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A Unified Model-Based Fault Diagnosis Scheme for Nonlinear Discrete-Time Systems with Additive and Multiplicative Faults

Hasan Ferdowsi and S. Jagannathan

Abstract—In this paper, a novel model-based fault diagnosis scheme is proposed for a class of uncertain nonlinear discrete-time systems which can be subjected to both additive and multiplicative faults. Faults are detected by using a novel (FD) observer consisting of two online approximators in discrete-time (OLAD) and a robust adaptive term. Upon detection, a fault diagnosis scheme is introduced to determine the fault type by monitoring the input residual generated via the first OLAD output. Then the appropriate OLAD is included in the observer while the other OLAD is switched off. Next, by using both the parameter update law of the active OLAD and user-selected failure thresholds, an online time-to-failure (TTF) scheme is introduced. Boundedness and asymptotic convergence of the residual and parameter estimation errors respectively are derived in the case of multiplicative and additive faults respectively. Finally a simulation example is used to demonstrate the proposed fault diagnosis scheme.

I. Introduction

Due to the high risk of failures, reliable fault diagnosis schemes are required to guarantee safe operation for complex industrial systems even in the presence of uncertainties and faults. If the faults can be detected early enough, further damage to the system could be prevented.

Fault diagnosis schemes are generally divided into data-driven and model-based methods. Data-driven fault diagnosis approaches need healthy and faulty data and an offline training session. As a result, these methods are not usually preferred since they are expensive and they may result in false alarms when the operating conditions change. On the other hand, model-based fault diagnosis methods [1] relax the need for a priori data and detect faults online. In this approach, an observer or estimator representative of the system is utilized for detecting faults [1,2]. These model-based fault detection (FD) methods have been implemented on both linear and nonlinear systems that have a linear representation [3,4].

As part of model-based FD framework, in [5], fault diagnosis schemes using adaptive estimators have been discussed, while neural network (NN)-based estimators and fuzzy observers have been utilized for the purpose of fault detection, in [6] and [7] respectively.

Hasan Ferdowsi (e-mail: hfqk6@mst.edu), and S. Jagannathan (e-mail: sarangap@mst.edu) are with the Department of Electrical & Computer Engineering, Missouri University of Science and Technology (formerly University of Missouri–Rolla), Rolla, MO, 65401, USA. Research supported in part by NSF I/UCRC on Intelligent Maintenance Systems and Intelligent Systems Center.

Numerous researchers have worked on detection of additive faults [5,8,9]. Fault diagnosis for systems with multiplicative actuator faults has also been done using parameter similarity measures [10], and NN-based method [7]. However in all these papers, only one type of fault is normally considered while practical nonlinear systems can be subject to both fault types.

Therefore, a class of nonlinear discrete-time system is considered in this paper that is subjected to both additive and multiplicative faults. A novel observer design is proposed wherein two OLADs, one for each fault type, is introduced. Detection residual is generated by comparing the estimated states of the observer with that of the nonlinear system. A deadzone operator is used to declare the presence of a fault when the detection residual exceeds a user defined threshold.

Upon detection, the first OLAD is activated to learn the fault dynamics and to generate an input residual which is subsequently utilized for identifying the fault type. A decision is then made to activate the appropriate OLAD. In other words, upon detection, only one OLAD will be active.

Next, the time-to-failure (TTF) is determined online by comparing the active OLAD parameter estimates against the designer specified limits since for most practical systems the parameters could be tied to physical quantities which have a safe range of values [8]. In this paper TTF determination is performed for the system both with additive or multiplicative faults provided a single fault type can occur at a given time.

Thus the contributions of this paper include the development of unified prognostic framework to handle additive and multiplicative faults in contrast with the literature [5,7,8,10] where a single fault type is normally handled. Fault type is identified via input residual and TTF determination scheme is then introduced online whereas such schemes are not available in the literature for model-based methods [5,6,9].

II. System Description

Consider the nonlinear discrete-time system

$$x(k+1)=\omega\big(x(k),u(k)\big)+\eta\big(x(k),u(k)\big)$$

where $u\in\mathbb{R}^m$ is the control input vector, $x\in\mathbb{R}^n$ is the system state vector, $\omega:\mathbb{R}^n\times\mathbb{R}^m\to\mathbb{R}^n$ represents the known nonlinear system dynamics, and $\eta:\mathbb{R}^n\times\mathbb{R}^m\to\mathbb{R}^n$ represents the system uncertainties.

Now consider the nonlinear system with additive fault term

$$x(k+1)=\omega\big(x(k),u(k)\big)+\eta\big(x(k),u(k)\big)+\Pi(k-k_0)h\big(x(k),u(k)\big)$$

where $h(x(k), u(k))$ represents a vector of possible additive fault dynamics, which are defined as $h(.) = [\theta_1^T f_1(x(k), u(k)), \dots, \theta_n^T f_n(x(k), u(k))]$. $\theta_i \in \mathbb{R}^{l_i}$, $i = 1, \dots, n$, is an unknown parameter vector, and $f_i: \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^{l_i}$, $i = 1, \dots, n$, is a known fault basis function. Each f_i represents the fault function of the i[th] fault affecting the i[th] state equation, and each θ_i is the unknown magnitude of the i[th] fault function. The time profile of a fault is given by $\Pi(k - k_0)$.

The system under consideration can also have multiplicative faults. Let $T_A(x(k), u(k)) = diag\{T_{Ai}(x(k), u(k))\}$, $i = 1, \dots, m$ denote the matrix of unknown multiplicative actuator faults [7], and $u_f(k) = T_A(x(k), u(k))u(k)$. If we replace $u(k)$ by $u_f(k)$ then the nonlinear system dynamics with the additive and multiplicative faults can be rewritten as

$$x(k+1) = \omega\left(x(k), u_f(k)\right) + \eta\left(x(k), u_f(k)\right) + \Pi(k - k_0)h\left(x(k), u_f(k)\right) \quad (1)$$

The time profile $\Pi(k - k_0)$ is modeled by

$$\Pi(k - k_0) = diag\{\Omega_1(k - k_0), \Omega_2(k - k_0), \dots, \Omega_n(k - k_0)\}$$

where

$$\Omega_i(k - k_0) = \begin{cases} 0, & if\ \tau < 0 \\ 1 - e^{-\bar{\kappa}_i \tau}, & if\ \tau \geq 0 \end{cases} \quad for\ i = 1, \dots, n$$

and $\bar{\kappa}_i$ is an unknown constant that represents the rate at which a fault occurs. A larger value of $\bar{\kappa}_i$ indicates that it is an abrupt fault. The use of such time profiles is common in fault diagnosis literature [5,8]. Next standard assumptions are needed in order to proceed.

Assumption 1: The modeling uncertainty is bounded, i.e. $|\eta(x(k), u(k))| \leq \eta_M$, $\forall (x, u) \in (\chi \times U)$, where η_M is a positive known constant.

Assumption 2: In order to perform the diagnosis online, it is assumed that only a single fault type (either multiplicative or additive) occurs in the system at any given time. This means that both additive and multiplicative faults cannot occur at the same time.

Remark 1: Assumption 1 is needed to distinguish between faults and system uncertainties while Assumption 2 is needed to identify the fault types.

Assumption 3: The nonlinear system dynamics $\omega(x, u)$ is Lipschitz in x and u, i.e., $\|\tilde{\omega}(k)\| \leq c_g \|e(k)\|$, where $c_g > 0$ is the Lipschitz constant.

Next the proposed fault diagnosis scheme is introduced.

III. Fault Diagnosis Scheme

In this section, the proposed fault diagnosis scheme for detecting additive and multiplicative faults will be described via the estimator. Consider the nonlinear FD estimator

$$\hat{x}(k+1) = A_d \hat{x}(k) + \omega\left(x(k), \hat{u}_f(u(k), \hat{T}_A(k))\right) + \hat{h}_d\left(x(k), u(k); \hat{\theta}_d(k)\right) - A_d x(k) + F(k) \quad (2)$$

$$\hat{u}_f(u(k), \hat{T}_A(k)) = \hat{T}_A\left(x(k), u(k); \hat{\theta}_A(k)\right)u(k)$$

where $\hat{x}(k) \in \mathbb{R}^n$ is the estimated state vector, $\hat{T}_A: \mathbb{R}^m \times \mathbb{R}^{q \times m} \to \mathbb{R}^m$ is the output of the first detection OLAD with $\hat{\theta}_A \in \mathbb{R}^{q \times m}$ being its set of adjustable parameters, $\hat{h}_d: \mathbb{R}^n \times \mathbb{R}^{p \times n} \to \mathbb{R}^n$ is the output of the second OLAD with $\hat{\theta}_d \in \mathbb{R}^{p \times n}$ being its set of unknown parameters, $F(k)$ denotes the robust adaptive term, and A_d is a user defined diagonal matrix, which must be selected in a way that the eigenvalues of the closed loop system lies within the unit circle [11]. Initial values of the FD estimator are taken to be $\hat{x}(0) = \bar{x}_0$, $\hat{\theta}_d(0) = \hat{\theta}_{d_0}$, $\hat{T}_A(0) = \hat{T}_{A_0}$, such that $\hat{h}(x, u, \hat{\theta}_{d_0}) = 0$, $\hat{T}_A(x, u, \hat{\theta}_{A_0}) = I \quad \forall x \in X, u \in U$.

Remark 2: The proposed observer/estimator uses two OLADs to identify additive and multiplicative faults in contrast with other FD schemes that use one OLAD in their observer [5,8].

In the proposed FD estimator, NNs are used as the OLADs. Both NN-based OLADs are off prior to the detection of a fault and thus their outputs are zero. Upon detection of a fault the first OLAD is turned on to estimate the input by assuming that it could be a multiplicative fault. A decision is made to identify the type of fault occurred by monitoring the input residual. Depending upon this decision, the appropriate OLAD is left on and the other is switched off. Next the process of detecting a fault is introduced next.

Define the detection residual as $e = x - \hat{x}$. Prior to the detection of a fault, the residual dynamics are given by

$$e(k+1) = A_d e(k) + \eta(x(k), u(k))$$

which is bounded with the appropriate selection of A_d. Now consider a dead-zone operator

$$D[e(k)] = \begin{cases} 0, & if\ |e(k)| \leq \rho \\ e(k), & if\ |e(k)| > \rho \end{cases}$$

where ρ is the FD threshold. A fault is detected, regardless of its type, when the FD residual exceeds the predefined threshold. However these thresholds must be chosen carefully in order to minimize false or missed alarms. Analytically, a time varying threshold $\rho = \frac{\beta \eta_M (1 - \mu^k)}{(1 - \mu)}$ or a constant threshold can be determined by $\rho = \frac{\beta \eta_M}{(1 - \mu)}$, where $= \beta_c \mu$, μ and β_c are some positive constants such that $\|A_d^k\|_F \leq \beta_c \mu^k$ can be derived.

When the detection residual exceeds the detection threshold, a fault is declared active through the dead-zone operator and the first OLAD that generates $\hat{T}_A(.)$, is initiated and tuned online using the following update law

$$\hat{\theta}_A(k+1) = \hat{\theta}_A(k) + \alpha_1 \phi_1(k) D[e^T(k+1)] - \gamma_1 \|I - \alpha_1 \phi_1(k) \phi_1^T(k)\| \hat{\theta}_A(k) \quad (3)$$

where $\alpha_1 > 0$ is the learning rate, $0 < \gamma_1 < 1$ is the forgetting factor, and $\phi_1(k) = \phi_1(x(k), u(k))$ is a basis function such as sigmoid or RBF. Then, the output of the first OLAD that estimates the multiplicative fault function is given by

$$\hat{T}_A(k) = \hat{\theta}_A^T(k) \phi_1(x(k), u(k))$$

The input residual is then computed online using actual input and its estimate from the first OLAD. On the other hand, if a fault is identified as additive based on the input residual, then the first OLAD will be turned off and the second OLAD will be initiated and tuned online using the following update law

$$\hat{\theta}_d(k+1) = \hat{\theta}_d(k) + \alpha_2 \phi_2(k) e^T(k+1) - \gamma_2 \|I - \alpha_2 \phi_2(k) \phi_2^T(k)\| \hat{\theta}_A(k) \quad (4)$$

where $\alpha_2 > 0$ is the learning rate, $0 < \gamma_2 < 1$ is the forgetting factor, and $\phi_2(k) = \phi_2(x(k), u(k))$ is a basis function like sigmoid or RBF. Moreover the output of the second OLAD to estimate the fault function will be given by

$$\hat{h}_d(k) = \hat{\theta}_d^T(k) \phi_2(x(k), u(k))$$

A robust adaptive term is also turned on with the additive fault. In the following theorem conditions for fault detectability are presented.

Theorem 1 (Fault Detectability): Consider the nonlinear system defined by (1) and the FD estimator (2). The fault will be detected, if there exists a time instant k_d, such that the fault function satisfy

$$\frac{1}{2 + \|A_d^{k_d}\|} \left\| \sum_{j=0}^{k_d - 1} A_d^{k_d - j - 1} \left(\omega\left(x(j), u_f(j)\right) - \omega\left(x(j), u(j)\right) \right) \right\| \geq \rho$$

or

$$\frac{1}{2 + \|A_d^{k_d}\|} \left\| \sum_{j=0}^{k_d - 1} A_d^{k_d - j - 1} \left(\Pi(k - k_0) h\left(x(k), u_f(k)\right) \right) \right\| \geq \rho$$

in the case of multiplicative or additive fault respectively.

Next the performance of the proposed fault diagnosis observer is evaluated on multiplicative faults first and then additive faults.

A. Multiplicative Fault Case

After detection, the FD estimator dynamics would be described by

$$\hat{\hat{x}}(k+1) = A_d \hat{\hat{x}}(k) + \omega\left(x(k), \hat{u}_f\left(u(k), \hat{T}_A(k)\right)\right) - A_d x(k) \quad (5)$$

$$\hat{u}_f\left(u(k), \hat{T}_A(k)\right) = \hat{T}_A\left(x(k), u(k); \hat{\theta}_A(k)\right) u(k)$$

Consequently, the detection residual dynamics are given by

$$e(k+1) = A_d e(k) + \omega\left(x(k), u_f(k)\right) - \omega\left(x(k), \hat{u}_f(k)\right) + \eta\left(x(k), u_f(k)\right) \quad (6)$$

Define the input residual $\tilde{u} = u - \hat{u}_f$. The next theorem will assure the boundedness of the detection residual dynamics upon detecting a multiplicative fault. Hence the multiplicative fault can be estimated by the first OLAD, which will result in a noticeable difference between the actual and estimated input in a finite time or when the input residual exceeds a user defined threshold. Using this input residual, the fault diagnosis is carried out to identify the fault type.

Theorem 2 (Fault Diagnosis Observer Performance with Multiplicative Faults): Let the proposed observer defined in (5) be used to monitor the system described by (2), with the first OLAD being turned on upon the detection of a fault. Let the update law in (3) be used to update the unknown parameter vector $\hat{\theta}_A$. In the case of multiplicative faults, the FD residual, $e(k)$, and the parameter estimation errors, $\tilde{\theta}_A^T(k)$, will be uniformly ultimately bounded (UUB). Moreover, the input residual will exceed the user-defined threshold.

Proof: Consider the following Lyapunov function candidate

$$V = e^T(k) e(k) + tr\{\tilde{\theta}_A^T(k) \tilde{\theta}_A(k)\}$$

where $\tilde{\theta}_A^T(k) = \theta_A - \hat{\theta}_A(k)$. The first derivative of the Lyapunov function is given by

$$\Delta V = \underbrace{\left(e^T(k+1) e(k+1) - e^T(k) e(k)\right)}_{\Delta V_1} + \underbrace{tr\{\tilde{\theta}_A^T(k+1) \tilde{\theta}_A(k+1) - \tilde{\theta}_A^T(k) \tilde{\theta}_A(k)\}}_{\Delta V_2} \quad (7)$$

By substituting $e(k+1)$ from the error dynamics (6), in ΔV_1 to get

$$\Delta V_1 = \left[\left(A_d e(k) + \tilde{\omega}(k) + \eta^T(x(k), u_f(k))\right)\left(A_d e(k) + \tilde{\omega}(k) + \eta(x(k), u_f(k))\right) - e^T(k) e(k)\right]$$

where $\tilde{\omega}(k) = \omega(x(k), u_f(k)) - \omega(x(k), \hat{u}_f(k))$.

By using the Cauchy-Schwarz inequality ($(s_1 + s_2 + \cdots + s_n)^T (s_1 + s_2 + \cdots + s_n) \leq n(s_1^T s_1 + s_2^T s_2 + \cdots + s_n^T s_n)$) we get

$$\Delta V_1 \leq 3 e^T(k) A_d^T A_d e(k) + 3 \tilde{\omega}^T(k) \tilde{\omega}(k) - e^T(k) e(k) + 3 \eta^T\left(x(k), u_f(k)\right) \eta\left(x(k), u_f(k)\right) \quad (8)$$

Now we substitute $\hat{\theta}_A(k+1)$ from the update law and $e(k+1)$, in ΔV_2

$$\begin{aligned}\Delta V_2 = tr\{&[(1 - \gamma_1 \|I - \alpha_1 \phi_1(k) \phi_1^T(k)\|) \tilde{\theta}_A(k) \\ &+ \gamma_1 \|I - \alpha_1 \phi_1(k) \phi_1^T(k)\| \theta_A \\ &- \alpha_1 \phi_1(k) e^T(k+1)]^T [(1 \\ &- \gamma_1 \|I - \alpha_1 \phi_1(k) \phi_1^T(k)\|) \tilde{\theta}_A(k) \\ &+ \gamma_1 \|I - \alpha_1 \phi_1(k) \phi_1^T(k)\| \theta_A \\ &- \alpha_1 \phi_1(k) e^T(k+1)] - \tilde{\theta}_A^T(k) \tilde{\theta}_A(k)\}\end{aligned}$$

Applying the Cauchy-Schwarz inequality on the above equation yields

$$\begin{aligned}\Delta V_2 \leq\ & tr\{4 \tilde{\theta}_A^T(k) \tilde{\theta}_A(k) \\ &- 10 \gamma_1 \|I - \alpha_1 \phi_1(k) \phi_1^T(k)\| \tilde{\theta}_A^T(k) \tilde{\theta}_A(k) \\ &+ 5 \gamma_1^2 \|I - \alpha_1 \phi_1(k) \phi_1^T(k)\|^2 \tilde{\theta}_A^T(k) \tilde{\theta}_A(k)\} \\ &+ tr\{5 \gamma_1^2 \|I - \alpha_1 \phi_1(k) \phi_1^T(k)\|^2 \theta_A^T \theta_A\} \\ &+ 5 \alpha_1^2\, e^T(k) A_d^T A_d e(k) \phi_1^T(k) \phi_1(k) \\ &+ 5 \alpha_1^2 \eta^T\left(x(k), u_f(k)\right) \eta\left(x(k), u_f(k)\right) \phi_1^T(k) \phi_1(k) \\ &+ 5 \alpha_1^2 \tilde{\omega}^T(k) \tilde{\omega}(k) \phi_1^T(k) \phi_1(k)\end{aligned} \quad (9)$$

By using equations (7),(8), and (9), the first difference of the Lyapunov function candidate, ΔV, can be found as

$$\begin{aligned}\Delta V = \Delta V_1 + \Delta V_2 \leq\ & \\ \leq\ & 3 e^T(k) A_d^T A_d e(k) + 3 \tilde{\omega}^T(k) \tilde{\omega}(k) \\ &+ 3 \eta^T\left(x(k), u_f(k)\right) \eta\left(x(k), u_f(k)\right) - e^T(k) e(k) \\ &+ tr\{5 \tilde{\theta}_A^T(k) \tilde{\theta}_A(k) \\ &- 10 \gamma_1 \|I - \alpha_1 \phi_1(k) \phi_1^T(k)\| \tilde{\theta}_A^T(k) \tilde{\theta}_A(k) \\ &+ 5 \gamma_1^2 \|I - \alpha_1 \phi_1(k) \phi_1^T(k)\|^2 \tilde{\theta}_A^T(k) \tilde{\theta}_A(k)\} \\ &+ tr\{5 \gamma_1^2 \|I - \alpha_1 \phi_1(k) \phi_1^T(k)\|^2 \theta_A^T \theta_A\} \\ &+ 5 \alpha_1^2 e^T(k) A_d^T A_d e(k) \phi_1^T(k) \phi_1(k) \\ &+ 5 \alpha_1^2 \tilde{\omega}^T(k) \tilde{\omega}(k) \phi_1^T(k) \phi_1(k) \\ &+ 5 \alpha_1^2 \eta^T\left(x(k), u_f(k)\right) \eta\left(x(k), u_f(k)\right) \phi_1^T(k) \phi_1(k)\end{aligned}$$

Taking the Frobenius norm, the above inequality and by using the result of Assumptions 1 and 3, we will have

$$\Delta V \le -(1-(3+5\alpha_1^2\phi_{1max}^2)(A_{d_{max}}^2+c_g^2))\|e(k)\|^2$$
$$-(10\gamma_1\|I-\alpha_1\phi_1(k)\phi_1^T(k)\|$$
$$-5\gamma_1^2\|I-\alpha_1\phi_1(k)\phi_1^T(k)\|^2-4)\|\tilde{\theta}_A(k)\|^2$$
$$+5\gamma_1^2\|I-\alpha_1\phi_1(k)\phi_1^T(k)\|^2\|\theta_A\|^2$$
$$+(3+5\alpha_1^2\phi_{1max}^2)\eta_M^2$$

Hence $\Delta V \le 0$, if the following conditions are satisfied

$$\|e(k)\| > \sqrt{\frac{(3+5\alpha_1^2\phi_{1max}^2)\eta_M^2}{1-(3+5\alpha_1^2\phi_{1max}^2)(A_{d_{max}}^2+c_g^2)}} \quad (10)$$

or

$$\|\tilde{\theta}_A(k)\| > \sqrt{\frac{\alpha}{\beta}} \quad (11)$$

where $\alpha = 5\gamma_1^2\|I-\alpha_1\phi_1(k)\phi_1^T(k)\|^2\|\theta_A\|^2$ and $\beta = 10\gamma_1\|I-\alpha_1\phi_1(k)\phi_1^T(k)\| - 5\gamma_1^2\|I-\alpha_1\phi_1(k)\phi_1^T(k)\|^2 - 4$.

Therefore the detection residual and parameter estimation errors are uniformly ultimately bounded, with the bounds given in (10) and (11).

Upon detection, if the input residual exceeds a predefined threshold, δ, in a finite time, T_δ, then the fault type is identified as multiplicative and the first OLAD is kept online while the second OLAD will never be turned on. By contrast, if the input residual stays below δ, within the interval of T_δ, then the fault type is declared as additive and the second OLAD is turned on and the first one is turned off.

Remark 3: The time interval, T_δ, is determined by multiplicative fault rates and magnitudes analytically.

B. Additive Fault Case

Since the first OLAD is designed to estimate the multiplicative fault function, it will not be compensating an additive fault. Therefore, in case of an additive fault the estimated input, $\hat{u}_f$, will be close to the actual input, u or the input residual will be below the threshold. So in this case, the second part of Theorem 2 will help identify the fault type after a finite time T_δ, once a fault is detected.

Since in this case only the second OLAD is online. The FD estimator dynamics are described by

$$\hat{\hat{x}}(k+1) = A_d\hat{\hat{x}}(k) + \omega(x(k),u(k))$$
$$+\hat{h}_d\left(x(k),u(k);\hat{\theta}_d(k)\right) - A_d x(k) + F(k) \quad (12)$$

where the robust adaptive term, $F(k)$, defined by

$$F(k) = \frac{\hat{\theta}_d^T(k)B}{B^T\hat{\theta}_d(k)\hat{\theta}_d^T(k)B + c}$$

is utilized with the OLAD. Here B is a constant vector and $c > 0$ denotes a positive constant. The following theorem guarantees the performance of the observer with additive faults.

Theorem 3 [8] (Fault Diagnosis Observer Performance with Additive Faults): Let the proposed observer in (12) be used to monitor the system in (1), with the second OLAD and the robust adaptive term are turned on upon identifying an additive fault. Let the update law in (4) be used to update the unknown parameter set $\hat{\theta}_d$. Then the FD residual, $e(k)$, and the parameter estimation errors, $\tilde{\theta}_d^T(k)$, converge to zero asymptotically.

So far, the detection of a fault and the fault type identification is done. The next section discusses the TTF scheme.

IV. Prediction Scheme

Time to failure (TTF) determination is necessary for prognostics. This is also referred to as remaining useful life of the system. After the detection of a fault, by comparing the estimated parameters obtained from the OLAD to the user defined limits, time to failure could be determined [8]. The TTF is defined as the time elapsed when the first parameter reaches its limit. Next the following assumption is asserted.

Assumption 4: For the purpose of TTF, fault functions can be expressed as linear in the unknown parameters (LIP) [11], i.e. both additive and multiplicative fault functions can be approximated by two-layer NN with bounded activation functions and weight parameters.

The following theorem provides an analytical formula for finding TTF.

Theorem 4 (TTF Determination): In the presence of multiplicative faults, TTF for the j^{th} parameter of the i^{th} fault, at the k^{th} time instant can be determined using

$$TTF_{i,j}(k) = \frac{\left|\log\left(\left|\frac{\gamma_1\|I-\alpha_1\phi_1\phi_1^T\|\theta_{A_{i,j\,max}} - \alpha_1\phi_{1_i}e_j^T}{\gamma_1\|I-\alpha_1\phi_1\phi_1^T\|\hat{\theta}_{A_{i,j}}(k) - \alpha_1\phi_{1_i}e_j^T}\right|\right)\right|}{|\log(1-\gamma_1\|I-\alpha_1\phi_1\phi_1^T\|)|}$$

where $\theta_{A_{i,j\,max}}$ is the failure limit in terms of maximum value of the system parameter, $\theta_{A_{i,j}}$, and $\hat{\theta}_{A_{i,j}}(k)$ is the estimated system parameter at the time instant k.

Similarly in the presence of additive faults, TTF for the j^{th} parameter of the i^{th} fault, at the k^{th} time instant can be determined using

$$TTF_{i,j}(k) = \frac{\left|\log\left(\left|\frac{\gamma_2\|I-\alpha_2\phi_2\phi_2^T\|\theta_{d_{i,j\,max}} - \alpha_2\phi_{2_i}e_j^T}{\gamma_2\|I-\alpha_2\phi_2\phi_2^T\|\hat{\theta}_{d_{i,j}}(k) - \alpha_2\phi_{2_i}e_j^T}\right|\right)\right|}{|\log(1-\gamma_2\|I-\alpha_2\phi_2\phi_2^T\|)|}$$

where $\theta_{d_{i,j\,max}}$ is the failure limit in terms of maximum value of the system parameter, $\theta_{d_{i,j}}$, and $\hat{\theta}_{d_{i,j}}(k)$ is the estimated system parameter at the time instant k.

Proof: Suppose that a fault is detected and identified as multiplicative. Let $a(k) = 1-\gamma_1\|I-\alpha_1\phi_1(k)\phi_1^T(k)\|$, and let $v(k) = \phi_{1_i}(k)e_j^T(k+1)$. Then the parameter update law in (3) can be rewritten as

$$\hat{\theta}_{A_{i,j}}(k+1) = a(k)\hat{\theta}_{A_{i,j}}(k) + \alpha_1 v(k) \quad (13)$$

which is in the form of the state equation of linear time-varying system with $\hat{\theta}_{A_{i,j}}$ being the state and $v(k)$ being the input.

We know that $0 < a < 1$ and $v(k)$ is bounded since the activation functions are bounded and the boundedness of the residual has been proven earlier. So we can assume that $a(k)$ and $v(k)$ are time invariant. Hence (13) can be rewritten as

$$\hat{\theta}_{A_{i,j}}(k+1) = a\hat{\theta}_{A_{i,j}}(k) + \alpha_1 v$$

which is in form of a linear time-invariant state equation.

At the time of failure, $k_{f_{i,j}}$, estimated parameter will be equal to $\theta_{A_{i,j\,\max}}$, which means $\hat{\theta}_{A_{i,j}}(k_{f_{i,j}}) = \theta_{A_{i,j\,\max}}$. Therefore by finding the solution to the linear time-invariant equation in hand, at the time instant $k_{f_{i,j}}$, we get

$$\theta_{A_{i,j\,\max}} = \hat{\theta}_{A_{i,j}}\left(k_{f_{i,j}}\right) = a^{k_{f_{i,j}}-k}\hat{\theta}_{A_{i,j}}(k) + \alpha_1 \sum_{h=k+1}^{k_{f_{i,j}}} a^{k_{f_{i,j}}-h} v$$

$$= a^{k_{f_{i,j}}-k}\hat{\theta}_{A_{i,j}}(k) + \alpha_1 v \frac{1-a^{k_{f_{i,j}}-k}}{1-a}$$

Since $TTF_{i,j} = k_{f_{i,j}} - k$, by simple mathematical manipulations, we will have

$$TTF_{i,j} = \frac{\left|\log\left(\left|\frac{(1-\mathrm{a})\theta_{A_{i,j\,\max}} - \alpha_1 v}{(1-\mathrm{a})\hat{\theta}_{A_{i,j}}(k) - \alpha_1 v}\right|\right)\right|}{|\log(a)|}$$

By replacing a and v by $(1-\gamma_1\|I-\alpha_1\phi_1\phi_1^T\|)$ and $\phi_{1_i}e_j^T$ respectively, the desired result will be obtained.

The theorem can be proven for the additive fault case by following the same argument as that of the multiplicative case.

At each time instant, after calculating the TTF for all of the system parameters, one should take the minimum of time to failure for all of the parameters, to get the overall TTF for the system. This is because the system will be unsafe even if only one of its parameters reaches its limit.

V. SIMULATION RESULTS

In this section, a three-tank water system [12] is used to verify the proposed fault diagnosis and prediction schemes. Fig. 1 depicts this system consisting of three tanks connected to each other with input pumps on tank 1 and tank 2 and one water outlet on tank 2.

The three-tank system dynamics are described by

$$x(k+1) = \omega\big(x(k), u(k)\big) + \eta\big(x(k)\big)$$

where $x = [x_1, x_2, x_3]^T$ is the state vector and $\omega\big(x(k), u(k)\big)$ is the known nonlinear dynamics of the system [12] given by $\omega\big(x(k), u(k)\big) =$

$$\begin{bmatrix} \frac{T}{A}\{-c_1 S_p sign\big(x_1(k)-x_3(k)\big)\sqrt{2g|x_1(k)-x_3(k)|} \\ +u_1(k)\} + x_1(k) \\ \frac{T}{A}\{-c_3 S_p sign\big(x_2(k)-x_3(k)\big)\sqrt{2g|x_2(k)-x_3(k)|} \\ -c_2 S_p\sqrt{2gx_2(k)} + u_2(k)\} + x_2(k) \\ \frac{T}{A}\{-c_1 S_p sign\big(x_1(k)-x_3(k)\big)\sqrt{2g|x_1(k)-x_3(k)|} \\ -c_3 S_p sign\big(x_3(k)-x_2(k)\big)\sqrt{2g|x_3(k)-x_2(k)|}\} + x_3(k) \end{bmatrix}$$

where T is the sampling time chosen to be 0.01 seconds, $A = 0.0154\, m^2$ is the cross section of the tanks, $S_p = 5\times 10^{-5}\, m^2$ is the cross section of the connecting pipes, $c_1 = 1, c_2 = 0.8$, and $c_3 = 1$ are the outflow coefficients, and $g = 9.8\, m/s^2$ is the standard gravity. Moreover $\eta\big(x(k)\big) = [10^{-3}\sin(0.7kT)\ 10^{-2}\cos(0.8kT)\ 10^{-1.65}\cos(0.5kT)]^T$ represents the modeling uncertainty.

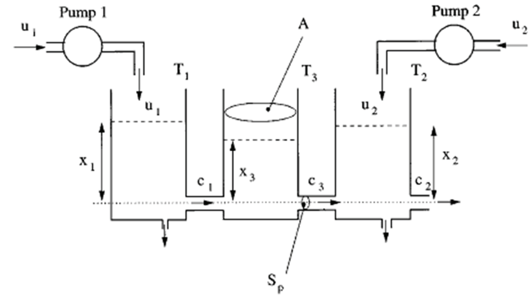

Fig. 1: Schematic view of the three-tank system

This system is subjected to additive faults which are given in terms of leakage in tank 1 and tank 2 and multiplicative actuator faults which can occur in pump 1 and pump 2. In this simulation we assume that either an additive or the multiplicative fault can occur at time $t_0 = 25$ sec. The additive and multiplicative fault functions are described by

$$h\big(x(k)\big) = \begin{bmatrix} 0.0154\big(1-e^{-0.5T(k-k_0)}\big)\sqrt{2gx_1(k)} \\ 0.0182\big(1-e^{-0.2T(k-k_0)}\big)\sqrt{2gx_2(k)} \\ 0 \end{bmatrix},$$

$$u_f(k) = \begin{bmatrix} 100\big(1-e^{-0.5T(k-k_0)}\big)u_1(k) \\ 50\big(1-e^{-0.5T(k-k_0)}\big)u_2(k) \end{bmatrix}$$

The FD estimator in (2) is used to detect the faults, where $A_d = 0.001 I_{3\times 3}$. The first OLAD output is given by $\hat{T}_A(k) = \hat{\theta}_A^T(k)\phi_1(V_1 x(k) + B_1)$, where $\hat{\theta}_A \in \mathbb{R}^{8\times 2}$ is the estimated parameter while $\phi_1 \in \mathbb{R}^8$ is a vector of sigmoid functions. The second OLAD output is given by $\hat{h}_d(k) = \hat{\theta}_d^T(k)\phi_2\big(x(k), u(k)\big)$, where $\hat{\theta}_d \in \mathbb{R}^{8\times 3}$ is the estimated parameters while $\phi_2 \in \mathbb{R}^8$ is a vector of sigmoid functions. Moreover V_1, B_1, V_2, and B_2 are selected randomly and the update law parameters are $\alpha_1 = 0.5, \gamma_1 = 10^{-4}$ and $\alpha_2 = 0.1, \gamma_2 = 10^{-4}$ respectively for the first and second OLADs. The detection threshold, ρ, is selected to be 0.05 while the identification threshold, δ, is chosen to be 0.01.

Fig. 2 shows the norm of the detection residual and the FD threshold when a multiplicative fault occurs. It is clearly seen that the residual remains below the detection threshold prior the occurrence of fault. After the fault occurs, the norm of residual starts to increase and it finally exceeds the threshold at a detection time $t = 25.44$ sec. At this point the first OLAD is activated and its update law will estimate the fault function. About 6 seconds after the detection of the fault, the FD residual falls below the threshold due to the OLAD function approximation property**.** This means that the OLAD has successfully estimated the fault function. As observed in Fig. 3, the norm of input residual $\|\tilde{u}\|$ crosses the identification threshold in the interval of $T_\delta = 2$ sec after the detection, indicating that the fault is of type multiplicative.

Fig. 4 shows the norm of the detection residual along with the detection threshold when additive faults are present. The residual reaches the detection threshold at time $t = 26.31$ sec. At this point a fault is declared active and the first OLAD is turned on, but since the fault is additive it cannot estimate the fault function. As seen in Fig. 5, norm of the input residual

$\|\tilde{u}\|$ remains below $\delta = 0.01$ within 2 seconds after the detection of fault. Therefore in this case the fault is identified as additive. Upon identifying the fault type, FD estimator uses the second OLAD alone to estimate the additive fault function, consequently, the FD residual converges to zero.

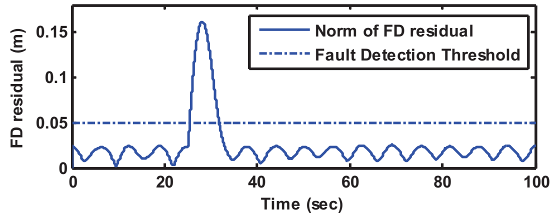

Fig. 2: Norm of FD residual when the fault is multiplicative.

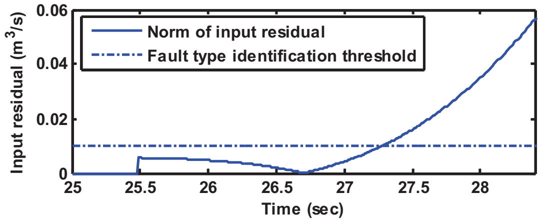

Fig. 3: Norm of input residual, $\|\tilde{u}\|$, when the fault is multiplicative.

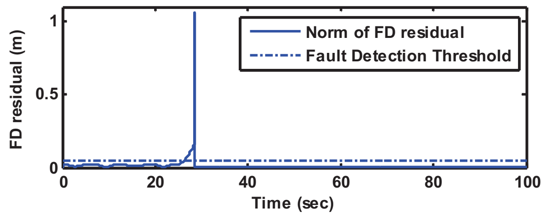

Fig. 4: Norm of FD residual when the fault is additive.

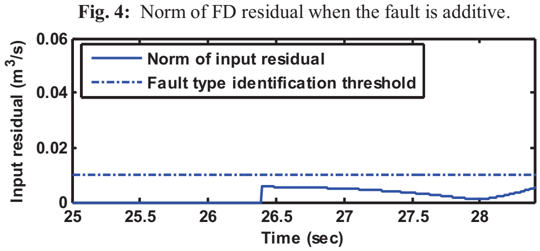

Fig. 5: Norm of input residual, $\|\tilde{u}\|$, when the fault is additive

TTF is determined for each of the multiplicative actuator faults as shown in Fig. 6 and Fig. 7. The initial estimates of TTF are not accurate due to the random selection of weights in the parameter update law. The time of failure is determined to be at 30.07seconds and 28.30 seconds, for the fault in the first and second inputs respectively. TTF estimation results for the additive fault case are not presented here.

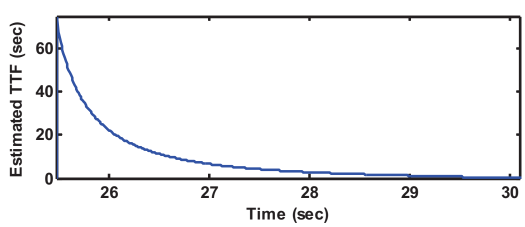

Fig. 6: TTF determination due to multiplicative fault in input 1.

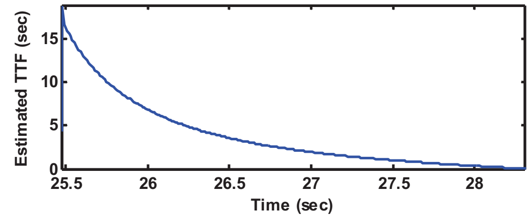

Fig. 7: TTF determination due to multiplicative fault in input 2

This example indicates that the proposed method of fault detection works for both additive and multiplicative fault types, the type of fault can be identified using the proposed method of fault type identification, and furthermore time to failure can be determined using Theorem 4.

VI. CONCLUSIONS

In this paper, a model-based fault detection scheme that detects both additive and multiplicative fault types and identifies fault type and TTF determination. Identification of fault type will help the process of finding the fault location for repairing and maintenance purposes. TTF estimation will in turn improve system availability. The proposed scheme does not need any a priori data or offline training and so it is generic and can be applied to a wide range of systems with a mathematical model available.

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