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Multisampled Hybrid Model Predictive Control for Pulse-Width Modulated Systems

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Abstract— **This paper deals with the model predictive control (MPC) of pulse-width modulated (PWM) systems. Hybrid models that predict the switched behaviour of PWM systems are investigated in order to formulate control objectives such as minimising the non-linear distortion or the power losses as it is typically desired in many power electronics applications. To obtain tractable problems, the PWM system dynamics are approximated by hybrid models where the time is quantised. The switching constraints are modelled employing binary variables. Two MPC schemes with different receding horizon policies are investigated to solve the resulting MILP or MIQP at each sampling period. Applying the proposed multisampled hybrid model predictive control scheme to a buck converter system, it is shown that recomputing the optimal solution to the problem at regular intervals during the switching period provides a better rejection of disturbances.**

I. Introduction

In model predictive control (MPC), the control objective is formulated as a finite time horizon constrained optimisation problem that needs to be solved at each sampling instant to find the control action to apply to the plant. As solving this optimisation problem is time consuming and requires appropriate hardware and software platforms, the application of MPC has long been restricted to the control of complex slow systems such as chemical plants [1]–[3]. The availability of more powerful hardware and of optimisation algorithms tailored to solve time invariant linearly constrained MPC problems off-line has permitted to envisage the application of MPC to much faster systems of reduced complexity where it is important to deal with constraints to obtain a satisfactory performance [4]–[6]. In particular, in the past few years MPC has been applied to enhance the dynamic performance of several power electronics systems. At the lowest level power electronics systems are controlled by appropriately manipulating semiconductor devices that can be seen as ideal switches from the control point of view. Due to their switched behaviour, power electronics systems are inherently hybrid. The switches are often controlled through pulse-width modulators (PWM) where the duty cycles are the manipulated variables. Continuous time dynamics are then obtained by employing model averaging techniques [7], which often allow for formulating the control objective as a linearly constrained MPC problem [8]. When the averaged model is nonlinear, a hybrid MPC problem needs to be solved [9], [10]. These types of problems quickly become intractable as the number of binary variables increases. Although the MPC formulations based on an averaged model allow to impose a satisfactory averaged dynamic behaviour, several situations occur in which the model mismatch deteriorates the system performance or even destabilises the system, or where it would be desirable to model the switched trajectory of the state and not only its averaged trajectory.

A hybrid model is employed in [11] to approximate the switched behaviour of the state in a buck converter with leading edge PWM. The trajectory of the state is predicted at regular time intervals T_q between two sampling instants. The approach is extended to more complex PWM systems in [12] where a tractable hybrid MPC scheme is proposed to improve the dynamic performance of AC-DC converters with resonant filters and to reduce the distortion by minimising the ripple. Improved dynamic performance is obtained, the achievable reduction of distortion is however limited, mostly because a one-norm cost criterion is employed for simplicity. The first problem with the one-norm cost criterion is that it results in a large number of constraints, which practically limits the achievable accuracy. The second problem is that the one-norm does not reflect the real objective, which is often to minimise the RMS value of the ripple. These problems are solved in [13], by introducing sampled data model predictive control to formulate an optimisation problem that minimises the integral of the quadratic tracking error. The problem is made tractable by employing a piecewise affine approximation of the cost criterion, which results in solving an MIQP at each sampling instant. The solution can be computed online since the number of binary variables remains small for the considered problems.

This paper proposes the multisampled hybrid model predictive control as a solution to improve the closed-loop system performance of PWM systems where it is possible to sample faster than the maximum admissible switching rate. The switched trajectory of the state is approximated by a hybrid model, extending the modelling approaches developed in [12], [13] to more general PWM schemes. In these approaches, the switching period is divided into N_q regular time intervals of duration $T_\mathrm{q} = \frac{T_\mathrm{p}}{N_\mathrm{q}}$. The value of the state is predicted at the end of each interval. In the multisampled hybrid MPC approach, the plant output is sampled and the plant input updated at each of these time intervals, which requires having switching constraints to prevent exceeding the admissible switching frequency. The motivation for a sampling rate higher than the possible rate of update of the PWM is that this allows to deal faster with disturbances and to reduce the effect of model mismatch by performing corrections of the control input

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when a deviation from the predicted switched behaviour is observed. An MPC problem that minimises a quadratic cost criterion is formulated. The switching constraints are integrated to the optimisation problem to enforce the desired PWM behaviour and constraints. This results in an MIQP that needs to be solved at each sampling instant following the same approach as in [13]. The difficulty is that the problem to be solved is time varying with the period T_p because the sampling rate is higher than the switching frequency.

The remainder of this paper is organised as follows. Section II derives an approximated hybrid control model that features the state ripple and includes the PWM switching constraints as binary variables. Section III formulates an MPC problem based on the approximated hybrid control model. It allows to formulate objectives featuring minimisation of converter losses and nonlinear distortion. Section IV introduces several receding horizon control concepts suitable for the control of PWM systems. Section V applies and compares the proposed concepts to the control of a buck converter. Section VI concludes.

II. Control model

A. Continuous time dynamics and switching constraints

The considered PWM dynamic systems are described as

$$\dot{x}(t) = F_0\, x(t) + G_\mathrm{w}\, w(t) + \sum_{i=1}^{N_\mathrm{b}} \left[s_i(t)\, F_i\, x(t) + s_i(t)\, G_i\, v(t) \right] \tag{1a}$$

$$s_i(t) \in \{0,1\} \tag{1b}$$

$$\lim_{N_\mathrm{q}\to\infty} \sum_{k=1}^{N_\mathrm{q}} \left| s_i\left(\frac{k\,T_\mathrm{p}}{N_\mathrm{q}}\right) - s_i\left(\frac{k\,T_\mathrm{p} - T_\mathrm{p}}{N_\mathrm{q}}\right) \right|_1 \leq 2 \tag{1c}$$

where N_b is the number of binary variables associated with the switches, $x \in \mathbb{R}^{n_\mathrm{x}}$ is the state vector, $v \in \mathbb{R}^{n_\mathrm{v}}$ and $w \in \mathbb{R}^{n_\mathrm{w}}$ are vectors of exogenous inputs, and $F \in \mathbb{R}^{n_\mathrm{x}\times n_\mathrm{x}}$, $G \in \mathbb{R}^{n_\mathrm{x}\times n_\mathrm{v}}$, and $G_w \in \mathbb{R}^{n_\mathrm{x}\times n_\mathrm{w}}$ are constant matrices. (1b) indicates the binary nature of the switching functions s_i, while (1c) constrains them to maximum two transitions per switching period T_p. (1) describes a wide range of power electronics system dynamics.

B. Discrete-time approximation of dynamics

Formulating an MPC problem that features the hybrid dynamics (1) generally yields an intractable problem. A simplified discrete-time model is required for the MPC problem formulation. The switching period is divided into N_q intervals of duration $T_\mathrm{q} = \frac{T_\mathrm{p}}{N_\mathrm{q}}$ over which the state evolution is approximated by the discrete time dynamics

$$x_{k+1} = A_0\, x_k + B_\mathrm{w}\, w_k + \sum_{i=1}^{N_\mathrm{b}} \left[s_{i,k}\, A_i\, x_k + s_{i,k}\, B_i\, v_k \right] \tag{2a}$$

$$A_0 = e^{F_0\,T_\mathrm{q}} \quad A_i = F_i\, T_\mathrm{q} \quad B_i = \int_0^{T_\mathrm{q}} e^{F_0 \tau}\, d\tau\, G_i$$
$$B_\mathrm{w} = \int_0^{T_\mathrm{q}} e^{F_i \tau}\, d\tau\, G_\mathrm{w} \qquad \forall i \in \{1,\ldots,N_\mathrm{b}\} \tag{2b}$$

where the subscript k denotes the time index, and the s_i take values in the interval $[0,1]$. $s_{i,k} = d$ indicates that the switch i is in on-state for a fraction d of the interval T_q (see Fig. 1).

C. PWM switching constraints

The binary and switching constraints (1b) and (1c) need to incorporated into the model (2) in order to describe the PWM dynamic behaviour

$$s_{i,k} \in \begin{cases} [0,1] : \neg\left(\delta^+_{i,k} \wedge \delta^-_{i,k}\right) \\ \{0\} : \neg\delta^0_{i,k} \wedge \delta^+_{i,k} \wedge \delta^-_{i,k} \\ \{1\} : \delta^0_{i,k} \wedge \delta^+_{i,k} \wedge \delta^-_{i,k} \end{cases} \tag{3a}$$

where three binary variables per switch are required. $\delta^+_{i,k}$ ($\delta^-_{i,k}$) is true if the switch i has already been turned on (respectively off) once during the current switching period. $\delta^0_{i,k}$ indicates the state of the switch at the beginning of the switching period. The boolean conditions (3a) inhibit multiple switching on and off during a switching period. Constraints (3a) can be rewritten as inequalities involving binary variables and the manipulated switched function. The binary variables $\delta_{i,k}$ are updated after each time interval T_q as follows

$$\delta^0_{i,k+1} = \begin{cases} 1 : \delta^\mathrm{b}_k \wedge \delta^+_{i,k} \wedge \neg\delta^-_{i,k} \wedge s_{i,k} = 1 \\ 0 : \delta^\mathrm{b}_k \wedge \neg\delta^+_{i,k} \wedge \delta^-_{i,k} \wedge s_{i,k} = 0 \\ \delta^0_{i,k} \text{ otherwise} \end{cases} \tag{3b}$$

where δ^b_k is true only at the last sample of the switching period. $\delta^0_{i,k}$ will only change its value if a switch was turned on but not off or vice versa during the previous switching period.

The variable $\delta^+_{i,k}$ that captures the transition to on-state is defined as

$$\delta^+_{i,k+1} = \begin{cases} 1 : \left(\delta^0_{i,k} \wedge \neg\delta^\mathrm{b}_k \wedge \delta^-_{i,k} \wedge s_{i,k} > 0\right) \\ \quad \vee \left[\neg\delta^0_{i,k} \wedge \neg\delta^\mathrm{b}_k \wedge \left(\delta^+_{i,k} \vee s_{i,k} > 0\right)\right] \\ 0 : \left[\delta^0_{i,k} \wedge \left(\delta^\mathrm{b}_k \vee \neg\delta^-_{i,k} \vee s_{i,k} = 0\right)\right] \\ \quad \vee \left[\neg\delta^0_{i,k} \wedge \left(\delta^\mathrm{b}_k \vee \left(\neg\delta^+_{i,k} \wedge s_{i,k} = 0\right)\right)\right] \end{cases} \tag{3c}$$

The initial switch state $\delta^0_{i,k}$ allows to decide which of the raising and falling transition is coming first. δ^b_k allows to reinitialise the binary variables at the beginning of the switching period. The variable $\delta^-_{i,k}$ that constrains the transition to off-state is similarly defined as

$$\delta^-_{i,k+1} = \begin{cases} 1 : \left[\delta^0_{i,k} \wedge \neg\delta^\mathrm{b}_k \wedge \left(\delta^-_{i,k} \vee s_{i,k} < 1\right)\right] \\ \quad \vee \left(\neg\delta^0_{i,k} \wedge \neg\delta^\mathrm{b}_k \wedge \delta^+_{i,k} \wedge s_{i,k} < 1\right) \\ 0 : \left[\delta^0_{i,k} \wedge \left(\delta^\mathrm{b}_k \vee \neg\delta^-_{i,k} \wedge s_{i,k} = 1\right)\right] \\ \quad \vee \left[\neg\delta^0_{i,k} \wedge \left(\delta^\mathrm{b}_k \vee \neg\delta^+_{i,k} \vee s_{i,k} = 1\right)\right] \end{cases} \tag{3d}$$

The hybrid model defined by dynamics (2) and switching constraints (3) approximates the switched behaviour of the system as depicted in Fig. 1. The integral of the real input and of the continuous time signal formed by the zero order hold approximation are equal.

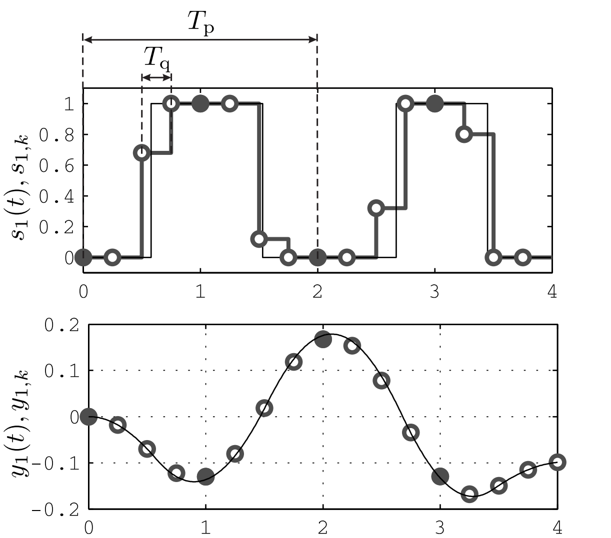

Fig. 1. PWM system input (top) and output (bottom). Continuous time signals of the real system (solid thin) and discrete-time approximations with a quantisation time T_q (circles).

III. MPC Problem formulation

The finite time constrained optimal control problem is formulated as follows:

$$\min \sum_{l=k+1}^{k+N} J(x_l, u_{l-1}) \tag{4a}$$

$$\text{s.t.} \quad \text{dynamics (2)} \tag{4b}$$

$$\text{switching contraints (3)} \tag{4c}$$

$$x_k = \bar{x}_k \tag{4d}$$

$$x_l \in \mathbb{X} \ , \ v_l \in \mathbb{U} \quad \forall l \in \{k, \ldots, k+N\} \tag{4e}$$

Due to the binary variables (3), the resulting optimisation problem requires solving a mixed integer program (MIP). As the hybrid system model (2) predicts the switched behaviour of the system, (4) allows to formulate a control objective related to the switched behaviour of the PWM system, such as for example minimising the nonlinear tracking distortion or minimising the losses. To keep the problem tractable, it is desirable to have a convex cost function $J(x_l, u_{l-1})$ that captures these properties and a linear model in (2).

IV. MPC schemes for PWM systems

The control model developed above features the switching constraints of the PWM system. Each switch control signal is constrained to make at most two transitions per switching period. In digital control, the sampling frequency must be synchronised with the PWM frequency in order to update the switch control signal parameters as fast as possible. Using an averaged model, it does not make sense to sample and update the control input of PWM systems faster than twice the PWM frequency due to the switching constraints. Moreover, it complicates the control as it is difficult to filter the ripple component without cutting dramatically the closed-loop bandwidth. Still, this has been done in order to increase the bandwidth of PI controllers by mimicking analog controllers [14]. Employing the hybrid model presented in the previous section, it is not necessary to remove the state ripple as it is predicted by the control model.

A. MPC with averaged model

If $N_q = 1$, all the binary constraints vanish. The hybrid model becomes linearly constrained and equivalent to an averaged model. The MPC problem results in an LP or QP that can be solved efficiently. In turn, the control objective can no longer feature the switched behaviour of the system. Depending on the switching constraints, the same situation occurs with $N_q = 2$.

B. MPC with intersampling

By intersampling, it is meant that the hybrid model predicts the switched behaviour of the system between the instants at which the plant output is sampled and the PWM input is updated. The sampling period T_s is therefore equal to the switching period T_p or twice the switching period when both transitions can be controlled. Based on the state measured at time k, the future sequence of states is predicted at N_q equidistant instants in each switching period. This results in a discretisation period that is a fraction N_q of the switching period, $T_q = \frac{T_p}{N_q}$.

Fig. 1 illustrates the case where the switching period T_p is divided in $N_q = 8$ periods of duration T_q. The constrained control input and the state are predicted at each interval of duration T_q featured by a circle, the PWM system input and output are sampled and updated only twice per switching period at the time intervals featured by the plain black disks.

C. Multisampled Hybrid MPC

By multisampled hybrid MPC, it is meant that the hybrid model predicts the switched behaviour of the system within a *switching period*. The conceptual difference with the intersampling approach is that the plant output, respectively the PWM input are sampled, respectively updated at each of these instants. The sampling period T_s is therefore equal to the time quantisation period T_q. For the same control objective, the hybrid model and the MPC problem are the same for intersampling and multisampling. The computed control inputs are therefore identical at the same time instances. As the control input is computed more often in the multisampling case, a (slightly) different input sequence is expected. The reason is that the horizon is shifted by a quantity that varies between T_q and $T_p - T_q$, such that the optimisation is recomputed with slightly different constraints and with a different state sequence. In absence of modelling errors and of disturbances this difference should however be little, especially if the horizon is sufficiently long.

In presence of disturbances, the system is expected to react faster, as much as allowed by the switching contraints. In presence of modelling errors, the multisampling mechanism is also expected to readjust the optimal control sequence to correct the system behaviour. These features obviously come at the expense of more computational power to allow solving the optimisation problem at higher sampling rates.

In Fig. 1 the PWM system input and output are sampled and updated at each of the instants featured by a circle or a plain black disk.

TABLE I
COMPARISON OF INTERSAMPLING AND MULTISAMPLING PARAMETERS AND PROPERTIES.

	Intersampling	Multisampling
switching period	T_p	T_p
number of intersamples	q	q
sampling period	$T_\mathrm{s} = T_\mathrm{p}$ or $\frac{T_\mathrm{p}}{2}$	$T_\mathrm{s} = \frac{T_\mathrm{p}}{q}$
prediction step size	$T_\mathrm{q} = \frac{T_\mathrm{p}}{q}$	$T_\mathrm{q} = \frac{T_\mathrm{p}}{q}$
system update	beginning/middle of switching period	at each prediction step
system dynamics	time-invariant	time-varying

V. CASE STUDY

In a case study, the approaches formalised in the above are compared to each other. As an example PWM system a buck converter is chosen. Buck converters belong to the class of DCDC converters. They are used to convert an unregulated input voltage to a desired lower output voltage. The schematic of the buck converter topology is given in Fig. 2. The system states are the inductor current i_L, the output capacitor voltage u_C and the output current i_o. The output current i_o cannot be influenced by the controller.

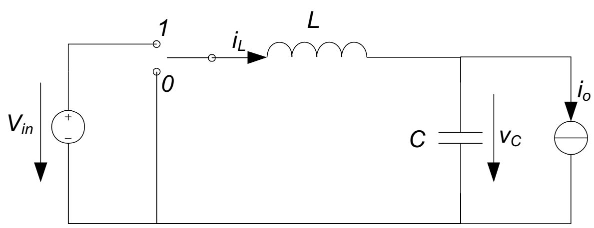

Fig. 2. Schematic of a buck converter.

A. System dynamics

When the switch is in position 1, i.e. $s(t) = 1$, the voltage across the inductor is positive, i_L increases. For $s(t) = 0$, the voltage across the inductor is negative, so i_L decreases. The output capacitor C accounts for the current ripple such that a constant output current can be achieved. Thus, a small ripple in the output voltage results.

In the standard PWM framework given in equation (1) with $N_\mathrm{b} = 1$ the respective vectors and matrices are:

$$F_0 = \begin{pmatrix} 0 & \frac{1}{C} \\ \frac{1}{L} & 0 \end{pmatrix},\ F_1 = 0,\ G_\mathrm{w} = \begin{pmatrix} -\frac{1}{C} \\ 0 \end{pmatrix},\ G_1 = \begin{pmatrix} 0 \\ \frac{1}{L} \end{pmatrix},$$

$$x = \begin{pmatrix} u_\mathrm{C} \\ i_\mathrm{L} \end{pmatrix},\ v = V_\mathrm{in},\ w = i_\mathrm{o}.$$

B. Control problem formulations

The system shall be controlled using leading edge PWM and three different MPC schemes: MPC using an averaged model, intersampling and multisampling. A quadratic cost function that penalises the deviation of the state from a given reference value is chosen, yielding an optimal tracking problem.

As leading edge PWM is applied, where turning a switch on is only allowed at the beginning of the switching period, the constraints for the hybrid model (3) simplify greatly as one can set $\delta^0_{i,k}$ to zero and $\delta^+_{i,k}$ to one $\forall k$.

$$s_{i,k} \in \begin{cases} [0,1] : \neg\delta^-_{i,k} \\ \{0\} \ : \delta^-_{i,k} \end{cases} \tag{5a}$$

$$\delta^-_{i,k+1} = \begin{cases} 1 : s_{i,k} < 1 \wedge \neg\delta^b_k \\ 0 : s_{i,k} = 1 \vee \delta^b_k \end{cases} \tag{5b}$$

The translation of (5a) to the according inequalities is:

$$\delta^-_{i,k} \leq 1 - s_{i,k} \leq 1 \tag{5c}$$

1) Averaging: In the state space averaged modelling approach, the system dynamics are averaged over a switching period, the state ripples are not taken into account. The sampling rate is equal to the switching frequency. The resulting control model is

$$\dot{x}(t) = F_0 x(t) + G_1 v d(t) + G_\mathrm{w} w(t) \tag{6}$$

where $d(t)$ is the fraction of the current switching period during which the switch is in position one. $d(t)$ is usually referred to as duty cycle. Here, it is the system's only control input.

To be able to implement MPC, the system has to be discretised. As the system dynamics are averaged over one switching period, the natural discretisation frequency is equal to the switching frequency. The only constraint on the system input is $d_k \in [0,1]$. Let A_avg, B_avg, and $B_\mathrm{w,avg}$ denote the discretised system matrices. The MPC tracking problem using an averaged model and a state reference x_ref can be written as

$$\min \sum_{k=1}^{N} \|x_k - x_\mathrm{ref}\|^2_Q \tag{7a}$$

$$\text{s.t.}\ \ x_{k+1} = A_\mathrm{avg} x_k + B_\mathrm{avg} v d_k + B_\mathrm{w,avg} w_k \tag{7b}$$

$$d_k \in [0,1] \tag{7c}$$

where $Q = \begin{pmatrix} 1 & 0 \\ 0 & 10 \end{pmatrix}$ a positive semidefinite matrix. This optimisation problem has to be solved once per switching period.

2) Intersampling: The number of prediction steps per switching period q is 5, yielding a discretisaton frequency for the PWA approximated system of $f_\mathrm{q} = 5 f_\mathrm{p}$. Let A_q, B_q, and $B_\mathrm{w,q}$ denote the discretised system matrices of the PWA model. The bilinearity in the general formulation (2) with $N_\mathrm{b} = 1$ is zero, i.e. $F_i = 0$, and the external input $v_k = v = V_\mathrm{in}$ is constant and scalar. Furthermore, the system here is only sampled and updated at the beginning of each switching period. Thus, the evolution of inputs, states and binaries during switching period k can be written in vector form:

$$X_k = \begin{pmatrix} x_k^{1\,T} & x_k^{2\,T} & \dots & x_k^{q\,T} \end{pmatrix}^T \quad (8a)$$

$$S_{i,k} = \begin{pmatrix} s_{i,k}^0 & s_{i,k}^1 & \dots & s_{i,k}^{q-1} \end{pmatrix}^T \quad (8b)$$

Δ_k^- and W_k are defined analogously. Thus, the general MPC problem stated in (4) simplifies significantly, and the MPC tracking problem for the intersampling scheme using leading edge PWM can be written as

$$\min \sum_{k=1}^{N} \|X_k - X_{ref}\|_{\tilde{Q}}^2 \quad (9a)$$

$$\text{s.t.} \quad X_{k+1} = \tilde{A}_\text{q} X_k(q) + \tilde{B}_\text{q} v S_k + \tilde{B}_{\text{w,q}} W_k \quad (9b)$$

$$\Delta_k^-(j) = \begin{cases} 0 : j = 1 \vee S_k(j-1) = 1 \\ 1 : S_k(j) < 1 \end{cases} \quad (9c)$$

$$\Delta_k^- \leq \mathbf{1}_{\text{q}\times 1} - S_k \leq \mathbf{1}_{\text{q}\times 1} \quad (9d)$$

where q is the number of intersampling instants, n_x is the dimension of the vector x. $X_\text{ref} = \begin{pmatrix} x_\text{ref}^T & \dots & x_\text{ref}^T \end{pmatrix}^T \in \mathbb{R}^{q n_\text{x}}$, x_ref and Q are as in the averaging approach, $\tilde{Q}$ is an $n_\text{x}q \times n_\text{x}q$-matrix with Q on its diagonal. The state update matrices are $\tilde{A}_\text{q} = \begin{pmatrix} A_\text{q} \\ A_\text{q}^2 \\ \vdots \\ A_\text{q}^q \end{pmatrix}$, $\tilde{B}_\text{q} = \begin{pmatrix} B_\text{q} & 0 & \dots & 0 \\ A_\text{q} B_\text{q} & B_\text{q} & \dots & 0 \\ \vdots & & \ddots & \vdots \\ A_\text{q}^{q-1} B_\text{q} & A_\text{q}^{q-2} B_\text{q} & \dots & B_\text{q} \end{pmatrix}$, and $\tilde{B}_{\text{w,q}}$ analogously to $\tilde{B}_\text{q}$. This optimisation problem also has to be solved only once per switching period.

3) Multisampling: For fair comparison, the number of prediction steps per switching period is $q = 5$ as in the intersampling approach. Again, the MPC cost function penalises the deviation of the rippling state x_k from a given constant reference value. The optimisation problem is

$$\min \sum_{k=1}^{N} \|x_k - x_\text{ref}\|_Q^2 \quad (10a)$$

$$\text{s.t.} \quad x_{k+1} = A_\text{q}\, x_k + B_\text{q} v\, s_k + B_{\text{w,q}}\, w_k \quad (10b)$$

$$\delta_{k+1}^- = \begin{cases} 1 : s_k < 1 \wedge \neg\delta_k^b \\ 0 : s_k = 1 \vee \delta_k^b \end{cases} \quad (10c)$$

$$\delta_k^- \leq 1 - s_k \leq 1 \quad (10d)$$

where x_ref and Q are as in the averaging approach. The system update rate is q-times higher than in the approaches above. Thus, q optimisation problems have to be solved per switching period.

C. Simulation results

The three control schemes are now implemented and their respective tracking performances are compared in a system that experiences a load jump. At the beginning of the simulation, the system is at steady state with a reference output voltage of $32V$ and an output current of $200mA$. After $250\mu s$ the load current jumps by 30% to $260mA$. The circuit parameters are summarised in table II.

TABLE II
BUCK CONVERTER PARAMETERS

Circuit parameter	Abbreviation	Value
switching frequency	f_p	$20kHz$
inductor size	L	$10mH$
capacitor size	C	$1\mu F$
input voltage	V_in	$100V$
output current	i_o	$200mA$ / $260mA$
reference voltage	$v_{\text{C,ref}}$	$32V$
reference current	$i_{\text{L,ref}}$	$200mA$ / $260mA$

A horizon of three switching periods is chosen for all schemes. Using the state-space averaged or the intersampling approach, this yields a control horizon of $N = 3$, in the multisampling scheme $N = 3q = 15$, as there are 5 prediction steps per switching period and the problem cannot be written in compact matrix form.

1) Averaging: In Fig. 3 the simulation results using the state-space averaged approach are displayed. The states are measured at the beginning of a switching period. Therefore, only the minimum value and not the average of i_L is measured. Thus, the controller tries to increase i_L. By

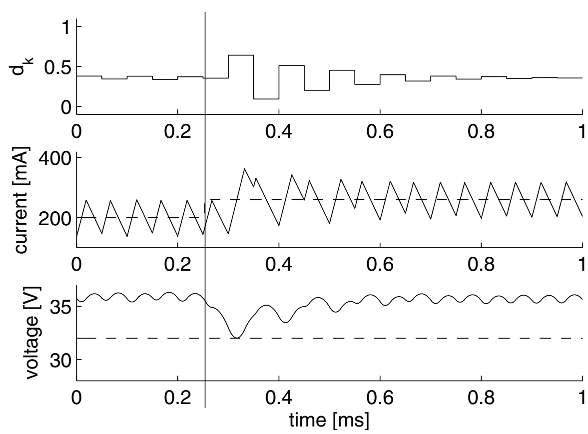

Fig. 3. Simulation results for the averaging approach. Top plot: duty cycle in zero-order-hold fashion. Bottom plots: inductor current and capacitor voltage. Dashed lines: reference values. Solid lines: simulation results. The solid vertical line marks the time of the load jump of 30%.

increasing i_L at a constant load current, however, the output capacitor is charged, which leads to a higher output voltage. The controller has two conflicting objectives: increasing i_L at constant load and at the same time decreasing u_C. This results in a substantial off-set in the output voltage. After the disturbance, the system takes several switching periods to settle at the new steady state.

2) Intersampling: Fig. 4 shows the simulation results for the intersampling approach. The performance of the controller using this more accurate model is clearly superior to the averaged approach. Good reference tracking is achieved. The reaction to the disturbance is considerably faster than in averaging. As in averaging it takes one switching period before the disturbance is detected. Then, however, the system settles at the new steady state within three switching periods.

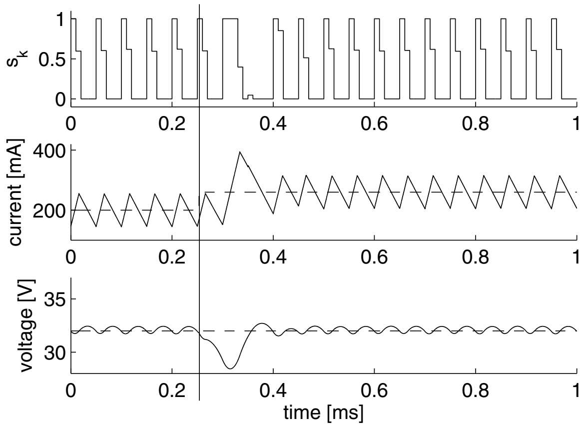

Fig. 4. Simulation results for the intersampling control approach. Top plot: manipulated variable in zero-order-hold fashion. Bottom plots: inductor current and capacitor voltage. Dashed lines: reference values. Solid lines: simulation results. The solid vertical line marks the time of the load jump of 30%.

3) Multisampling: The multisampling simulation results are given in Fig. 5. As expected, as long as no disturbance is present, the system yields the same steady state results as in the intersampling approach. After the disturbance, the system recovers faster than in the other approaches. As the state update is performed five times more often, the disturbance is detected sooner, thus the controller can react better and is able to enter the new steady state after one sampling period.

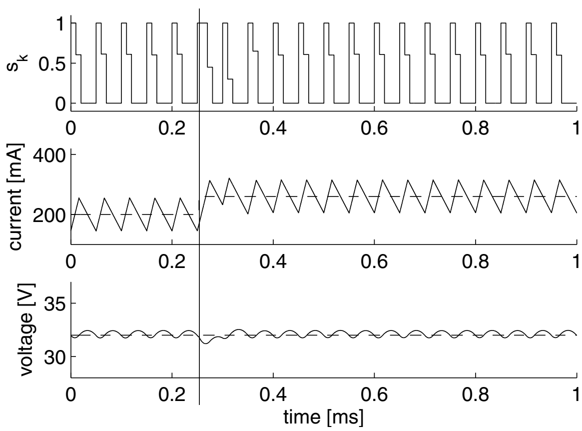

Fig. 5. Simulation results for the multisampling control approach. Top plot: manipulated variable in zero-order-hold fashion. Bottom plots: inductor current and capacitor voltage. Dashed lines: reference values. Solid lines: simulation results. The solid vertical line marks the time of the load jump of 30%.

VI. Conclusion

Different MPC schemes dedicated to the control of PWM systems have been investigated. The receding horizon schemes are different in the way the plant output is sampled and the PWM system input is updated. A hybrid model that allows to model most PWM system constraints as binary variable is employed. The time is discretised and the switched behaviour is predicted at regular intervals in the switching period. This extend previous results in the field in two ways. The proposed formulation allows to incorporate most PWM schemes. It is proposed to sample at the same rate as the time discretisation interval to better deal with disturbances and model mismatch. The optimisation problem is time varying, with the same period as the switching period and results in either an MILP or an MIQP. By setting the discretisation interval to the switching period, the well known averaged model is obtained. The effectiveness of the proposed hybrid multisampled MPC scheme is demonstrated by simulation on a simple case study.

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