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A simple nonlinear filter for low-cost ground vehicle localization system

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Abstract— **This paper introduces a simple and intuitive nonlinear observer for low-cost ground vehicle localization system, using measurements from an inertial measurement unit, two wheel speed sensors, and a GPS. Taking advantage of the nonholonomic constraints, the design of the observer takes into account imperfections of the embedded sensors measurements, such as slowly time-varying gyroscope biases or some uncertainty on the angle between the vehicle's frame and the road, to estimate the attitude, velocity and position of a ground vehicle. Thanks to a simple nonlinear structure based on the theory of symmetry-preserving observers, the estimator is easy to tune, easy to implement, and well-behaved even at very low speed. Moreover, the proposed filter presents some guaranteed convergence properties when GPS is available. Simulations and experiments in urban area illustrate the good performances of this simple algorithm.**

I. INTRODUCTION

The Global Position System (GPS) has become a widespread device that is used in most of Vehicle Localization System (VLS) algorithms. Standard commercial GPS have a (only) relative accuracy (approximately 2 meters Circular Error of Probability in position estimation) and low update rate (1 to 4 Hz). Lack of GPS is also very common in urban environments (indoors, in tunnels) and the signals are frequently degraded in the so-called urban canyons where the buildings tend to either mask the GPS transmitted signal, or to reflect it along multipaths. For those reasons, the GPS data are often fused with inertial measurements. Recent technological developments of low-cost embedded sensors (MEMS) have lead to a wide-spread use of low-cost accelerometers and gyroscopes in VLS. The severe drifts of those sensors are then corrected by GPS estimations, which are, in the long run, accurate on average.

In the last decade, numerous on-board system designs have been proposed for ground vehicle localization: see e.g. [1], [7], [8], which use Inertial Measurement Unit (IMU), speedometers and GPS. The VLS rely on the fusion of the several sensors measurements and a trusted model of the vehicle dynamics. The trusted dynamical model can be entirely based on general kinematics equations, considering the system as a material point to which a frame is attached (orientation of the vehicle). In this case, any algorithm designed for any moving system (e.g. an aerial vehicle) may be used (e.g. see [14], [16], [15], or any commercial off-the-shelf system, often called "aided Attitude and Heading Reference Systems" (aided AHRS), e.g. MIDG II from Microbotics, or MTi-G from Xsens). However, a specific model of terrestrial vehicles may improve the estimation performances. In particular, [8] have shown that considering nonholonomic constraints significantly improves the precision of the VLS. This kind of roll without slip model has been advocated in several recent publications such as [7], [9].

The data fusion is generally based on popular filtering techniques such as the well-established extended Kalman filter (EKF). When properly tuned and implemented, this algorithm yields remarkably good results. However it suffers several drawbacks: it is not easy to choose the numerous parameters of the filter, and an expensive computation board is needed to run the algorithm in real-time. Another main drawback is that the models involved are nonlinear, so it is in general very difficult to prove the convergence of the estimation error to zero, even on the first order expansion of the error around any trajectory, since the linearized error equation is time-varying. If the system is badly initialized or if the estimation differs much from the true state value after a long GPS loss (e.g. in a tunnel), there is absolutely no theoretical guarantee that the filter behaves well. This is the reason why some recent works have aimed at developing nonlinear observers with guaranteed convergence properties for localization (e.g. see [11], see also [2], [13]).

In this paper we consider a vehicle moving in three dimensions, and equipped with 3 low-cost biased gyroscopes, 3 low-cost accelerometers, a stand-alone GPS, and a speedometer. Moreover the longitudinal angle between the IMU (the car steel frame) and the ground is supposed to be unknown. The precision sought for the attitude estimation is typically a couple of degrees, and a clear improvement of GPS velocity and position estimates.

The trusted model of the dynamics relies on a specific car model (nonholonomic constraints), and a simple nonlinear observer, which provides vehicle state estimations in real-time, is proposed. Its structure is based on the theory of symmetry-preserving observers [2], [3]. The proposed filtering algorithm has the following advantages:

1) *Convergence properties:* the estimation of the yaw angle possesses some global convergence properties.
2) *Precision at low speed:* the estimation of the yaw angle generally relies on the *arctan* of the ratio between North and East GPS velocity measurements. Such techniques drastically simplify the analysis but they lead to very poor results at low speed [1]. The proposed observer bypasses this limitation. In the same way, the GPS velocity measurements are never divided by the

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norm of the velocity that can be possibly small.

3) *Altitude estimation:* the quality of the vertical GPS velocity measurement is generally poor when compared to North and South velocity measurements. Thus, the algorithm does not use this component at all.
4) *Simplicity:* the observer is easy to tune (only a few parameters to choose), easy to implement and computationally economic (very few scalar operations).

The paper is organized as follows. In Section II, the problem is presented, and the considered model is simplified under some realistic assumptions. In Section III, a nonlinear observer is proposed, and its convergence properties are analyzed. Finally, in Section IV, simulations and real experiments illustrate the nice properties of the observer.

Preliminary results can be found in [4], which considers a slightly different problem in the particular case where the vehicle is moving only in a horizontal plane.

II. GROUND VEHICLE LOCALIZATION PROBLEM

A. Considered model

We consider the localization problem of a vehicle moving on a road in the three dimensional space. The Earth is considered flat and defining an inertial frame $\mathscr{R}_0$. It is also assumed there is no sideslip, i.e. the vehicle velocity vector is always tangent to the trajectory (as in [2], [8], [9]). The frame attached to the vehicle is $\mathscr{R} = (e_x, e_y, e_z)$.

Three kinds of sensors are used: a low-cost inertial measurement unit (IMU) with three orthogonal accelerometers and three orthogonal gyroscopes, a GPS and two wheel speed sensors. The wheel speed sensors (speedometers) measure the average speed of the wheels multiplied afterwards by the radius of the wheel to provide the vehicle longitudinal velocity. Note that the wheel speed can be easily measured from the signals of the Anti-lock Braking System (ABS)-dedicated sensors via the Analog to Digital converter.

Each gyroscope is supposed to be biased. Such biases tend to slowly drift with temperature and must be estimated online, especially when low-cost gyroscopes are used. Indeed, they lead to errors that grow linearly with time, and should be well estimated at any time, in case of loss of GPS. Moreover, a constant small pitch harmonization angle θ_0 is also considered in the model. This longitudinal misalignment between the IMU and the road vehicle frames is essentially due to the specific geometry of the car (the steel frame of the car is not parallel to the ground), and variations of load in the vehicle. The motion of the vehicle is described by the following nonholonomic equations

$$
\begin{aligned}
\dot{\phi} &= \omega_x - b_x + \tan(\theta)\big((\omega_y - b_y)\sin\phi + (\omega_z - b_z)\cos\phi\big)\\
\dot{\theta} &= (\omega_y - b_y)\cos\phi - (\omega_z - b_z)\sin\phi\\
\dot{\psi} &= \big((\omega_z - b_z)\cos\phi + (\omega_y - b_y)\sin\phi\big)/\cos(\theta)\\
\dot{p} &= v_s\big(\cos(\theta-\theta_0)\cos\psi, \cos(\theta-\theta_0)\sin\psi, \sin(\theta-\theta_0)\big)^T\\
\dot{b}_x &= \dot{b}_y = \dot{b}_z = \dot{\theta}_0 = 0,
\end{aligned}
$$

where ϕ, θ, ψ are the Euler angles (resp. roll, pitch, yaw angles), v_s is the (scalar) velocity measurement provided by the wheel speed sensors, $p = (p^n, p^e, z)^T$ and $v = (v^n, v^e, v^z)^T = (\dot{p}^n, \dot{p}^e, \dot{z})^T$ are the position and the velocity vectors of the midpoint of the rear axle with respect to the Earth-fixed frame, in coordinates that correspond to North, East, and Up (altitude) respectively. The known input (from gyroscope and speedometer measurements, which are assumed to be always available) is

$$
\begin{pmatrix} u_\omega \\ u_s \end{pmatrix} = \begin{pmatrix} \omega_m \\ v_s \end{pmatrix}, \tag{1}
$$

where $\omega_m = (\omega_x, \omega_y, \omega_z)^T$. The gyroscopes bias vector is $b = (b_x, b_y, b_z)^T$. GPS measurements yield (when they are available)

$$
\begin{pmatrix} y_p \\ y_v \end{pmatrix} = \begin{pmatrix} p \\ v \end{pmatrix}; \tag{2}
$$

$v = v_s(\cos(\theta-\theta_0)\cos\psi, \cos(\theta-\theta_0)\sin\psi, \sin(\theta-\theta_0))^T$.

Those outputs can be completed by accelerometers measurements. Indeed, first of all, speedometer is a special type of sensor, where the time between two pulses indicates the angular velocity of the wheel. This signal is theoretically noiseless and can be differentiated. As a result, $\frac{d}{dt}v_s$ can be computed. Moreover we have $\dot{v} = (\omega_m - b)\times v + a_m + g_b$, where g_b is the gravity expressed in the vehicle (body) frame and a_m is the accelerometers measurements. The nonholonomic constraint writes $v = v_s e_x$. This implies $a_m.e_x = \dot{v}_s + g\sin\theta$, $a_m.e_y = v_s(\omega_z - b_z) + g\sin\phi$, and $a_m.e_z = -v_s(\omega_y - b_y) + g\cos\theta\sin\phi$. As a result a combination between $\dot{v}_s$ and the (noisy) accelerometers measurements yield

$$
\begin{pmatrix} y_{a_x} \\ y_{a_y} \\ y_{a_z} \end{pmatrix} = \begin{pmatrix} g\sin\theta \\ -v_s b_z + g\sin\phi \\ v_s b_z + g\cos\theta\sin\phi \end{pmatrix}. \tag{3}
$$

Accelerometers are sometimes assumed to provide roll and pitch angles under quasi-static approximation (see, e.g. [13]). With additional information from the speedometer, we see how the approximation can be made exact, as θ and ϕ can be deduced from (3). This idea seems to have never been much used, but it can yield decent results, as advocated by real experiments in Section IV.

Remark 1: a constant scaling corresponding to the uncertainty on the wheel radius, s, could be added in the model $v_s \rightarrow s v_s$. This scaling depends on slowly changing parameters such as pressure in the wheel and tire wear. It can be easily estimated through the following non-linear filter $\dot{\hat{s}} = k_s(\|V_{GPS}\|^2 - \hat{s}^2 v_s^2)$, which is such that $\hat{s} - s$ tends to zero as we have $\|V_{GPS}\| = s v_s$. Such an algorithm uses the vertical component of V_{GPS}. This is not really a problem since the gain k_s can be very small (typically 0.1 min^{-1}, as s is approximately known and drifts slowly) and such long-run identification procedures filter efficiently GPS noise.

B. Simplified model

The design and convergence analysis are based on the following assumptions:

- Linearization around the horizontal position: θ and ϕ are assumed to be small. This is correct as the maximum

slopes (for θ and ϕ) are rarely above 15 degrees (30%) on human-made roads. It corresponds to a cosine of at least 0.97 (which is very close to 1). On the contrary, ψ does not remain necessarily small.
- Small biases: in low-cost systems, the magnitude of the biases is typically of order $0.1^\circ/s$. The terms in b_t^2 or $b_t\phi$, where b_t is any gyroscope bias, are considered second order. We will also assume $-(v_s b_z)/g + \sin\phi \approx \sin\phi$ as for a maximum speed of $40m/s$, neglecting the term $(v_s b_z)/g$ will result in an error of at most 0.4° on the estimation of ϕ, that we consider as negligible for the precision of estimation sought.

Under those assumptions, the linearized system around the horizontal position writes

$$\dot{\phi} = \omega_x - b_x + \theta\omega_z \tag{4}$$
$$\dot{\theta} = \omega_y - b_y - \phi\omega_z \tag{5}$$
$$\dot{\psi} = \omega_z - b_z + \omega_y\phi \tag{6}$$
$$\dot{p} = v_s(\cos\psi, \sin\psi, \theta - \theta_0)^T \tag{7}$$
$$\dot{b}_x = \dot{b}_y = \dot{b}_z = \dot{\theta}_0 = 0, \tag{8}$$

with output $y_p = p, y_v = v, y_\theta = \theta, y_\phi = \phi$. By only considering the attitude and velocity estimation problem, there are 4 unknown biases, and 4 effective measurements (as the norm of the GPS velocity is already known). It is thus hopeless to try to identify the two other accelerometers biases (θ_0 can be viewed as an accelerometer bias), unless the vehicle is assumed to follow a sequence of specific trajectories (see [5]). Note that we do not want to use the altitude velocity measurement and we replace it with the altitude estimation z of the GPS (which is a less sensitive procedure as it is only used to identify θ_0 in the long run, i.e. the corresponding gain is small).

III. PROPOSED NONLINEAR OBSERVER

When the GPS signal is available, we propose the following nonlinear observer defined via several output injections:

$$\frac{d}{dt}\hat{\phi} = \omega_x - \hat{b}_x + y_\theta\omega_z - k_1^\phi(\hat{\phi} - y_\phi) \tag{9}$$
$$\frac{d}{dt}\hat{b}_x = k_2^\phi(\hat{\phi} - y_\phi) \tag{10}$$
$$\frac{d}{dt}\hat{\theta} = \omega_y - \hat{b}_y - y_\phi\omega_z - k_1^\theta(\hat{\theta} - y_\theta) \tag{11}$$
$$\frac{d}{dt}\hat{b}_y = k_2^\theta(\hat{\theta} - y_\theta) \tag{12}$$
$$\frac{d}{dt}\hat{\psi} = \omega_z - \hat{b}_z + y_\phi\omega_y + k_\psi\big(\cos(\hat{\psi})y_{v^e} - \sin(\hat{\psi})y_{v^n}\big) \tag{13}$$
$$\frac{d}{dt}\hat{b}_z = -k_b v_s\big(\cos(\hat{\psi})y_{v^e} - \sin(\hat{\psi})y_{v^n}\big) \tag{14}$$
$$\frac{d}{dt}\hat{p}^n = v_s\cos\hat{\psi} + k_p(y_{p^n} - \hat{p}^n) \tag{15}$$
$$\frac{d}{dt}\hat{p}^e = v_s\sin\hat{\psi} + k_p(y_{p^e} - \hat{p}^e), \tag{16}$$
$$\frac{d}{dt}\hat{z} = v_s(y_\theta - \hat{\theta}_0) - v_s k_1^z(\hat{z} - y_z) \tag{17}$$
$$\frac{d}{dt}\hat{\theta}_0 = v_s k_2^z(\hat{z} - y_z). \tag{18}$$

The velocity estimation is given by

$$v = v_s\big(\cos(\hat{\theta} - \hat{\theta}_0)\cos\hat{\psi}, \cos(\hat{\theta} - \hat{\theta}_0)\sin\hat{\psi}, \sin(\hat{\theta} - \hat{\theta}_0)\big)^T.$$

When GPS measurements are not available, the GPS output cannot be used anymore to estimate the state variables. In this case, equations (13)–(18) are updated without any correction terms (open-loop), as the corresponding subsystem becomes unobservable.

The algorithm obviously possesses the advantages 1)-4) listed in the Introduction. Indeed, it is easy to implement, computationally thrifty, and easy to tune (10 gains to choose that boil down to 5 if tuned as recommended below). Furthermore, this algorithm can still be used at low speed. Indeed, there is no division between measurements having a possibly degraded signal-to-noise ratio (especially, the term $arctan(y_{v^e}/y_{v^n})$ is never computed), and no division by the (possibly very small) term v_s is involved. The motivation is that such simplifications lead to poor performances at low speed. For instance, suppose that y_{v^n} is noisy and close to zero, then $arctan(y_{v^e}/y_{v^n})$ can switch from $\pi/2$ to $-\pi/2$ a great number of times in small amount of time. It is then common to define a threshold in the norm of the velocity (typically 2m/s) under which the estimated yaw angle is not corrected anymore, see e.g. [6].

Another important property of the observer is its convergence guarantees around a large set of trajectories. Indeed the error system breaks into 2 main independent subsystems. The heading subsystem (13)–(16) is concerned with the estimation of the yaw angle and the North and East components of the position and velocity vectors, whereas the pitch, roll and vertical subsystem (9)–(12) and (17)–(18) is concerned with the corresponding Euler angles and the vertical component of the velocity and position vectors.

A. *Convergence analysis of the heading subsystem*

The nice nonlinear structure of the observer is based on the theory of symmetry-preserving observers, and it can be linked to invariant observer design on Lie groups [3], [12]. Here, the (Lie) symmetry group is the group of rotations around the vertical axis. The design is based on a so-called "invariant" state-error that considerably simplifies convergence analysis. Indeed consider the state error

$$\begin{pmatrix} \tilde{\psi} \\ \tilde{b}_z \\ \tilde{p}^n \\ \tilde{p}^e \end{pmatrix} = \begin{pmatrix} \hat{\psi} - \psi \\ \hat{b}_z - b_z \\ \hat{p}^n - p^n \\ \hat{p}^e - p^e \end{pmatrix},$$

the (nonlinear) state error equation has the following nice structure:

$$\frac{d}{dt}\tilde{\psi} = -\tilde{b}_z - k_\psi v_s\sin(\tilde{\psi}) \tag{19}$$
$$\frac{d}{dt}\tilde{b}_z = k_b v_s^2\sin(\tilde{\psi}) \tag{20}$$
$$\frac{d}{dt}\tilde{p}^n = (v_s\cos\hat{\psi} - v_s\cos\psi) - k_p(\tilde{p}^n) \tag{21}$$
$$\frac{d}{dt}\tilde{p}^e = (v_s\sin\hat{\psi} - v_s\sin\psi) - k_p(\tilde{p}^e), \tag{22}$$

and we see it is invariant to rotations around a vertical axis (for instance, if the first axis is pointing North or East): $\psi \mapsto \psi + \psi_0$ and $\hat{\psi} \mapsto \psi + \hat{\psi}_0$. The error system breaks into two subsystems: (19)-(20) and (21)-(22). Convergence of the subsystem (21)-(22) is straightforward as soon as $\tilde{\psi}$ has converged. Thanks to the use of an invariant error, the first system is independent of the trajectory $(\psi(t), \omega(t))$. Indeed, although the system is nonlinear, the error only depends on the errors $\tilde{\psi}$, $\tilde{b}$, and v_s (instead of $\tilde{\psi}, \psi, \tilde{b}, \omega_m$, and v_s as it is usual for nonlinear systems). This property plays a key role in the following convergence properties of the observer.

1) Local convergence around a very large set of trajectories: the following theorem proves that the proposed observer converges around *any trajectory* followed by the system, under some assumptions on the vehicle speed that are practically relevant in normal driving conditions.

Proposition 1: Consider subsystem (19)–(22). Assume there exist three scalars $M_1, \underline{M}_2, \bar{M}_2 > 0$, such that the speed v_s satisfies

$$\frac{d}{dt} v_s < M_1 v_s^2, \quad \underline{M}_2 \leq v_s^2 \leq \bar{M}_2. \tag{23}$$

Take $k_p, k_b > 0$ and $k_\psi > M_1$. The nonlinear observer (13)-(16) converges around any trajectory of the time-varying corresponding subsystem.

Proof: This proof is inspired by [10]. The main goal is to prove convergence of the subsystem (19)-(20). When it has converged, the convergence of subsystem (21)-(22) is obvious (indeed if $\tilde{\psi}, \tilde{b}_z$ converge to zero, it implies that (21)-(22) are linear equations in the variables $\tilde{p}^n, \tilde{p}^e$ driven by bounded terms, and thus there is no peaking). The linearized subsystem (19)-(20) writes

$$\frac{d}{dt} \tilde{\psi} = -\tilde{b}_z - k_\psi v_s \tilde{\psi} \tag{24}$$

$$\frac{d}{dt} \tilde{b}_z = k_b v_s^2 \tilde{\psi}. \tag{25}$$

Consider the Lyapunov function $V = \frac{1}{2}(k_b v_s^2 \tilde{\psi}^2 + \tilde{b}_z^2)$. The derivative of V writes

$$\frac{d}{dt} V = k_b v_s [\frac{d}{dt} v_s - k_\psi v_s^2] \tilde{\psi}^2.$$

$\dot{V} \leq 0$ as $\frac{d}{dt} v_s \leq M_1 v_s^2$ implies that $\frac{d}{dt} v_s \leq k_\psi v_s^2$. As V is not increasing, it is bounded and $\frac{d}{dt} V$ is integrable. As $\frac{d}{dt} v_s - k_\psi v_s^2 \leq (M_1 - k_\psi)\underline{M}_2$, which is a fixed negative scalar, it implies that $\tilde{\psi}^2$ is integrable. $\tilde{\psi}$ and $\tilde{b}_z$ are bounded as V is bounded and $v_s^2 > \underline{M}_2 > 0$. Thus $\frac{d}{dt}(\tilde{\psi})^2$ is bounded from (24), and applying Barbalat's lemma, $(\tilde{\psi})^2 \to 0$ and thus $\tilde{\psi} \to 0$.

Suppose that $V(t)$ does not tend to zero. As V is monotonically decreasing, it means there exists $\delta > 0$ such that for all $t \geq 0$, $V(t) \geq \delta$. As $(\tilde{\psi})^2 \to 0$ it implies there exists $t_1 \geq 0$ such that for all $t \geq t_1$ one has $\tilde{b}_z^2(t) \geq \delta/2$. As $\tilde{b}_z$ is continuous, it implies that $|\tilde{b}_z|$ is lower bounded by a strictly positive scalar. It yields a contradiction as it implies $|\tilde{\psi}(t)|$ tends to infinity according to (24). Thus $V(t)$ tends to 0 and $\tilde{b}_z(t) \to 0$. ∎

2) Almost global convergence in case of constant velocity: the following proposition ensures that the observer converges almost globally when the norm of the vehicle speed is constant, which is often the case in normal driving conditions.

Proposition 2: Assume the speed v_s is constant over the time. Set $k_\psi, k_b, k_p > 0$. The nonlinear observer (13)–(16) is such that:

- the error $(\tilde{\psi}, \tilde{p}^n, \tilde{p}^e, \tilde{b}_z)$ is locally exponentially stable to 0;
- for almost any initial conditions, the error asymptotically converges to 0.

Proof: Consider the error subsystem (19)-(20) and the candidate Lyapunov function $V = \left(1 - \cos(\tilde{\psi})\right) + \frac{1}{2k_b v_s^2} \tilde{b}_z^2$. One has

$$\frac{d}{dt} V = -k_\psi v_s \sin(\tilde{\psi})^2. \tag{26}$$

A standard application of Barbalat's lemma proves that $\sin \tilde{\psi}$ tends to zero and thus $\tilde{\psi} \to k\pi$, with $k \in \mathbb{Z}$. Studying the linearized system, one proves that $(\tilde{\psi}, \tilde{b}_z) = (0 + 2k\pi, 0)$ is an exponentially stable equilibrium, and $(\tilde{\psi}, \tilde{b}_z) = (\pi + 2k\pi, 0)$ is an unstable equilibrium, as the linear subsystem has one eigenvalue with strictly positive real part. There are trajectories that converge along the stable center manifold associated with the stable direction of the linearization around $(\pi + 2k\pi, 0)$. From center manifold theory, the set of the trajectories that converge to this unstable equilibrium point is of null measure in overall space. Finally, exponential convergence of the error subsystem (21)-(22) is straightforward for $k_p > 0$ as soon as $\tilde{\psi}$ has converged to 0. ∎

Note that "almost any" means that the set of initial conditions such that the error does *not* converge to zero is of null measure in the overall space. Thus, if the observer is initialized inside this set, a small perturbation (such as gyroscope measurement noise) will make it move out of the set. So global convergence is always expected in practice.

B. Convergence analysis of the pitch, roll and vertical subsystem

The following proposition ensures that the rest of the system converges asymptotically.

Proposition 3: Take $k_1^\theta, k_2^\theta, k_1^\phi, k_2^\phi, k_1^z, k_2^z > 0$. Then

- $\hat{\theta} - \theta, \hat{\phi} - \phi, \hat{b}_x - b_x, \hat{b}_y - b_y$ converge exponentially to zero.
- $\hat{z} - z$ and $\hat{\theta}_0 - \theta_0$ tend to zero as soon as $\int_0^\infty v_s dt = \infty$.

Proof: Convergence of $\hat{\theta} - \theta, \hat{\phi} - \phi, \hat{b}_x, \hat{b}_y$ is guaranteed by the fact that the corresponding error subsystems are linear and time-invariant. The convergence of subsystem (17)-(18) relies on the Lyapunov function $V = \tilde{\theta}_0^2 + k_2^z \tilde{z}^2$ whose time derivative is $\dot{V} = -v_s k_1^z \tilde{z}^2 k_2^z \tilde{z}^2$. A change of time scale and a standard application of LaSalle's theorem proves convergence as soon as $\int_0^\infty v_s dt = \infty$. ∎

C. Gain tuning

When the velocity v_s is constant the linearized heading error subsystem (24)-(25) writes : $\frac{d^2}{dt^2} \tilde{\psi} + k_\psi v_s \frac{d}{dt} \tilde{\psi} + k_b v_s^2 \tilde{\psi} = 0$. Letting $k_\psi = 2\xi \omega_\psi$ and $k_b = \omega_\psi^2$, the error system is a

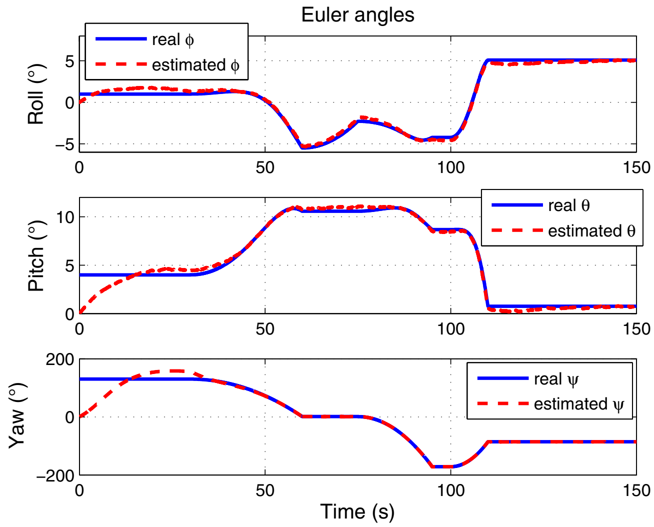

Fig. 1. Estimated Euler angles (simulation)

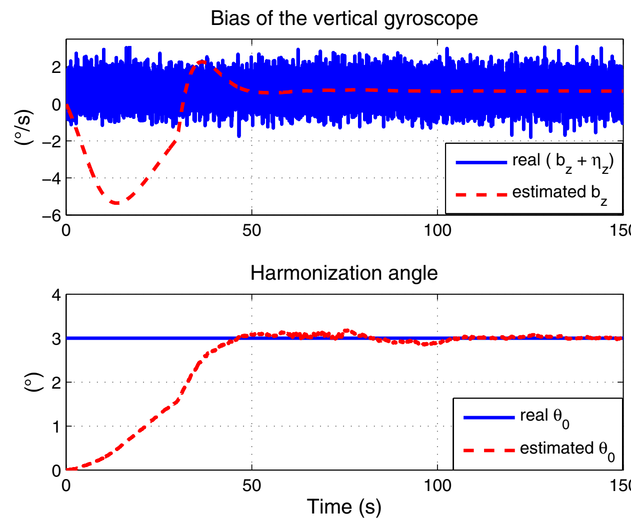

Fig. 2. Estimated bias and harmonization angle (simulation)

damped linear oscillator with pulsation $v_s\omega_\psi$. This choice is very natural, as the convergence speed $v_s\omega_\psi$ naturally adapts to the car velocity and thus to the signal-to-noise ratio (if the velocity measurement noise of the GPS is supposed to be additive). Moreover, given a convergence speed $v_s\omega_\psi$, the choice $\xi = \sqrt{2}/2$ minimizes the ITAE criterion.

The rest of the system can be tuned independently from the measured velocity v_s. In order to minimize the ITAE criterion once again, we propose the following relations: $k_1^\theta = 2\xi\omega_\theta$, $k_2^\theta = \omega_\theta^2$, $k_1^\phi = 2\xi\omega_\phi$, $k_2^\phi = \omega_\phi^2$, $k_1^z = 2\xi\omega_z$, $k_2^z = \omega_z^2$, where $\xi = \sqrt{2}/2$. Finally, the horizontal position gain k_p can be chosen in accordance with the quality of the GPS position estimation.

IV. Simulations and Experiments

A. Simulations

The proposed estimation algorithm is first validated through simulation. To simulate a realistic setup, noise is added to the several sensor measurements, as illustrated in Fig. 2 (η_z is the noise added on the gyroscope measurement ω_z). The vehicle velocity is pretty low, around 2m/s. Equations of nonlinear observer are given by (9)–(18). The gains of the estimator are set to $k_1^\phi = k_1^\theta = k_\psi = 0.14$, $k_2^\phi = k_2^\theta = k_b = 0.01$, $k_1^z = 0.71$, and $k_2^z = 2.5e-2$.

To illustrate the large domain of convergence of the proposed filtering algorithm, the estimator is badly initialized (e.g. with an initial heading estimation error equal to 130°). Figures 1-2 show that the observers converges despite very wrong initial values (only a few variables are plotted due to lack of space). Notice that the yaw angle is well estimated, even if the speed of the vehicle is very low.

B. Experimental validation

1) Experimental setup: the proposed nonlinear observer is now validated through real experiments. The vehicle used for the experiments belongs to Mines ParisTech, and is shown on Fig. 3, as well as the IMU and GPS antenna on the roof. The trajectory followed by the vehicle is a turn around the “Place du Châtelet”, at the center of Paris (France), in order to illustrate the good behavior of the filter in urban areas, where GPS measurements are likely to be inaccurate or even unavailable. The vehicle moves at very low speed (around 10km/h). The vehicle trajectory is almost planar. Therefore, only two dimensional plots are shown, despite the use of the complete 3D nonlinear observer (9)–(18). Measurements from Crossbow VG600 IMU (update rate 84Hz) and GPS Trimble AG132 antenna (update rate 10Hz) are used. In addition, two wheel speed sensors provide measurements at 10Hz. A computer gathers all information and gives estimations in real-time at 84Hz, using a simple Euler explicit approximation for the integration scheme.

Fig. 3. Used vehicle and IMU/GPS on the roof

2) Results: Figure 4 validates the approach adopted in the paper: it is possible to identify the pitch angle through the accelerometer and speedometer measurements. Indeed, the differentiated measurements from the wheel speed sensors are compared to the data from the longitudinal accelerometer on an horizontal surface. They overlap pretty well. These signals are filtered with a low-pass filter to get rid of high

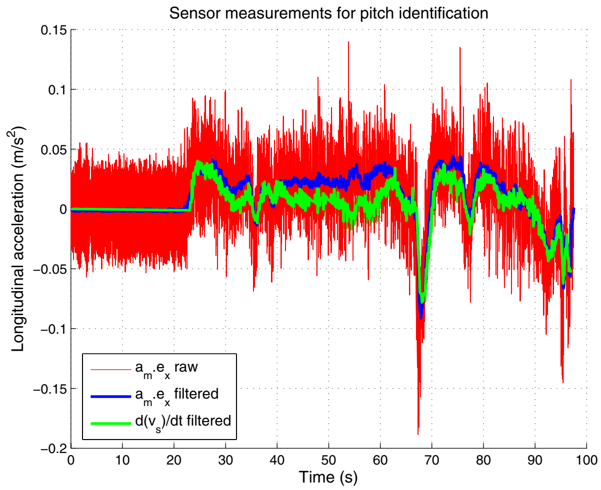

Fig. 4. Plot of the longitudinal accelerometer measurement and the time derivative of speedometer measurements on a flat road (experiment).

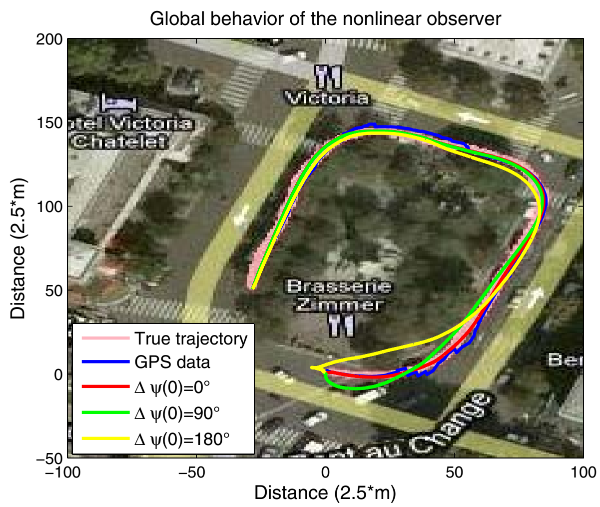

Fig. 5. Global behavior of the nonlinear observer (experiment).

frequency noise. Figure 5 illustrates two important properties of the proposed nonlinear filter:

- its good behavior, despite a low speed of the vehicle and inaccurate GPS measurements due to the surrounding buildings and trees,
- its large domain of convergence, here with an error in the initial estimated yaw angle, $\Delta\psi(0)$, equal to $90°$ and even to $180°$.

V. CONCLUSION

In this article, we propose a simple easy-to-tune nonlinear observer for VLS, which takes into account several sensor measurements imperfections. It can be seen as a credible alternative to Kalman-based filtering algorithms usually used for vehicle localization. Beyond several nice features, the main advantage of the proposed observer is that it has guaranteed convergence properties for a very large set of trajectories, and provides an online estimation of the gyroscopes biases that tend to slowly drift. Such theoretical guarantees allow the observer to be robust to GPS losses, and from an industrial viewpoint, it can be of great interest for safety, especially if the estimator is used for feedback control of vehicle. Another main advantage is the fact that contrarily to many state of the art VLS, the pitch and roll angles are estimated. They give information on the longitudinal and lateral inclination of the road, which can be useful in several applications (such as mobile mapping systems).

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