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A lane control mechanism with fault tolerant control capabilities

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***Abstract*— The paper discusses the problem of lane departure avoidance for a vehicle. A corrective mechanism imposes its control action whenever the vehicle is no longer inside a nominal region centered along the middle of the lane. Set theoretic methods are used in order to design this control action and to guarantee global stability. Additionally, for the same lane departure avoidance system, a fault tolerant control mechanism is proposed in order to discard faulty sensors in a redundant measurement setting, thus guaranteeing stability even in the presence of faults.**

I. Introduction

Lane departure avoidance represents a topic of interest in today's automotive control applications. It concerns a class of systems intrinsically more complex than fully automation components as the one described in [1] since their aim is to design a switched control mechanism. That is, the control is provided either by the driver in normal conditions either through an assistance mechanism which takes control in abnormal condition and/or when the driver is deemed inattentive or incapacitated. Due to intermittent switching and interaction with the driver, the complexity of the scheme is greatly increased. We note previous results in this area, e.g., [2] and [3] which propose as actuator for vehicle lateral control a DC motor mounted on the steering column, whereas in [4] a differential braking approach is advocated. Notably, in [5] a combination of the two aforementioned methods is provided.

Another topic of wide interest which emerged in the last decades as a main challenge in control is the detection and isolation of faults and the subsequent fault tolerant control (FTC) of applications. The presence of faults in closed-loop control systems creates severe practical challenges with potentially disastrous consequences (e.g., processes in chemical plants or aviation catastrophes). As a result, various FTC schemes are deployed to counteract faults affecting the subsystems of a plant: using information provided by a fault detection and isolation (FDI) block, a reconfiguration mechanism (RC) reconfigures the control action in order to minimize or discard the influence of the fault occurrences [6].

The lane departure avoidance system is an assistance system which aims to negate faults on the part of the driver (such as lapses in attention or temporary incapacity). It is then natural to complete the scheme by adding a fault tolerant control layer which detects and counteracts faults in the physical components of the scheme (and in particular in the sensors since they are the components most prone to faults). Several similar approaches exist in the literature: [7] discusses fault detection and reconfiguration mechanisms for lateral control in automated highway systems; [8] shows that, in the event of a sensor (from a bank of redundant sensors) fault, the system keeps tracking the lane.

As a novelty, in the present paper we propose the use of a set theoretic framework for both control and fault detection. Polyhedral sets will be used to guarantee stability in the absence of faults and to implement a simplified model of control switching with the help of the redundant information provided by a bank of sensors. The goal is that whenever the vehicle dynamics exit a nominal region, the corrective mechanism will be able to return the state to its nominal region without violating given safety bounds.

The fault tolerant layer considers and manages the possibility of faults in the bank of sensors which are used to recuperate the system state. We design a FDI mechanism by comparing the expected mathematical model with the actual results under a set theoretic framework based on previous results presented in [9] and [10]. Sets which describe the healthy and faulty behavior of a fault sensitive signal are computed and the fault detection reduces to set membership testings for the aforementioned signal. Note that the use of sets, although not unique (see for example [11]), differs from other approaches by the use of invariance notions which simplifies the computations and offers stability guarantees under mild conditions: by exact fault detection, a sensor under fault can always be detected and the information it provides is discarded from the reconfiguration mechanism.

Notation

For a vector or a set denoted as x the notations, x, x^-, x^+ denote, respectively, the current, previous and successor values $x(k)$, $x(k-1)$ and $x(k+1)$ for some integer $k > 0$. The Minkowski sum of two sets, A and B is denoted as $A \oplus B = \{x : \, x = a + b, \, a \in A, b \in B\}$, whereas, the Pontryagin difference is denoted as $A \ominus B = \{a : \, a + b \in A, \, a \in A, b \in B\}$. We define for further use the bounding set $\mathbb{B}(\alpha) = \{x : \, |x| \leq \alpha\}$ parameterized after the elementwise positive vector α.

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II. Vehicle lateral dynamics

For the design of the vehicle lateral control, a fourth-order discrete linear "bicycle model" ([12]) has been used[1]:

$$x^+ = Ax + Bu + B_\rho \rho_{ref} \tag{1}$$

where $x = \begin{bmatrix}\beta & r & y_L & \psi_L\end{bmatrix}^T$ denotes the state with β the sideslip angle, r the yaw rate, y_L the lateral offset and ψ_L the relative yaw angle. Input u is the steering angle of the front wheels and ρ_{ref} denotes the road curvature (considered here as a disturbance).

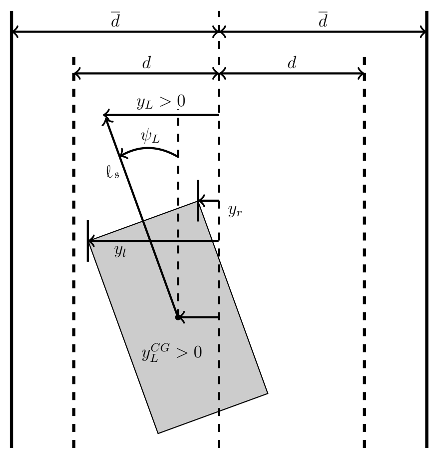


Fig. 1: Vehicle lane model

Matrices $A_c \in \mathbb{R}^{n\times n}$, $B_c \in \mathbb{R}^{n\times m}$ and $B_{c,\rho} \in \mathbb{R}^{n\times m_\rho}$ which describe the continuous counterpart of system (1) are given as follows:

$$A_c = \begin{bmatrix} a_{11} & a_{12} & 0 & 0 \\ a_{21} & a_{22} & 0 & 0 \\ 0 & 1 & 0 & 0 \\ v & l_s & v & 0 \end{bmatrix}, B_c = \begin{bmatrix} b_1 \\ b_2 \\ 0 \\ 0 \end{bmatrix}, B_{c,\rho} = \begin{bmatrix} 0 \\ 0 \\ -v \\ 0 \end{bmatrix}$$

where the parameters used above depend on vehicle characteristics and can be retrieved from [13].

The system $(A_c, B_c, B_{c,\rho})$ is discretized into (A, B, B_ρ) through a fixed step $h = 2.5ms$.

A. Sensors and estimators dynamics

For measuring purposes we associate to the vehicle a bank of sensors S_i, $i = 1, \dots, N$. The sensors are assumed to be static (i.e., with very fast dynamics relative to the vehicle dynamics) and to satisfy, under healthy functioning:

$$y_i = C_i x + \eta_i \tag{2}$$

with $y_i \in \mathbb{R}^{p_i}$ the sensor output, $C_i \in \mathbb{R}^{p_i \times n}$ the output matrix and $\eta_i \in \mathbb{R}^{p_i}$ the bounded measurement noise[2] belonging to a compact set.

[1] To simplify the problem, the system was linearized by considering small angles and a constant velocity.

[2] For the following numerical examples the manipulated sets will be considered to be polyhedral for their numerical reliability.

The information provided independently by each sensor, together with the system known input, are used to construct N state estimators:

$$\hat{x}_i^+ = A\hat{x}_i + Bu + L_i(y_i - C_i\hat{x}_i), \tag{3}$$

with matrices L_i chosen such that $A - L_iC_i$ have their eigenvalues strictly inside the unit circle.

The estimation errors are defined as

$$\tilde{x}_i \triangleq x - \hat{x}_i, \quad i = 1, \dots, N \tag{4}$$

and using (2), (3) and (4) we can write

$$n\tilde{x}_i^+ = \big(A - L_iC_i\big)\,\tilde{x}_i + B_\rho \rho_{ref} - L_i\eta_i. \tag{5}$$

III. Control mechanism

A. Preliminaries

The control objective for the vehicle is to remain inside a predefined strip with respect to the center of the lane. These limits are described by the constraints imposed to the values y_l and y_r, the offsets of the left, respectively the right, side of the vehicle. These values can be expressed as a linear (see footnote [1]) combination of components of the state, y_L and ψ_L and the parameters l_f and l_s:

$$y_l = y_L + (l_f - l_s)\psi_L + \frac{a}{2},\ y_r = y_L + (l_f - l_s)\psi_L - \frac{a}{2}. \tag{6}$$

For further use, by exploiting (6) we define the polyhedral region $R(\lambda) \subset \mathbb{R}^4$:

$$R(\lambda) = \left\{ x : \left| \begin{bmatrix} 0 & 0 & l_f - l_s & 1 \end{bmatrix} x \right| \leq \frac{2\lambda - a}{2} \right\} \tag{7}$$

parameterized after a positive scalar λ which constrains y_l and y_r to be inside a predefined strip of $\pm\lambda$ width.

We are now able to describe the nominal and safety regions of interest. By considering the nominal set as defined by a strip of $\pm d$ width around the center of the lane and nominal bounds x_N on the state we obtain the following set description of the nominal region:

$$S = R(d) \cap \mathbb{B}\left(x_N\right). \tag{8}$$

Whenever the vehicle violates these constraints, a control action is provided by a corrective mechanism which aims to steer the vehicle inside the aforementioned bounds whilst in the same time respecting safety constraints (it must contain the offsets y_l, y_r inside a span of $\pm L/2$ around the center of the lane and respect safety bounds x_S upon the state). The set describing the admissible state is given as follows:

$$\bar{S} = R(L/2) \cap \mathbb{B}\left(x_S\right). \tag{9}$$

Ideally, for a known value of the state, the control action is provided by the following switch mechanism:

$$u = \begin{cases} u^d, & x \in S \\ u^a, & x \in \bar{S} \setminus S \end{cases} \tag{10}$$

where inputs u^d and u^a denote the input provided by the driver, respectively by the corrective mechanism.

However, the system state is not directly accessible and as such, the sensor estimations (3) have to be used to construct an artificial estimate x^*. This may be realized by selecting one of the available estimations or by considering a convex combination of them. This in turn permits to rewrite (10) as

$$u = \begin{cases} u^d, & \hat{x}^* \in S^* \\ u^a, & \hat{x}^* \in \bar{S}^* \setminus S^* \end{cases} \tag{11}$$

with notation

$$S^* = S \oplus \bigcup_{i \in \mathcal{I}} \tilde{S}_i, \; \bar{S}^* = \bar{S} \ominus \bigcup_{i \in \mathcal{I}} \tilde{S}_i. \tag{12}$$

Note that sets S, $\bar{S}$ used (10) are replaced with sets S^*, $\bar{S}^*$ in (11) to counterbalance the influence of the measurement noises. This allows for the driver to control the steering as long as there exists the possibility that the state is still in S and, additionally, for the assisting mechanism, to guarantee that the state remains at all times inside $\bar{S}$.

B. Control strategies

The sensor selection scheme considered in this paper selects a sensor-estimator pair at each sampling time upon an optimization based procedure

$$\hat{x}^* = \arg\min_{\hat{x}_i} \hat{x}_i^T P \hat{x}_i, \quad i = 1, \ldots, N \tag{13}$$

with $P > 0$, solution of the Lyapunov equation $P = (A - BK)' P (A - BK) + Q$ for a given feedback gain K and a given matrix $Q > 0$.

The control action provided by the corrective mechanism is obtained from $u^a = K\hat{x}^*$. Using (3), (4) and (13) and supposing that, at a given time instant, the minimum is achieved at the subindex $\ell \in \{1, \ldots, N\}$ one rewrite the control law as:

$$u^a = K\left(x - \tilde{x}_l\right) \tag{14}$$

which, together with (1), gives the closed loop system

$$x^+ = (A + BK)x - BK\tilde{x}_l + B_\rho \rho_{ref}. \tag{15}$$

For further use let recall some basic notions of set invariance.

Definition 1 (RPI set)**.** The set $\Omega \subset \mathbb{R}^n$ is a *robust positively invariant (RPI) set* of dynamics $x^+ = \Psi x + \delta$ with $\delta \in \Delta$ if and only if $\Psi\Omega \oplus \Delta \subseteq \Omega$. ♦

The *minimal robust positive invariant (mRPI)* set is defined as the RPI set contained in any closed RPI set and the *maximal robust positively invariant (MRPI)* is defined as the maximal RPI set contained in a given bounding set.

As seen from the switch mechanism in (11), whenever the state is no longer included in the nominal region S, a corrective mechanism takes control and provides an action which aims to keep the state inside the safety region $\bar{S}$ and eventually to steer it inside the nominal region. These requirements can be formally presented as:

$$S^{*,+} \subseteq \Omega_M \tag{16}$$

$$\Omega_m \subseteq S^* \tag{17}$$

where $S^{*,+}$ denotes the successor value of set S^* mapped through dynamics (1) ($S^{*,+} = AS^* \oplus B\mathbb{U} \oplus B_\rho P_{ref}$) and Ω_M, Ω_m denote the MRPI, respectively the mRPI sets of dynamics (15).

The corrective mechanism is activated only when the state "jumped" outside the nominal region S^*. As long as this one step reachable set $S^{*,+}$ respects condition (16) we can guarantee that all the future states will remain in $\bar{S}^*$ (by the very definition of the MRPI set Ω_M). Condition (17) guarantees that the state will return inside the nominal region S^* in a finite time.

C. Numerical considerations

Usually, the mRPI set cannot be explicitly determined. There are however various techniques in the literature which provide RPI approximations. An interesting approach is provided in [14] where ultimate bounding invariant sets are used. For MRPI sets, there exist iterative algorithms which guarantee a solution in a finite number of steps, see [15].

The feasibility of relations (16), (17) as a function of the control law given in (14) can be addressed by convex optimization arguments. For example, using ellipsoidal approximations of Ω_m, Ω_M we are able to analyze the existence of a feedback gain K as discussed in [16].

For a greater flexibility, the control law (14) can be generalized to a piecewise affine function. This will lead to a larger set Ω_M, respectively a smaller set Ω_m which in turn means that we have greater leeway in choosing the nominal region (8). The control law will be then obtained as the result of an optimization problem under a receding horizon.

IV. Fault tolerant control scheme

In this section we describe the components of a FTC mechanism which interact as a whole and present the necessary conditions for exact fault detection. A schematic view is given in Fig. 2 where the FTC components are added to the closed loop dynamics of system (15) (sensors S_i, estimators F_i and feedback gain K appear explicitly).

A. Fault description

The faults considered here are *abrupt* total[3] sensor output outages. The failure is then represented by the

[3]The reasoning can be readily extended for the case of partial outages but we rest in the framework of total outages for the sake of simplicity.

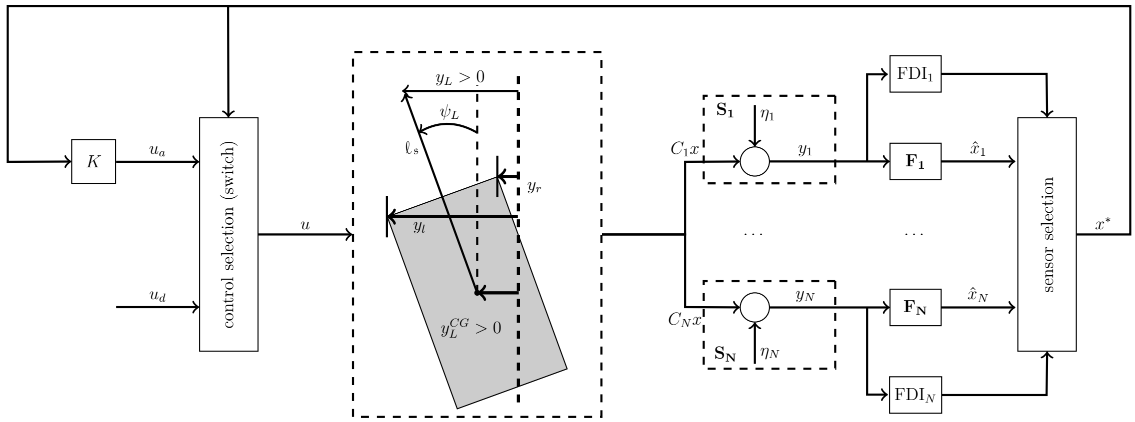


Fig. 2: Multisensor fault tolerant control scheme

following switch in the structure of the observation equation:

$$y_i = C_i x + \eta_i \xrightleftharpoons[RECOVERY]{FAULT} y_i = 0 \cdot x + \eta_i^F. \qquad (18)$$

The noise affecting the observation channel during the fault, η_i^F, may be different from the one during the healthy functioning, η_i. All the noises and disturbances affecting plant and sensors are considered to be bounded. As such, $\rho \in P_{ref}$ and $\eta_i \in N_i$, $\eta_i^F \in N_i^F$ for $i = 1, \dots, N$ with $\Phi \subset \mathbb{R}^4$ and $N_i, N_i^F \subset \mathbb{R}$ bounded polyhedral sets.

B. Fault detection and isolation

A residual signal ([17]) is by construction sensitive to fault occurrences and with a manageable dependence upon the measurement noises. In our framework we consider the best choice to be the sensor output itself.

For a set theoretic decision the residuals will be characterized by either "healthy" or "faulty" polyhedral sets. The fault detection reduces then to the study of the relationship between sets Y_i^H and Y_i^F of all the possible values under healthy, respectively faulty, functioning of signal y_i:

$$Y_i^H = C_i X \oplus N_i,\ Y_i^F = N_i^F \qquad (19)$$

where X denotes a set of admissible system states. These sets can be described offline and the actual FDI is a fast online set membership evaluation which differentiates between the healthy/faulty functioning for the i^{th} sensor as long as the following assumption holds:

Assumption 1 (Discernability). The reference set X, dynamics and physical characteristics defining sets N_i and N_i^F are such that the "separation" condition

$$Y_i^H \cap Y_i^F = \emptyset. \qquad (20)$$

is verified. ♦

As seen from relation (20), exact fault detection and isolation are possible under certain boundedness assumptions for noises and plant state. Usually, the noise bounds are fixed and the only part left to deal with is X. Therefore, a maximal set (usually nonconvex), which contains all the values of the state for which (20) is validated is given as follows:

$$X^o = \bigcap_{i \in \mathcal{I}} \left\{ x :\ \{C_i x\} \oplus N_i \cap N_i^F = \emptyset \right\}. \qquad (21)$$

In the aforementioned scheme, the detection and isolation of faulty sensors and the use of their estimations for constructing the control action (14) are required only over the region $\bar{S}^* \setminus S^{*,+}$. Using (21) we can conclude that condition

$$\bar{S}^* \setminus S^{*,+} \subseteq X^o \qquad (22)$$

together with conditions (16) and (17) suffice for a complete FTC scheme with global stability guarantees.

C. Control reconfiguration

Considering a functioning FDI mechanism we can now partition the sensors into the healthy subset $\mathcal{I}_H$, respectively the faulty subset $\mathcal{I}_F$:

- $\mathcal{I}_H$, all the sensors acknowledged healthy (i.e. with healthy functioning (2) and estimation error (4) inside its invariant set):

$$\mathcal{I}_H = \left\{ i \in \mathcal{I}_H^- : y_i \in Y_i^H \right\} \cup \left\{ i \in \mathcal{I}_F^- : \tilde{x}_i \in \tilde{S}_i,\ y_i \in Y_i^H \right\}$$

- $\mathcal{I}_F$, all the sensors acknowledged faulty (i.e. with faulty functioning (18) or with estimation error (4) outside its invariant set):

$$\mathcal{I}_R = \mathcal{I} \setminus \mathcal{I}_H$$

such that $\mathcal{I} = \mathcal{I}_H \cup \mathcal{I}_F$ and $\mathcal{I}_H \cap \mathcal{I}_F = \emptyset$ with the assumption that $\mathcal{I}_H$ is not empty along the closed loop functioning (in order to guarantee the existence of reliable information for feedback).

Remark 1. Note that as long as condition (22) holds, the subset $\mathcal{I}_H$ contains only healthy sensors thus making the FDI mechanism exact. The analysis of inclusion of unknown values $\tilde{x}_i$ into set $\tilde{S}_i$ is required only when a sensor previously fallen regains its healthy functioning. Extensive details are to be found in [18]. ♦

Using the above partitioning, we recast the control law (13) as follows:

$$\hat{x}^* = \arg\min_{\hat{x}_i} \hat{x}_i^T P \hat{x}_i, \quad i \in \mathcal{I}_H \tag{23}$$

which will allow for the FTC scheme to negate any harmful effects of a sensor fault.

V. Illustrative example

A. Test environment and numerical data

For the illustrative example depicted here we take the numerical values given in [13]. The vehicle dynamics are considered for a constant velocity of 20m/s.

The bounds x_N and x_S upon the state for the nominal and safety case, respectively, are given in Table I. Further, typical values for the nominal and safety strips around the center of the lane are given by $2\underline{d} = 2$m and $2\bar{d} = 3.5$m. We consider that ρ_{ref} is bounded by $P_{ref} = \mathbb{B}(0.1\text{m}^{-1})$, with 0.01m^{-1} corresponding to a radius of 100m (lateral acceleration at 20m/s is 0.4g). The steering angle is bounded by $\mathbb{U} = \mathbb{B}(10°)$ and we apply the feedback gain

	β	r	ψ_L	y_L
nominal case	$2°$	$5°/s$	$5°$	0.5m
safety case	$6°$	$15°/s$	$10°$	1m

TABLE I: State bounds for nominal and safety case.

$$K = \begin{bmatrix} -0.2079 & -0.0699 & -0.7696 & -0.0489 \end{bmatrix}.$$

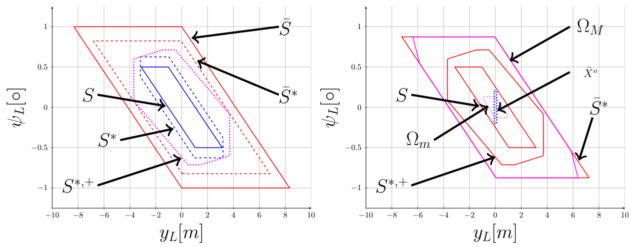
Throughout the paper it was implicitly assumed that the sensors are observable. Due to the state dynamics, this property is verified only for sensors which measure (at least) the state components y_L and Ψ_L. In our practical setting, realist sensors are: i) estimations through computer vision algorithms and ii) GPS RTK (Real Time Kinetic) systems with the following physical characteristics (output matrix, noise bounds in healthy, respectively faulty case):

$$C_1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, N_1 = \mathbb{B}(\begin{bmatrix} 0.10m \\ 0.5° \end{bmatrix}), N_1^F = \mathbb{B}(\begin{bmatrix} 0.10m \\ 0.5° \end{bmatrix})$$

$$C_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, N_2 = \mathbb{B}(\begin{bmatrix} 0.05m \\ 0.25° \end{bmatrix}), N_2^F = \mathbb{B}(\begin{bmatrix} 0.05m \\ 0.25° \end{bmatrix}).$$

Note that the illustrative example is academic since at the present, GPS RTK systems (which require additional road infrastructure) are found only in experimental facilities and not in every day use.

For both of the sensors we take a gain matrix $L_1 = L_2 = L$ such that the poles of the closed loop estimator system (3) are $\begin{bmatrix} 0.9 & 0.1 & 0.01 & 0.2 \end{bmatrix}$. We are now able to depict the sets of interest mentioned throughout the paper. In Fig. 3 (a) we show S, S^* (blue solid and dashed lines, respectively), $\bar{S}, \bar{S}^*$ (red solid and dashed lines, respectively) and $S^{*,+}$ (magenta dotted line). We observe here that condition $S^{*,+} \subset \bar{S}^*$, which is a prerequisite for conditions (16) and (17), holds. In Fig. 3 (b) we show the maximal and minimal RPI sets Ω_M (solid magenta line), respectively Ω_m (dashed magenta line) together with the complement of the admissible reference set, $\bar{X}^o$ (dotted blue line).



(A) Nominal and safety sets (B) Set conditions for stability and fault detection

FIG. 3: Sets of interest.

We observe that the gain matrix K in conjunction with the aforementioned constraints leads to sets which respect conditions (16), (17) and (22) thus making the problem feasible from the point of view of control and fault tolerance.

B. System simulations

For a practical application we consider a road with curvature profile given in Fig. 4 and take two segments (as highlighted in the figure) upon which we run the simulations. The first segment corresponds to a curved section of the road, whereas the second describes a straight line.

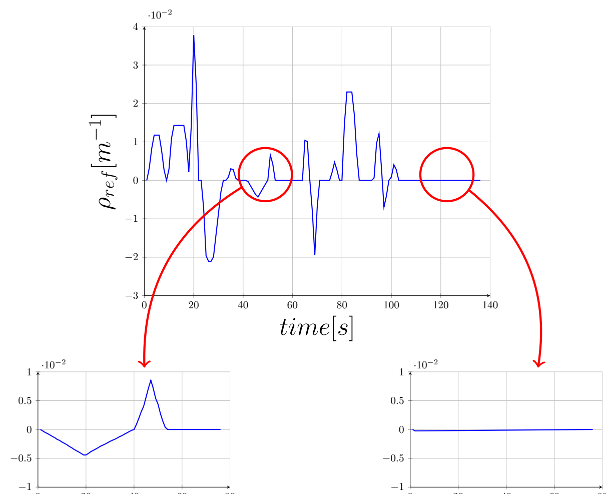
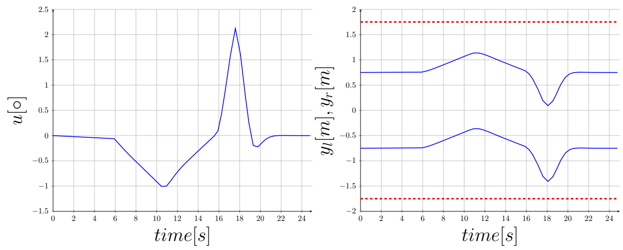


FIG. 4: Profile of road curvature with curved and straight segments of the road detailed.

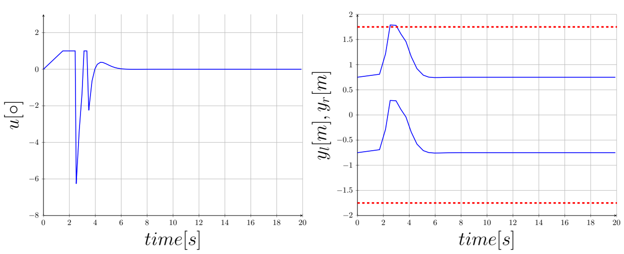
In the first simulation we analyze a curved portion of the road of maximum curvature $\rho_{ref} = 0.009m^{-1}$. We presume that the inattentive driver drives straight ignoring the curvature. Consequently, the nominal bounds of region S^* are violated and the corrective mechanism assumes control. As it can be seen in Fig. 5 the corrective control action steers the vehicle inside the nominal region without tresspasing the safety region as seen in Fig. 5 (b). Moreover, the steering angle, as shown in Fig. 5 (a) lies between $-1^\circ \ldots 2.25^\circ$, well below the bounds of $-10^\circ \ldots 10^\circ$.



(A) Steering angle (B) Front wheels of the vehicle on the lane

FIG. 5: Simulation for the curved road segment.

The same simulation is carried for the second segment of road which covers a straight line. Here the innatentive driver starts to drift, until, as in the previous case, the constraints are broken and the corrective mechanism proposes a corrective control action. In Fig. 6 (b) we see the offsets of the front wheels and in Fig. 6 (a) the values of the steering angle.



(A) Steering angle (B) Front wheels of the vehicle on the lane

FIG. 6: Simulation for the straight road segment.

Note that both simulation reflect the "proof of concept" nature of the discussion. For example, once the driver exits the nominal region, the corrective mechanism takes control until the state is returned inside the nominal region. In practice this is unacceptable as it renders the driver powerless even if s/he is again attentive. Additionally, if the state of inattention of the driver is prolonged, we may have a "chattering" at the boundary of region S^* where the corrective mechanism cedes control only to regain it after a few instants of time. A more realist implementation would require for example the use of an alarm signal which makes the driver attentive once the nominal region is trespassed.

VI. Conclusions

The present paper detailed a discrete time lane control mechanism. Global stability guarantees where analyzed with the help of set invariance notions. Additionally, a fault tolerant control scheme was implemented in order to detect and accomodate abrupt faults in the measuring sensors.

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