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Recursive continuous-time subspace identification using Laguerre filters

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Abstract— **This paper deals with the problem of model identification in continuous-time using subspace techniques. More precisely, a recently presented continuous-time predictor-based subspace identification algorithm which relies on a system transformation using the Laguerre basis is considered and a recursive counterpart is developed.**

I. Introduction

Subspace Model Identification (SMI) algorithms provide an extremely useful approach to deal with the estimation of discrete-time state space models for MIMO systems. The problem of developing dedicated SMI methods for the identification of continuous-time systems has been studied in a number of contributions, see, e.g., [1], [2], [3], where a detailed overview of this research area is provided, In particular, in the cited papers a continuous-time version of the Predictor-Based Subspace IDentification (PBSID) algorithm (see [4]) is provided, based on the adoption of orthonormal basis functions as originally proposed in [16], [14].

Similarly, the problem of recursive subspace model identification (RSMI) in discrete-time has been an active area of research in recent years (see, e.g., [18], [6], [7], [11], [17], [12]). Most RSMI algorithms are inspired by offline versions of SMI techniques and therefore rely on the availability of efficient updating methods for the numerical linear algebra algorithms used in batch SMI. While the above cited papers are concerned with the derivation of recursive versions for the MOESP class of SMI algorithms, more recently, two recursive implementations for the discrete-time PBSID algorithm have been proposed, in [8], [5].

Recursive implementation is particularly important in connection to continuous-time subspace methods in view of the significant computational burden associated with their implementation. In this respect, the recursive implementation of the continuous-time version of PO-MOESP first proposed in [16], [14] has been presented in [10], together with a novel set of orthonormal, Laguerre-like basis functions with the specific feature of being compactly supported. In view of the above discussion, the aim of this paper is to propose a novel algorithm for RSMI in continuous-time, building on related results presented in [1], [2], [3] for the continuous-time counterpart of the PBSID method and on the recent advances in recursive identification presented in the above cited papers. More precisely, a recursive version of the algorithm in [1], [2], [3] is presented, and a comparison between conventional and compactly supported basis functions is carried out and discussed.

The paper is organised as follows. In Section II the problem statement is given and some definitions are provided. Section III provides a summary of the approach based on Laguerre projections used to convert continuous-time problems to equivalent discrete-time ones; subsequently, in Section IV the batch algorithm is first briefly summarised and the proposed recursive counterpart is presented. Finally, some simulation results are presented in Section V to illustrate the performance of the proposed method.

II. Problem statement and preliminaries

Consider the linear, time-invariant continuous-time system

$$\begin{aligned} dx(t) &= Ax(t)dt + Bu(t)dt + dw(t),\ x(0) = x_0 \\ dz(t) &= Cx(t)dt + Du(t)dt + dv(t) \\ y(t)dt &= dz(t) \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$ and $y \in \mathbb{R}^p$ are, respectively, the state, input and output vectors and $w \in \mathbb{R}^n$ and $v \in \mathbb{R}^p$ are the process and the measurement noise, respectively, modelled as Wiener processes with incremental covariance given by

$$E\left\{\begin{bmatrix} dw(t) \\ dv(t)\end{bmatrix}\begin{bmatrix} dw(t) \\ dv(t)\end{bmatrix}^T\right\} = \begin{bmatrix} Q & S \\ S^T & R\end{bmatrix} dt.$$

The system matrices A, B, C and D, of appropriate dimensions, are such that (A, C) is observable and $(A, [B, Q^{1/2}])$ is controllable. Assume that a dataset $\{u(t_i), y(t_i)\}$, $i \in [1, N]$ of sampled input/output data (possibly associated with a non equidistant sequence of sampling instants) obtained from system (1) is available. Then, the problem is to provide a recursive estimator of the state space matrices A, B, C and D (up to a similarity transformation) on the basis of the available data.

In the following Sections a number of definitions will be used, which are summarised hereafter for the sake of clarity. See, e.g., [19], [9], [13] for further details.

Let $\mathcal{L}_2(0,\infty)$ denote the space of square integrable and Lebesgue measurable functions of time $0 < t < \infty$, with the inner product defined as $< f, g >= \int_0^\infty f(t)g(t)dt$, for $f, g \in \mathcal{L}_2(0,\infty)$; the space $\mathcal{H}_2$ is the closed subspace of $\mathcal{L}_2(i\mathbb{R})$ with functions analytic in the open right half plane, with norm

$$\|U\|_2^2 = \sup_{\sigma>0} \frac{1}{2\pi}\int_{-\infty}^{\infty} |U(\sigma+j\omega)|^2 d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} |U(j\omega)|^2 d\omega. \tag{2}$$

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In view of Parseval's relation, the spaces $\mathcal{L}_2(0,\infty)$ and $\mathcal{H}_2$ are related by the isometric isomorphism defined by the bilateral Fourier transform, so if $U \in \mathcal{H}_2$, the inverse Fourier transform $u = \mathcal{F}^{-1}[U]$ is in $\mathcal{L}_2(0,\infty)$ and $\|U\|_2 = \|u\|_2$.

A scalar transfer function $w(s)$ is called inner if it is a bounded analytic function in the open right half plane (i.e., $w(j\omega) \in \mathcal{H}_\infty$), such that $|w(j\omega)| = 1$ or $w^\sim(j\omega)w(j\omega) = 1$ almost everywhere on the imaginary axis, where $w^\sim(j\omega) = w^T(-j\omega)$ is the para-conjugate (i.e., w is an all-pass transfer function). We further denote by Λ_w the multiplication operator $\mathcal{L}_2(0,\infty) \mapsto \mathcal{L}_2(0,\infty)$ defined as

$$\Lambda_w u(t) = \mathcal{F}^{-1}[w\mathcal{F}[u(t)]]. \tag{3}$$

In the following the focus will be on the first order inner function

$$w(s) = \frac{s-a}{s+a}, \tag{4}$$

$a > 0$, together with the associated realisation

$$w(s) = \frac{c_w b_w}{s - a_w} + d_w, \tag{5}$$

where $a_w = -a$, $b_w = -\sqrt{2a}$, $c_w = \sqrt{2a}$, $d_w = 1$. Then, it can be shown that $w(s)\mathcal{H}_2$ is a proper closed subspace of $\mathcal{H}_2$, the orthogonal complement of which is denoted as $S = \mathcal{H}_2 \ominus w(s)\mathcal{H}_2$, that

$$\mathcal{L}_0(s) = \frac{c_w}{s+a} = \frac{\sqrt{2a}}{s+a} \tag{6}$$

is a basis of the (one-dimensional) subspace S and that the set

$$\left\{\mathcal{L}_0, w\mathcal{L}_0, \ldots, w^k\mathcal{L}_0, \ldots\right\} \tag{7}$$

is an orthonormal basis of $\mathcal{H}_2$, i.e., $\mathcal{H}_2 = \bigoplus_{k=0}^{\infty} w^k S$. Equivalently, letting $\ell_0 = \mathcal{F}^{-1}[\mathcal{L}_0]$, the set

$$\left\{\ell_0, \Lambda_w\ell_0, \ldots, \Lambda_w^k\ell_0, \ldots\right\} \tag{8}$$

is an orthonormal basis of $\mathcal{L}_2(0,\infty)$, i.e., $\mathcal{L}_2(0,\infty) = \bigoplus_{k=0}^{\infty} \Lambda_w^k S$.

The transfer function of the k-th (order $k+1$) Laguerre filter is then defined as

$$\mathcal{L}_k(s) = w^k(s)\mathcal{L}_0(s) = \sqrt{2a}\frac{(s-a)^k}{(s+a)^{k+1}}. \tag{9}$$

III. From continuous-time to discrete-time using Laguerre projections

The continuous-time algorithms discussed in this paper are based on the results first presented in [16], [14], [13], and further expanded in [10], [15], which allow to obtain a discrete-time equivalent model starting from the continuous-time system (1), along the following lines.

A. Laguerre projections

First note that under the assumptions stated in Section II, (1) can be written in innovation form as

$$\begin{aligned} dx(t) &= Ax(t)dt + Bu(t)dt + Kde(t) \\ dz(t) &= Cx(t)dt + Du(t)dt + de(t) \\ y(t)dt &= dz(t), \end{aligned} \tag{10}$$

and it is possible to apply the results of [16], [14] to derive a discrete-time equivalent model, as follows. Consider the first order inner function $w(s)$ and apply to the input u, the output y and the innovation e of (10) the transformations

$$\begin{aligned} \tilde{u}(k) &= \int_0^\infty \Lambda_w^k \ell_0(t)u(t)dt \\ \tilde{y}(k) &= \int_0^\infty \Lambda_w^k \ell_0(t)y(t)dt \\ \tilde{e}(k) &= \int_0^\infty \Lambda_w^k \ell_0(t)de(t), \end{aligned} \tag{11}$$

where $\tilde{u}(k) \in \mathbb{R}^m$, $\tilde{e}(k) \in \mathbb{R}^p$ and $\tilde{y}(k) \in \mathbb{R}^p$. Then (see [16], [14] for details) the transformed system has the state space representation

$$\begin{aligned} \xi(k+1) &= A_o\xi(k) + B_o\tilde{u}(k) + K_o\tilde{e}(k),\ \xi(0) = 0 \\ \tilde{y}(k) &= C_o\xi(k) + D_o\tilde{u}(k) + \tilde{e}(k) \end{aligned} \tag{12}$$

where the state space matrices are given by

$$\begin{aligned} A_o &= (A - aI)^{-1}(A + aI) \\ B_o &= \sqrt{2a}(A - aI)^{-1}B \\ C_o &= -\sqrt{2a}C(A - aI)^{-1} \\ D_o &= D - C(A - aI)^{-1}B. \end{aligned} \tag{13}$$

B. Signal transformation with finitely-supported filter kernels

As remarked in [10] the above described projections based on Laguerre basis functions can only be computed in an approximate sense as the indefinite integrals in (11) need to be truncated to finite intervals. In order to circumvent this difficulty, a novel set of basis functions, for which it can be proved that they are compactly supported, has been proposed in the same paper, where it has been shown that for suitably chosen scalars α_0, α_1, ..., α_ρ (see [10] for details) the functions

$$\tilde{\ell}_k(t) = \sum_{i=0}^{\rho} \alpha_i \Lambda_w^k \ell_0(t - i\tau) \tag{14}$$

can be used to define the system transformations

$$\begin{aligned} \tilde{u}(k) &= \int_0^\infty \tilde{\ell}_k(t)u(t)dt \\ \tilde{y}(k) &= \int_0^\infty \tilde{\ell}_k(t)y(t)dt \\ \tilde{e}(k) &= \int_0^\infty \tilde{\ell}_k(t)de(t), \end{aligned} \tag{15}$$

with the property that the signals obtained with the transformations (15) satisfy the discrete-time system (12) for suitably chosen initial states. The support of the functions is compact if $\rho > k$.

IV. Continuous-time recursive predictor-based subspace model identification

In this Section a summary of the batch continuous-time PBSID algorithm proposed in [1], [3] is provided, and its recursive implemention is discussed, both using Laguerre basis functions according to Section III-A and their compactly supported counterparts described in III-B.

A. Batch estimation

Starting from system (10), in this Section a sketch of the derivation of a PBSID-like approach to the estimation of the state space matrices A_o, B_o, C_o, D_o, K_o is presented. Considering the sequence of sampling instants t_i, $i = 1, \dots, N$, the input u, the output y and the innovation e of (10) are subjected to the transformations

$$\begin{aligned} \tilde{u}_i(k) &= \int_0^\infty (\Lambda_w^k \ell_0(\tau)) u(t_i + \tau) d\tau \\ \tilde{e}_i(k) &= \int_0^\infty (\Lambda_w^k \ell_0(\tau)) de(t_i + \tau) \\ \tilde{y}_i(k) &= \int_0^\infty (\Lambda_w^k \ell_0(\tau)) y(t_i + \tau) d\tau \end{aligned} \tag{16}$$

(or to the equivalent ones derived from (15)), where $\tilde{u}_i(k) \in \mathbb{R}^m$, $\tilde{e}_i(k) \in \mathbb{R}^p$ and $\tilde{y}_i(k) \in \mathbb{R}^p$. Then (see [16], [14] for details) the transformed system has the state space representation

$$\begin{aligned} \xi_i(k+1) &= A_o \xi_i(k) + B_o \tilde{u}_i(k) + K_o \tilde{e}_i(k), \; \xi_i(0) = x(t_i) \\ \tilde{y}_i(k) &= C_o \xi_i(k) + D_o \tilde{u}_i(k) + \tilde{e}_i(k) \end{aligned} \tag{17}$$

where the state space matrices are given by (13).

Letting now

$$\tilde{z}_i(k) = \begin{bmatrix} \tilde{u}_i^T(k) & \tilde{y}_i^T(k) \end{bmatrix}^T$$

and

$$\begin{aligned} \bar{A}_o &= A_o - K_o C_o \\ \bar{B}_o &= B_o - K_o D_o \\ \widetilde{B}_o &= \begin{bmatrix} \bar{B}_o & K_o \end{bmatrix}, \end{aligned}$$

system (17) can be written as

$$\begin{aligned} \xi_i(k+1) &= \bar{A}_o \xi_i(k) + \widetilde{B}_o \tilde{z}_i(k), \; \xi_i(0) = x(t_i) \\ \tilde{y}_i(k) &= C_o \xi_i(k) + D_o \tilde{u}_i(k) + \tilde{e}_i(k), \end{aligned} \tag{18}$$

to which the PBSID$_{opt}$ algorithm can be applied to compute estimates of the state space matrices A_o, B_o, C_o, D_o, K_o. To this purpose note that iterating $p-1$ times the projection operation (i.e., propagating $p-1$ forward in the index k the first of equations (18), where p is the so-called past window length) one gets

$$\begin{aligned} \xi_i(k+2) &= \bar{A}_o^2 \xi_i(k) + \begin{bmatrix} \bar{A}_o \widetilde{B}_o & \widetilde{B}_o \end{bmatrix} \begin{bmatrix} \tilde{z}_i(k) \\ \tilde{z}_i(k+1) \end{bmatrix} \\ &\vdots \\ \xi_i(k+p) &= \bar{A}_o^p \xi_i(k) + \mathcal{K}^p Z_i^{0,p-1} \end{aligned} \tag{19}$$

where

$$\mathcal{K}^p = \begin{bmatrix} \bar{A}_o^{p-1} \widetilde{B}_0 & \dots & \widetilde{B}_o \end{bmatrix} \tag{20}$$

is the extended controllability matrix of the system in the transformed domain and

$$Z_i^{0,p-1} = \begin{bmatrix} \tilde{z}_i(k) \\ \vdots \\ \tilde{z}_i(k+p-1) \end{bmatrix}.$$

Under the considered assumptions, $\bar{A}_o$ has all the eigenvalues inside the open unit circle, so the term $\bar{A}_o^p \xi_i(k)$ is negligible for sufficiently large values of p and we have that

$$\xi_i(k+p) \simeq \mathcal{K}^p Z_i^{0,p-1}.$$

As a consequence, the input-output behaviour of the system is approximately given by

$$\begin{aligned} \tilde{y}_i(k+p) &\simeq C_o \mathcal{K}^p Z_i^{0,p-1} + D_o \tilde{u}_i(k+p) + \tilde{e}_i(k+p) \\ &\vdots \\ \tilde{y}_i(k+p+f) &\simeq C_o \mathcal{K}^p Z_i^{f,p+f-1} + D_o \tilde{u}_i(k+p+f) + \\ &\quad + \tilde{e}_i(k+p+f), \end{aligned} \tag{21}$$

so that introducing the vector notation

$$\begin{aligned} Y_i^{p,f} &= \begin{bmatrix} \tilde{y}_i(k+p) & \tilde{y}_i(k+p+1) & \dots & \tilde{y}_i(k+p+f) \end{bmatrix} \\ U_i^{p,f} &= \begin{bmatrix} \tilde{u}_i(k+p) & \tilde{u}_i(k+p+1) & \dots & \tilde{u}_i(k+p+f) \end{bmatrix} \\ E_i^{p,f} &= \begin{bmatrix} \tilde{e}_i(k+p) & \tilde{e}_i(k+p+1) & \dots & \tilde{e}_i(k+p+f) \end{bmatrix} \\ \Xi_i^{p,f} &= \begin{bmatrix} \xi_i(k+p) & \xi_i(k+p+1) & \dots & \xi_i(k+p+f) \end{bmatrix} \\ \bar{Z}_i^{p,f} &= \begin{bmatrix} Z_i^{0,p-1} & Z_i^{1,p} & \dots & Z_i^{f,p+f-1} \end{bmatrix} \end{aligned} \tag{22}$$

equations (19) and (21) can be rewritten as

$$\begin{aligned} \Xi_i^{p,f} &\simeq \mathcal{K}^p \bar{Z}_i^{p,f} \\ Y_i^{p,f} &\simeq C_o \mathcal{K}^p \bar{Z}_i^{p,f} + D_o U_i^{p,f} + E_i^{p,f}. \end{aligned} \tag{23}$$

Considering now the entire dataset for $i = 1, \dots, N$, the data matrices become

$$\begin{aligned} Y^{p,f} = [&\tilde{y}_1(k+p) \; \dots \; \tilde{y}_N(k+p) \; \dots \\ &\tilde{y}_1(k+p+f) \; \dots \; \tilde{y}_N(k+p+f)], \end{aligned} \tag{24}$$

and similarly for $U_i^{p,f}$, $E_i^{p,f}$, $\Xi_i^{p,f}$ and $\bar{Z}_i^{p,f}$. The data equations (23), in turn, are given by

$$\begin{aligned} \Xi^{p,f} &\simeq \mathcal{K}^p \bar{Z}^{p,f} \\ Y^{p,f} &\simeq C_o \mathcal{K}^p \bar{Z}^{p,f} + D_o U^{p,f} + E^{p,f}. \end{aligned} \tag{25}$$

From this point on, the algorithm can be developed along the lines of the discrete-time PBSID$_{opt}$ method, i.e., by carrying out the following steps. Considering $p = f$, estimates for the matrices $C_o \mathcal{K}^p$ and D_o are first computed by solving the least-squares problem

$$\min_{C_o \mathcal{K}^p, D_o} \| Y^{p,p} - C_o \mathcal{K}^p \bar{Z}^{p,p} - D_o U^{p,p} \|_F. \tag{26}$$

Defining now the extended observability matrix Γ^p as

$$\Gamma^p = \begin{bmatrix} C_o \\ C_o\bar{A}_o \\ \vdots \\ C_o\bar{A}_o^{p-1} \end{bmatrix} \tag{27}$$

and noting that the product of Γ^p and $\mathcal{K}^p$ can be written as

$$\Gamma^p\mathcal{K}^p \simeq \begin{bmatrix} C_o\bar{A}^{p-1}\widetilde{B}_o & \dots & C_o\widetilde{B}_o \\ 0 & \dots & C_o\bar{A}\widetilde{B}_o \\ \vdots & & \\ 0 & \dots & C_o\bar{A}^{p-1}\widetilde{B}_o \end{bmatrix}, \tag{28}$$

such product can be computed using the estimate $\widehat{C_o\mathcal{K}^p}$ of $C_o\mathcal{K}^p$ obtained by solving the least squares problem (26).

Recalling now that

$$\Xi^{p,p} \simeq \mathcal{K}^p\bar{Z}^{p,p} \tag{29}$$

it also holds that

$$\Gamma^p\Xi^{p,p} \simeq \Gamma^p\mathcal{K}^p\bar{Z}^{p,p}. \tag{30}$$

Therefore, computing the singular value decomposition

$$\Gamma^p\mathcal{K}^p\bar{Z}^{p,p} = U\Sigma V^T \tag{31}$$

an estimate of the state sequence can be obtained as

$$\widehat{\Xi}^{p,p} = \Sigma_n^{1/2}V_n^T = \Sigma_n^{-1/2}U_n^T\Gamma^p\mathcal{K}^p\bar{Z}^{p,p}, \tag{32}$$

from which, in turn, an estimate of C_o can be computed by solving the least squares problem

$$\min_{C_o} \|Y^{p,p} - \widehat{D}_oU^{p,p} - C_o\widehat{\Xi}^{p,p}\|_F. \tag{33}$$

The final steps consist of the estimation of the innovation data matrix $E^{p,p}$

$$E^{p,p} = Y^{p,p} - \widehat{C}_o\widehat{\Xi}^{p,p} - \widehat{D}_oU^{p,p} \tag{34}$$

and of the entire set of the state space matrices for the system in the transformed domain, which can be obtained by solving the least squares problem

$$\min_{A_o,B_o,K_o} \|\widehat{\Xi}^{p+1,p} - A_o\widehat{\Xi}^{p,p-1} - B_oU^{p,p-1} - K_oE^{p,p-1}\|_F. \tag{35}$$

B. Recursive estimation

In discrete-time RSMI schemes the recursion is implemented directly with respect to the new discrete input-output sample acquired at the current sampling time. When dealing with the corresponding continuous-time counterpart the first step to be carried out is the (approximate) computation of the projections (16) (or of the equivalent ones derived from (15)). In this respect, since the index k in the transformed system represents the order of the basis function on which the data has been projected, while the index i is related to the sampling instants t_i, the arrival of a new input-output sample leads to the addition of a new time instant at which the projections (16) have to be computed, which leads, in turn, to a new column to be added to the data matrices defined according to (24). In order to compute the projections (16) when dealing with the conventional (i.e., with infinite support) Laguerre filters, the following approximation is introduced:

$$\begin{aligned} \tilde{u}_i(k) &= \int_0^\infty \Lambda_w^k\ell_0(\tau)u(t_i+\tau)d\tau \\ &= \int_{t_i}^\infty \Lambda_w^k\ell_0(\tau-t_i)u(\tau)d\tau = \\ &\simeq \int_{t_i}^{t_F+t_i} \Lambda_w^k\ell_0(\tau-t_i)u(\tau)d\tau, \end{aligned} \tag{36}$$

where t_F is the instant where the impulse response of the filter of maximum order can be considered approximately equal to zero. In other words

$$t_F = arg\max_t t \tag{37}$$

$$s.t.\ \|\Lambda_w^{2p-1}\ell_0(t)| \geq \epsilon, \tag{38}$$

where ϵ is a sufficiently small number. On the other hand, the approximation can be avoided by modifying the basis functions as described in Section III-B. In particular, the modified functions have compact support if $\rho > k$, therefore in the following it will be assumed that $\rho > p + f$.

As far as the actual implementation of the recursive algorithm is concerned, the following steps have to be implemented (along the lines of the general template proposed in [5] and of the algorithm in [8]):

- Recursive update of the solution of the least squares problem (26), using a conventional RLS scheme.
- Update of the estimate of the state sequence, i.e., of the state estimate given by (32). In this respect, note that this is the most critical step in the implementation, as one has to ensure that the recursive state estimates are expressed in a consistent state space basis. One way of guaranteeing this is given by, e.g., the scheme proposed in [8], which is based on the so-called propagator method for the recursive update of the state sequence (see also [12] for details).
- Recursive estimate of the state space matrices of the system, i.e., update of the solution of the least squares problems (33) and (35), again by means of RLS.

The overall recursive algorithm is summarised in Table I.

V. Simulation examples

In this Section some simulation results obtained with the application of the proposed algorithms (recursive PBSID both with conventional and modified Laguerre bases) and of the algorithm in [10] to data generated by systems operating both in open-loop and in closed-loop are presented and discussed.

A. Open-loop case

The considered example is the SISO continuous-time system given by the state space matrices

$$A = \begin{bmatrix} -0.5 & a_1 & 0 \\ -a_1 & -0.5 & 0 \\ 0 & 0 & a_2 \end{bmatrix} \qquad B = \begin{bmatrix} 1.5 \\ 1.8 \\ 1.6 \end{bmatrix} \tag{39}$$

$$C = \begin{bmatrix} 1.5 & 1.3 & 1.6 \end{bmatrix} \qquad D = 0; \tag{40}$$

where a_1 and a_2 are initially equal to 10 and -2 respectively and change to 15 and -5 after 150 s. The simulated data has been collected by applying to the input of the system a piece-wise constant signal with base period $T_p = 0.01$ s, for a duration of 300 s. The input level is chosen randomly according to a Gaussian distribution with zero mean and unit variance. White Gaussian noise of increasing variance has been added to the output in order to assess the influence of decreasing signal-to-noise ratio on the quality of the computed estimates. For the input and output variables the sampling interval $\Delta t = 0.001$ s has been considered. The pole of the Laguerre filters has been chosen as $a = 10$, while different choices have been made concerning the implementation of the projection operators associated with the system transformations. In the case of the conventional Laguerre basis, windows of 2.6 s have been used, while for the modified basis a shorter window of 0.8 s only (in view of the compact support) has been employed. Figures 1-2 illustrate the estimation errors of the real and imaginary parts of the system eigenvalues as computed by running the algorithm in [10] (black line) and the one proposed herein, both using conventional Laguerre bases (pale gray line) and finitely supported ones (dark gray line), for two different values of the signal-to-noise ration at the output. Apart from occasional overshoots in transients, which are likely to be reduced by a more refined implementation, it appears that algorithms based on the modified Laguerre basis can provide good performance while requiring a shorter time window for the integration, which leads to a potentially faster operation.

B. Closed-loop case

In the second example the same system is considered, but data are now collected during closed-loop operation, subject to the control law $u = Ky$, $K = 1$. Again, Figures 3-4 illustrate the estimation errors of the real and imaginary parts of the system eigenvalues as computed by running the algorithm in [10] (black line) and the one proposed herein, both using conventional Laguerre bases (pale gray line) and finitely supported ones (dark gray line), for two different values of the signal-to-noise ration at the output. Similar comments as in the open-loop case apply to the results obtained in this situation.

VI. Concluding Remarks

The problem of recursive continuous-time subspace model identification has been considered and an updating scheme for a batch algorithm based on Laguerre projections of the input-output variables followed by a PBSID identification

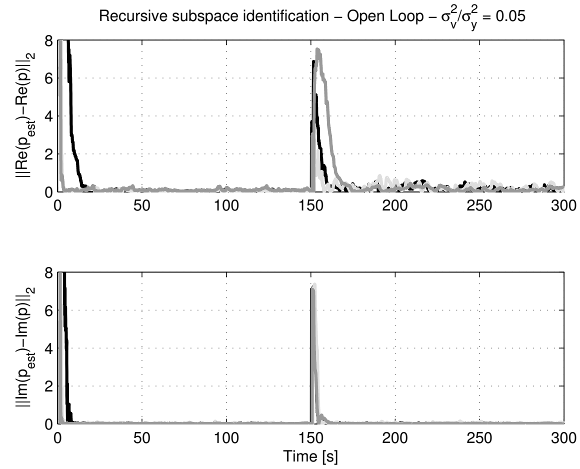

Fig. 1. Real and imaginary part of the estimated eigenvalues - open loop experiment - $\frac{\sigma_v^2}{\sigma_y^2} = 0.05$.

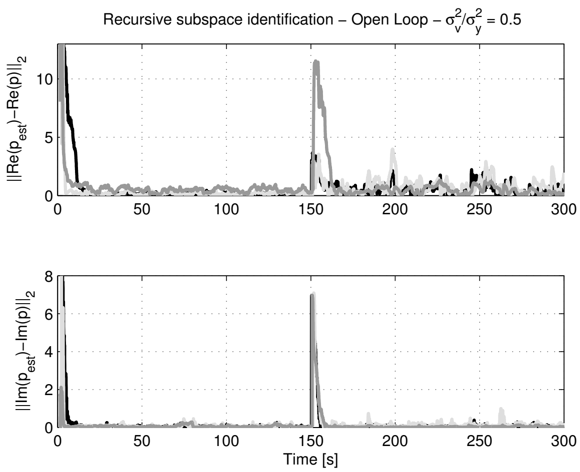

Fig. 2. Real and imaginary part of the estimated eigenvalues - open loop experiment - $\frac{\sigma_v^2}{\sigma_y^2} = 0.5$.

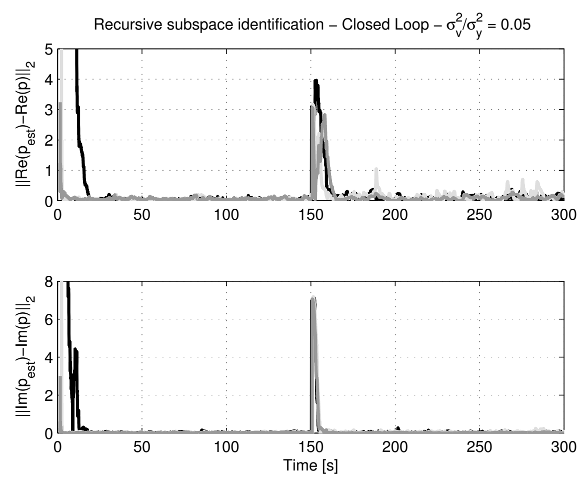

Fig. 3. Real and imaginary part of the estimated eigenvalues - closed loop experiment - $\frac{\sigma_v^2}{\sigma_y^2} = 0.05$.

Algorithm RPBSID$_o$

For t_i with $i = 1, ..., N$

1) Compute $\tilde{u}_i(k)$ and $\tilde{y}_i(k)$ for $k = 0, .., 2p$ using (36).
2) Build the matrices $Y_i^{p,p}$, $U_i^{p,p}$ and $\bar{Z}_i^{p,p}$ according to (22).
3) Update the recursive least-squares version of the problem (26) obtaining $C_o\mathcal{K}^p$ and D_o.
4) According to (28) an estimate of $\Gamma^p\mathcal{K}^p$ is obtained using $C_o\mathcal{K}^p$.
5) Obtain an estimate of the state sequence from the recursive estimate of $\Gamma^p\mathcal{K}^p$ along the lines of [8].
6) Update the recursive least-squares version of the problem (33) obtaining C_o.
7) Compute $E_i^{p,p}$ with (34).
8) Update the recursive least-squares version of the problem (35) obtaining A_o, B_o and K_o.
9) Use the matrix relations (13) to obtain A, B, C and D.

end

TABLE I

SUMMARY OF THE RPBSID$_o$ ALGORITHM.

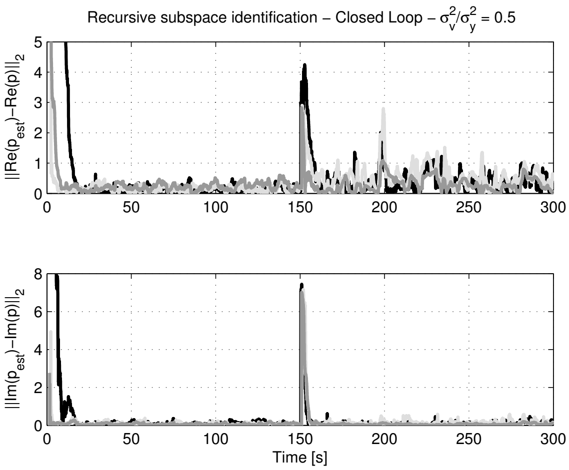

Fig. 4. Real and imaginary part of the estimated eigenvalues - closed loop experiments - $\frac{\sigma_v^2}{\sigma_y^2} = 0.5$.

step has been proposed. In particular, the role of classical and compactly supported Laguerre basis functions has been considered. Simulation results show that the proposed schemes are viable and can be therefore applied to either relieve the computational burden and memory storage requirements of the corresponding batch algorithms when dealing with large scale and/or fast sampling problems or to compute on-line updates of the system matrices for slowly varying systems.

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