

Oil Reservoir Production Optimization using Optimal Control

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Abstract—Practical oil reservoir management involves solution of large-scale constrained optimal control problems. In this paper we present a numerical method for solution of large-scale constrained optimal control problems. The method is a single-shooting method that computes the gradients using the adjoint method. We use an Explicit Singly Diagonally Implicit Runge-Kutta (ESDIRK) method for the integration and a quasi-Newton Sequential Quadratic Programming (SQP) algorithm for the constrained optimization. We use this algorithm in a numerical case study to optimize the production of oil from an oil reservoir using water flooding and smart well technology. Compared to the uncontrolled case, the optimal operation increases the Net Present Value of the oil field by 10%.

I. INTRODUCTION

Petroleum reservoirs are subsurface formations of porous rocks with hydrocarbons trapped in the pores. Initially, the reservoir pressure may be sufficiently large to push the fluids to the production facilities. However, as the fluids are produced the pressure declines and production reduces over time. When the natural pressure becomes insufficient, the pressure must be maintained artificially by injection of water. Conventional technologies for recovery leaves more than 50% of the oil in the reservoir. Wells with adjustable downhole flow control devices coupled with modern control technology offer the potential to increase the oil recovery significantly. [1] introduces optimal control of smart wells. In these applications, downhole sensor equipment and remotely controlled valves are used in combination with large-scale subsurface flow models and gradient based optimization methods in a Nonlinear Model Predictive Control framework to increase the production and economic value of an oil reservoir [2]–[6]. Whether the objective is to maximize recovery or some financial measure like Net Present Value, the increased production is achieved by manipulation of the well rates and bottom-hole pressures of the injection and production wells. The optimal water injection rates and production well bottom-hole pressures are computed by solution of a large-scale constrained optimal control problem.

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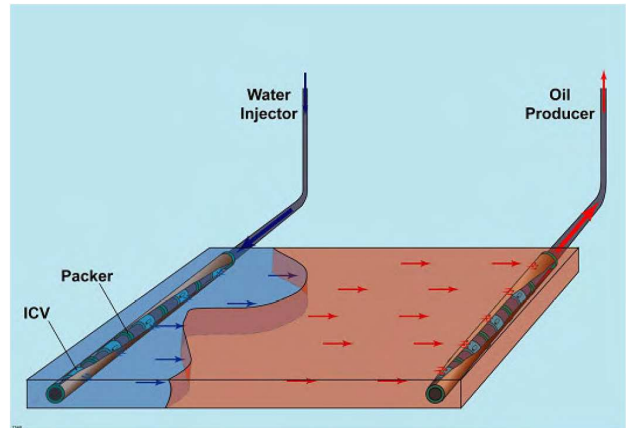


Fig. 1. Schematic view of horizontal wells in the water flooding problem [1].

In this paper, we focus on maximizing the economical value of an oil field and describe the gradient based method to compute the optimal control strategy. An Explicit Singly Diagonally Implicit Runge-Kutta (ESDIRK) method with an adaptive step size control is used for computationally efficient solution of the model [7], [8]. The gradients are computed by the adjoint method [9]. The adjoint equations associated with the integration scheme are solved by integrating backwards in time. The necessary information for the adjoint computation is computed and stored during the forward solution of the model. The backward adjoint computation assembles this information to compute the gradients [9], [10]. We demonstrate the optimal control strategy using a 2-dimensional water-flooding example as illustrated in Fig. 1.

The paper is organized as follows. Section II briefly introduces the two phase flow model. Section III states the general constrained optimal control problem using a novel representation of the system dynamics. The numerical methods for the constrained optimal control problem are described in Section IV. Section V describes specific details related to the objective function and the constraints for the water flooding production optimization problem. Section VI describes a numerical case study illustrating the method. Conclusions are presented in Section VII.

II. TWO-PHASE FLOW MODEL

In this section we briefly state the governing equations of an oil reservoir model. We consider isothermal two-

phase flow of oil and water in a porous media. We assume complete immiscibility of the reservoir fluids, zero capillary pressure and we neglect gravity effects. Let $P = P(t, r)$ be the pressure in the reservoir and $S = S(t, r)$ be the saturation of water, as function of time $t \geq 0$ and position $r \in \Omega \subset \mathbb{R}^3$ with Ω being the domain of the reservoir. Let $C_w = C_w(P, S)$ and $C_o = C_o(P, S)$ be the mass concentrations of water and oil, respectively. Then the mass balances for water and oil in the reservoir are expressed by

$$\frac{\partial}{\partial t} C_w = -\nabla \cdot F_w + Q_w \quad (1a)$$

$$\frac{\partial}{\partial t} C_o = -\nabla \cdot F_o + Q_o \quad (1b)$$

$F_w = F_w(P, S)$ and $F_o = F_o(P, S)$ are the fluxes of water and oil through the porous media. The source/sink terms of water and oil are denoted $Q_w = Q_w(P, S)$ and $Q_o = Q_o(P, S)$. They describe the flow rate of water from the injection wells into the reservoir and the flow rates of oil and water from the reservoir into the production wells. [11]–[13] provide more detailed descriptions of the model.

III. CONTINUOUS-TIME OPTIMAL CONTROL

Process models are based on conservation of mass, energy and momentum. It is desirable to preserve such properties upon numerical integration in time. Such problems related to flow in porous media can be represented by the system of differential equations [13]

$$\frac{d}{dt} g(x(t)) = f(x(t), u(t)) \quad (2)$$

with the initial condition $x(t_0) = x_0$. The left-hand side $g(x(t))$ are the properties conserved, $x(t)$ are the system states, $u(t)$ are the manipulated variables, while the right-hand side $f(x(t), u(t))$ has the usual interpretation. Considering (2) we formulate the water flooding problem as a continuous time Bolza problem

$$\min_{\{x(t), u(t)\}_{t_0}^{t_f}} \int_{t_0}^{t_f} J(t, x(t), u(t)) dt \quad (3a)$$

$$\text{s.t.} \quad \frac{d}{dt} g(x(t)) = f(x(t), u(t)), \quad x(t_0) = x_0 \quad (3b)$$

$$u_{min} \leq u(t) \leq u_{max} \quad (3c)$$

$$-u_{min}^{\Delta} \leq \frac{d}{dt} u(t) \leq u_{max}^{\Delta} \quad (3d)$$

The algorithm developed for solution of this problem is suitable for production optimization of oil reservoirs. We use a zero-order-hold parameterization for $u(t)$. This implies that the constraints (3d) should be interpreted as the movement constraints (6d).

IV. NUMERICAL METHODS

In this section, we describe a single-shooting algorithm for solution of (3). An ESDIRK method is used for the integration, the gradients are computed using the adjoint method, and the constrained optimization is performed using a quasi-Newton SQP method.

To convert the infinite-dimensional problem (3) into a numerically tractable finite-dimensional problem, we divide the temporal domain $[t_0, t_f]$ into K control steps and each control step into N_k time steps for the integration of the differential equations. We then define a set of control step indices $\mathcal{K}_i = \{i, i+1, \dots, K-1\}$ and a set of time step indices $\mathcal{N}_k = \{0, 1, \dots, N_k-1\}$ for all $k \in \mathcal{K}_0$. The number of control steps is known in advance due to the zero-order-hold parametrization of the manipulated variables. A control step $k \in \mathcal{K}_0$ is defined as an interval between the times $t_{n_k=0}$ and $t_{n_k=N_k}$. Note that $t_{n_0=0} = t_0$ and $t_{n_{K-1}=N_{K-1}} = t_f$. For a given control interval k , the number of time steps are not known in advance as we use an adaptive step length controller in the numerical integrator [7], [8]. This indexing of the control steps and the time steps are illustrated in Fig. 2.

Using the ESDIRK12 scheme for temporal discretization of (2), we can compute the trajectory $\{\{x_{n_k+1}\}_{n_k=0}^{N_k-1}\}_{k=0}^{K-1}$ as the solution of the system of difference equations [13]

$$g(x_{n_k+1}) = g(x_{n_k}) - f(x_{n_k+1}, u_k) h_{n_k} \quad (4)$$

in which $x(t_{n_k}) = x_{n_k}$ and $u(t_{n_k}) = u_k$ for $n_k \in \mathcal{N}_k$ and $k \in \mathcal{K}_0$. For notational convenience we define the residual function

$$R_{n_k+1}(x_{n_k+1}, x_{n_k}, u_k) = g(x_{n_k+1}) - g(x_{n_k}) - f(x_{n_k+1}, u_k) h_{n_k} = 0 \quad (5)$$

for $n_k \in \mathcal{N}_k$ and $k \in \mathcal{K}_0$. The continuous-time optimal control problem (3) can be formulated as the following discrete-time optimal control problem

$$\min_{\{\{x_{n_k+1}\}_{n_k=0}^{N_k-1}, u_k\}_{k=0}^{K-1}} \sum_{k=0}^{K-1} \sum_{n_k=0}^{N_k-1} J_{n_k}(x_{n_k}, u_k) \quad (6a)$$

$$\text{s.t.} \quad R_{n_k+1}(x_{n_k+1}, x_{n_k}, u_k) = 0 \quad (6b)$$

$$u_{min} \leq u_k \leq u_{max} \quad (6c)$$

$$-u_{min}^{\Delta} \leq \Delta u_k \leq u_{max}^{\Delta} \quad (6d)$$

where $\Delta u_k = u_k - u_{k-1}$ and

$$J_{n_k}(x_{n_k}, u_k) = \int_{t_{n_k}}^{t_{n_k+1}} J(x(t), u_k) dt, \quad n_k \in \mathcal{N}_k, \quad k \in \mathcal{K}_0 \quad (7)$$

A. Single-Shooting Optimization

The discrete-time optimal control problem (6) can be solved using single-shooting, multiple-shooting, and the simultaneous method. Reservoir models are large-scale and the number of states are easily in the order of magnitude of $10^5 - 10^6$ for realistic problems.

To keep the dimension of the optimization problem small and to be able to use adaptive temporal step size, we use the single-shooting method in this paper. In the single-shooting method, the manipulated variables, u , are fixed at each iteration and used to solve the difference equations (6b) numerically. Knowledge of the

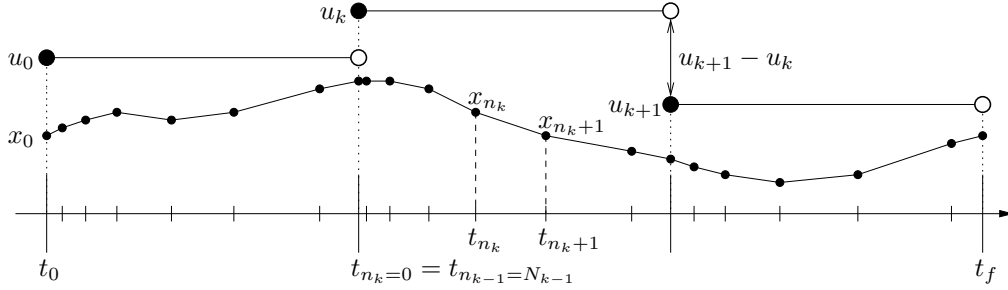


Fig. 2. The zero order hold parametrization and the relation between the control steps and the time steps. For a given time step t_{n_k} in a given control step k the optimal control problem can be described by the system states x_{n_k} and the control settings u_k .

initial state, x_0 , the manipulated variables, $\{u_k\}_{k=0}^{K-1}$, and the requirement that the systems dynamics are satisfied determines the states $\{x_{n_k+1}\}_{n_k=0}^{N_k-1}\}_{k=0}^{K-1}$. In practical computations, the system constraints (6b) are satisfied by solving (5), i.e. by doing a system simulation. In this way, a single-shooting method for (6) can be stated as the optimization problem

$$\min_{\{u_k\}_{k=0}^{K-1}} \psi(\{u_k\}_{k=0}^{K-1}, x_0) \quad (8a)$$

$$\text{s.t. } u_{min} \leq u_k \leq u_{max} \quad (8b)$$

$$-u_{min}^{\Delta} \leq \Delta u_k \leq u_{max}^{\Delta} \quad (8c)$$

with the objective function

$$\psi(\{u_k\}_{k=0}^{K-1}, x_0) = \left\{ \sum_{k=0}^{K-1} \sum_{n_k=0}^{N_k-1} J_{n_k}(x_{n_k}, u_k) : R_{n_k+1}(x_{n_k+1}, x_{n_k}, u_k) = 0, n_k \in \mathcal{N}_k, k \in \mathcal{K}_0 \right\} \quad (9)$$

B. Sequential Quadratic Programming

We solve the reduced problem (8) using sequential quadratic programming (SQP) with line-search and modified BFGS approximations, B , of the Hessian of the Lagrangian [14]. In each iteration, we solve the convex quadratic program

$$\min_{\Delta u} \frac{1}{2} \Delta u^T B \Delta u + \nabla_u \psi^T \Delta u \quad (10a)$$

$$\text{s.t. } \nabla_u c(u)^T \Delta u \geq -c(u) \quad (10b)$$

in which $u = [u_0, u_1, \dots, u_{K-1}]^T$. The optimal solution of (10), $\Delta u = \{\Delta u_k\}_{k=0}^{K-1}$, combined with a line-search method based on Powell's exact penalty function are used to determine the next iterate

$$u^{(i+1)} = u^{(i)} + \alpha \Delta u^{(i)} \quad (11)$$

α is the line search parameter.

C. Gradient Computation by the Adjoint Method

In computing the search direction, i.e. solving (10), we must compute the gradient $\nabla_{u_k} \psi$. The system states in dynamic optimization problems are dependent on the

control variables, in the sense that any past change of the control variables has an influence on all subsequent system states. Consequently, the gradient information of (9) is not directly accessible. The necessary information for computing $\nabla_{u_k} \psi$ is obtained during the simulation step at each optimization iteration in the single-shooting approach. The adjoint method uses this information efficiently to compute the gradients.

Assume that the current iterate, $u^{(i)}$, satisfies the input constraints (8b-8c). The adjoint method can be derived using parts of the first order necessary conditions and the Lagrangian [9]

$$\mathcal{L}(\{\{x_{n_k+1}\}_{n_k=0}^{N_k-1}, u_k, \{\lambda_{n_k+1}\}_{n_k=0}^{N_k-1}\}_{k=0}^{K-1}) = \sum_{k=0}^{K-1} \sum_{n_k=0}^{N_k-1} [J_{n_k}(x_{n_k}, u_k) - \lambda_{n_k+1}^T R_{n_k+1}(x_{n_k+1}, x_{n_k}, u_k)] \quad (12)$$

When the Lagrange multipliers (adjoint variables) $\{\{\lambda_{n_k+1}\}_{n_k=0}^{N_k-1}\}_{k=0}^{K-1}$ and the state variables $\{\{x_{n_k+1}\}_{n_k=0}^{N_k-1}\}_{k=0}^{K-1}$ satisfy certain parts of the KKT conditions, we have

$$\psi(\{u_k\}_{k=0}^{K-1}) = \left\{ \mathcal{L}(\{\{x_{n_k+1}\}_{n_k=0}^{N_k-1}, u_k, \{\lambda_{n_k+1}\}_{n_k=0}^{N_k-1}\}_{k=0}^{K-1}) : R_{n_k+1}(x_{n_k+1}, x_{n_k}, u_k) = 0, n_k \in \mathcal{N}_k, k \in \mathcal{K}_0 \right\} \quad (13)$$

such that we can compute the sensitivity $\nabla_{u_k} \psi$ as the sensitivity $\nabla_{u_k} \mathcal{L}$. The KKT condition corresponding to the state derivative of (12) yields

$$\begin{aligned} \nabla_{x_{n_k}} \mathcal{L} &= \nabla_{x_{n_k}} J_{n_k}(x_{n_k}, u_k) - \\ &\nabla_{x_{n_k}} R_{n_k+1}(x_{n_k+1}, x_{n_k}, u_k) \lambda_{n_k+1} - \\ &\nabla_{x_{n_k}} R_{n_k}(x_{n_k}, x_{n_k-1}, u_k) \lambda_{n_k} = 0 \end{aligned} \quad (14)$$

for $n_k \in \mathcal{N}_k$ and $k \in \mathcal{K}_0$. Substituting the definition of the residuals (5) into (14) and taking derivatives gives

$$\begin{aligned} \left[\nabla_{x_{n_k}} g(x_{n_k}) - \nabla_{x_{n_k}} f(x_{n_k}, u_k) h_{n_k-1} \right] \lambda_{n_k} = \\ \nabla_{x_{n_k}} J_{n_k}(x_{n_k}, u_k) + \nabla_{x_{n_k}} g(x_{n_k}) \lambda_{n_k+1} \end{aligned} \quad (15)$$

from which we can compute the adjoint variables λ_{n_k} marching backwards. The Lagrange multiplier at the final time is $\lambda_{N_{K-1}} = 0$ since the cost-to-go function is

zero. $\lambda_{N_{k-1}} = 0$ is used to initialize the backward march for computation of the adjoint variables λ_{n_k} . Special attention must be given when computing $\lambda_{N_{k-1}}$ at the transition between u_{k-1} and u_k for $k \in \mathcal{K}_1$. This is because the first term and the second term on the right-hand side in (14) both belong to control step u_k , while the third term belongs to control step u_{k-1} .

The partial derivatives of (12) with respect to the manipulated variables are

$$\begin{aligned} \nabla_{u_k} \mathcal{L} = & \nabla_{u_k} \mathcal{L} + \nabla_{u_k} J_{n_k}(x_{n_k}, u_k) - \\ & \nabla_{u_k} R_{n_k+1}(x_{n_k+1}, x_k, u_k) \lambda_{n_k+1} \end{aligned} \quad (16)$$

for $k \in \mathcal{K}_0$. Using (5) and $\psi = \mathcal{L}$ we arrive at the following expression for $\nabla_{u_k} \psi$

$$\begin{aligned} \nabla_{u_k} \psi = & \nabla_{u_k} \mathcal{L} = \nabla_{u_k} \mathcal{L} + \nabla_{u_k} J_{n_k}(x_{n_k}, u_k) + \\ & \nabla_{u_k} f(x_{n_k+1}, u_k) h_{n_k} \lambda_{n_k+1} \end{aligned} \quad (17)$$

for $k \in \mathcal{K}_0$. Consequently, the gradients $\nabla_{u_k} \psi$ may be computed using (17) in combination with solution of the adjoint equations (15) marching backwards.

V. WATER FLOODING PRODUCTION OPTIMIZATION

The objective of oil reservoir management is to maximize the economic value of the oil reservoir. Essentially, we want to produce as much oil as possible while keeping the operational cost at a minimum. We do this by maximizing the Net Present Value (NPV). Consequently the stage cost $J(t) = J(t, x(t), u(t))$ in (3) becomes

$$\begin{aligned} J(t) = & -e^{-dt} \left[\sum_{j \in \mathcal{N}_{pro}} (r_{op} Q_{o,j}(t) - r_{wp} Q_{w,j}(t)) \right. \\ & \left. - \sum_{j \in \mathcal{N}_{inj}} r_{wi} Q_{w,j}(t) \right] \end{aligned} \quad (18)$$

The factor e^{-dt} accounts for the time value of capital. The terms contributing to $J(t)$ are the value of the produced oil, the cost of separating water from the produced oil, and the cost of water injection. r_{op} is the oil price, r_{wp} is the water separation cost, and r_{wi} is the water injection cost. $Q_{o,j}(t)$ is the oil production and $Q_{w,j}(t)$ is the water production at production wells, $j \in \mathcal{N}_{pro}$, at time t . $Q_{w,j}(t)$ is water injection rate as the water injectors, $j \in \mathcal{N}_{inj}$. d is the continuous discount rate (cost of capital per unit time).

For water flooding using multiple injectors and producers, the well rates and pressures are adjusted by the optimal control problem (3) such that the NPV is maximized [1], [5].

The inequality constraints in (3) are bound constraints (3c) and rate-of-movement constraints (3d). The bound constraints corresponds to constraints on the water injection rates at the injectors and the bottom hole pressures (BHPs) at the production wells. The lower bounds on the water injection rates are zero, while the upper bound is computed such that no more than PV_{max} pore volumes

TABLE I
RESERVOIR PROPERTIES.

Symbol	Description	Value	Unit
ϕ	Porosity	0.2	-
c_r	Rock compressibility	0	Pa ⁻¹
ρ_o	Oil density (at 1 atm)	800	kg/m ³
ρ_w	Water density (at 1 atm)	1000	kg/m ³
c_o	Oil compressibility	10 ⁻⁵	Pa ⁻¹
c_w	Water compressibility	10 ⁻⁵	Pa ⁻¹
μ_o	Oil viscosity (dynamic)	1	cP
μ_w	Water viscosity (dynamic)	1	cP
S_{or}	Residual oil saturation	0.15	-
S_{wc}	Connate water saturation	0.20	-
$k_{ro,wc}$	End-point rel. perm., oil	0.8	-
$k_{rw,or}$	End-point rel. perm., water	0.6	-
n_o	Corey exponent, oil	2.0	-
n_w	Corey exponent, water	1.5	-
P_{init}	Initial reservoir pressure	200	atm
S_{init}	Initial water saturation	0.3	-

of water are injected over the time horizon considered, $[t_0, t_f]$. These bound constraints implies that we will implicitly satisfy

$$0 \leq \sum_{k=0}^{K-1} \sum_{n_k=0}^{N_k-1} \sum_{j=1}^{N_{inj}} Q_{n_k,j}^{wi} h_{n_k} \leq PV_{max} \quad (19)$$

The reservoir fluids are trapped inside the pores of a porous medium. The total void space of a reservoir is defined by the fraction (the porosity) of the porous medium that is not occupied by rock. Ideally we would replace and thus produce all the reservoir fluids by injecting one PV of water into the reservoir. The BHP's in the production wells are restricted to be lower than the initial pressure of the reservoir. The lower bound of the BHP's is chosen such that the pressure in the well is high enough to push the produced fluids to the production facilities. The rate-of-change constraints of both the injection rates and BHP's are chosen such that the controller is able to change e.g. the injection rate from maximum to minimum within a predefined number of control steps.

VI. NUMERICAL CASE STUDY

In this section, we apply our algorithm for the constrained optimal control problem (3) to maximize the Net Present Value of a horizontal 2D reservoir using water flooding and smart well technology. The permeability field of the reservoir is illustrated in Fig. 3 [1]. The reservoir dimensions are 450 × 450 × 10 m and it is discretized into 45 × 45 × 1 grid blocks. One horizontal injector (white squares at $x = 5$ m) and one horizontal producer (white circles at $x = 445$ m) are divided into 45 segments each. With this setup each grid block that is penetrated by a well represents a well segment. Table I lists the geological and fluid properties of the reservoir. The economical data are listed in Table II [2] [5]. The discount rate is zero, $d = 0$ [4]. Table III provides the constraints of the injection rates and the BHP's as well as the maximum allowed number of PV's to be injected over the period of production.

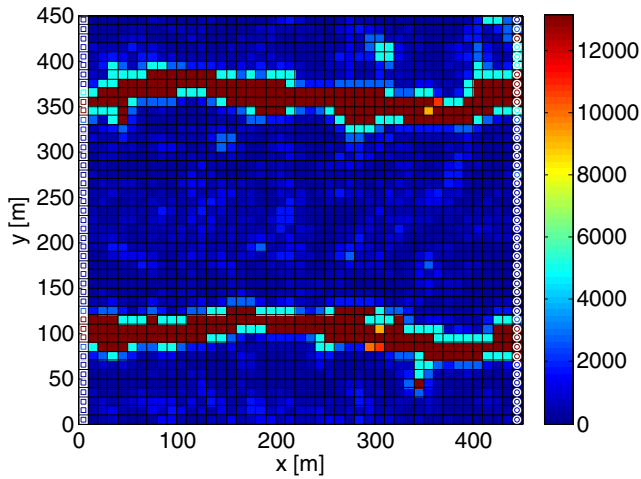


Fig. 3. Permeability field (mDa) with two high permeable streaks, 45 injector segments (white squares at $x = 5$ m) and 45 producer segments (white circles at $x = 445$ m) ([1]).

TABLE II
ECONOMIC DATA.

Symbol	Description	Value	Unit
r_{op}	Oil price	283,0	\$/m ³
r_{wp}	Cost of water separation	31,5	\$/m ³
r_{wi}	Cost of water injection	5,0	\$/m ³
d	Discount rate	0	-

We apply two different production strategies. In the first approach, we use fixed injection rates and fixed BHP's. In this case, 2 PV's are injected over a period of 728 days (2 years). In the second approach, we apply optimized well rates and pressures that we update every 28 days (4 weeks). This strategy leads to 1.00 PV injected over the optimal production period of 374 days. Fig. 4(a) illustrates the injected pore volumes as function of time. Fig. 4(b) illustrates that the recovery factor (produced oil related to the initial mass of oil in the reservoir) and the water cut (produced oil related to the total mass of produced reservoir fluids) as function of time. Fig. 5 shows the NPV as function of the time in which we develop the reservoir. Without control, the optimal development period is 484 days. In the case with optimized water injections and BHPs, the optimal development period is 374 days. NPV increases by approximately 10% by

TABLE III
CONTROLLER SETTINGS.

Symbol	Description	Value	Unit
Q_{min}^{wi}	Min. injection rate	0	m ³ /day
Q_{max}^{wi}	Max. injection rate	50	m ³ /day
BHP_{min}	Min. BHP in producers	150	atm
BHP_{max}	Max. BHP in producers	200	atm
ΔQ_{min}^{wi}	Max. rate of change	-3.85	m ³ /day
ΔQ_{max}^{wi}	Max. rate of change	3.85	m ³ /day
ΔBHP_{min}	Max. rate of change	-3.85	atm
ΔBHP_{max}	Max. rate of change	3.85	atm
PV_{max}	Max. PV's allowed	4	-
T	period of production	728	days

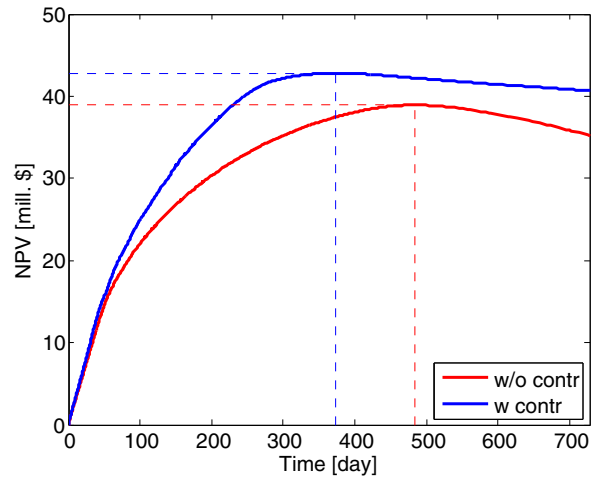


Fig. 5. NPV over 728 days of production. Red curve: Fixed injection rates and BHP's. Blue curve: Optimized injection rates and BHP's.

adjusting the water injections and BHPs compared to the uncontrolled case with a development period of 484 days. The recovery factor corresponding to optimal operation in the controlled case is 65%. In the uncontrolled case, the optimal recovery factor is 63%. The corresponding optimal water cuts are 62% in the controlled case and 72% in the uncontrolled case. Thus, the 10% increase in NPV for the controlled case is due to 2% increased oil recovery, a 10% decrease in produced water, and a reduction in injected water from 1.33 PV to 1.00 PV.

Fig. 7 illustrates the optimal water injection rates and the optimal BHPs for the controlled case. The water injection rates are increased in the injectors located at regions with low permeabilities. Similarly, the water injection rates are decreased for the injectors located in areas with high permeability. The BHPs are adjusted such that the back pressures are increased at locations with high water breakthrough. Fig. 8 illustrates the corresponding oil saturations of the reservoir at time 50, 125, 200, 374 days for the optimally controlled case. Fig. 6 shows the oil saturations for the uncontrolled case after 484 days of production.

VII. CONCLUSION

We have implemented a numerical method for solution of large-scale constrained optimal control problems (3). The implementation uses a novel formulation of the system dynamics that is relevant to describe flow in porous media. We use Explicit Singly Diagonally Implicit Runge-Kutta (ESDIRK) methods for the integration along with adaptive temporal step sizes. The optimization is based on single-shooting, the SQP optimization algorithm with line-search and BFGS approximations of the Hessian, and the adjoint method for computation of the gradients.

We use this algorithm to maximize the Net Present Value of an oil reservoir case study. In this case study,

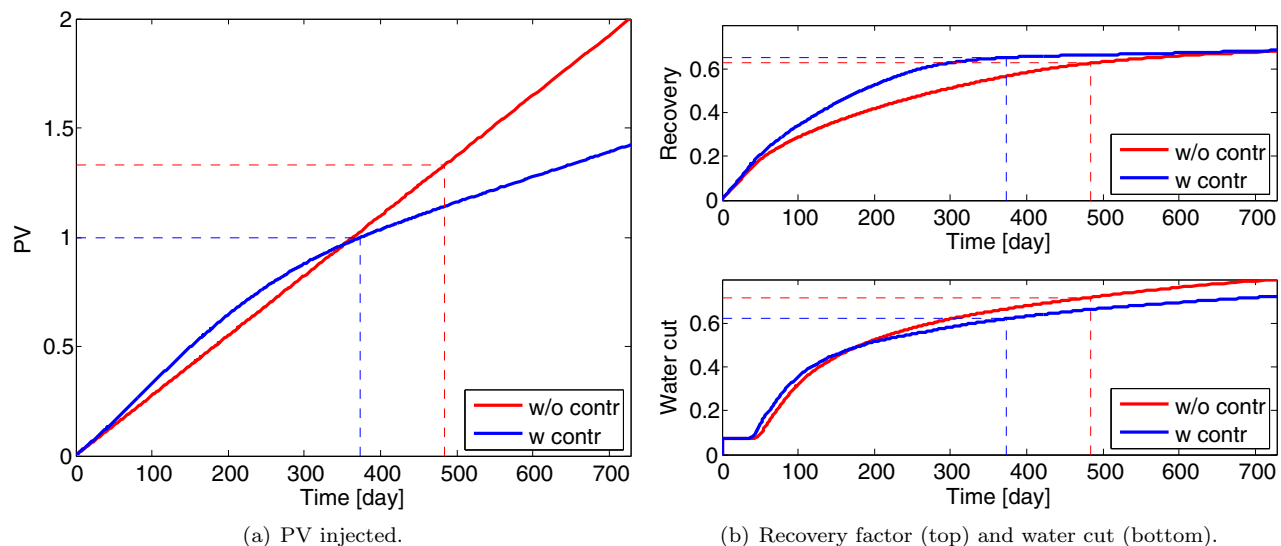


Fig. 4. Production data over 728 days of production. Red: Fixed injection rates and BHP's. Blue: Optimized injection rates and BHP's.

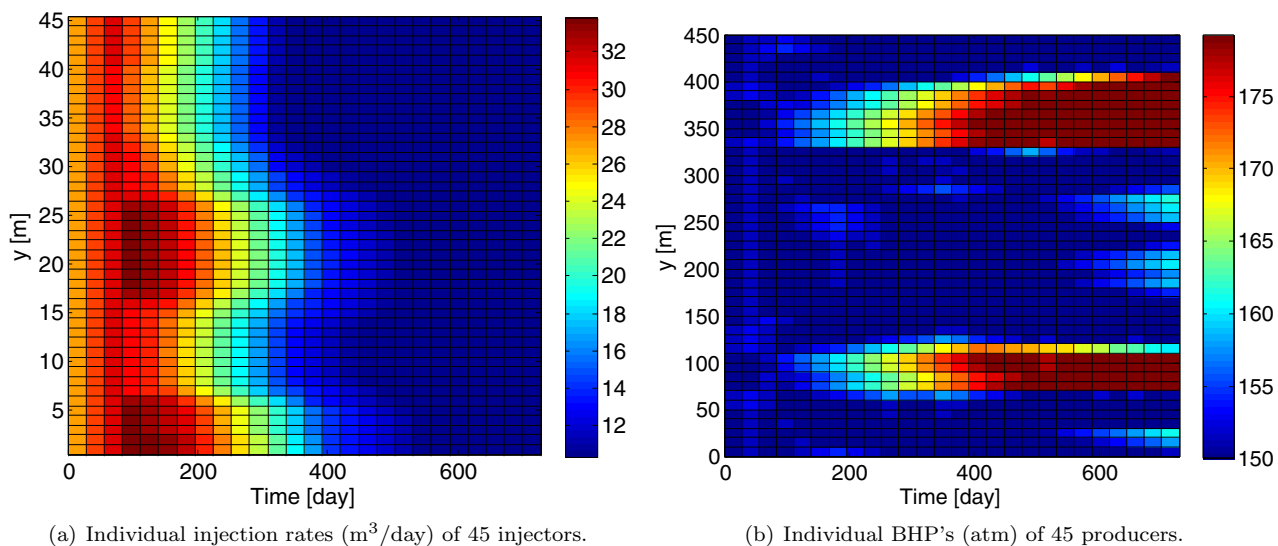


Fig. 7. Injection rates and BHP's over 728 days of production, updated each 28 days. The injectors and producers are depicted in Figure 3.

we use water flooding to produce the oil. The developed large-scale constrained optimal control algorithm computes the optimal profiles of water injection rates and the bottom hole pressures. Compared to the uncontrolled case, the Net Present Value in the controlled case increases by 10%. This figure demonstrates a significant economic potential of applying smart well technology along with constrained optimal control in oil reservoir management.

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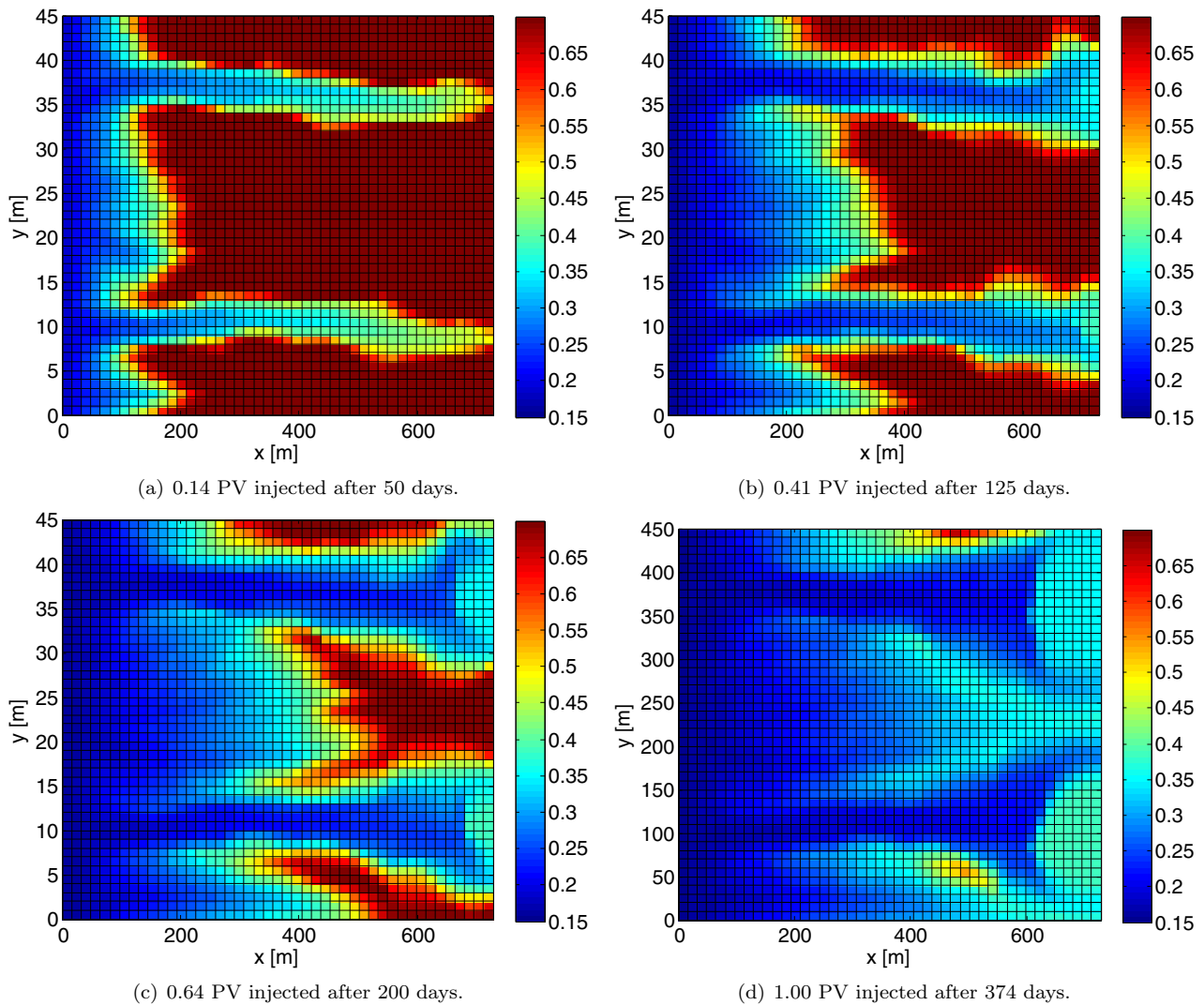


Fig. 8. Oil saturations after 50, 125, 200 and 374 days of production using optimized injection rates and BHP's.

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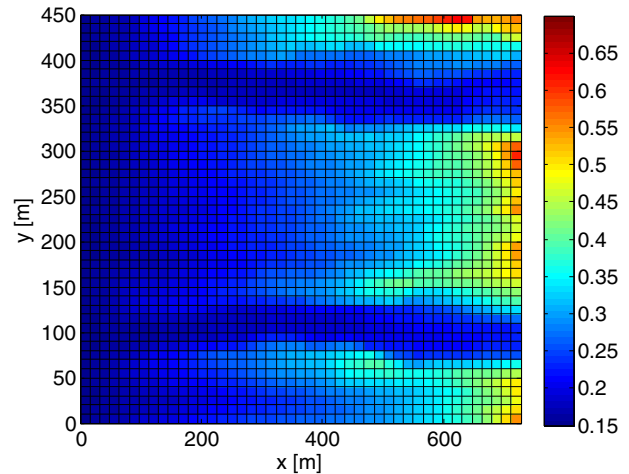


Fig. 6. Oil saturation 484 days of production using fixed injection rates and BHP's.