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New Results on Robust State Estimation in Spacecraft Attitude Control

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Abstract— **In this paper, an integrated attitude estimation and control algorithm is addressed and implemented to a spacecraft dynamical model subject to observation (sensor) losses. Rigid body equations of motion for modeling and control of spacecraft model is obtained from both kinematic and dynamic equations. An earlier version of the so-called closed-loop estimation scheme presented in [9] is extended and implemented to the spacecraft model subject to observation losses. Compensated observation signals are reconstructed based on linear prediction subsystem and utilized at measurement update steps. Simulation results verify that the proposed robust estimation algorithm applied to the rigid body spacecraft model significantly outperforms existing open-loop filtering algorithms and could attack many other practical applications with intermittent output measurement losses.**

I. INTRODUCTION

Many spacecraft systems rely on the ground-based data processing and communication which may be affected by many factors such as inherent time-delay and measured data loss [13]. A well-established communication is an integral part of the successful completion of a spacecraft mission. However, there might be frequently encountered scenarios where the observation data packets are lost due to a number of reasons, such as limited bandwidth of communication channels, confined memory capability of buffer registers and congestion of networks channels to name but a few. Despite of rapid advancements in the spacecraft technology, their intense reliability on the ground-based data may be suffered due to any of the above disturbing factors results of which could lead towards a communication delay and even failure in the spacecraft mission.

On the other hand, delay-free and uncorrupted communication plays a key role when a failure occurs. To overcome such failures, hardware redundancy approaches such as duplicated, triplicated and voting schemes are used to handle some classes of Fault Detection and Isolation (FDI) problems to name a few [8]. However, several facts consisting of complexity, cost and weight of added (hardware) components have turned the attention towards Model-based FDI approaches which can overcome the above drawbacks by using mathematical models of the plant [17].

In the event of loss of observation (LOOB), there is a vital need for a robust estimation algorithm to provide reliable and superior estimation performance with bounded estimation errors if an output data loss is occurred. Towards this end, theoretical concepts of the robust estimation technique proposed in [9], [10] is discussed in this paper and implemented for the attitude estimation and control problem of a rigid body spacecraft dynamical model subject to intermittent observation losses where a Kalman filter is utilised to carry out real-time estimation of the roll, pitch and yaw attitudes. Typically, spacecraft attitude state vector is obtained by employing the kinematic equations - see e.g. [11], [4] and other references therein. In this paper, both kinematic and dynamics models are considered in order to compute the full state-vector of the spacecraft model. In addition, uncertainties arising from the external torques and distribution of momentum (e.g. due to rotating instruments) are assumed negligible for the sake of this study which is focused on attitude measurement losses.

The rest of the paper is organized as follows: In Section II, dynamics of the spacecraft are presented based on nonlinear Euler equations in modified Rodriguez parameterizations. Section III outlines the control system design used to stabilize the closed-loop system. A brief overview of the open-loop Kalman filtering and compensated closed-loop estimation techniques is presented in Section IV. The effectiveness of the proposed estimation scheme, based on various performance indexes of the spacecraft model, subject to LOOB, is illustrated through a numerical example in Section V followed by our conclusions in Section VI.

II. SPACECRAFT RIGID BODY

To overcome the limitations of the quaternion and Euler angles parameterizations, Modified Rodriguez Parameters (MRPs) are recently found an elegant enhancement compared to the family of attitude parameters [14]. For this reason, MRP representation is also employed in this paper for studying the nonlinear spacecraft model as discussed subsequently.

A. Nonlinear Plant Dynamics

The dynamics of the spacecraft can be described based on its *Kinematic* equations, see e.g. [5], [11]. However, it has been shown to achieve superior performance if the spacecraft is modeled as a rigid body where its states are described by two set of equations, “Euler equations of rotational dynamics" and “Kinematic equations" using MRP representation [11]. The MRP structure is used in this paper to explore a complete insight of the spacecraft dynamics as follows:

1) Kinematic Equations: The Kinematic equations in terms of MRP are

$$\dot{\sigma} = T(\sigma)\overline{\omega} \tag{1}$$

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where $T(\sigma)$ is a Jacobian matrix defined as

$$T(\sigma) = \frac{1}{2}\left[\left(\frac{1-\sigma^T\sigma}{2}\right) I_{3\times 3} + S(\sigma) + \sigma\sigma^T\right] \quad (2)$$

and $S(\sigma)$ is the skew symmetric matrix which represents the cross product operation of vector, σ. The MRP vector, σ, and noisy angular velocity vector, $\overline{\omega}$, are both of dimension 3×1 where $\overline{\omega}$ is defined as follows:

$$\overline{\omega} = \begin{bmatrix} \overline{\omega}_1 \\ \overline{\omega}_2 \\ \overline{\omega}_3 \end{bmatrix} = \begin{bmatrix} \omega_1 + n_1 \\ \omega_2 + n_2 \\ \omega_3 + n_3 \end{bmatrix} \quad (3)$$

where n_i $(i = 1, 2, 3)$ represent gyroscope noises which may include scale factor error and drift. The plant disturbances are assumed to be Gaussian zero-mean white noises, i.e.

$$n_i \sim \mathcal{N}(0, \Lambda) \qquad ; \quad i = 1, 2, 3. \quad (4)$$

where Λ is the variance of bias.

2) Dynamic Equations: Euler's equations of rotational dynamics are described as

$$J\dot{\overline{\omega}} = -S(\overline{\omega})J\overline{\omega} + \tau \quad (5)$$

where $J_{3\times 3}$ is the spacecraft inertia matrix, $\tau_{3\times 1}$ is the control input torque and $S(\overline{\omega}) = \overline{\omega} \times \overline{\omega}'$ is the skew symmetric matrix representing the cross product operation as

$$S(\omega) = \begin{bmatrix} 0 & -\overline{\omega}_3 & \overline{\omega}_2 \\ \overline{\omega}_3 & 0 & -\overline{\omega}_1 \\ -\overline{\omega}_2 & \overline{\omega}_1 & 0 \end{bmatrix} \quad (6)$$

The *kinematic* and *dynamic* equations from Eqs. (1) and (5), produce the augmented state vector as

$$\dot{x}(t) = f(x(t), \tau(t)) + \xi(t) \quad (7)$$

where $x = [\sigma \quad \omega]^T$ and ξ are the state vector and process noise vector. After obtaining the spacecraft plant model, an stable closed-loop system is required for the implementation of the estimation algorithm, wherein the effects of intermittent measurement losses could be analyzed. Towards this end, an overview of a straightforward method to design the stable closed-loop system using an output feedback control law is presented in the subsequent section.

III. Control System Design

In the conventional design methods for the spacecraft control applications, the controllers may require two components of angular velocity and attitude – see e.g. [19], [20]. Ref. [1] addresses an output feedback control law to stabilize the closed-loop. We shall employ this method in this paper assuming that we do not have any access to $\dot{\sigma}$ and ω measurements. The control scheme consists of two loops; an inner loop with a transfer function and an outer loop with an unity (negative) feedback. The control system design is summarized as follows[1]:

$$\tau = T^T(S_p\tilde{\sigma} - \sigma^*) \quad (8)$$

where

$$\begin{aligned} S_p &= diag(s_{p1}, s_{p2}, s_{p3}), \\ \tilde{\sigma} &= \sigma_d - \hat{\sigma}, \\ \sigma^* &= N\sigma, \\ N &= diag\Big(s_{d1}\frac{\alpha_1 s}{s+\alpha_1}, s_{d2}\frac{\alpha_2 s}{s+\alpha_2}, s_{d1}\frac{\alpha_3 s}{s+\alpha_3}\Big) \end{aligned} \quad (9)$$

where S_p and N are controller design parameters and are positive definite matrices. The candidate Lyapunov function is chosen to be [1]

$$V(\sigma^*, \dot{\sigma}, \tilde{\sigma}) = \frac{1}{2}(\dot{\sigma}^T H^* \dot{\sigma} + \tilde{\sigma}^T S_p \tilde{\sigma} + \sigma^{*T}\{\alpha S_d\}^{-1}\sigma^*) \quad (10)$$

where H^* and S_d are defined as

$$\begin{aligned} H^* &= (T(\sigma)^{-1})^T J T(\sigma)^{-1} \\ S_d &= diag(s_{d1}, s_{d2}, s_{d3}) \end{aligned} \quad (11)$$

The time derivative of the above Lyapunov function is computed as

$$\begin{aligned} \dot{V} &= -\dot{\sigma}^T\sigma^* + \sigma^{*T}\{\alpha S_d\}^{-1}(S_d\alpha\dot{\sigma} - \alpha\sigma^*) \\ &= -\sigma^{*T}S_d^{-1}\sigma^* \end{aligned} \quad (12)$$

To stabilize the nonlinear plant, the design elements (s_{pi}, s_{di} and α_i) must be selected so that $\dot{V} \leq 0$ to yield guaranteed stable closed-loop. It is also worthwhile to emphasize that we do not intend to discuss proof of the control system stability since our main intention is to attain the state estimation of the spacecraft system in the event of measurement loss. However, to be able to obtain appropriate estimation results, one is in need of a stabilizing controller design - for a detailed discussion on the asymptotic stability associated with the output feedback control law the interested readers are referred to [1].

IV. Proposed Robust Kalman Filtering

In practice, factors such as intermittent sensor faults, limited bandwidth of communication channels, confined memory space, congestion of network, etc may produce adverse scenarios for any estimation and filtering algorithms including Kalman filtering (linear or non-linear) due to there being heavily dependant on the measured data [6], [2]. Any of such unfavorable conditions may lead Kalman filter to diverge rapidly. To overcome those shortcomings, one approach for the problem of state estimation is to utilize the so-called Open-Loop Estimation (OLE) algorithm – see e.g. [15], [12], [18], [16]. In OLE, LOOB cases are considered via running the Kalman filter in an Open-Loop fashion, that is whenever

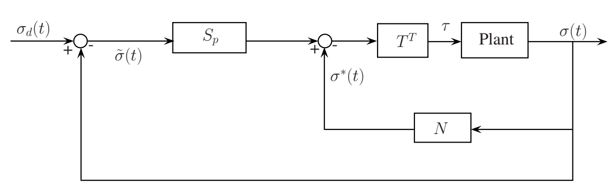

Fig. 1. Attitude stabilization using output feedback control [1].

an observation loss is detected, the predicted quantities are processed for the next iteration with no update. It might be useful to outline a brief summary of OLE and its relevant drawbacks as discussed below.

A. Open-Loop Estimation

The Open-Loop Kalman Filtering (OLKF) or simply Open-Loop Estimation (OLE) is effectively a straightforward approach to accommodate short-period data losses. In this approach, if, say at time step *k*, a measurement loss is occurred, Kalman filter gain K_k is set to zero, and hence no update step is performed. Therefore, the filter is to be run with the measurement sensitivity matrix of $C = 0$ [7]. Consider, a discrete time nonlinear model of the form

$$x_{k+1} = f(x_k, u_k) + \xi_k \quad \& \quad z_k = g(x_k) + v_k \tag{13}$$

Using the concept of extended Kalman filter (EKF), the OLE algorithm is summarized with appropriate comments as discussed below.

1) Prediction step: The predicted state and covariance are as follows[1]:

$$_o x_{k+1|k} = f(\hat{x}_{k|k}, u_k) \tag{14}$$

$$_o P_{k+1|k} = \frac{\partial f}{\partial x}\Big|_{\hat{x}_{k|k}} P_{k|k} \frac{\partial f}{\partial x}\Big|^T_{x\hat{|}k_k} + Q_k \tag{15}$$

2) Accommodating measurement vector: The unique adopted observation vector in OLE scheme is

$$_o z_{k+1} = \frac{\partial g}{\partial x}\Big|_{\hat{x}_{k|k}} {}_o x_{k+1|k} \equiv \hat{z}_{k+1} \tag{16}$$

This will cause the residual vector to be zero (2-norm) and consequently,

$$_o K_{k+1} = 0 \tag{17}$$

3) Measurement update step: Since no correction is performed at the predicted step of OLE, the *a posteriori* step quantities will be

$$_o x_{k+1|k+1} \leftarrow {}_o x_{k+1|k} \tag{18}$$

$$_o P_{k+1|k+1} \leftarrow {}_o P_{k+1|k} \tag{19}$$

Therefore, in the OLKF method the *a posteriori* state and error covariance matrix strictly follow the *a priori* state and error covariance matrix respectively.

B. Drawbacks of the Open-Loop Kalman Filtering (OLKF)

In practice, the OLKF approach may diverge in the presence of large data loss duration which may end up with an unstable estimation configuration. Besides, there are other shortcomings associated with the OLKF approach as follows:

1) High divergence-rate of state estimation and error covariance due to the fact that no update is performed at the measurement update step,
2) Undesired transient (sharp spikes) and oscillatory estimation results,
3) After resuming output observation, the state estimation and error covariance may not fully attain failure-free (nominal) steady-state values since the process is simply running based on only the prediction.

[1]The leading subscript 'o' denotes Open-Loop KF approach.

Under loss of observations for a sufficiently large period of time, there seems a vital need for an optimal estimation technique which could provide robust estimation with minimised error covariance.

C. CCLKF Estimation Scheme

Due to the shortcomings associated with the OLKF approach, a robust estimation technique based on the linear prediction concept is employed here. This technique referred to as the "Compensated Closed-Loop Kalman Filtering (CCLKF)" [10], [9] could reconstruct missing observation data through a linear prediction dynamics. In CCLKF, loss of data could be detected through residual based FDI methods [17], [22] from which missing data signals are reconstructed as follows:

$$\bar{z}_k = \sum_{i=1}^{p} \alpha_k z_{k-i} \tag{20}$$

where α_i's are the Modified Linear Prediction Coefficients (MLPC) representing the weights assigned to past observations in relation to their correlation to the missing data and p is the linear prediction filter order (LPFO). Both α_i's and LPFO in Eq. (20) are dominant factors for the design of robust filtering algorithm [10]. Recall that a larger number of LPFO does not necessarily guarantee optimal compensated signals [3]. Hence, an attentive consideration is required in order to obtain the optimal parameters. Algorithm **1** is our straightforward algorithm used to provide the optimal values of MLPC and LPFO which is discussed in detail as follow.

Algorithm 1 : Selection of the LP filter order

1: **Select** R_{th} set threshold autocorrelation value
2: **Recursion** $j = 1, 2, \ldots M$
3: **Compute** $R[j]$ using Equation (26)
4: **Check**: Is $R[j] \leq R_{th}$,
  Yes Stop further computation of $R[j]$ & Select order of LP filter ($p \leftarrow j$)
  Else update $j \longleftarrow j + 1$
5: **Repeat** Step 3

Once CCLKF is designed and developed, the switching mechanism between normal filtering operation and the CCLKF is carried out for the estimation purposes subject to loss of measurements;if data loss is detected, output will be switched so as to compute the compensated observation signal, $\bar{z}_k$ using CCLKF, and resume normal operation if failure (due to loss) is cleared.

Although EKF is considered in this paper, in order to grasp a clear idea, we shall consider a discrete LTI plant dynamics, for the sake of simplicity, as described by the following state-space equations:

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k + \xi_k \\ y_k &= \gamma_k\big(Cx_k\big) + v_k \end{aligned} \tag{21}$$

where γ_k is characterized as follows:

$$\gamma_k = \begin{cases} 0; & \text{if LOOB is detected} \\ 1; & \text{otherwise} \end{cases} \tag{22}$$

Assuming LOOB has been detected at time step 'k', the CCLKF approach is summarized below:

- **Prediction cycle** at time step $(k-1)$, we have

$$\begin{aligned} {}_c x_{k|k-1} &= A_c x_{k-1|k-1} + Bu_{k-1} \\ {}_c P_{k|k-1} &= A_c P_{k-1|k-1} A^T + Q_k \end{aligned} \tag{23}$$

  where $E[\xi_k \xi_k^T] = Q_k$ is the process noise covariance matrix[2].
- **Check** for data-loss detection:
  if $\gamma_k = 1 \rightarrow$ no LOOB is occurred.
  $\Rightarrow$ Run conventional Kalman filter [6].
  if $\gamma_k = 0 \rightarrow$ an abnormal condition is detected (LOOB case)
  $\Rightarrow$ The actual observation is not available to which the prediction step is updated. Run Robust Kalman filter summarized below.
- **Select** a suitable window size of the previous available observations (n) which is modeled through the linear prediction filter order (p) with a constraint of $n \geq 2p$, as Nyquist Shannon sampling condition [21].
- **Compute** autocorrelation matrix R_γ as

$$R_\gamma = \begin{bmatrix} R[0] & R[1] & R[2] & \cdots & R[n-1] \\ R[1] & R[0] & R[1] & \cdots & R[n-2] \\ R[2] & R[1] & R[0] & \cdots & R[n-3] \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ R[n-1] & R[n-2] & R[n-3] & \cdots & R[0] \end{bmatrix} \tag{24}$$

  and the modified autocorrelation array r_γ is

$$r_\gamma = \begin{bmatrix} r[1] & r[2] & r[3] & \cdots & r[n] \end{bmatrix}^T \tag{25}$$

  where

$$\begin{aligned} E\big[z_{k-i}^T z_{k-j}\big] &= \begin{cases} R[0], & \text{if i = j} \\ R\big[|i-j|\big], & \text{if i} \neq \text{j} \end{cases} \\ E\big[z_k^T z_{k-j}\big] &= r[j]. \end{aligned} \tag{26}$$

- **Compute** MLPC as $A_\alpha = [\alpha_j]^T = R_\gamma^{-1} \cdot r_\gamma$
- **Calculate** compensated measurement vector as

$$\bar{z}_k = \sum_{j=1}^{p} \alpha_j z_{k-j} \equiv C\bar{x}_k + \bar{v}_k \tag{27}$$

- **Obtain** compensated residual vector, $\bar{z}_k - \hat{z}_k$.
- **Calculate** the CCLKF gain
  ${}_c K_k = {}_{2c}P_k C^T (C_{3c} P_k C^T + R_k)^{-1}$.

[2]It should be clarified that if LOOB has been detected at time instant k, there is no need to insert the initial subscript 'c' to the prediction step entities (state and covariance), as they are not affected by LOOB. However, a leading subscript 'c' is used for consistency of our notations.

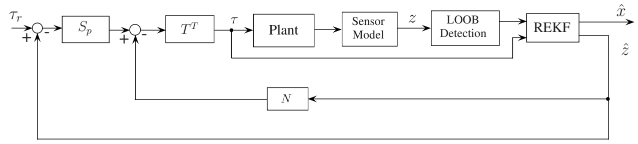

Fig. 2. Robust attitude estimation in spacecraft control using the proposed REKF.

- **Measurement update** step will be proceeded as:

$$\begin{aligned} &{}_c x_{k|k} = {}_c x_{k|k-1} + {}_c K_k (\bar{z}_k - C_c x_{k|k-1}) \\ &{}_c P_{k|k} = {}_{1c}P_k - {}_{2c}P_k C^T (C\ {}_{3c}P_k C^T + R_k)^{-1} C_{2c} P_k \end{aligned} \tag{28}$$

- **Increase** time step, k=k+1.
- **Return** to prediction cycle.

The error covariance matrices are computed as follows[3]:

$$\begin{aligned} {}_{1c}P_k = {}_{1c}P_{k|k-1} &\triangleq E[e_{k|k-1}(e_{k|k-1})^T] \\ {}_{2c}P_k = {}_{2c}P_{k|k-1} &\triangleq E[e_{k|k-1}(\bar{e}_{k|k-1})^T] \\ {}_{3c}P_k = {}_{3c}P_{k|k-1} &\triangleq E[(\bar{e}_{k|k-1})(\bar{e}_{k|k-1})^T] \end{aligned} \tag{29}$$

with $e_{k|k-1} = x_k - {}_c x_{k|k-1}$ and $\bar{e}_{k|k-1} = \bar{x}_k - {}_c x_{k|k-1}$.

The schematic diagram of the designed robust algorithm for a general nonlinear plant, referred to as as Robust Extended Kalman Filter (REKF) is shown in Fig. 2.

D. Features of CCLKF Approach

The CCLKF approach has been found to provide satisfactory state estimation results which are superior than those of open-loop approaches. However, using such an advanced estimation technique would increase the algorithm complexity, and hence there should be a trade-off of the system performance between the additional computational burdensome and required efficiency level. The added cost of memory needed for storing previous measurement data and extra computational efforts can be compromised for an added complexity to spacecraft model so as to achieve a successful completion of mission in the event of loss of measurements.

V. Numerical Simulation Results

In this section, the extended version of the aforementioned estimation approaches in the form of extended Kalman filter are applied to the spacecraft model subject to loss of measurements. Also, the drawbacks of the OLKF approach along with the added advantages of the CCLKF scheme are illustrated through our typical simulation results.

A. Spacecraft model

An extended Kalman filter is employed to estimate the attitude of the nonlinear spacecraft model. As the system is derived in MRP representation, an output model could be derived in Euler angles computed through sensors such as rate-integrated gyro or accelerometers. Having Euler angles at input model and MRP elements at output model, a precise

[3]${}_c P_{1k} = P_{k|k-1}$ is the normal predicted error covariance matrix.

relationship is required to be established between the two distinct representations. The output model in terms of MRP is deduced by comparing two Direction Cosine Matrices (DCM) associated with Euler angels and MRP of the same sequences. Therefore, at measurement update step in Kalman filtering we have

$$\hat{z} = C\hat{x}(t) = \begin{bmatrix} \frac{\partial\phi}{\partial\sigma_1} & \frac{\partial\phi}{\partial\sigma_2} & \frac{\partial\phi}{\partial\sigma_3} & \\ \frac{\partial\theta}{\partial\sigma_1} & \frac{\partial\theta}{\partial\sigma_2} & \frac{\partial\theta}{\partial\sigma_3} & \mathbf{0}_{3\times3} \\ \frac{\partial\psi}{\partial\sigma_1} & \frac{\partial\psi}{\partial\sigma_2} & \frac{\partial\psi}{\partial\sigma_3} & \end{bmatrix} \hat{x}(t) \tag{30}$$

where $\hat{x}(t) = [\sigma_1\ \sigma_2\ \sigma_3\ \omega_1\ \omega_2\ \omega_3]^T$.
As mentioned above, by comparing the two associated DCM of Euler angles and MRP (of sequence 3-2-1) and carrying out some algebra, we have

$$\begin{aligned}
\partial\phi/\partial\sigma_1 &= H_1\sigma_1\big[1-\pi\big] \\
\partial\phi/\partial\sigma_2 &= H_1\big[\sigma_2\{1-\pi\}\big] \\
\partial\phi/\partial\sigma_3 &= H_1\big[-\sigma_3\{\sigma^2+\pi\}\big] \\
\partial\theta/\partial\sigma_1 &= H_2\big[2\sigma_3\big[(3-\sigma^2)\sigma_1+(1-4\sigma_1^2)\big]\big] \\
\partial\theta/\partial\sigma_2 &= H_2\big[2\{\sigma_2\sigma_3(3-\sigma^2)+2\sigma_1(1+\sigma^2-4\sigma_3^2)\}\big] \\
\partial\theta/\partial\sigma_3 &= H_2\big[\sigma^4+6\sigma^2-2\sigma^2\sigma_2^2-8\sigma_1\sigma_2\sigma_3-1\big] \\
\partial\psi/\partial\sigma_1 &= H_3\Big[2\sigma_1\Big\{(\sigma^4+1)+2(\sigma^2-2\sigma_1^2)\Big\}\Big] \\
\partial\psi/\partial\sigma_2 &= H_3\Big(-2\sigma_2\big[1+2(2\sigma_1^2-\sigma^2)\big]\Big) \\
\partial\psi/\partial\sigma_3 &= H_3\Big(-2\sigma_3\big[1+2(2\sigma_1^2-\sigma^2)\big]\Big)
\end{aligned} \tag{31}$$

where $\pi = -\sigma_1^2 - \sigma_2^2 + \sigma_3^2$ and

$$\begin{aligned}
H_1 &= \frac{4}{\big[8\sigma_2\sigma_3+4\sigma_1(1-\sigma^2)\big](1+\sigma^2)} \\
H_2 &= \frac{4}{(1+\sigma^2)\sqrt{\big[8\sigma_2\sigma_3+4\sigma_1(1-\sigma^2)\big]^2+\big[-4\pi+(1-\sigma^2)^2\big]^2}} \\
H_3 &= \frac{4}{(1+\sigma^2)[8\sigma_1\sigma_2+4\sigma_3(1-\sigma^2)]}
\end{aligned}$$

Through the above differentiation elements of ϕ, θ and ψ, the output Jacobian matrix C can be constructed. In the subsequent section, the estimation algorithms are applied to the spacecraft model and the results are discussed.

B. Simulation Results

In this section, a representative set of simulation results consisting of EKF with data loss (Blind EKF or BEKF), CCLEKF and OLEKF, is shown for our spacecraft case-study model results of which are compared with those of fault-free (normal operation) EKF (NEKF). It is assumed that LOOB has occurred due to the measurement transmission channel failure at time $t = 30$secs and is remained for 15secs - this is a tractable assumption as such intermittent losses occur in many real-time applications. The sampling time used is 0.01secs. In our simulations, the initial state vector is selected as $x_0 = \begin{bmatrix}0.2 & 0.2 & 0.2 & -0.3 & -0.4 & 0.2\end{bmatrix}^T$. The plant and output noise covariance matrices are also assumed to be as $Q = 0.01I_{6\times6}$ and $R = 0.05I_{3\times3}$ where I is identity matrix.

1) Modified Rodriguez Parameters: Figs. 3 and 4 show the performance in terms of attitude signals estimation (σ_1, σ_2 and σ_3). In Fig. 3, the complete simulation period is shown while in Fig. 4 the LOOB period is remarkably highlighted. As clarified in Fig. 4, the BEKF and OLE diverge rapidly from the nominal steady-state values during the 15secs of LOOB. However, the results of the proposed CCLEKF approach illustrated are very promising which show that this algorithm is comparatively robust to LOOB while it is not deviated from the normal-operation results.

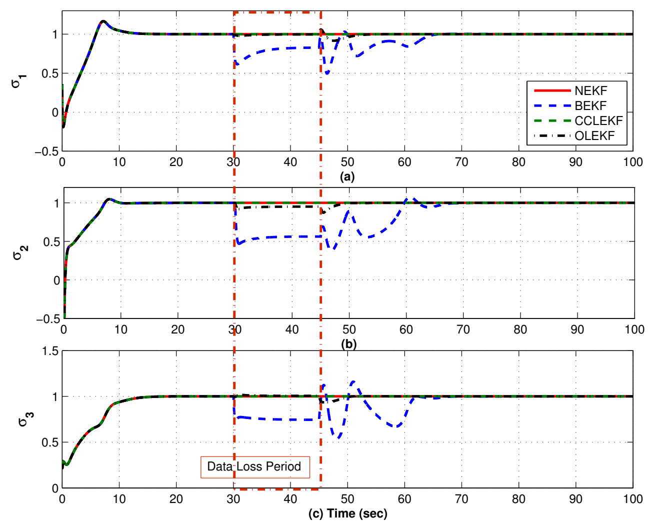

Fig. 3. Estimated attitude parameter σ.

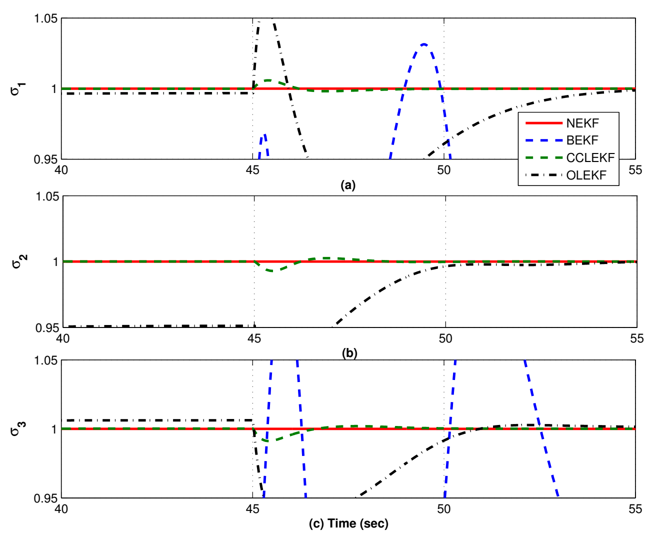

Fig. 4. Highlighted estimated attitude parameter σ. Recall that BEKF and OLEKF are exceeded from the divergence limits if provided.

2) Angular Velocity: In addition to the attitude signals, three states of angular velocities associated with the rigid body spacecraft model are also analysed. Fig. 5 shows the distinctions of the different approaches in the event of

LOOB as an index of angular velocities. It can be seen that LOOB makes OLE a poor estimation solution during the loss period and even after the observation is resumed – it is also important to stress that there are abrupt spikes and undesired oscillations at the angular velocity estimation results associated with the OLE method. On the other hand, the measurement update in CCLEKF approach provides less chattering in the state estimation which makes the CCLEKF (and CCLKF for linear cases in general) very effective and robust estimation algorithm to observation losses.

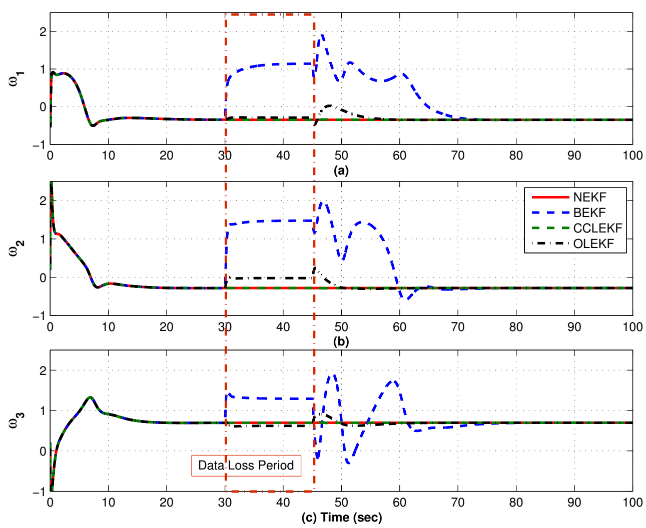

Fig. 5. Estimated angular velocity ω using different algorithms. Recall that the proposed CCLEKF outperforms other estimation methods in the presence of loss of observation.

VI. CONCLUSIONS

In this paper, a robust estimation algorithm was addressed and applied to a rigid body spacecraft model subject to measurement losses. Only typical simulation results were shown in this paper. But, our conclusions are based on many other simulation results (such as control effort, error analysis and computational capabilities, etc) which were not shown in the paper due to space limitation. Simulation results using a numerical case study illustrated a comprehensive analysis of the proposed CCLEKF approach vs other methods including the OLE. Whilst the conventional Kalman filter approach could potentially fail in providing satisfactory attitude estimation in the event of loss of observation, the proposed Robust Kalman Filtering approach is found to be a successful approach for the rigid body spacecraft application.

We plan to consider a more sophisticated dynamical model of the spacecraft dynamics for our future studies by deviating from the nominal operation and assumption of plant stationarity. More quantitative stability analysis together with employing fast and low-memory-demand approaches, such as Levinson-Durbin or Leroux-Gueguen algorithms [7] to reduce computational burdensome of the proposed estimation algorithm, are also intended to be covered in our future studies.

REFERENCES

[1] R. Ahmed, D. Gu, and I. Postelwaite. A Case Study on Spacecraft Attitude Control. In *IEEE Control and Decision Conference*, pages 7345 – 7350, December 2009.

[2] B. O. Anderson and J. B. Moore. *Optimal Filtering*. Printice Hall, Inc., 1979.

[3] W. C. Chu. *Speech Coding Algorithms: Foundation and Evolution of Standardized Coders*, chapter Linear Prediction, pages 91–142. John Wily and Sons Inc., 2003.

[4] T. Gao, Z. Gong, J. Lue, W. Ding, and W. Feng. An Attitude Determination System For A Small Unmmanned Helicopter Using Low-Cost Sensors. In *IEEE International Conference on Robotics and Biomimetics*, pages 1203–1208, December 17-20 2006.

[5] T. Gao, Z. Gong, J. Luo, W. Ding, and W. Feng. An Attitude Determination System For A small Unmanned Helicopter Using Low-Cost Sensors. In *Proceedings of the IEEE, International Conference on Robotics and Biomimetics*, pages 1203–1208, Kunming, China, 17-20 December 2006.

[6] A. Gelb. *Applied Optimal Estimation*. The M.I.T. Press, 1974.

[7] M. S. Grewal and A. P. Andrews. *Kalman Filtering: Theory and Practice using MATLAB*. John Wiley & Sons, Inc., third edition, 2008.

[8] P. R. J. and C. J. Observer-based fault detection and isolation: Robustness and applications. *Control Engineering Practice*, 5(5):671–682, 1997.

[9] N. Khan, S. Fekri, and D. Gu. Optimal State Estimation for Discrete-Time LTI Systems subject to a Measurement Loss. *Journal of Measurement*, 43(2010):1609–1622, September 2010.

[10] N. Khan, S. Fekri, and D.-W. Gu. A Sub-Optimal Kalman Filtering for Discrete-Time LTI Systems with Loss of Data. In *The 7th IFAC Conference on Intelligent Control Automation and Robotics*. ICINCO, Portugal, June 2010.

[11] E. J. Lefferts, F. L. Markley, and M. D. Shuster. Kalman Filtering for Spacecraft Attitude Estimation. *AIAA Journal of Guidence, Control and Dynamics*, 5(5):417–429, September-October 1982.

[12] X. Liu and A. Goldsmith. Kalman Filtering with Partial Observation Loss. In *Proceeding of the 43^{rd} IEEE Conference on Decision and Control*, pages 4180 – 4186, December 2004.

[13] R. J. Patton, F. J. Uppal, S. Simani, and B. Polle. Robust FDI applied to thruster faults of a satellite system. *Control Engineering Practice*, 2010(18):1093–1109, July 2010.

[14] H. Schaub and J. L. Junkins. *Analytical Mechanics of Space System*, pages 96 – 101. American Institute of Aeronautics and Astronautics, Inc., 2003.

[15] L. Schenato. Kalman Filtering for Network Control System with Random Delay and Packet Loss. In *Conference of Mathematical Theory of Networks and Systems (MTNS 06)*, Kyoto, Japan, July 2006.

[16] L. Schenato, B. Sinopoli, M. Franceschetti, K. Poolla, and S. S. Sastry. Foundation of Control and Estimation Over Lossy Network. *Proceeding of The IEEE*, 95(1):163–187, January 2007.

[17] S. Simani, C. Fantuzzi, and R. J. Patton. *Model Based Fault Diagnosis in Dynamic Systems Using Identificaiton Techniques*. Advances in Industrial Control. Springer, 2003.

[18] B. Sinopoli, L. Schenato, M. Franceschetti, K. Poolla, M. I. Jordon, and S. S. Sastry. Kalman Filtering with Intermittent Observations. *IEEE Transactions on Automatic Control*, 49:1453 – 1464, 2004.

[19] J. J. E. Slotine and M. D. Benedetto. Hamiltonian adaptive control of spacecraft. *IEEE Transactions on Automatic Contorl*, 35(7):848 – 852, July 1990.

[20] J.-J. E. Slotine and W. Li. *Applied Nonlinear Control*. Prentice Hall, 1991.

[21] M. UNSER. Sampling - 50 Years After Shannon. *In Proceeding of IEEE*, 88(4):569–587, April 2000.

[22] W. Xue, Y.-Q. Guo, and X.-D. Zhang. Application of A bank of Kalman filters and A Robust Kalman Filter for Aircraft Engine Sensor/Actuator Fault Diagnosis. *International Journal of Innovative Computing, Information and Control*, 4(12):3161–3168, December 2008.