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Multiple-Model Adaptive Switching Control for Uncertain Multivariable Systems

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***Abstract*— This paper addresses the problem of controlling an uncertain multi-input multi-output (MIMO) system by means of adaptive switching control schemes. In particular, the paper aims at extending the approach of multiple-model unfalsified adaptive switched control, so far restricted to single-input single-output systems, to a general multivariable setting. The proposed scheme relies on a data-driven "high-level" unit, called the supervisor, which at any time can switch on in feedback with the uncertain plant one controller from a finite family of candidate controllers. The supervisor performs routing and scheduling tasks by monitoring suitable test functionals which, based on the measured data, provide a measure of mismatch between the potential loop made up by the uncertain plant in feedback with the candidate controller and the nominal "reference loop" related to the same candidate controller.**

I. INTRODUCTION

In many control systems, such as industrial plants, aircrafts, and communication networks a large number of actuator and/or sensors may be employed in order to achieve the desired control task. In these kind of control problems, the inputs and outputs cannot usually be grouped into pairs and treated as if they were separate single-input single-output (SISO) problems, because the interactions between the multiple inputs and outputs are non-negligible. In this case, one has to tackle control design as a genuine multiple-input multi-output (MIMO) problem. The problem is even more complicated when the multivariable system to be controlled is completely or partially unknown. One of the approaches for controlling uncertain plants is the introduction of feedback adaptation. The extension of adaptive control algorithms developed for SISO systems to a MIMO setting is not trivial. Some MIMO adaptive control algorithms based on the model reference approach and the pole placement approach can be found in [1], [2]. In recent years, adaptive switching control (ASC) has emerged as an alternative to conventional continuous adaptation, providing an attractive framework for combining tools from adaptive and robust control [3], [4], [5], [6], [7]. The supervisory architecture comprises a *multicontroller* consisting of a family of precomputed candidate controllers and a *supervisor* that orchestrates the switching by selecting at any time one among the candidate controllers based on processed plant input/output data [8]. Although the literature on ASC is quite vast, most of the works deal with the SISO case with notable exceptions being [9], [10], [11], [12].

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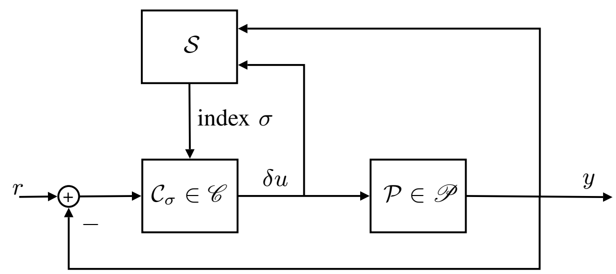

Fig. 1. Typical ASC arrangement.

This paper aims at extending multi-model unfalsfied ASC (MMUASC) of [7], [13] to a MIMO setting. In MUASSC, the supervisor performs in real-time the scheduling task (when to switch) and the routing task (which controller select), by monitoring suitable test functionals, pairwise associated with the given candidate controllers, as indicators of controller suitability. Each test functional, computed on the basis of the measured plant input and output, provides a measure of percentage discrepancy between the potential loop, made up by the uncertain plant in feedback with the candidate controller, and a nominal "reference loop" related to the same candidate controller.

While, in the SISO case, such a discrepancy can in principle be defined by resorting to the concept of a virtual reference, the situation becomes more intricate in a MIMO setting since in this case the virtual reference need not exist. Nevertheless, it will be constructively shown that, irrespectively of the existence of the virtual reference, the interpretation based on discrepancy can be recovered and the inferred stability information on the potential loop remains unaltered.

II. SWITCHING CONTROL FRAMEWORK

Let the "switched" system be represented as follows

$$\left.\begin{aligned} y(t) &= \mathcal{P}(\delta u)(t) \\ \delta u(t) &= \mathcal{C}_{\sigma(t)}(r-y)(t) \end{aligned}\right\} \tag{1}$$

where $t \in \mathbb{Z}_+$, $\mathbb{Z}_+ := \{0, 1, \cdots\}$, $\mathcal{P} : \delta u \mapsto y$ denotes the uncertain plant with input-increment $\delta u(t) := u(t) - u(t-1) \in \mathbb{R}^m$ and output $y(t) \in \mathbb{R}^p$; $r(t) \in \mathbb{R}^p$ denotes the reference to be tracked by the plant output and $\sigma(t)$

the subscript identifying the candidate controller connected in feedback to the plant at time t. It is assumed that the uncertain plant consists of a discrete-time strictly causal MIMO LTI dynamic system with matrix fraction descriptions (MFDs, for short)

$$\mathcal{P}:\ A^{-1}(d)\,B(d)=N(d)\,D^{-1}(d) \tag{2}$$

where $A(d)=\mathscr{A}(d)\Delta(d)$, $\mathscr{A}(d)=I_p+\mathscr{A}_1 d+\ldots+\mathscr{A}_{n_a}d^{n_a}$, $\Delta(d)=(1-d)I_p$ and $B(d)=\mathscr{B}_1 d+\ldots+\mathscr{B}_{n_b}d^{n_b}$ are polynomial matrices in the unit backward-shift operator d with strictly Schur greatest common left divisor (g.c.l.d.). Similar definitions apply to the right MFD $N(d)\,D^{-1}(d)$.

A so-called switching supervisory unit handles the plant I/O records in order to generate the sequence σ specifying the switching controller $\mathcal{C}_{\sigma(t)}$. Specifically, at each time t, the controller $\mathcal{C}_{\sigma(t)}$ is one element from a finite family of N one-degree-of-freedom LTI controllers $\mathcal{C}_i$ $\mathscr{C}=\{\mathcal{C}_i, i\in \overleftarrow{N}\}$, $\overleftarrow{N}:=\{1,2,\cdots,N\}$, with MFDs

$$\mathcal{C}_i:\ R_i^{-1}(d)\,S_i(d)=Y_i(d)\,X_i^{-1}(d) \tag{3}$$

where $R_i(d)=I_m+\mathscr{R}_{i1}d+\ldots+\mathscr{R}_{in_r}d^{n_r}$ and $S_i(d)=\mathscr{S}_{i0}+\mathscr{S}_{i1}d+\ldots+\mathscr{S}_{in_s}d^{n_s}$ are polynomial matrices with strictly Shur g.c.l.d.. As beforehand, a similar definition applies to the right MFDs $Y_i(d)\,X_i^{-1}(d)$. Plant input increments are considered throughout this paper so as to address the common practice of controllers equipped with an "integral action".

In the remainder of the paper, the linear time-varying feedback system (1) will be denoted by $(\mathcal{P}/\mathcal{C}_{\sigma(t)})$. Let now $\mathbb{S}$ denote the linear space of all the real-valued sequences on $\mathbb{Z}_+$. Then, given a vector-valued sequence $x\in\mathbb{S}$ of dimension n, x^t denotes its time truncation up to time t, *i.e.*, $x^t:=\{x(0),x(1),\ldots,x(t)\}$, with $x(k)\in\mathbb{R}^n$. Furthermore, $\|x^t\|^2:=\sum_{k=0}^{t}|x(k)|^2$ where $|\cdot|$ denotes Euclidean norm.

Definition 1: The switched system (1) is said to be stable relatively to r (*r-stable*, for short) if, for every input $r\in\mathbb{S}$, there exist finite positive reals c_i, $i=1,2$, such that

$$\left\|z^t\right\|\le c_1+c_2\left\|r^t\right\|,\quad \forall t\in\mathbb{Z}_+ \tag{4}$$

where $z(k):=[\,\delta u(k)'\,y(k)'\,]'$.

Let $\mathscr{P}$ denote the plant uncertainty set. In other terms, $\mathscr{P}$ represents the set of possible plant configurations, *e.g.* a range of parametric uncertainty. In order for the problem to be well-posed, the following requirement is assumed.

Definition 2: The adaptive switching control problem is said to be *feasible* if, for every $\mathcal{P}\in\mathscr{P}$, there is at least an index $i\in\overleftarrow{N}$, such that $(\mathcal{P}/\mathcal{C}_i)$ is internally stable.

Before proceeding some comments are in order. For clarity of exposition, in the remainder of this paper, the analysis will be carried out assuming zero plant initial conditions and zero noises/disturbances. Nonetheless, the results to be presented can be readily extended to the general case along the same lines as those of [7], [14]. In accordance with the mentioned restrictions, next definition is introduced in order to avoid possible ambiguities.

Definition 3: Given an LTI dynamic system with transfer matrix $F(d)$, and left MFD, $F(d)=G^{-1}(d)H(d)$, with input u and output y, by the notation $y(t)=F(d)u(t)$ we mean that the sequence $y(t)$, $t\in\mathbb{Z}_+$, is computed via the following difference equation ($\det G_0\neq 0$)

$$\begin{aligned}&\sum_{k=0}^{n_G}G_k\,y(t-k)=\sum_{k=0}^{n_H}H_k\,u(t-k)\\ &y(k)=0,\quad u(k)=0,\quad k=-1,-2,\cdots\end{aligned} \tag{5}$$

if $G(d)=\sum_{k=0}^{n_G}G_k\,d^k$ and $H(d)=\sum_{k=0}^{n_H}H_k\,d^k$.

In order to decide whether or not, and, in the affirmative, how to change the controller, the supervisor embodies a family $\Pi:=\left\{\Pi_i,\, i\in\overleftarrow{N}\right\}$ of test functionals such that, in broad terms, $\Pi_i(t)$ quantifies the suitability of the i-th *potential loop* (P/C_i) given the data up to time t. In the hysteresis switching logic considered hereafter, at each step, one computes the least index $i_*(t)$ in $\overleftarrow{N}$ such that $\Pi_{i_*(t)}(t)\le\Pi_i(t)$, $\forall i\in\overleftarrow{N}$. Then, the switching index sequence σ is given by

$$\begin{aligned}&\sigma(t+1)=l(\sigma(t),\Pi(t)),\qquad \sigma(0)=i_0\in\overleftarrow{N}\\ &l(i,\Pi(t))=\begin{cases} i, & \text{if}\quad \Pi_i(t)<\Pi_{i_*(t)}(t)+h\\ i_*(t), & \text{otherwise}\end{cases}\end{aligned} \tag{6}$$

where $h>0$ is the hysteresis constant.

The next Hysteresis Switching Logic (HSL) lemma establishes the limiting behavior of $(\mathcal{P}/\mathcal{C}_{\sigma(\cdot)})$ subject to (6). Let Σ denote the class of all possible switching sequences σ giving rise to the switched system (1). Consider the assumptions:

A1. For each $\sigma\in\Sigma$ and $i\in\overleftarrow{N}$, $\Pi_i(t)$ admits a limit (even infinite) as $t\to\infty$;

A2. For each $\sigma\in\Sigma$, there exist integers $\mu\in\overleftarrow{N}$ such that $\Pi_\mu(\cdot)$ is bounded.

HSL Lemma [3] *Let z denote the vector-valued sequence of I/O plant data, and σ the switching sequence resulting from (1) and (6). Then, for any initial condition and reference r, if **A1** and **A2** hold, there is a finite time $t_f\in\mathbb{Z}_+$, after which no more switching occurs. Moreover, $\Pi_{\sigma(t_f)}(\cdot)$ is bounded.*

III. Reference Loop Identification in Multivariable Systems

As discussed in [7], the combination of *Multiple-Models* and *Unfalsified Control* has appealing features and intuitive advantages. Indeed, the resulting approach, called MMUASC (multi-model unfalsified adaptive switching control), provides good performance, as is typical in multiple-models schemes, in case a set $\mathscr{M}$ of nominal models is available which tightly approximates $\mathscr{P}$. On the other hand, in MMUASC, stability depends only on the problem feasibility, not on specific choices of $\mathscr{M}$.

The remaining part of this section will be devoted to briefly recall the main concepts underlying MMUASC as well as to discuss the main issues which appear in the MIMO case and require careful interpretation.

Let $\mathscr{M} := \{\mathcal{M}_i, i \in \overleftarrow{N}\}$ be a set of N discrete-time strictly causal MIMO LTI dynamic systems with MFDs

$$\mathcal{M}_i : \quad A_i^{-1}(d)\, B_i(d) = N_i(d)\, D_i^{-1}(d) \tag{7}$$

where $A_i(d) = \mathscr{A}_i(d)\Delta(d)$, $\mathscr{A}_i(d) = I_p + \mathscr{A}_{i1}d + \ldots + \mathscr{A}_{in_a}d^{n_a}$ and $B_i(d) = \mathscr{B}_{i1}d + \ldots + \mathscr{B}_{in_b}d^{n_b}$ are polynomial matrices with strictly Schur greatest g.c.l.d.. Similar definitions apply to the right MFDs $N_i(d)\, D_i^{-1}(d)$.

In connection with $\mathscr{M}$, the controllers $\mathcal{C}_i$'s are chosen so as to form, along with the associated $\mathcal{M}_i$'s, a finite family $\mathscr{F} = \{(\mathcal{M}_i/\mathcal{C}_i), i \in \overleftarrow{N}\}$ of *internally* stable feedback loops, each designed to fulfill desirable prescriptions (which in general need not be optimal relatively to any specific performance index). Hereafter, $(\mathcal{M}_i/\mathcal{C}_i)$ will be referred to as the i-th *reference loop*. Given the uncertain plant $\mathcal{P}$, the aim is to carry out a *reference loop identification* task, *viz.*, select a candidate controller $\mathcal{C}_\sigma$ in such a way that $(\mathcal{P}/\mathcal{C}_\sigma)$ behaves as closest as possible to one of the candidate reference loops in $\mathscr{F}$. Hence, roughly speaking, the ideal goal of the switching supervisor, can be envisaged as follows. Given an uncertain plant $\mathcal{P} \in \mathscr{P}$, find an index $\sigma \in \overleftarrow{N}$ such that:

i) $(\mathcal{P}/\mathcal{C}_\sigma)$ be stable;
ii) The behavioral data produced by $(\mathcal{P}/\mathcal{C}_\sigma)$ in response to r be as closest as possible to the ones produced by $(\mathcal{M}_\sigma/\mathcal{C}_\sigma)$ in accordance to the reference loop identification criterion

$$\sigma := \arg\min_{i\in\overleftarrow{N}} \sup_{r\neq 0} \frac{\|(\mathcal{P}/\mathcal{C}_i)r - (\mathcal{M}_i/\mathcal{C}_i)r\|}{\|(\mathcal{M}_i/\mathcal{C}_i)r\|} \tag{8}$$

where $(\mathcal{P}/\mathcal{C}_i)\, r$ and $(\mathcal{M}_i/\mathcal{C}_i)\, r$ denote the behavioral data produced by $(\mathcal{P}/\mathcal{C}_i)$ and $(\mathcal{M}_i/\mathcal{C}_i)\, r$, respectively, in response to r: by simplicity no time-indication is shown.

Remark 1: As elaborated in detail in [15], [16], the reason for using percentage criteria is essentially that, in case of large uncertain plant dynamic range, we can have a different cost associated to each index i. Selection criteria in normalized form like (8) thus help to avoid possible biases associated with the controller selection.

Unfortunately, on line implementation of (8) is impossible without using logics like pre-routing, which in general have to be ruled out because typically cause large and long-lasting learning transients. The Unfalsified Control approach introduced in [17] provides, under certain conditions, a way to side-step this problem. At each time and for each candidate controller, one computes (if possible) the variable $v_i(t)$ which solves the difference equation

$$S_i(d)v_i(t) = R_i(d)\delta u(t) + S_i(d)y(t). \tag{9}$$

In words, v_i^t equals the *virtual reference* sequence which would reproduce the recorded I/O sequence $(\delta u^t, y^t)$ should the plant $\mathcal{P}$ be fed-back by the candidate controller $\mathcal{C}_i$, irrespective of the way the plant data is generated. This means that, if $(\mathcal{P}/\mathcal{C}_{\sigma(\cdot)})$ is intended as the linear (time-varying) transformation (1) mapping the reference r into z, one has $z = \left(\mathcal{P}/\mathcal{C}_{\sigma(\cdot)}\right) r = (\mathcal{P}/\mathcal{C}_i)\, v_i$.

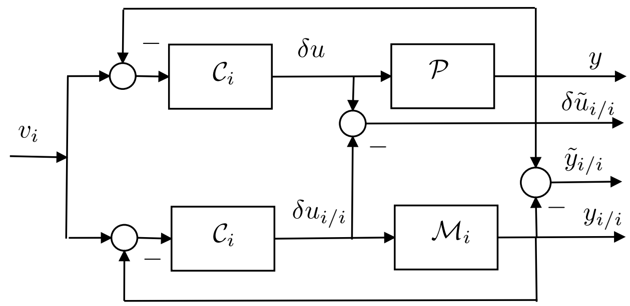

Fig. 2. Detail of a multiple-model switching control.

In MMUASC, the virtual reference concept is used as follows. For each candidate reference loop $(\mathcal{M}_i/\mathcal{C}_i)$, we define the closed loop response of $(\mathcal{M}_i/\mathcal{C}_i)$ to v_i as

$$\left.\begin{array}{ll} y_{i/i}(t) & = \mathcal{M}_i(\delta u_{i/i})(t) \\ \delta u_{i/i}(t) & = \mathcal{C}_i(v_i - y_{i/i})(t) \end{array}\right\} \tag{10}$$

Accordingly, by letting $(\mathcal{M}_i/\mathcal{C}_i)\, v_i := [\delta u'_{i/i}\, y'_{i/i}]'$ (Figure 2), the reference loop identification criterion (8) can be modified in the following on-line implementable form

$$\sigma := \arg\min_{i\in\overleftarrow{N}} \sup_{v_i\neq 0} \frac{\|(\mathcal{P}/\mathcal{C}_i)v_i - (\mathcal{M}_i/\mathcal{C}_i)v_i\|}{\|(\mathcal{M}_i/\mathcal{C}_i)v_i\|} \tag{11}$$

As discussed in [7], test functionals related to the identification criterion (11) can be obtained with no need of computing v_i. In fact, in the section IV we will show that even in the MIMO case the numerical computation of v_i (which would require $\mathcal{C}_i$ to be stably invertible) can be avoided and substituted by a suitably filtered prediction error based on the nominal model. Then, from a practical viewpoint, one could simply ask whether a similar result exists also for MIMO systems. However, from a theoretical point of view, the introduction for MIMO systems of identification criteria like (11) is more intricate than in the SISO case, since the virtual reference in (9) need not even exist. Such an issue is addressed in detail in the following.

A. Virtual Implementation of the Control Loop

To begin with, some results of linear algebra are briefly recalled for the reader's benefit [18].

Lemma 1: Consider the linear equation

$$Gx = L\,, \tag{12}$$

where $G \in \mathbb{R}^{m\times p}$, and $L \in \mathbb{R}^m$ are given matrices. Then, the following statements are equivalent:

i) there exists a solution $x \in \mathbb{R}^p$;
ii) the columns of $L \in \operatorname{Im} G$.

Furthermore, the solution, if it exists, is unique if and only if G has full column rank, *i.e.*, $\operatorname{rank} G = p$. $\square$

Lemma 2: Consider the linear equation (12), where $m \leq p$ and G has full row rank, *i.e.*, $\operatorname{rank} G = m$. Then, the solution, provided it exists, can be expressed as

$$x = G^{\dagger} L + (I_p - G^{\dagger} G)\gamma \tag{13}$$

where $\gamma \in \mathbb{R}^p$ while $G^{\dagger} := G' \, (G\,G')^{-1}$ denotes right pseudo-inverse of G. $\square$

Now, consistent with Section II, the difference equation (9) can be suitably rewritten as

$$\begin{aligned}\mathscr{S}_{i0} v_i(t) &= R_i(d)\delta u(t) + S_i(d)y(t) + [\mathscr{S}_{i0} - S_i(d)]\, v_i(t) \\ &:= \xi_i(t)\,. \end{aligned} \tag{14}$$

Then, three possible cases arise.

1) Let $m = p$. Then, if $\mathscr{S}_{i0}$ has full column rank, exists unique the virtual reference $v_i(t) = \mathscr{S}_{i0}^{-1}\xi_i(t)$;
2) Let $m < p$. Then, if $\mathscr{S}_{i0}$ has full row rank, $(\mathscr{S}_{i0}\mathscr{S}_{i0}')$ is invertible and all the possible v_i's are given by
$$v_i(t) = \mathscr{S}_{i0}^{\dagger}\xi_i(t) + (I_p - \mathscr{S}_{i0}^{\dagger}\mathscr{S}_{i0})\nu(t) \tag{15}$$
where $\nu(\cdot)$, $\nu(t) \in \mathbb{R}^p$, is an arbitrary signal;
3) Let $m > p$. Then, the virtual reference v_i need not exist unless $\xi_i(t) \in \operatorname{Im} \mathscr{S}_{i0}$.

This analysis forces one to use identification criteria alternative to those based on the virtual reference as defined in (9). To this end, let $w(t) := S_{\sigma(t)}(d)r(t)$. Then, (1) is equivalent to the switched system of Figure 3 where $R_{\sigma(t)}^{-1}(d)$ and $S_{\sigma(t)}(d)$ are in the forward path and, respectively, in the backward path of control loop. Consequently, in place of v_i, one can define (see Figure 4)

$$w_i(t) = R_i(d)\delta u(t) + S_i(d)y(t)\,. \tag{16}$$

Note that, in contrast with (9), the difference equation (16) can always be solved in real-time with respect to w_i. In the light of (16), assuming that the control scheme is implemented as in Figure 3, a convenient reference loop identification-criterion turns out to be the following

$$\sigma := \arg\min_{i \in \overleftarrow{N}} \sup_{w_i \neq 0} \frac{\left\| \widehat{(\mathcal{P}/\mathcal{C}_i)} w_i - (\widehat{\mathcal{M}_i/\mathcal{C}_i}) w_i \right\|}{\|(\widehat{\mathcal{M}_i/\mathcal{C}_i}) w_i\|}\,, \tag{17}$$

where, with obvious meaning of the symbols, $\widehat{(\mathcal{P}/\mathcal{C}_i)}$ and $(\widehat{\mathcal{M}_i/\mathcal{C}_i})$ denote the i-th potential loop and the i-th reference loop, respectively.

Hence, in case the virtual reference v_i does not exist, the criterion (17) can be considered as a possible substitute of (11). In fact, as it will be shown in the next section, (17) provides the same information on stability of the switched system (1) as given by (11) and so, no reconfiguration of the control loop in accordance with the arrangement of Figure 3 is actually needed.

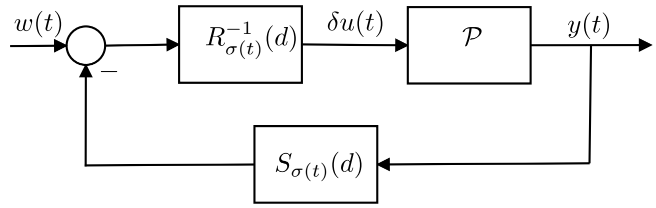

Fig. 3. Virtual implementation of the switched-on control loop.

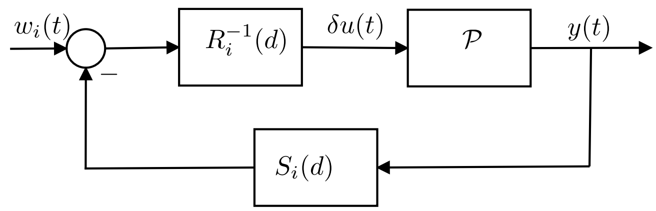

Fig. 4. Potential loop associated to definition of w_i.

IV. Stability inference via prediction errors filtering

A convenient test functional related to the identification criterion (17) is as follows

$$\Pi_i(t) := \max \Lambda_i^t \tag{18}$$

$$\Lambda_i^{1/2}(t) := \frac{\| \tilde{z}_i^t \|}{\| (z - \tilde{z}_i)^t \|} \tag{19}$$

with $\tilde{z}_i := z - z_i$ and, $z_i = (\widehat{\mathcal{M}_i/\mathcal{C}_i}) w_i := [\,\delta u_i' \, y_i'\,]'$, $i \in \overleftarrow{N}$.

Effectiveness of (19) stems from the fact that $(\mathcal{P}/\mathcal{C}_i)$ is r-stable if and only if Λ_i takes on finite values, $\forall r \in \mathbb{S}$ and $\forall \sigma \in \Sigma$. To see this, let the plant $\mathcal{P}$ be represented in terms of a coprime factor uncertainty as follows

$$\begin{aligned}\Delta_{*/i}(d) \;&:=\; [\,\Delta_{B_i}(d) \;\; \Delta_{A_i}(d)\,] \\ &=\; [\,B(d) - B_i(d) \;\; A_i(d) - A(d)\,]\,. \end{aligned} \tag{20}$$

Consistent with Sections II and III, define two polynomial matrices

$$\Xi_{i/i}(d) \;:=\; A_i(d)X_i(d) + B_i(d)Y_i(d) \tag{21}$$

$$\Xi_{*/i}(d) \;:=\; A(d)X_i(d) + B(d)Y_i(d) \tag{22}$$

whose determinants equal the characteristic polynomials of the i-th reference loop and, respectively, the i-th potential loop.

Lemma 3: Let us define the following two matrices

$$Q_{i/i}(d) := \begin{bmatrix} Y_i(d) \\ X_i(d) \end{bmatrix} \Xi_{i/i}^{-1}(d) \tag{23}$$

$$Q_{*/i}(d) := \begin{bmatrix} Y_i(d) \\ X_i(d) \end{bmatrix} \Xi_{*/i}^{-1}(d)\,. \tag{24}$$

Then,

$$Q_{*/i}(d) - Q_{i/i}(d) = Q_{*/i}(d)\,\Delta_{*/i}(d)\,Q_{i/i}(d)\,. \tag{25}$$

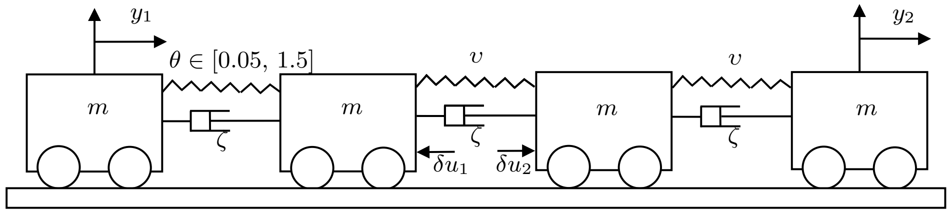

Fig. 5. Four carts plant.

Hence, from Lemma 3 it is immediate to conclude that

$$\begin{aligned}\tilde{z}_i(t) &= Q_{i/i}(d)\,\Delta_{*/i}(d)\,z(t)\\ &= Q_{*/i}(d)\,\Delta_{*/i}(d)\,\left[I - Q_{i/i}(d)\Delta_{*/i}(d)\right] z(t)\\ &= Q_{*/i}(d)\,\Delta_{*/i}(d)\,\left(z(t) - \tilde{z}_i(t)\right)\,. \qquad (26)\end{aligned}$$

Remark 2: It follows from equation (26) that the boundedness of (19) depends on the stability of $(\mathcal{P}/\mathcal{C}_i)$. In fact, test functional (19) can take on unbounded value only for indices corresponding to destabilizing controllers. We further notice that, under Problem Feasibility, there always exists at least one candidate controller such that the associated test functional is bounded for every possible switching sequence. Thus, the validity conditions of the HSL lemma hold true.

Next result shows that (19) can be obtained by suitable filtering the prediction error related to the i-th model $\mathcal{M}_i$.

Theorem 1: Consider the vector valued sequence $\tilde{z}_i^t$ in (19). Then,

$$\tilde{z}_i(t) = \begin{bmatrix} -Y_i(d) \\ X_i(d) \end{bmatrix} \Xi_{i/i}^{-1}(d)\ \epsilon_i(t) \qquad (27)$$

where

$$\epsilon_i(t) := A_i(d)y(t) - B_i(d)\delta u(t) \qquad (28)$$

is the prediction error based on $\mathcal{M}_i$ given the switched-on loop $(\mathcal{P}/\mathcal{C}_{\sigma(t)})$.

V. Main results

Given any $\mathcal{P} \in \mathscr{P}$, let $S(\mathcal{P}) \subseteq \overleftarrow{N}$ be the set of all indices $s \in \overleftarrow{N}$ such that $(\mathcal{P}/\mathcal{C}_s)$ is stable. Note that, under Problem Feasibility, $S(\mathcal{P}) \neq \{\varnothing\}$. Whenever the plant uncertainty set $\mathscr{P}$ is compact and a priori known, $\mathscr{M}$ can be designed dense enough in $\mathscr{P}$ so as to ensure that, for any $\mathcal{P} \in \mathscr{P}$, there exist indices $i \in \overleftarrow{N}$, yielding stable loops $(\mathscr{P}/\mathcal{C}_i)$ such that $\max_{\mathcal{P}\in\mathscr{P}} \min_{i\in S(\mathcal{P})} \left\|Q_{*/i}\,\Delta_{*/i}\right\|_\infty < \beta$, given the desired accuracy β. This is captured in the following.

Proposition 1: Let Θ be a compact set, and $\theta \to P(\theta)$ continuous on Θ. Then, for any positive real β, there always exists a finite model family such that:

$$\max_{\mathcal{P}\in\mathscr{P}} \min_{i\in S(\mathcal{P})} \left\|Q_{*/i}\,\Delta_{*/i}\right\|_\infty := \overline{\beta} < \beta\,. \qquad (29)$$

A model distribution, for which such a property holds, will be denoted by $\mathscr{M}(\overline{\beta})$. The following theorem is the main result of this section.

Theorem 2: Consider the switched system $(\mathcal{P}/\mathcal{C}_{\sigma(\cdot)})$ (1), $\mathcal{P} \in \mathscr{P}$, under zero plant initial conditions. Let $\sigma(t)$ be selected in accordance with the (6), with $\Pi_i(t)$ as in (18)-(19). Then, provided that the problem feasibility holds, for any reference $r \in \mathbb{S}$, the HSL lemma holds, and the switched system $(\mathcal{P}/\mathcal{C}_{\sigma(\cdot)})$ is r-stable. Further, under a nominal model distribution $\mathscr{M}(\overline{\beta})$ the total number of switches N_σ is bounded as follows

$$N_\sigma \le N \left\lceil \frac{\overline{\beta}}{h} \right\rceil, \qquad (30)$$

where $\lceil\alpha\rceil$ denotes the smallest integer greater than or equal to $\alpha \in \mathbb{R}_+$.

VI. Example

In this section, a MIMO plant will be considered. Let $\mathcal{P}$ be the plant of Figure 5 made up by four carts, all having mass $m = 1\ Kg$. The carts are mechanically coupled by springs and dampers: the latter ones have a viscous damping coefficient ζ equal to 0.2 Ns/m; only the spring connecting the carts on the left has an uncertain stiffness parameter $\theta \in \Theta = [0.05,\ 1.5]\ N/m$, the other two have a known stiffness coefficient v equal to 0.7 N/m. The control problem is to position external carts by applying manipulable forces to internal carts. Hence, this is a 2-inputs/2-outputs problem.

Three different one-degree-of-freedom continuous LTI controllers were designed in order to guarantee the stability and performances requirements on the whole uncertain interval. The controllers were designed relatively to plant models corresponding the following three stiffness values: $\theta_1 = 0.2\ N/m$, $\theta_2 = 0.5\ N/m$ and $\theta_3 = 1.0\ N/m$. Let $M_i(s) = A_i^{-1}(s)B_i(s)$ be the plant model with stiffness θ_i from $\delta u = [\delta u_1\, \delta u_2]'$ to $y = [y_1\, y_2]'$, and $C_i(s) = Y_i(s)X_i(s)^{-1}$ the corresponding tuned controllers, $i \in \overleftarrow{3}$, selected among all stabilizing controllers $\tilde{C} = \tilde{Y}(d)\tilde{X}^{-1}(d)$ according to a weighted H_∞ mixed-sensitivity criterion [19]:

$$C_i(s) = \arg\inf_{\tilde{C}} \sup_{\omega} \overline{\sigma}[\Phi_W(j\omega, \theta_i)]$$

where $\overline{\sigma}$ denotes the maximum singular value and $\Phi_W(s,\theta_i)$ the W-weighted mixed sensitivity matrix

$$\Phi_W(s,\theta_i) = \begin{bmatrix} \{\Psi_i^{\delta u}\}^{-1/2} W_i(s)\tilde{Y}(s) \\ \{\Psi_i^{y}\}^{-1/2} W_i(s)\tilde{X}(s) \end{bmatrix} \tilde{\Xi}_i^{-1}(s)A_i(s)$$

where $\tilde{\Xi}_i(s) := A_i(s)\tilde{X}(s) + B_i(s)\tilde{Y}(s)$. The weighting polynomial matrices and the positive real-valued matrices have been chosen as follows: $W_i(s) = 0.01/(s+0.01)I_2$, $\Psi_i^y = I_2$, $i \in \overleftarrow{3}$, and, $\Psi_1^{\delta u} = 2.85 \times 10^{-3} I_2$, $\Psi_2^{\delta u} = 2.14 \times 10^{-3} I_2$, $\Psi_3^{\delta u} = 1.00 \times 10^{-3} I_2$. Then, nominal models

	$\theta = 0.10$	$\theta = 0.30$	$\theta = 0.40$	$\theta = 0.65$	$\theta = 0.70$	$\theta = 0.80$	$\theta = 1.10$
Maximum values of $\lvert y_1\rvert$ and $\lvert y_2\rvert$	2.87 and 2.13	2.85 and 1.98	3.19 and 2.12	2.70 and 1.92	2.78 and 1.78	2.74 and 1.97	2.72 and 1.72
Maximum values of $\lvert\delta u_1\rvert$ and $\lvert\delta u_2\rvert$	5.98 and 5.82	5.98 and 5.81	6.33 and 5.70	6.34 and 5.70	4.69 and 5.01	4.71 and 5.12	4.70 and 5.12
Final controller index	1	1	2	2	3	3	3
Final switching instant	0 s	0 s	2.9 s	1.6 s	4.8 s	1.5 s	1.3 s

TABLE I

SIMULATION RESULTS IN THE IDEAL CASE WITH $\sigma(0) = 1$.

and relative controllers are discretized by the use of an input zero-order holder with sample time equal to 0.1 s. Each controller yields closed loop stability in the following subintervals: $\Theta_1 = [0.05, 0.48)$ N/m, $\Theta_2 = (0.15, 1.01)$ N/m, $\Theta_3 = (0.65, 1.5]$ N/m. Simulation results reported hereafter refers to the hysteresis constant h equal to 0.05 and reference signals $r_1(t)$ and $r_2(t)$ given by square-waves of amplitudes and periods of ± 5 m and 150 s, and, respectively, ± 3 m and 100 s. In Table I, experiments relative to different values of parameter θ and $\sigma(0) = 1$ are summed up. Such experiments are carried out in a ideal case configuration, *viz.* zero plant initial condition and zero noises/disturbances. In particular, Figure 6 depicts the plant output and the controller selection as function of time for the case $\theta = 0.4$.

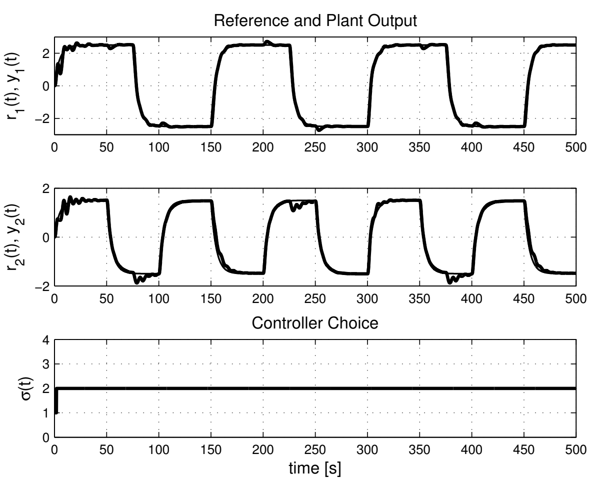

Fig. 6. Tracking and controller selection for $\theta = 0.40$ and $\sigma(0) = 1$.

VII. CONCLUSIONS

The paper has extended the MMUASC approach to a MIMO setting. In fact, consideration has been given on how to infer in real-time stability of a potential loop made up by a given candidate controller interconnected in feedback with an uncertain MIMO plant, while the latter is possibly driven by a different controller. As in a SISO setting [7], even in the MIMO case a sufficient condition on the stability of the potential loop can be provided and, moreover, irrespectively of the existence of the virtual reference, the consequent test functional can be computed by suitably filtering the prediction error based on the nominal model.

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