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Sliding Mode Exponential H$_\infty$ Synchronization of Markovian Jumping Master-Slave Systems with Time-Delays and Nonlinear Uncertainties

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Abstract— **This paper investigates the problem of exponential H$_\infty$ synchronization for a class of master-slave systems with both discrete and distributed time-delays, norm-bounded nonlinear uncertainties and Markovian switching parameters. Using an appropriate Lyapunov-Krasovskii functional, some delay-dependent sufficient conditions and a synchronization law which include the master-slave parameters are established for designing a delay-dependent mode-dependent sliding mode exponential H$_\infty$ synchronization control law in terms of linear matrix inequalities. The controller guarantees the H$_\infty$ synchronization of the two coupled master and slave systems regardless of their initial states. A numerical example is given to show the effectiveness of the method.**

***Index Terms*—Synchronization; master-slave systems; sliding mode; Delay; H$_\infty$ performance; Nonlinear uncertainties.**

I. Introduction

The sliding mode method has been recognized as one of the efficient tools to design robust controllers for the complex high-order nonlinear dynamic system operating under uncertainty conditions. The research in this area were initiated in the former Soviet Union about 40 years ago, and then the sliding mode control methodology has been receiving much more attention from the international control community within the last two decades. The major advantage of sliding mode is low sensitivity to plant parameter variations and disturbances which eliminates the necessity of exact modeling. Sliding mode control enables the decoupling of the overall system motion into independent partial components of lower dimension and, as a result, reduces the complexity of feedback design [1]-[2].

In recent years, more attention has been devoted to the study of stochastic hybrid systems, where the so-called Markov jump systems. These systems represent an important class of stochastic systems that is popular in modeling practical systems like manufacturing systems, power systems, aerospace systems and networked control systems that may experience random abrupt changes in their structures and parameters [3]-[6]. Random parameter changes may result from random component failures, repairs or shut down, or abrupt changes of the operating point. Many such events can be modeled using a continuous time finite-state Markov chain, which leads to the hybrid description of system dynamics known as a Markov jump parameter system [7]. Furthermore, the delay effects problem on the stability of systems is a problem of recurring interest since the delay presence may induce complex behaviors for the schemes, see for instance [8]. The problem of nonlinear filtering for state delayed systems with Markovian switching is proposed in [9]-[11].The problem of robust mode-dependent delayed state feedback H_∞ control is investigated for a class of uncertain time-delay systems with Markovian switching parameters and mixed discrete, neutral and distributed delays in [12]. Moreover, the sliding mode control problem for uncertain systems with time delays and stochastic jump systems are also investigated in [13]-[15], respectively. Recently, the problem of sliding mode control for a class of nonlinear uncertain stochastic systems with Markovian switching is studied in [16]. More recently, in [17], sliding mode control of nonlinear singular stochastic systems with Markovian switching is proposed.

On another research front line, synchronization is a basic motion in nature that has been studied for a long time, ever since the discovery of Christian Huygens in 1665 on the synchronization of two pendulum clocks. The results of chaos synchronization are utilized in biology, chemistry, secret communication and cryptography, nonlinear oscillation synchronization and some other nonlinear fields. The first idea of synchronizing two identical chaotic systems with different initial conditions was introduced by Pecora and Carroll in [18], and the method was realized in electronic circuits. The methods for synchronization of the chaotic systems have been widely studied in recent years, and many different methods have been applied theoretically and experimentally to synchronize chaotic systems; see for instance [19]-[22]. On the synchronization problems of systems with time-delays and nonlinear perturbation terms, we see that there have been some research works; see for instance [23]-[28] and the references therein. So the development of synchronization methods for master-slave systems with Markovian switching parameters and time-varying delays is important and has not been fully investigated in the past and remains to be important and challenging. This motivates the present study.

In this paper, the problem of exponential H$_\infty$ synchronization is studied for a class of master-slave systems with both discrete and distributed time-delays, norm-bounded nonlinear uncertainties and Markovian switching parameters. Using an appropriate Lyapunov-Krasovskii functional, some delay-dependent sufficient conditions and a synchronization law which include the master-slave parameters are established for designing a delay-dependent mode-dependent sliding mode exponential H$_\infty$ synchronization control law in terms of linear matrix inequalities (LMIs). The controller guarantees the H$_\infty$ synchronization of the two coupled master and slave systems

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regardless of their initial states. A numerical example is given to show the effectiveness of the method.

Notation: The notations used throughout the paper are fairly standard. I and 0 represent identity matrix and zero matrix; the superscript $'T'$ stands for matrix transposition. $\|.\|$ refers to the Euclidean vector norm or the induced matrix 2-norm. $\mathrm{diag}\{\cdots\}$ represents a block diagonal matrix and the operator $\mathrm{sym}(A)$ represents $A + A^T$. Let $\Re^+ = [0, \infty)$ and $\mathcal{E}\{.\}$ denotes the expectation operator with respect to some probability measure $\mathcal{P}$. If $x(t)$ is a continuous $\Re^n$-valued stochastic process on $t \in [-\kappa, \infty)$, we let $x_t = \{x(t+\theta): -\kappa \le \theta \le 0\}$ for $t \ge 0$ which is regarded as a $C([-\kappa, 0]; \Re^n)$-valued stochastic process. The notations g_{ji} stand for $g_j(i)$. The notation $P > 0$ means that P is real symmetric and positive definite; the symbol $*$ denotes the elements below the main diagonal of a symmetric block matrix.

II. PROBLEM DESCRIPTION

Consider a model of master and slave systems with Markovian switching parameters and mixed discrete and distributed time-varying delays and nonlinear perturbations in the form of

$$\begin{cases} \dot{x}_m(t) = A_1(r(t))x_m(t) + A_2(r(t))\, x_m(t-h(t)) + A_3(r(t)) \int\limits_{t-\tau(t)}^{t} x_m(s)\, ds \\ \quad + N_1(r(t))f_1(t; x_m(t)) + N_2(r(t))\, f_2(t, x_m(t-h(t))), \\ x_m(t) = \phi(t), \qquad t \in [-\kappa, 0], \\ z_m(t) = C_1(r(t))x_m(t) + C_2(r(t))x_m(t-h(t)) + C_3(r(t)) \int\limits_{t-\tau(t)}^{t} x_m(s)\, ds \end{cases} \tag{1a-c}$$

$$\begin{cases} \dot{x}_s(t) = A_1(r(t))x_s(t) + A_2(r(t))x_s(t-h(t)) + A_3(r(t)) \int\limits_{t-\tau(t)}^{t} x_s(s)\, ds \\ \quad + N_1(r(t))f_1(t; x_s(t)) + N_2(r(t))f_2(t, x_s(t-h(t))) \\ \qquad + B(r(t))u(t) + D(r(t))\, w(t), \\ x_s(t) = \varphi(t), \qquad t \in [-\kappa, 0], \\ z_s(t) = C_1(r(t))x_s(t) + C_2(r(t))x_s(t-h(t)) + C_3(r(t)) \int\limits_{t-\tau(t)}^{t} x_s(s)\, ds \end{cases} \tag{2a-c}$$

where $\kappa := \max\{h_M, \tau_m\}$, $x_m(t), x_s(t)$ are the $n \times 1$ state vector of the master and slave systems, respectively and $u(t)$ is the $r \times 1$ control input. $A_i(r(t)), B(r(t)), D(r(t))$ and $C_i(r(t))$ are matrix functions of the random jumping process $\{r(t)\}$. $\{r(t), t \ge 0\}$ is a right-continuous Markov process on the probability space which takes values in a finite space $S = \{1, 2, \ldots, s\}$ with generator $\Pi = [\pi_{ij}]$ $(i, j \in S)$ given by

$$P\{r(t+\Delta) = j | r(t) = i\} = \begin{cases} \pi_{ij}\Delta + o(\Delta), & \text{if } i \ne j \\ 1 + \pi_{ii}\Delta + o(\Delta), & \text{if } i = j \end{cases} \tag{3}$$

where $\Delta > 0$, $\lim_{\Delta \to 0} o(\Delta)/\Delta = 0$ and $\pi_{ij} \ge 0$, for $i \ne j$, is the transition rate from mode i at time t to mode j at time $t + \Delta$ and $\pi_{ii} = -\sum_{j=1, j \ne i}^{j=s} \pi_{ij}$. The vector valued initial functions $\phi(t)$ and $\varphi(t)$ are continuously differentiable functionals and $f_i(.,.)$ are also time-varying vector-valued functions. The time delays are satisfying

$$0 < h(t) \le h_M, \qquad \dot{h}(t) \le h_D \tag{4a}$$

$$0 < \tau(t) \le \tau_M, \qquad \dot{\tau}(t) \le \tau_D \tag{4b}$$

Assumption 1. The continuous functions $f_i : \Re^+ \times \Re^n \to \Re^{n'}$ are unknown and satisfy $f_i(t, 0) = 0$ and the Lipschitz conditions, i.e., $\|f_i(t, x_0) - f_i(t, y_0)\| \le \|\Gamma_i(x_0 - y_0)\|$ for all t and for all $x_0, y_0 \in \Re^n$ such that Γ_i are some known matrices.

Definition 1. The Markovian systems (1)-(2) are said to be globally exponentially stable in the mean square sense if, when $u(t) = 0$, for any finite $\varphi(t), \phi(t) \in \Re^n$ defined on $[-\kappa, 0]$, and $r_0 \in S$ the following condition is satisfied

$$\mathcal{E}\{\|e(t)\|^2\} \le c e^{-\alpha t} \sup_{-\kappa \le s \le 0} \|e(s)\|^2, \quad t > 0$$

where $e(t) = x_m(t) - x_s(t)$ is the synchronization error of the master and slave systems (1)-(2) system from initial system state $\phi(0) - \varphi(0)$ and initial mode r_0, and c is a positive constant.

Definition 2. The H_∞ performance measure of the system (1)-(2) is defined as $J_\infty = \mathcal{E}\left(\int_0^\infty [z_e(t)^T z_e(t) - \gamma^2 w^T(t) w(t)]\, dt\right)$, where $z_e(t) = z_m(t) - z_s(t)$ and the positive scalar γ is given.

Remark 1. The model (1)-(2) can describe a large amount of well-known dynamical systems with time-delays, such as the delayed Logistic model, the chaotic models with time-delays and the artificial neural network model with discrete time-delays. In real application, these coupled systems can be regarded as interacting dynamical elements in the entire system, such as physical particles, biological neurons, ecological populations, genic oscillations, and even automatic machines and robots. A feasible coupling design for successful synchronization leads us to fully command the intrinsic mechanism regulating the evolution of real systems, to fabricate emulate systems, and even to remotely control the machines and nodes in networks with large scales [23], [29].

Assumption 2. The full state variables $x_s(t)$ and $x_m(t)$ are available for measurement.

Now, it is required to synchronize the slave system with the master system at the same time. Let $\hat{e}(t) = e^{\alpha t} e(t)$ and α is called the exponential decay rate. Then the error dynamics, namely synchronization error system, can be expressed by

$$\begin{cases} \dot{\hat{e}}(t) = (A_1(r(t)) + \alpha I)\hat{e}(t) + e^{\alpha h(t)} A_2(r(t))\hat{e}(t-h(t)) \\ \quad + A_3(r(t)) \int\limits_{t-\tau(t)}^{t} e^{\alpha(t-s)} \hat{e}(s)\, ds + N_1(r(t))\hat{f}_1(t; \hat{e}(t)) \\ \quad + N_2(r(t))\hat{f}_2(t; \hat{e}(t-h(t))) - B(r(t))\hat{u}(t) - D(r(t))\hat{w}(t), \\ \hat{e}(t) = \phi(t) - \varphi(t), \qquad t \in [-\kappa, 0], \\ \hat{z}_e(t) = C_1(r(t))\hat{e}(t) + e^{\alpha h(t)} C_2(r(t))\hat{e}(t-h(t)) \\ \quad + C_3(r(t)) \int\limits_{t-\tau(t)}^{t} e^{\alpha(t-s)} \hat{e}(s)\, ds \end{cases} \tag{5a-c}$$

where $\hat{u}(t) = e^{\alpha t} u(t)$, $\hat{w}(t) = e^{\alpha t} w(t)$, $\hat{z}_e(t) = \hat{z}_m(t) - \hat{z}_s(t) = e^{\alpha t}(z_m(t) - z_s(t))$, $\hat{f}_1(t; \hat{e}(t)) := e^{\alpha t}(f_1(t; x_m(t)) - f_1(t; x_m(t) - e(t)))$, $\hat{f}_2(t; \hat{e}(t-h(t))) := e^{\alpha t}(f_2(t; x_m(t-h(t))) - f_2(t; x_m(t-h(t)) - e(t-h(t))))$.

From Assumption 1, the corresponding uncertainty set is denoted by

$$\Xi_i(\hat{e}(t)) := \{\hat{f}_i(t,\hat{e}(t)) : \left\|\hat{f}_i(t,\hat{e}(t))\right\| \le \left\|\Gamma_i \hat{e}\right\|\} \quad (6)$$

Now the original synchronization problem can be replaced by the equivalent problem of stabilizing the system (5) by a suitable choice of the sliding mode control. In the following, the sliding mode controller will be designed using variable structure control and sliding mode control methods [1]. Let us introduce the sliding surface as

$$S_i(t) = V_i\hat{e}(t) - V_i\int_0^t[(A_{1i} + \alpha I - B_iK_{1i})\hat{e}(s) + e^{\alpha h(s)}(A_{2i} - B_iK_{2i})\hat{e}(s - h(s))]\,ds - V_i\int_0^t\int_{s-\tau(s)}^{s} e^{\alpha(s-r)}(A_{3i} - B_iK_{3i})\hat{e}(r)\,dr\,ds \quad (7)$$

where $V_i \in \mathcal{R}^{m\times n}$, $K_{ji} \in \mathcal{R}^{r\times n}, j = 1,2,3$ are real matrices to be designed. It is clear that $\dot{S}_i(t) = 0$ is a necessary condition for the state trajectory to stay on the switching surface $S_i(t) = 0$. Therefore, by $\dot{S}_i(t) = 0$ and (5), we get

$$0 = V_i[B_iK_{1i}\hat{e}(t) + e^{\alpha h(t)} B_iK_{2i}\hat{e}(t - h(t)) + \int_{t-\tau(t)}^{t} e^{\alpha(t-r)} B_iK_{3i}\,\hat{e}(r)\,dr + N_{1i}\hat{f}_1(t,\hat{e}(t)) + N_{2i}\hat{f}_2(t,\hat{e}(t - h(t))) - B_i\,\hat{u}_i(t) - D_i\,\hat{w}(t)] \quad (8)$$

Solving equation (8) for $\hat{u}_i(t)$ yields the equivalent control

$$\hat{u}_{i_{eq}}(t) = K_{1i}\hat{e}(t) + e^{\alpha h(t)} K_{2i}\hat{e}(t - h(t)) + \int_{t-\tau(t)}^{t} e^{\alpha(t-r)}K_{3i}\,\hat{e}(r)\,dr + \tilde{V}_iN_{1i}\hat{f}_1(t,\hat{e}(t)) + \tilde{V}_iN_{2i}\hat{f}_2(t,\hat{e}(t - h(t))) - \tilde{V}_iD_i\,\hat{w}(t) \quad (9)$$

where $\tilde{V}_i = (V_iB_i)^{-1}V_i$. By using (9) in (5), the error dynamics in sliding mode is given as follows:

$$\begin{cases} \dot{\hat{e}}(t) = (A_{1i} + \alpha I - B_i K_{1i})\hat{e}(t) + e^{\alpha h(t)}(A_{2i} - B_i K_{2i})\hat{e}(t-h(t)) \\ \quad + (A_{3i} - B_i K_{3i})\int_{t-\tau(t)}^{t} e^{\alpha(t-s)}\hat{e}(s)\,ds + (N_{1i} - B_i\tilde{V}_iN_{1i})\hat{f}_1(t;\hat{e}(t)) \\ \quad + (N_{2i} - B_i\tilde{V}_iN_{2i})\hat{f}_2(t;\hat{e}(t-h(t))) - (I + B_i\tilde{V}_i)D_i\,\hat{w}(t), \\ \hat{e}(t) = \phi(t) - \varphi(t), \qquad t \in [-\kappa, 0], \\ \hat{z}_e(t) = C_1(r(t))\hat{e}(t) + e^{\alpha h(t)}C_2(r(t))\hat{e}(t-h(t)) + C_3(r(t))\int_{t-\tau(t)}^{t} e^{\alpha(t-s)}\hat{e}(s)\,ds \end{cases} \quad (10a\text{-}c)$$

The problem to be addressed in this paper is formulated as follows: given the master-slave systems (1)-(2) with both discrete and distributed time-delays and Markovian switching parameters, find a mode-dependent sliding mode exponential H_∞ synchronization control $u(t)$ with any $r(t) = i \in S$ for the slave system (2) such that the resulting closed-loop system is stochastically stable and satisfies an H_∞ norm bound γ, i.e. $J_\infty < 0$.

III. MAIN RESULTS

In this section, we propose sufficient conditions for the stochastic stability of the sliding error motion (10) using the Lyapunov method.

3.1 H_∞ performance analysis

Define the following Lyapunov-Krasovskii functional

$$V(\hat{e}(t),t,i) = V_1(\hat{e}(t),t,i) + V_2(\hat{e}(t),t,i) + V_3(\hat{e}(t),t,i), \quad (11)$$

with $V_1(\hat{e},t,i) = \hat{e}(t)^T P_{1i}\,\hat{e}(t)$, $V_2(\hat{e},t,i) = \int_{t-h(t)}^{t} \hat{e}(\xi)^T S_i\,\hat{e}(\xi)\,d\xi + \int_{t-h(t)}^{t}\int_{\xi}^{t} \hat{e}(s)^T R_1\,\hat{e}(s)\,ds\,d\xi$, $V_3(\hat{e},t,i) = \int_{t-\tau(t)}^{t}[\int_s^t \hat{e}(\theta)^T\,d\theta]U_1[\int_s^t \hat{e}(\theta)\,d\theta]\,ds + \int_0^{\tau(t)}\int_{t-s}^{t}(\theta - t + s)\hat{e}(\theta)^T U_1\,\hat{e}(\theta)\,d\theta\,ds$, where P_{1i}, S_i, R_i and U_i are mode-dependent matrix functions. The weak infinitesimal operator $\mathcal{L}V(.)$ of the stochastic process $\{(e_t, r(t)), t \ge 0\}$, acting on $V \in C(\Re^n \times \Re^+ \times S)$ at the point $\{t, \hat{e}(t), r(t) = i\}$, is given by (see Lemma 3.1, [30])

$$\begin{aligned}\mathcal{L}V(\hat{e},t,i) &= \lim_{\Delta\to0^+}\frac{1}{\Delta}\{\mathcal{E}[V(\hat{e}(t+\Delta),\hat{e}_{t+\Delta},t+\Delta,r(t+\Delta)|\hat{e}(t),\hat{e}_t,t,r(t)=i)] - V(\hat{e},t,i)\} \\ &= V_t(\hat{e},t,i) + \dot{\hat{e}}(t)^T V_e(\hat{e},t,i) + \sum_{j=1}^{s}\pi_{ij}V(\hat{e},t,j)\end{aligned}$$

where $V_t(\hat{e},t,i) = \frac{\partial V(\hat{e},t,i)}{\partial t}$, $V_e(\hat{e},t,i) = (\frac{\partial V(\hat{e},t,i)}{\partial \hat{e}_1}, \cdots, \frac{\partial V(\hat{e},t,i)}{\partial \hat{e}_n})^T$. Differentiating $V_1(\hat{e},t,i)$ in t we obtain

$$\begin{aligned} LV_1(\hat{e},t,i) &= 2\hat{e}(t)^T P_{1i}((A_{1i} + \alpha I - B_i K_{1i})\hat{e}(t) \\ &+ e^{\alpha h(t)}(A_{2i} - B_i K_{2i})\hat{e}(t-h(t)) + (A_{3i} - B_i K_{3i})\int_{t-\tau(t)}^{t} e^{\alpha(t-s)}\hat{e}(s)\,ds \\ &+ (N_{1i} - B_i\tilde{V}_iN_{1i})\hat{f}_1(t;\hat{e}(t)) + (N_{2i} - B_i\tilde{V}_iN_{2i})\hat{f}_2(t;\hat{e}(t-h(t))) \\ &- (I + B_i\tilde{V}_i)D_i\,\hat{w}(t)) + \sum_{j=1}^{s}\hat{e}(t)^T\pi_{ij}P_{1j}\,\hat{e}(t) \end{aligned} \quad (12)$$

Differentiating other Lyapunov terms in (11) give

$$\begin{aligned} LV_2(\hat{e},t,i) &= h(t)\hat{e}(t)^T R_1\,\hat{e}(t) - (1-\dot{h}(t))\int_{t-h(t)}^{t}\hat{e}(s)^T R_1\,\hat{e}(s)\,ds + \hat{e}(t)^T S_i\,\hat{e}(t) \\ &- (1-\dot{h}(t))\hat{e}(t-h(t))^T S_i\,\hat{e}(t-h(t)) + \sum_{j=1}^{s}\int_{t-h(t)}^{t}\hat{e}(\xi)^T \pi_{ij}S_j\,\hat{e}(\xi)\,d\xi \\ &\le \hat{e}(t)^T(h_M R_1 + S_i)\,\hat{e}(t) + \int_{t-h(t)}^{t}\hat{e}(s)^T(\sum_{j=1}^{s}\pi_{ij}S_j - (1-h_D)R_1)\,\hat{e}(s)\,ds \\ &- (1-h_D)\hat{e}(t-h(t))^T S_i\,\hat{e}(t-h(t)) \end{aligned} \quad (13)$$

and

$$\begin{aligned} LV_3(\hat{e},t,i) &= -(1-\dot{\tau}(t))[\int_{t-\tau(t)}^{t}\hat{e}(\theta)^T\,d\theta]U_1[\int_{t-\tau(t)}^{t}\hat{e}(\theta)\,d\theta] \\ &+ 2\int_{t-\tau(t)}^{t}(\theta - t + \tau(t))\hat{e}(t)^T U_i\,\hat{e}(\theta)\,d\theta \\ &+ \int_0^{\tau(t)} s\hat{e}(t)^T U_1\,\hat{e}(t)\,ds - \int_0^{\tau(t)}\int_{t-s}^{t}\hat{e}(\theta)^T U_1\,\hat{e}(\theta)\,d\theta\,ds \\ &\le \int_{t-\tau(t)}^{t}(\theta - t + \tau(t))[\hat{e}(t)^T U_1\hat{e}(t) + \hat{e}(\theta)^T U_1\hat{e}(\theta)]\,d\theta \\ &- (1-\dot{\tau}(t))[\int_{t-\tau(t)}^{t}\hat{e}(\theta)^T\,d\theta]U_1[\int_{t-\tau(t)}^{t}\hat{e}(\theta)\,d\theta] \\ &+ \int_0^{\tau(t)} s\hat{e}(t)^T U_1\,\hat{e}(t)\,ds - \int_{t-\tau(t)}^{t}(\theta - t + \tau(t))\hat{e}(\theta)^T U_1\hat{e}(\theta)\,d\theta \\ &= \tau_M^2\,\hat{e}(t)^T U_1\,\hat{e}(t) - (1-\tau_D)[\int_{t-\tau(t)}^{t}\hat{e}(\theta)^T\,d\theta]U_1[\int_{t-\tau(t)}^{t}\hat{e}(\theta)\,d\theta] \end{aligned} \quad (14)$$

According to Assumption 1, ones read

$$-\hat{f}_1(t,\hat{e}(t))^T\,\hat{f}_1(t,\hat{e}(t)) + \hat{e}(t)^T\Gamma_1^T\Gamma_1\,\hat{e}(t) \ge 0, \quad (15a)$$

$$-\hat{f}_2(t;\hat{e}(t-h(t)))^T \hat{f}_2(t;\hat{e}(t-h(t)))+\hat{e}(t)^T \Gamma_2^T \Gamma_2 \hat{e}(t) \geq 0 \quad (15b)$$

On the other hand, for a prescribed $\gamma > 0$ and under zero initial conditions, J_∞ can be rewritten as

$$J_\infty \leq \mathcal{E}\left(\int_0^\infty e^{-2\alpha t}[\hat{z}_e(t)^T\hat{z}_e(t) - \gamma^2\hat{w}^T(t)\hat{w}(t)]\ dt + V(\hat{e},t,i)|_{t\to\infty} - V(\hat{e},t,i)|_{t=0}\right)$$

$$\leq \mathcal{E}\left(\int_0^\infty e^{-2\alpha\kappa}[\hat{z}_e(t)^T\hat{z}_e(t) - \gamma^2\hat{w}^T(t)\hat{w}(t) + \mathcal{L}V(\hat{e},t,i)]\ dt\right)$$

From the obtained derivative terms in (12)-(14) and adding the left-hand side of the equation (15) into $\mathcal{L}V(\hat{e},t,i)$, we obtain

$$J_\infty \leq \mathcal{E}\left(\int_0^\infty \tilde{\chi}_e(t)^T\tilde{\Xi}_{ei}\tilde{\chi}_e(t)\ dt\right) \quad (16)$$

where

$$\tilde{\chi}_e(t) = [\hat{e}(t)^T, \hat{e}(t-h(t))^T, \int_{t-\tau(t)}^t e^{\alpha(t-\kappa)}\hat{e}(\kappa)^T d\kappa, \hat{f}_1(t,\hat{e}(t))^T, \hat{f}_2\left(t,\hat{e}(t-h(t))\right)^T, \hat{w}(t)^T]^T$$

and

$$\tilde{\Xi}_{ei} = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} & \Sigma_{13} & \Sigma_{14} & \Sigma_{15} & -P_{1i}(I+B_i\tilde{V}_i)D_i \\ * & \Sigma_{22} & e^{\alpha h_M}e^{-2\alpha\kappa}C_{2i}^TC_{3i} & 0 & 0 & 0 \\ * & * & -(1-\tau_D)U_1 + e^{-2\alpha\kappa}C_{3i}^TC_{3i} & 0 & 0 & 0 \\ * & * & * & -I & 0 & 0 \\ * & * & * & * & -I & 0 \\ * & * & * & * & * & -\gamma^2e^{-2\alpha\kappa}I \end{bmatrix} \quad (17)$$

with $\Sigma_{11} := \mathrm{sym}(P_{1i}(A_{1i}+\alpha I - B_iK_{1i})) + \sum_{j=1}^s \pi_{ij}P_{1j} + h_MR_1 + S_i + \tau_M^2 U_1 + \Gamma_1^T\Gamma_1 + \Gamma_2^T\Gamma_2 + e^{-2\alpha\kappa}C_{1i}^TC_{1i}$, $\Sigma_{12} := e^{\alpha h_M}P_{1i}(A_{2i} - B_iK_{2i}) + e^{\alpha h_M}e^{-2\alpha\kappa}C_{1i}^TC_{2i}$, $\Sigma_{22} := -(1-h_D)S_i + e^{2\alpha h_M}e^{-2\alpha\kappa}C_{2i}^TC_{2i}$, $\Sigma_{13} := P_{1i}(A_{3i} - B_iK_{3i}) + e^{-2\alpha\kappa}C_{1i}^TC_{3i}$, $\Sigma_{1(3+j)} := P_{1i}(N_{ji} - B_i\tilde{V}_iN_{ji})$, $j=1,2$.

Then, the conditions

$$\tilde{\Xi}_{ei} < 0, \quad (18a)$$

$$\sum_{j=1}^s \pi_{ij}S_j - (1-h_D)R_1 \leq 0, \quad (18b)$$

mean that the condition $J_\infty < 0$ is satisfied. Moreover, the condition $J_\infty < 0$ for $w(t) = 0$ implies $\mathcal{E}\{\mathcal{L}V(\hat{e},t,i)\} < 0$. Then, we have

$$\mathcal{E}\{\mathcal{L}V(\hat{e},t,i)\} \leq -\sigma_1\mathcal{E}\{\hat{e}(t)^T\hat{e}(t)\} \quad (19)$$

where $\sigma_1 = \min\{\lambda_{\min}(-\tilde{\Xi}_{ei}), i \in S\}$, then $\sigma_1 > 0$. By Dynkin's formula, we have

$$\mathcal{E}\{V(\hat{e},t,i)\} - \mathcal{E}\{V(\phi(0)-\varphi(0),r_0,0)\} \leq -\sigma_1\mathcal{E}\left\{\int_0^t \hat{e}(s)^T\hat{e}(s)\ ds\right\}$$

or

$$\mathcal{E}\left\{\int_0^t \hat{e}(s)^T\hat{e}(s)\ ds\right\} \leq \sigma_1^{-1}V(\phi(0)-\varphi(0),r_0) \quad (20)$$

Moreover, from LKF (11) the following condition holds

$$\mathcal{E}\{V(\hat{e},t,i)\} \geq \sigma_2\mathcal{E}\{\hat{e}(t)^T\hat{e}(t)\} \quad (21)$$

where $\sigma_2 = \min\{\lambda_{\min}(P_{1i}), i \in S\}$. From (19)-(21), we obtain

$$\mathcal{E}\{\hat{e}(t)^T\hat{e}(t)\} \leq -\sigma_1\sigma_2^{-1}\mathcal{E}\left\{\int_0^t \hat{e}(s)^T\hat{e}(s)\ ds\right\} + \sigma_2^{-1}V(\phi(0)-\varphi(0),r_0).$$

Hence

$$\mathcal{E}\left\{\int_0^t \hat{e}(s)^T\hat{e}(s)\ ds\right\} \leq \sigma_1^{-1}[1 - e^{-\sigma_1\sigma_2^{-1}t}]\ V(\phi(0)-\varphi(0),0,r_0)$$

or

$$\lim_{t\to\infty}\mathcal{E}\left\{\int_0^t \hat{e}(s)^T\hat{e}(s)\ ds\right\} \leq \sigma_1^{-1}V(\phi(0)-\varphi(0),0,r_0) \quad (22)$$

Moreover, from the Chebyshev integral inequality and considering $e^{\alpha t} \geq 1+\alpha t$, it is clear that

$$\int_0^t \hat{e}(s)^T\hat{e}(s)\ ds \geq \frac{1}{t}\int_0^t e^{2\alpha s}\ ds\ .\int_0^t e(s)^Te(s)\ ds \geq \frac{1}{2}e^{\alpha t}\ .\int_0^t e(s)^Te(s)\ ds$$

Therefore, from (22) and the inequality above, we have

$$\lim_{t\to\infty}\mathcal{E}\left\{\int_0^t e(s)^Te(s)\ ds\right\} \leq 2\sigma_1^{-1}e^{-\alpha t}\ V(\phi(0)-\varphi(0),0,r_0) \quad (23)$$

which indicates that, from Definition 1, the system in (10) with Markovian switching parameters in (3) is globally exponentially stable in the mean square sense and has the exponential decay rate α. The following result is now concluded for the H_∞ performance analysis of the error dynamics (10) with Markovian switching parameters.

Theorem 1. Let the matrices V_i, K_{ji} $(i = 0,1,\cdots,N; j = 1,2,3)$ with $\det(V_iB_i) \neq 0$ be given. The master-slave time-delay systems (1)-(2) with Markovian switching parameters in (3) is synchronized exponentially with a degree α and an H_∞ performance level $\gamma > 0$ at least in the sense of Definition 1, if there exist some positive definite matrices P_{1i}, U_1, S_i satisfying the LMIs (18).

Remark 2. If the switching modes are not considered, i.e. $S = \{1\}$, the jumping master-slave systems (1)-(2) are simplified into a general linear system with nonlinearities and time delays. Then it is easy to conclude a criterion from Theorem 1, which can be used to determine the stability of such master-slave systems.

Now we are in the position to solve the synchronization problem of the systems (1)-(2). Based on Theorem 1, we can obtain a mode-dependent delayed H_∞ synchronization law in the form of (9) in the following theorem.

Theorem 2. Under Assumptions 1-2, a synchronization law given in the form (9) exists such that the Markovian jumping synchronization error system (5) with time-varying delays in (4) is stochastically exponentially stable with a degree α and an H_∞ performance level $\gamma > 0$ at least in the sense of Definition 1, if there exist some matrices $\tilde{K}_{ji}, \tilde{V}_i$ and positive definite matrices $\bar{P}_{1i}, \tilde{U}_1, \tilde{R}_1, \tilde{S}_i$ $(i = 1,\cdots,s; j = 1,2,3)$ satisfying the following LMIs

$$\begin{bmatrix} \Sigma_{11} & \Sigma_{12} & \Sigma_{13} & \Sigma_{14} & \Sigma_{15} & e^{-\alpha\kappa}\bar{P}_{1i}C_{1i}^T & \Sigma_{17} \\ * & -(1-h_D)\tilde{S}_i & 0 & 0 & 0 & e^{\alpha(h_M-\kappa)}\bar{P}_{1i}C_{2i}^T & 0 \\ * & * & -(1-\tau_D)\tilde{U}_1 & 0 & 0 & e^{-\alpha\kappa}\bar{P}_{1i}C_{3i}^T & 0 \\ * & * & * & \Sigma_{44} & 0 & 0 & 0 \\ * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & \Sigma_{77} \end{bmatrix} < 0 \quad (24a)$$

$$\sum_{j=1}^s \pi_{ij}\tilde{S}_j - (1-h_D)\tilde{R}_1 \leq 0, \quad (24b)$$

$$\tilde{V}_iB_i = I, \quad (24d)$$

where $\Sigma_{11} := \mathrm{sym}((A_{1i}+\alpha I)\bar{P}_{1i} - B_i\tilde{K}_{1i}) + h_M\tilde{R}_1 + \tilde{S}_i + \tau_M^2\tilde{U}_1$, $\Sigma_{12} := e^{\alpha h_M}(A_{2i}\bar{P}_{1i} - B_i\tilde{K}_{2i})$, $\Sigma_{13} := A_{3i}\bar{P}_{1i} - B_i\tilde{K}_{3i}$, $\Sigma_{15} := \left[\bar{P}_{1i}\Gamma_1^T \quad \bar{P}_{1i}\Gamma_2^T\right]$, $\Sigma_{14} := [N_{1i} - B_i\tilde{V}_iN_{1i}, N_{2i} - B_i\tilde{V}_iN_{2i}, -(I+B_i\tilde{V}_i)D_i]$, $\Sigma_{44} := \mathrm{diag}\{-I,-I,-\gamma^2e^{-2\alpha\kappa}I\}$, $\Sigma_{17} := \left[\sqrt{\pi_{i1}}\bar{P}_{1i} \quad \cdots \quad \sqrt{\pi_{ii-1}}\bar{P}_{1i} \quad \sqrt{\pi_{ii+1}}\bar{P}_{1i} \quad \cdots \quad \sqrt{\pi_{is}}\bar{P}_{1i}\right]$ and $\Sigma_{77} := \mathrm{diag}\{-\sqrt{\pi_{i1}}\bar{P}_{11},\cdots,-\sqrt{\pi_{ii-1}}\bar{P}_{11},-\sqrt{\pi_{ii+1}}\bar{P}_{1s},\cdots,-\sqrt{\pi_{is}}\bar{P}_{1s}\}$.

Then, the equivalent control in (9) is given by

$$\hat{u}_{i_{eq}}(t) = \tilde{K}_{1i}\bar{P}_{1i}^{-1}\hat{e}(t) + e^{\alpha h(t)}\tilde{K}_{2i}\bar{P}_{1i}^{-1}\hat{e}(t-h(t)) + \int_{t-\tau(t)}^{t} e^{\alpha(t-r)}\tilde{K}_{3i}\bar{P}_{1i}^{-1}\hat{e}(r)\,dr + \tilde{V}_i N_{1i}\hat{f}_1(t,\hat{e}(t)) + \tilde{V}_i N_{2i}\hat{f}_2(t,\hat{e}(t-h(t))) \quad (25)$$

Proof. By performing a congruence transformation $\mathrm{diag}\{\bar{P}_{1i}^T, I, \dots, I\}$, where $\bar{P}_{1i} := P_{1i}^{-1}$, to both sides of (18a), applying Schur complements and considering $\tilde{K}_{li} = K_{li}\bar{P}_{li}$ result in (24a). The other two conditions (24b)-(24c) are easily concluded from Theorem 1. ■

Remark 3. By setting $\delta = \gamma^2$ and minimizing δ subject to (24), we can obtain the optimal H_∞ performance index γ^* (by $\gamma = \sqrt{\gamma}$) and the corresponding control gains as well.

3.2 Sliding model control design

After designing the switching surface, in this section, an appropriate control law will be constructed such that the system state trajectories from arbitrary initial values are globally attracted to the switching surface in a finite time and maintain them on the surface afterwards.

Theorem 3. Under Assumptions 1-2, it is supposed that the sliding surface function is given as (7), where V_i is chosen to satisfy $V_i(I - B_i\tilde{V}_i) = 0$ and V_iB_i is nonsingular. Then, the trajectories of the error dynamics (5) can be driven onto the sliding mode surface if the control is designed as follows

$$\hat{u}_i(t) = \tilde{K}_{1i}\bar{P}_{1i}^{-1}\hat{e}(t) + e^{\alpha h(t)}\tilde{K}_{2i}\bar{P}_{1i}^{-1}\hat{e}(t-h(t)) + \int_{t-\tau(t)}^{t} e^{\alpha(t-r)}\tilde{K}_{3i}\bar{P}_{1i}^{-1}\hat{e}(r)\,dr + \tilde{V}_i\,[\tau_i + \epsilon_i\|\hat{e}(t)\| + \epsilon_{2i}\|\hat{e}(t-h(t))\| + 2\,\rho_i\|\hat{w}(t)\|]\,\mathrm{sign}(V_i^T S_i(t)) \quad (26)$$

where τ_i is a positive constant, $\epsilon_{ji} := \|N_{ji}\|\,\|\Gamma_j\|$, $j = 1,2$ and $\rho_i := \max_{i\in S}(\lambda_{max}(D_iD_i^T))^{0.5}$ with $\tilde{K}_{ji}$, $\tilde{V}_i$ can be found from (24).

Proof. Choose the following Lyapunov function

$$W_i(t) = 0.5\,S_i^T(t)\,S_i(t) \quad (27)$$

By considering the time derivate of the sliding mode surface $S_i(t)$ and (7), we obtain

$$\begin{aligned}
\dot{W}_i(t) &= S_i^T(t)\,\dot{S}_i(t) \\
&= S_i^T(t)[V_i\dot{\hat{e}}(t) - V_i(A_{1i} + \alpha I - B_iK_{1i})\hat{e}(t) - e^{\alpha h(t)}V_i(A_{2i} - B_iK_{2i})\hat{e}(t-h(t)) - V_i\int_{t-\tau(t)}^{t} e^{\alpha(t-r)}(A_{3i} - B_iK_{3i})\hat{e}(r)\,dr\,] \\
&= S_i^T(t)\,[V_i\,N_{1i}\hat{f}_1(t,\hat{e}(t)) + V_i\,N_{2i}\hat{f}_2(t,\hat{e}(t-h(t))) - 2V_i\,D_i\hat{w}(t) \\
&\quad - V_i\,[\tau_i + \epsilon_i\|\hat{e}(t)\| + 2\rho_i\|\hat{w}(t)\|]\,\mathrm{sign}(V_i^TS_i(t))] \\
&\le \|S_i(t)V_i\|\,[\|N_{1i}\|\|\Gamma_1\|\|\hat{e}(t)\| + \|N_{2i}\|\|\Gamma_2\|\|\hat{e}(t-h(t))\| \\
&\quad + 2\|D_i\|\|\hat{w}(t)\|] \\
&\quad - \|S_i(t)V_i\|[\tau_i + \epsilon_{1i}\|\hat{e}(t)\| + \epsilon_{2i}\|\hat{e}(t-h(t))\| \\
&\quad + 2\rho_i\|\hat{w}(t)\|] \\
&\le -\tau_i\,\|S_i(t)V_i\| \\
&= -\sqrt{2}\tau_i\,\|V_i\|\,W_i(t)^{0.5}
\end{aligned} \quad (28)$$

It is shown from (28) that the system trajectories can be driven onto the predefined sliding surface in a finite time, $t_i^* = \sqrt{2}\,W_i(0)^{0.5}/(\tau_i\|V_i\|)$. In other words, the sliding mode surface $S_i(t)$ must be reachable. ■

Remark 4. In order to eliminate the chattering behavior caused by $\mathrm{sign}(V_i^TS_i(t))$, a boundary layer is introduced around each switch surface by replacing $\mathrm{sign}(V_i^TS_i(t))$ in (26) by saturation function. Hence, the control law (26) can be expressed as

$$\hat{u}_i(t) = \tilde{K}_{1i}\bar{P}_{1i}^{-1}\hat{e}(t) + e^{\alpha h(t)}\tilde{K}_{2i}\bar{P}_{1i}^{-1}\hat{e}(t-h(t)) + \int_{t-\tau(t)}^{t} e^{\alpha(t-r)}\tilde{K}_{3i}\bar{P}_{1i}^{-1}\hat{e}(r)\,dr + \tilde{V}_i\,[\tau_i + \epsilon_i\|\hat{e}(t)\| + \epsilon_{2i}\|\hat{e}(t-h(t))\| + \rho_i\|\hat{w}(t)\|]\,\mathrm{sat}\left(\frac{V_i^TS_i(t)}{\delta}\right) \quad (29)$$

The j-th element of $\mathrm{sat}(\frac{V_i^TS_i(t)}{\delta})$ is described as

$$\mathrm{sat}\left(\frac{[V_i^TS_i(t)]_j}{\delta_j}\right) = \begin{cases} [\mathrm{sign}(V_i^TS_i(t))]_j\,, & \text{if } [V_i^TS_i(t)]_j > \delta_j\,, j = 1,\dots,m \\ \dfrac{[V_i^TS_i(t)]_j}{\delta_j}, & \text{otherwise} \end{cases}$$

where δ_j is a measure of the boundary layer thickness around the j-th switching surface.

IV. SIMULATION RESULTS

In this section, we will verify the proposed methodology by giving an illustrative example.

Consider a continuous-time master-slave system (1)-(2) with two Markovian switching modes and the following state-space matrices

Mode 1:

$$A_1(1) = \begin{bmatrix} -5 & 0.6 & -2.4 \\ 0 & -2 & -0.8 \\ 0 & 0 & 0.5 \end{bmatrix};\ A_2(1) = 10A_3(1) = \begin{bmatrix} 0.1 & -0.1 & 1 \\ 0.2 & 0.1 & 0.1 \\ -1 & 1 & 0.1 \end{bmatrix};$$

$$B(1) = \begin{bmatrix} 0.25 \\ 0.1 \\ 0.15 \end{bmatrix};\ D(1) = \begin{bmatrix} 0.2 \\ 0.1 \\ 0.3 \end{bmatrix};\ C_1(1) = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix};\ N_1(1) = N_2(1) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix};$$

Mode 2:

$$A_1(2) = \begin{bmatrix} -1 & 0.8 & 1 \\ 0 & -1 & -0.6 \\ 0 & 0 & -2.5 \end{bmatrix};\ A_2(2) = 10A_3(2) = \begin{bmatrix} 0.1 & -0.04 & 0.1 \\ 0.1 & 0.02 & 0.1 \\ 0.01 & 0.02 & -0.01 \end{bmatrix};$$

$$B(2) = \begin{bmatrix} 2 \\ 3 \\ 2 \end{bmatrix};\ D(2) = \begin{bmatrix} 0.5 \\ 0.1 \\ 1 \end{bmatrix};\ C_1(2) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix};\ N_1(2) = N_2(2) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix};$$

with nonlinear functions $f_1(t,x(t)) = f_2(t,x(t)) = 0.5(\|x(t)+1\| - \|x(t)-1\|)$. The delays $h(t) = \tau(t) = (1-e^{-t})/(1+e^{-t})$ satisfy $0 \le h(t) = \tau(t) \le 1$ and $\dot{h}(t) = \dot{\tau}(t) \le 0.5$. The following transition matrix is considered

$$\pi = \begin{bmatrix} -0.33 & 0.33 \\ 0.53 & -0.53 \end{bmatrix},$$

for the system with two operating modes and the initial mode $r_0 = 1$. It is required to design the sliding mode exponential H_∞ synchronization signal (26) such that the trajectories of the slave subsystem and master subsystem (1)-(2) can be synchronized. To this end, in light of Theorem 3, we solved the LMIs (24) for $\gamma = 0.25, \alpha = 0$ and obtained

$$K_{11} = [52.2850 \quad 49.8348 \quad 377.7906];\ K_{12} = [5.8019 \quad 6.1163 \quad 7.6750];$$
$$K_{21} = [-0.1336 \quad 0.1643 \quad 0.4344];\ K_{22} = [-0.0067 \quad 0.0121 \quad 0.0138];$$
$$K_{31} = [-0.0120 \quad 0.0145 \quad 0.0484];\ K_{32} = [0.0029 \quad 0.0001 \quad 0.0029].$$

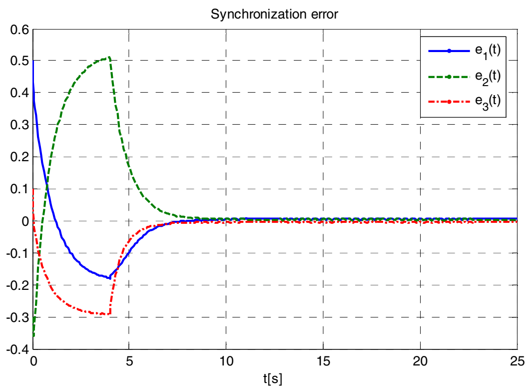

Fig. 1. Synchronization error signals.

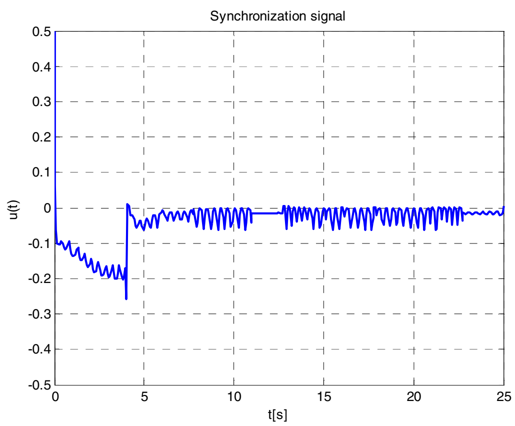

Fig. 2. Synchronization control signal.

Now, by applying the synchronization signal (26) and the parameters above, the temporal evolution of the master-slave synchronization errors, i.e., $e(t) = x_s(t) - x_m(t)$, are shown in Figure 1. Moreover, the synchronization control signal $u(t)$ is depicted in Figure 2.

V. COCLUSION

In this paper, the problem of exponential H_∞ synchronization was studied for a class of master-slave systems with both discrete and distributed time-delays, norm-bounded nonlinear uncertainties and Markovian switching parameters. Using an appropriate Lyapunov-Krasovskii functional, some delay-dependent sufficient conditions and a synchronization law which include the master-slave parameters were established for designing a delay-dependent mode-dependent sliding mode exponential H_∞ synchronization control law in terms of linear matrix inequalities. The controller guarantees the H_∞ synchronization of the two coupled master and slave systems regardless of their initial states. A numerical example was given to show the effectiveness of the method.

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