

To establish the input-output relation, the following application is defined,

$$\begin{array}{l} \mathcal{M}(\imath, nN) \times \mathcal{M}(n, \jmath N) \rightarrow \mathcal{M}\left(\imath, \jmath N^2\right) \\ [V_1 \; \ldots \; V_N] \bigodot [R_1 \; \ldots \; R_N] \mapsto [V_1R_1 \; V_1R_2 \; \ldots \; V_NR_N] \end{array} \tag{7}$$

where the matrices $V_\gamma \in \mathcal{R}^{\imath \times n}$ and $R_\gamma \in \mathcal{R}^{n \times \jmath}$ and $\mathcal{M}(a_1, a_2)$ represents the set of $(a_1 \; rows, \; a_2 \; columns)$ matrices. The scalar N represents the number of local model. Some properties are defined here to clarify the function's application and to simplify further developments. Let $R_0 = [R_1 \; R_2 \ldots \; R_N]$

- $\underbrace{R_0 \odot \ldots \odot R_0}_{i} = \prod\limits_{\substack{\odot \\ q=0}}^{i} R_0 = \wp^i(R_0) \quad \forall i \in \mathcal{Z}_+^* \setminus \{1\}$
- $\wp^i(R_0) = \zeta \quad for \; i \leq \; 0.$
- $\wp^1(R_0) = R_0 \odot \zeta = R_0$
  where $\zeta = [I_j \; \ldots \; I_j]$ and I_j is the identity matrix of size j.

Remark: For the simplicity sake, we will not use the model uncertainties $(\Delta A, \Delta B)$ in the following, but the relations established properties are trues for uncertainties systems too by replacing A by $A + \Delta A$ and B by $B + \Delta B$ further in proposition 1.

Proposition 1:
The following equations (8) is equivalent to equations (9):

$$\begin{cases} x_{k+1} = \sum\limits_{\gamma=1}^{N} \omega_k^{\gamma}(Z)\left(A_\gamma x_k + B_\gamma u_k\right) \\ y_k = \sum\limits_{\gamma=1}^{N} \omega_k^{\gamma}(Z)\left(C_\gamma x_k + D_\gamma u_k\right) + e_k \end{cases} \tag{8}$$

$$\begin{cases} x_{k+1} = \left(\wp^{p+1}(A_0)\right)\Psi_k^p x_{k-p} \\ \qquad + \sum\limits_{r=0}^{p} \left(\wp^{p-r}(A_0) \odot B_0\right)\Psi_k^{p-r} u_{k-p+r} \\ y_k = \left(C_0 \odot \wp^p(A_0)\right)\Psi_k^p x_{k-p} \\ \quad + \sum\limits_{r=0}^{p-1} \left(C_0 \odot \wp^{p-r-1}(A_0) \odot B_0\right)\Psi_k^{p-r} u_{k-p+r} \\ \quad + D_0 \left(\Psi_k^0\right)^T u_k + e_k \end{cases} \tag{9}$$

with $\Psi_\beta^\alpha = \left(\prod\limits_{\substack{\odot \\ q=0}}^{\alpha} \omega_{\beta-q}^0\right)^T$, $A_0 = [A_1 \; A_2 \ldots A_N]$, $B_0 = [B_1 \; B_2 \ldots B_N]$, $C_0 = [C_1 \; C_2 \ldots C_N]$, $D_0 = [D_1 \; D_2 \ldots D_N]$ and $\omega_\beta^0 = \left[\omega_\beta^1 \; \omega_\beta^2 \ldots \omega_\beta^N\right]$.

Proof:
(i) Basis step
The proof is given by induction on the power p. The equivalence is straightforward for $p = 0$, using equation (9):

$$\begin{aligned} x_{k+1} &= \underbrace{\wp^1(A_0)}_{A_0} \underbrace{\Psi_k^0}_{\left(\omega_k^0\right)^T} x_k + \sum_{r=0}^{0} \underbrace{\left(\wp^0(A_0) \odot B_0\right)}_{\zeta \odot B_0} \underbrace{\Psi_k^0}_{\left(\omega_k^0\right)^T} u_k \\ &= A_0 \underbrace{\Psi_k^0}_{\left(\omega_k^0\right)^T} x_k + B_0 \underbrace{\Psi_k^0}_{\left(\omega_k^0\right)^T} u_k \end{aligned} \tag{10}$$

Then, for the output y_k we have:

$$\begin{aligned} y_k &= \underbrace{\left(C_0 \odot \wp^0(A_0)\right)}_{C_0 \odot \zeta} \underbrace{\Psi_k^0}_{\left(\omega_k^0\right)^T} x_k + D_0 \underbrace{\Psi_k^0}_{\left(\omega_k^0\right)^T} u_k + e_k \\ &= C_0 \underbrace{\Psi_k^0}_{\left(\omega_k^0\right)^T} + D_0 \underbrace{\Psi_k^0}_{\left(\omega_k^0\right)^T} u_k + e_k \end{aligned} \tag{11}$$

(ii) Inductive step
Here, we show that if statement (9) holds for p, then it holds also for $p + 1$:
In (8), we have:

$$x_{k+1} = \sum_{\gamma=1}^{N} \omega_k^{\gamma}(Z)\left(A_\gamma x_k + B_\gamma u_k\right) \tag{12}$$

If we assume that the equivalence (9) holds at stage p, then we have:

$$x_k = \wp^p(A_0)\,\Psi_{k-1}^{p-1}x_{k-p} + \sum_{r=0}^{p-1}\left(\wp^{p-1-r}(A_0)\odot B_0\right)\Psi_{k-1}^{p-1-r}u_{k-p+r} \tag{13}$$

If we substitute the above expression of x_k in (12), we have:

$$\begin{aligned}
x_{k+1} &= \sum_{\gamma=1}^{N}\omega_k^{\gamma}(Z)\,A_\gamma\wp^p(A_0)\,\Psi_{k-1}^{p-1}x_{k-p} \\
&+\sum_{\gamma=1}^{N}\sum_{r=0}^{p-1}A_\gamma\left(\wp^{p-1-r}(A_0)\odot B_0\right)\Psi_{k-1}^{p-1-r}\omega_k^{\gamma}u_{k-p+r} \\
&+\sum_{\gamma=1}^{N}\omega_k^{\gamma}B_\gamma u_k \\
&= \underbrace{\left[A_1\wp^p(A_0)\,A_2\wp^p(A_0)\ldots A_N\wp^p(A_0)\right]}_{A_0\odot\wp^p(A_0)=\wp^{p+1}(A_0)}
\end{aligned}$$

$$\underbrace{\left[\omega_k^1(\Psi_{k-1}^{p-1})^T\,\omega_k^2(\Psi_{k-1}^{p-1})^T\ldots\omega_k^N(\Psi_{k-1}^{p-1})^T\right]^T}_{\omega_k^0\odot\Psi_{k-1}^{p-1}=\Psi_k^p}x_{k-p}$$

$$\begin{aligned}
&+\sum_{r=0}^{p-1}\sum_{\gamma=1}^{N}A_\gamma\wp^{p-1-r}(A_0)\odot B_0\Psi_{k-1}^{p-1-r}\omega_k^{\gamma}u_{k-p+r} \\
&+\sum_{\gamma=1}^{N}B_\gamma\omega_k^{\gamma}u_k \\
&= \wp^{p+1}(A_0)\underbrace{\left(\sum_{\gamma=1}^{N}\omega_k^{\gamma}\Psi_{k_1}^{p-1}\right)}_{\omega_k^0\odot\left(\Psi_{k-1}^{p-1}\right)^T=\Psi_k^p}x_{k-p} \\
&+\sum_{r=0}^{p-1}\left(\wp^{p-r}(A_0)\odot B_0\right)\times \\
&\left[\left(\Psi_{k-1}^{p-1-r}\right)^T\omega_k^1\left(\Psi_{k-1}^{p-1-r}\right)^T\omega_k^2\ldots\left(\Psi_{k-1}^{p-1-r}\right)^T\omega_k^N\right]\times \\
&u_{k-p+r}+B_0\Psi_k^0u_k \\
&= \wp^{p+1}(A_0)\,\Psi_k^px_{k-p} \\
&+\sum_{r=0}^{p-1}\left(\wp^{p-r}(A_0)\odot B_0\right)\Psi_k^{p-r}u_{k-p+r} \\
&+\underbrace{B_0\Psi_k^0u_k}_{\left(\wp^0(A_0)\odot B_0\right)\Psi_k^0u_k}
\end{aligned}$$

$$x_{k+1} = \wp^{p+1}(A_0)\,\Psi_k^px_{k-p} + \sum_{r=0}^{p}\left(\wp^{p-r}(A_0)\odot B_0\right)\Psi_k^{p-r}u_{k-p+r} \tag{14}$$

That proves the equivalence holds at stage $p+1$.
One can prove in the same way that the output y_k is equal to:

$$\begin{aligned}
y_k &= \left(C_0\odot\wp^p(A_0)\right)\Psi_k^px_{k-p} \\
&+\sum_{r=0}^{p-1}\left(C_0\odot\wp^{p-1-r}(A_0)\odot B_0\right)\Psi_k^{p-r}u_{k-p+r} \\
&+D_0\Psi_k^0u_k+e_k
\end{aligned} \tag{15}$$

■

Proposition 2: For a large p, the contribution of the initial state can be neglected under stability condition of all local models . That implies $\|\wp^p(A_0)\,\Psi_k^px_{k-p}\| \simeq 0$. Therefore, the output equation (11) will be expressed as:

$$\begin{aligned}
y_k &\approx \sum_{r=0}^{p-1}\left(C_0\odot\wp^{p-1-r}(A_0)\odot B_0\right)\Psi_k^{p-r}u_{k-p+r} \\
&+D_0\Psi_k^0u_k+e_k
\end{aligned} \tag{16}$$

Proof: In the following, we prove that each terms of $\wp^p(A_0)$ is negligible.

Let consider, first each term of the result of $\|\wp^p(A_0)\| = \|\underbrace{A_1^p}_{term_1}|\underbrace{A_1^{p-1}A_2^1}_{term_2}|A_1^{p-2}A_2^1A_3^1|\ldots|\underbrace{A_N^p}_{term_{N^p}}\|$.

Each term is in form: $\|A_1^{S_1}A_2^{S_2}A_3^{S_3}\ldots A_N^{S_N}\|$, with $\sum_{i=0}^{N}S_i = p$. Then, by applying the sub-multiplicative norm theorem, we have:

$$\begin{aligned}
\|A_1^{S_1}A_2^{S_2}A_3^{S_3}\ldots A_N^{S_N}\| &\le \|A_1^{S_1}\|\|A_2^{S_2}\|\|A_3^{S_3}\|\ldots\|A_N^{S_N}\| \\
&\le \left(max_\gamma\left(\|A_\gamma\|\right)\right)^p \\
and\|\wp^p(A_0)\| \le \left(max_\gamma\left(\|A_\gamma\|\right)\right)^{Np} &\quad 1\le\gamma\le N
\end{aligned} \tag{17}$$

Since each local model is stable, we have $\|A_\gamma^S\|\to 0$ when $S\to\infty$. Then $\|A_1^{S_1}A_2^{S_2}A_3^{S_3}\ldots A_N^{S_N}\| \simeq 0$ if S is great enough. ■

Finally, by staking the output $Y_k = (y_k\, y_{k+1}\ldots y_{k+L-1}) \in \mathcal{R}^{\ell\times L}$ on a horizon L, the expression equation (16) will be expressed as:

$$Y_k \approx H\Phi_k + E_k \tag{18}$$

with $\Phi_k = \Omega_k\circ U_k$ is the Hadamard product of the inputs matrix by the weighting matrix.
where

$$H = \left[C_0\odot\wp^{p-1}(A_0)\odot B_0\,|\ldots|\,C_0\odot\wp^0(A_0)\odot B_0\,|D_0\right] \tag{19}$$

$$\Omega_k = \begin{bmatrix}\Psi_k^{p-1} & \ldots & \Psi_k^1 & \Psi_k^0\\ \Psi_{k+1}^{p-1} & \ldots & \Psi_{k+1}^1 & \Psi_{k+1}^0\\ \vdots & \ldots & \vdots & \vdots\\ \Psi_{k+L-1}^{p-1} & \ldots & \Psi_{k+L-1}^1 & \Psi_{k+L-1}^0\end{bmatrix} \tag{20}$$

$$U_k = \begin{bmatrix} u_{k-p}\mathbf{1}_{mN^p} & u_{k-p+1}\mathbf{1}_{mN^p} & \dots & u_{k-p+L-1}\mathbf{1}_{mN^p} \\ \vdots & \vdots & \vdots & \vdots \\ u_{k-1}\mathbf{1}_{mN} & u_k\mathbf{1}_{mN} & \cdots & u_{k+L-2}\mathbf{1}_{mN} \\ u_k\mathbf{1}_m & u_{k+1}\mathbf{1}_m & \cdots & u_{k+L-1}\mathbf{1}_m \end{bmatrix} \quad (21)$$

with $\mathbf{1}_r = \left(1 \dots 1\right)^T \in \mathcal{R}^{r\times 1}$.

$$E_k = [e_k \; e_{k+1} \; \dots \; e_{k+L-1}] \quad (22)$$

- Choice of p:

The determination of integer p should be done in initial stage, off-line, using a sufficient excited input (like Pseudo Random Binary Signal. The minimisation of the criterion $J(p)$ gives the value of p (see [24]):

$$J(p) = \|Y_k \Phi_k^{\perp}\| \quad (23)$$

with $\Phi_k^{\perp}$ is a projection matrix.

B. Robust Residual Generation

The proposed data-based residual is obtained by projecting the matrix of input-output relation in (18) onto the right kernel of Φ_k. The projection matrix $\Phi_k^{\perp}$ is given as follows:

$$\Phi_k^{\perp} = I - \Phi_k^T \left(\Phi_k \Phi_k^T\right)^{-1} \Phi_k \quad (24)$$

A necessary condition that allows the existing of the projection matrix $\Phi_k^{\perp}$ is given by:

$$L > m\left(\sum_{r=1}^{p} N^r\right) + 1 \quad (25)$$

The residual expression is obtained by right-multiplying equation (18) by the projection matrix $\Phi_k^{\perp}$ in equation (24):

$$\varepsilon(k) = Y_k \Phi_k^{\perp} S \quad \approx \underbrace{H\Phi_k \Phi_k^{\perp} S}_{=0} + E_k \Phi_k^{\perp} S = E_k \Phi_k^{\perp} S \quad (26)$$

where $S = (0 \; \dots \; 0 \; 1) \in \mathcal{R}^{\ell \times 1}$ is a vector that selects the last column of the generated residual. It represents the current residual at time-instant k.

C. Sensitivity to fault

The sensitivity of the residual is analyzed here.
If the fault f_k occurs on a time-window $[k, \; k+L-1]$, then we have:

$$Y_k = H\Phi_k + F_k + E_k \quad (27)$$

where F_k is defined in the same way as E_k in (22). The residual becomes:

$$\begin{aligned} \varepsilon(k) &= Y_k \Phi_k^{\perp} S \\ &= F_k \Phi_k^{\perp} S + E_k \Phi_k^{\perp} S \end{aligned} \quad (28)$$

If the row space of the fault matrix F_k is not included in the right Kernel of Φ_k, then the term $F_k \Phi_k^{\perp}$ is different from zero and the residual is sensitive to the fault. Indeed, its mathematical expectation is:

$$\begin{aligned} E[\varepsilon(k)] &= E\left[F_k \Phi_k^{\perp} + E_k \Phi_k^{\perp}\right] \\ &= F_k \Phi_k^{\perp} \neq 0 \end{aligned} \quad (29)$$

But in no fault situation, $F_k = 0$ and its mathematical expectation is null.

D. Robustness of residual

We prove here the robustness of the residual to constant parameters uncertainties. Equation (3) gives the uncertain TS model of the system. If there are uncertainties $(\Delta A_\gamma, \Delta B_\gamma)$, the equation (18) becomes:

$$Y_k \approx [H + \Delta H]\,\Phi_k + E_k \quad (30)$$

and the residual is:

$$\varepsilon(k) = \underbrace{[H + \Delta H]}_{\overline{H}} \Phi_k \Phi_k^{\perp} S + E_k \Phi^{\perp} S \quad (31)$$

As a result, the projection residuals $\varepsilon(k) = Y_k\left(\Phi_k^{\perp}\right) + E_k$ are insensitive to $\bar{H}$ and consequently to constant model uncertainties.

IV. Example

To illustrate the proposed method, we consider a nonlinear mass-damper-spring system used in [22]. We consider constant parameters uncertainties to show the robustness of the proposed technique. The considered nonlinear system is the mass-damper-spring system as shown in Fig. (1).

A. TS model determination for robust FDI

Two parametric uncertainties are considered as in ([22] and [23]). The dynamic equation of system with parametric uncertainties is described as:

$$M\ddot{x}(t) + g(x(t), \dot{x}(t)) + f(x(t)) = \phi(\dot{x}(t))\,u(t) \quad (32)$$

where M is the mass and $u(t)$ is the force. The functions $f(x(t))$, $g(x(t), \dot{x}(t))$ and $\phi(\dot{x}(t))$ are a nonlinear or uncertain terms with respect respectively to the spring, to the damper and to the input term. Here it is assumed that $x \in [-1.5, 1.5]$, $\dot{x} \in [-1.5, 1.5]$, $g(\dot{x}, x) = D(c_1 x + c_2 \dot{x})$, $f(x) = (c_3 + \Delta_1(t))\,x$ and $\phi(\dot{x}) = c_4 + \Delta_2(t) + c_5 \dot{x}^3$.
The above parameters are set as follows:
$M = 1$, $D = 1$, $c_1 = 0$, $c_2 = 1$, $c_3 = 1.13$, $c_4 = 1$, $c_5 = 0.13$ and the terms of parametric uncertainties are

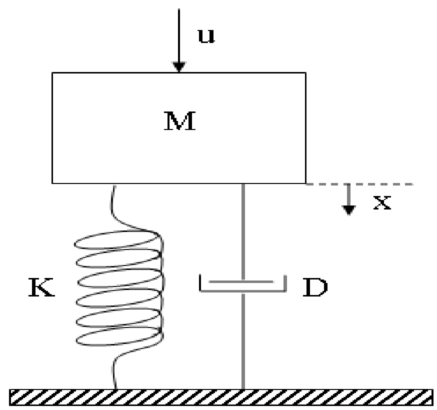

Fig. 1. Mass-Damper-Spring System.

$\Delta_1(t) \in [-1.07,\ 0.9]$ and $\Delta_2(t) \in [-0.54,\ 2]$. Then, the uncertain nonlinear system in (32) can be rewritten as:

$$\begin{aligned} \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} &= \left(\begin{pmatrix} 0 & 1 \\ -1.13 & -1 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ -\Delta_1(t) & 0 \end{pmatrix} \right) \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \\ &\quad + \left(\begin{pmatrix} 0 \\ 1 + 0.13x_2^3 \end{pmatrix} + \begin{pmatrix} 0 \\ \Delta_2(t) \end{pmatrix} \right) u(t) \\ y(t) &= x_1 \end{aligned} \tag{33}$$

where the vector of state represents the position and the velocity respectively to x_1 and x_2. Therefore, based on the sector nonlinearity approach described in section I, the following two-rules TS fuzzy model with two parametric uncertainties can approximate the uncertain nonlinear system in Fig. (1) as follows:

$$\begin{aligned} \dot{x}(t) = \sum_{\gamma=1}^{2} \omega^{\gamma}(z(t)) \left[(A_\gamma + \Delta_1(t)) x(t) \right. \\ \left. + (B_\gamma + \Delta_2(t)) u(t) \right] \end{aligned} \tag{34}$$

where $z(t) = (x_2)^3 \in [-3.375,\ 3.375]$ is the nonlinear term and matrices are: $A_1 = A_2 = \begin{pmatrix} 0 & 1 \\ -1.13 & -1 \end{pmatrix}; B_1 = \begin{pmatrix} 0 \\ 1.43875 \end{pmatrix}; B_2 = \begin{pmatrix} 0 \\ -0.56125 \end{pmatrix}; C = (1\ 0)$.

Finally, the discrete TS model is given by the following expression:

$$\begin{aligned} &Rule1: \ If \quad z(k)\ is\ \mathcal{O}_1^1\ Then \\ &x_{k+1} = (A_1 + \Delta_1) Te\, x_k + x_k + (B_1 + \Delta_2) Te\, u_k \\ &Rule2: \ If \quad z(k)\ is\ \mathcal{O}_1^2\ Then \\ &x_{k+1} = (A_2 + \Delta_1) Te\, x_k + x_k + (B_2 + \Delta_2) Te\, u_k \end{aligned} \tag{35}$$

For the discrete TS model , we used Euler method as follows:

$$\frac{dx(t)}{dt} = \frac{x(k+1) - x(k)}{Te} \tag{36}$$

where $Te = 0.1s$ is the sampling time used for the discretization procedure. The nonlinear term $z(k)$ has been divided in two regions according to the membership depicted in Fig. (2). The weighting functions are given by:

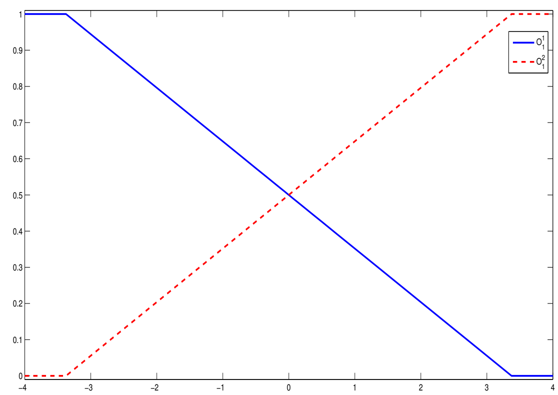

Fig. 2. Membership function in TS model (28).

$$\begin{aligned} \omega_k^1 &= \omega^1(z(k)) = \tfrac{3.375 - z(k)}{6.75} \\ \omega_k^2 &= 1 - \omega^1(z(k)) = \tfrac{-3.375 + z(k)}{6.75} \end{aligned} \tag{37}$$

The parameters uncertainties are as follows:
$\Delta_1 = \begin{pmatrix} 0 & 0 \\ -0.5 & 0 \end{pmatrix}; \Delta_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

B. Fault detection and isolation

We have simulate the nonlinear system for $240e$ seconds with zero initial state and white Gaussian unity-variance measurement noise and the sensor fault occurs at the interval time $[2000Te, 2400Te]$. The system output $y_k = x_k^1$ is shown in Fig. (3).

Robust residual is computed with $p = 8$ and $L = 1000Te$. Fig. (4) shows the effective detection of the fault with the computed residual. The sensor fault is detected in time-instant $T_f = 2000Te$.

V. Conclusions

In this paper, a design framework for robust fault detection and isolation is developed for a class of discrete-time nonlinear systems described by a TS fuzzy model. The framework includes only input-output data and weighting functions. Residuals generated by data-based projection method are insensitive to the constant model uncertainties. Simulation results of a discrete-time nonlinear system mass-damper-spring

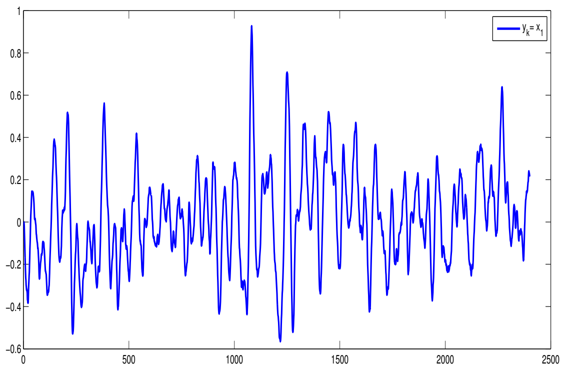

Fig. 3. System's output.

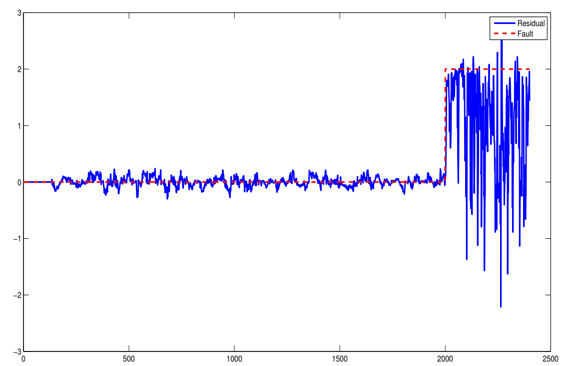

Fig. 4. Residual.

model are used to show the effectiveness of the obtained results. The example shows the robustness of the method to constant uncertainties. Future work will extend the proposed method to nonlinear systems subject to variable parameters uncertainties and to TS fuzzy models with unmeasurable premise variables.

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