2011 50th IEEE Conference on Decision and Control and
European Control Conference (CDC-ECC)
Orlando, FL, USA, December 12-15, 2011

Robust Optimization of Operations in Energy Hub

A. Parisio, C. Del Vecchio and G. Velotto

Abstract— **In this paper a robust optimization problem of an energy hub operations is presented. An energy hub is a multi-generation system where multiple energy carriers input to the hub are converted, stored and distributed in order to satisfy energy demands. The solution to energy hub operation problem determines the energy carriers to be purchased and stored in order to satisfy the energy requests while minimizing a cost function. A control approach using Robust Optimization (RO) techniques is proposed; bounded uncertainties on energy hub parameters are taken into account and RO methods are exploited to gain robust solutions which are feasible for all values, or for a selected subset, of uncertain data. Simulation results underline the benefits resulting from the application of the proposed approach to an energy hub structure located in Waterloo, Canada.**

I. Introduction

In the last few years several innovations have been introduced in the energy sector driven by fast evolution of the technologies. In this light, the scientific community is addressing the analysis and planning of distributed energy resources with widespread approaches, taking into account technical, environmental, economic and social issues (see [1] for an exhaustive review).

A relevant number of recent works (see [2] and reference therein) deal with characterization, planning, evaluation and optimization of a class of decentralized multi-generation energy systems named *energy hubs*, which can be considered functional units where multiple energy carriers are converted, stored, and dissipated [3]. From a system point of view, an energy hub is a unit supplied by multiple energy carriers at its input ports and provides required energy services (i.e. electricity, heating, cooling, compressed air), also referred to as energy hub loads, at the output ports [3]. Figure 1 illustrates an example of an energy hub exchanging electricity, natural gas and heat through three converters; output energy carriers (electricity and heat) could be stored in two devices. In this paper the modeling and the operational scheduling of an energy hub in an uncertain environment is studied. The energy hub operational scheduling addresses the optimal energy carriers purchase and storage utilizations over short time periods [4]. Uncertainties are mainly due to the energy costs dynamics, the energy demand fluctuations, components' availability and efficiencies. Literature on energy hub operational scheduling is rich ([3]). The optimization problem is frequently set up to minimize the total energy cost in the system, within a deterministic framework of load demands, prices, efficiencies and constraints ([3], [2]). If uncertainty is taken into account the solution is not guaranteed to be optimal and its achievement can be computationally demanding ([5], [6]).

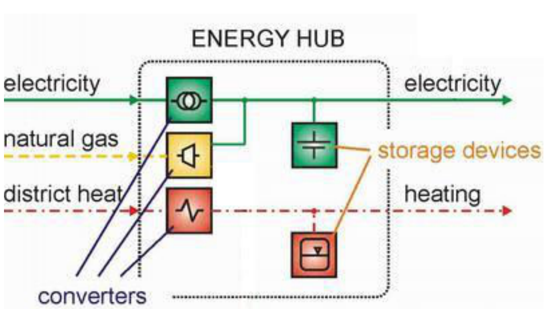

Fig. 1. General scheme of an energy hub: a multi-source multi-product system.

A. Parisio, C. Del Vecchio and G. Velotto are with Dipartimento di Ingegneria, Università del Sannio, Benevento, Italy c.delvecchio,aparisio,gevelotto@unisannio.it

In this paper a control-oriented approach to modeling and optimization of an energy hub is presented. We first provide a general modeling framework for energy hub which leads to a mixed integer dynamic model. Then a short term operation scheduling problem is posed considering both a deterministic environment and uncertain scenario. We then introduce parameter uncertainty and we provide a *robust* solution to the hub scheduling problem. The provided solution is feasible in all scenarios that uncertain parameters' variations could define, although it is more costly. Thus, we also solve an optimization problem where the level of robustness is controlled by a set of parameters which regulates the degree of uncertainty in the problem data. This approach is based on the idea of Bertsimas and Sim [7].

Although we remark that the main contribution of our work is the robust solution to the energy hub operation scheduling problem, we also consider the hub modeling framework we here propose an interesting contribution which extends the work of [4], [3] to more complex hub structures enabling couplings and interactions between converters while keeping a relatively simple mathematical structure (the resulting model is a mixed integer linear model). Simulations show the benefits resulting from the application of the proposed approach to an energy hub located in Waterloo, Canada.

II. Energy Hub Model

We consider an energy hub made of C devices converting P_α, $\alpha = 1, \ldots, N$, input power flows into L_β, $\beta = 1, \ldots, M$, output power flows. Without loss of generality we will consider storing devices located only on the output power flows; therefore the number of storing devices will be equal to M. Moreover we suppose that B converters (out of the C converters) are located in series to the C converters, that is the input to the B converters are output of the C converters.

- the system is considered to be in steady state;
- within energy hubs, energy losses occur only in converters and storage elements.

A. Converter Model

The most general form of a converter has multi inputs and multi outputs power flows. For each input power flow we introduce as many variables as converters. Thus for the input power flow P_α we introduce $P_\alpha^1 \dots P_\alpha^N$ variables with

$$P_\alpha = \sum_{j=1}^{C} P_\alpha^j.$$

Moreover we denote with $\eta_{\alpha\beta}^j$ the j^{th} converter efficiency when transforming the input power of type α into output power of type β.

Summarizing in a column vector all variables denoting input power flow $\mathbf{P}$=$[\mathbf{P}_1^{'}, \dots, \mathbf{P}_\alpha{}^{'}, \dots, \mathbf{P_N}{}^{'}]^{'}$, where $\mathbf{P}_\alpha$=$[P_\alpha^1 \dots P_\alpha^C]^{'}$, and the output power flow in a column vector $\mathbf{L}$=$[L_1, \dots, L_M]^{'}$, the resulting formulation for the multi-input multi-output converter is

$$\begin{bmatrix} L_1 \\ \vdots \\ L_M \end{bmatrix} = \mathbf{\Theta} \begin{bmatrix} \mathbf{P_1} \\ \vdots \\ \mathbf{P_N}. \end{bmatrix} \tag{1}$$

where matrix $\mathbf{\Theta}$ is called the converter coupling matrix; it is an MxNC matrix whose elements could be zeros, efficiencies or product of efficiencies.

A 'zero' in the $\theta_{i,j}$ element implies that no conversion exists between P_j and L_i. If the input power P_j is transformed in the output power L_i in one of the C converter, say c, then the corresponding element in the $\mathbf{\Theta}$ matrix is equal to the efficiency $\eta_{i,j}$. Finally if the input power P_j is transformed in the output power L_i through c (one of the C converters) and b (one of the B converters in series to C), then the $\theta_{i,j}$-th element of matrix $\mathbf{\Theta}$ is equal to the product of converters efficiencies c and b.

We remark that the converter model we here propose differs from the in [3], [8] where the *dispatch factors* are introduced. Since the total input of one energy carrier may split up to several converters (at input junction) the dispatch factors define the fraction of the energy carrier input to the converter. We refer to [3], [8] for further details. Our idea is to define a variable for each power input to each converter; in this way the hub model is linear. Moreover the power flows can become decision variables in an optimization problem aimed at deciding how to split input power at each converter. The input powers are constrained by minimum and maximum capacity limits

$$\mathbf{P}^{min} \leq \mathbf{P} \leq \mathbf{P}^{max} \tag{2}$$

where inequalities are meant component-wise.

B. Storage Model

Energy hubs may include storage elements to store any input or output energy of any converter.

If storing elements are considered in the hub model, the dependance on time of all model variables must be taken into account. We model storing devices through a discrete time system. The equation governing the power flow dynamic of the m-th storing device is:

$$E_m(k+1) = E_m(k) + \eta_m^{ch} Q_m^{ch}(k) - \eta_m^{dis} Q_m^{dis}(k) - E_m \tag{3}$$

with the uniform sampling time equal to $\Delta T = t_{k+1} - t_k$. We denote by $E_m(k)$ the level of the energy stored in the m-th device at time k. We also denote by $Q_m^{ch}(k)$ the power flowing through the m-th storing device at time k if, in ΔT, energy is stored into the device and by $Q_m^{dis}(k)$ the power exchange with the m-th storing device at time k if, in ΔT, energy is discharged from the device. We model charging and discharging energy storing 'efficiencies', respectively η_m^{ch} and η_m^{dis}, to consider losses due to the transformation from the energy carrier to the energy stored. Finally we denote by E_m a constant stored energy degradation in the sampling interval. In this work a scaled model will be used, i.e. $E_m(k)$ denotes energy stored in the m-th device at time k divided by ΔT. We denote by the column vectors $\mathbf{Q}^{ch}(k)$=$[Q_1^{ch}(k), \dots, Q_M^{ch}(k)]^{'}$ and $\mathbf{Q}^{dis}(k)$=$[Q_1^{dis}(k), \dots, Q_M^{dis}(k)]^{'}$ the power exchanged with M storing devices at time interval k, $\mathbf{E}(k)$=$[E_1(k), \dots, E_M(k)]^{'}$ the energy stored at time k, and with $\mathbf{E}$=$[E_1, \dots, E_M]^{'}$ the energy loss per time unit. We also introduce a diagonal matrix $\mathbf{A}^{ch}$= diag$(\eta_1 \dots \eta_M)$ for the charging efficiency of each storing device and a diagonal matrix $\mathbf{A}^{dis}$= diag$(1/\eta_1 \dots 1/\eta_M)$ for the discharging 'efficiency'. The equation describing the storage dynamics in matrix form is:

$$\mathbf{E}(k+1) = \mathbf{E}(k) + \mathbf{A}^{ch}\mathbf{Q}^{ch}(k) - \mathbf{A}^{dis}\mathbf{Q}^{dis}(k) - \mathbf{E}. \tag{4}$$

Since a storage cannot be charged and discharged at the same time, we introduce two binary variables $\delta_i^{ch}(k)$ and $\delta_i^{dis}(k)$, for each storing device and for each time k, subject to the following logical conditions:

$$Q_i^{ch}(k) > 0 \Longleftrightarrow \delta_i^{ch}(k) = 1$$

$$Q_i^{dis}(k) > 0 \Longleftrightarrow \delta_i^{dis}(k) = 1$$

with $i = \dots, M$. Then we pose the constraint $\delta_i^{ch}(k) + \delta_i^{dis}(k) \leq 1$ in order to force only one of the two variables $Q_i^{ch}(k)$ and $Q_i^{dis}(k)$ to be greater than zero at the same time k. In addition, we need to assume the following constraints on the capacity and exchange power of each storage:

$$\mathbf{C}^{min} \leq \mathbf{E}(k) \leq \mathbf{C}^{max}, \tag{5}$$

$$0 \leq Q_i^{ch}(k) \leq \delta_i^{ch}(k) Q_i^{max}(k) \tag{6}$$

$$0 \leq Q_i^{dis}(k) \leq \delta_i^{dis}(k) Q_i^{max}(k) \tag{7}$$

with $i = \dots, M$.

C. Complete Energy Hub Model

Based on the previous considerations, the flows through an energy hub at a given time k are modeled by the following discrete time, mixed integer, dynamical system:

$$\mathbf{L}(k) = \mathbf{\Theta}\mathbf{P}(k) - \mathbf{Q}^{ch}(k) + \mathbf{Q}^{dis}(k), \tag{8}$$

$$\mathbf{E}(k+1) = \mathbf{E}(k) + \mathbf{A}^{ch}\mathbf{Q}(k)^{ch} - \mathbf{A}^{dis}\mathbf{Q}(k)^{dis} - \mathbf{E}. \tag{9}$$

where we denote as $\mathbf{P}(k)$ and $\mathbf{L}(k)$ the power input and output vector at time k. These equations, combined with technical constraints, (2), (5), (6), (7) are the basis for operational and structural hub optimization.

D. Example

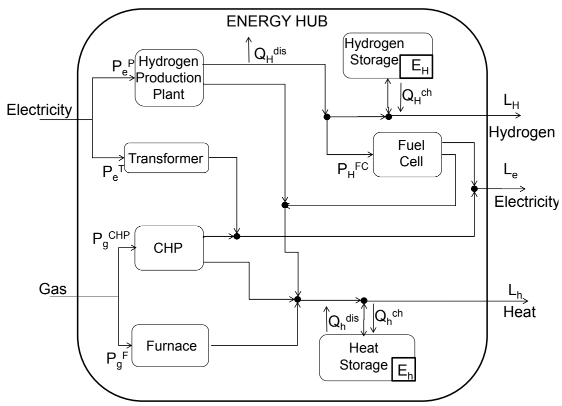

Fig. 2. Example of an energy hub located in Waterloo, Canada, for the supply of commercial load. Note that both Q_H^{ch}, Q_H^{dis} and Q_h^{ch}, Q_h^{dis} cannot be greater than zero simultaneously.

Consider the hub shown in Figure 2 (see [8]); it represents the structure of an energy hub located in Waterloo, Canada. It consists of five converters and two storage devices. The input power flows are the electricity and the natural gas carriers: the electricity is an input to the hydrogen production plant and to the transformer; the corresponding variables are denoted with P_e^P and P_e^T. The natural gas is an input to the cogeneration system (i.e. gas turbine), P_g^{CHP}, and to the furnace, P_g^F. Output power flows are hydrogen, L_H, electricity, L_e, and heat, L_h. Note that the hydrogen power plant and the fuel cell converters are connected in cascade, but the input to the fuel cell cannot be derived directly from the node balance equation due to the portion of power flowing as hydrogen. In this peculiar case, we need to introduce a new decision variable, P_H^{FC}.

The hydrogen production plant transforms the electricity into hydrogen, oxygen and heat; it is characterized by its electric-heat and electric-hydrogen efficiencies η_{eh}^P and η_{He}^P respectively. The fuel cell transforms the hydrogen into electricity and heat with efficiencies η_{He}^{FC} and η_{Hh}^{FC} respectively. We denote with P_H^{FC} the hydrogen input power flow to the fuel cell and with Q_H the hydrogen input to the storing device with efficiency η_H. The following equation relates output hydrogen power flow to hub input and to the stored quantity in the storing device:

$$L_H = \eta_{eH}^P P_e^P - P_H^{FC} - Q_H^{ch} + Q_H^{dis}. \tag{10}$$

The electricity power flow is input to the transformer; its output is the electricity power flow (reduced in voltage magnitude); its efficiency is denoted with η_{ee}^T. The gas turbine (CHP) is characterized by its gas-electric and gas-heat efficiencies η_{ge}^{CHP} and η_{gh}^{CHP} respectively. The furnace transforms natural gas in heat and operates with efficiency η_{gh}^F. Finally produced heat can be split in the hub output or stored in the heat storing device with efficiency η_h. Portion of stored heat is denoted with Q_h. We can pose equations regulating input power flow and electricity and heat output power flow (note that we drop the dependency from time to simplify the notation):

$$L_e = \eta_{ee}^T P_e^T + \eta_{ge}^{CHP} P_g^{CHP} + \eta_{eH}^{FC} P_H^{FC}, \tag{11}$$

$$\begin{aligned} L_h = {} & \eta_{eh}^P \eta_h P_e^P + \eta_{gh}^{CHP} \eta_h P_g^{CHP} + \eta_{gh}^F \eta_h P_g^F + \\ & \eta_{Hh}^{FC} \eta_h P_H^{FC} - Q_h^{ch} + Q_h^{dis}. \end{aligned} \tag{12}$$

The vectors for the input and output flows and power stored can be strictly derived according to the notation adopted in the model section.

The converter coupling matrix derived from equations (10), (11) and (12) is equal to:

$$\Theta = \begin{bmatrix} 0 & \eta_{ee}^T & \eta_{ge}^{CHP} & 0 & \eta_{eH}^{FC} \\ \eta_{eH}^P & 0 & 0 & 0 & -1 \\ \eta_{eh}^P \eta_h & 0 & \eta_{gh}^{CHP} \eta_h & \eta_{gh}^F \eta_h & \eta_{Hh}^{FC} \eta_h \end{bmatrix}.$$

III. Operation Scheduling in Energy Hub

The design of optimal energy hub operations consists in determining how much of each energy carrier should be bought/generated depending on the current load situation and on the energy carriers costs. Moreover if the hub includes storage elements, decisions on the quantity of stored energy should be taken and they will affect successive time periods. In these cases the design of optimal operations should be performed over multiple time periods and should provide decisions on energy quantities to be purchased and stored at each point in time. We consider a planning horizon of T periods; at each point in time ($k = 1 \dots T$), decision variables are the purchase of energy input carriers and dispatching between converters ($\mathbf{P}(k)$) as well as energy to be stored in storing devices ($\mathbf{E}(k)$).

We consider energy purchasing costs for each energy input varying at each point in time k of the planning horizon; costs are measured in monetary unit (mu) per unit and are denoted with the row vector $\mathbf{c}(k)$. Problem constraints are hub and energy storage equations (8), storage capacity constraints (5), power and energy limits (2), (6), (7). We also impose the equality of the stored energy at the beginning and at the end of the planning horizon. Thus we pose the following scheduling problem:

Problem 1 (Energy hub scheduling problem):

$$\min \sum_{k=0}^{T-1} \mathbf{c}(k)^{'}\mathbf{P}(k)$$

$s.t.$

$$\begin{aligned}
&\mathbf{E}(k+1) = \mathbf{E}(k) + \mathbf{A}^{ch}\mathbf{Q}^{ch}(k) - \mathbf{A}^{dis}\mathbf{Q}^{dis}(k) - \mathbf{E} \\
&\mathbf{L}(k) = \mathbf{\Theta}\mathbf{P}(k) - \mathbf{Q}^{ch}(k) + \mathbf{Q}^{dis}(k) \\
&\mathbf{P}(k)^{min} \leq \mathbf{P}(k) \leq \mathbf{P}(k)^{max} \\
&0 \leq Q_i^{ch}(k) \leq \delta_i^{ch}(k) Q_i^{max}(k) \quad i = 1, \dots, M \\
&0 \leq Q_i^{dis}(k) \leq \delta_i^{dis}(k) Q_i^{max}(k) \quad i = 1, \dots, M \\
&\delta_i^{ch}(k) + \delta_i^{dis}(k) \leq 1 \quad i = 1, \dots, M \\
&\mathbf{E}(k)^{min} \leq \mathbf{E}(k) \leq \mathbf{E}(k)^{max} \\
&\mathbf{E}_0 = \mathbf{E}_T
\end{aligned}$$

where $\mathbf{E}_0$ is a vector denoting the quantities in the storing devices at time $k = 0$. The output of the problem is a plan specifying for each point in time over the planning horizon, the energy carriers purchases, their dispatch among converters and the energy stored.

IV. Robust Operation Scheduling in Energy Hub

So far we have solved the problem of determining optimal input energy purchases and storage in order to fulfill hubs' loads. If some parameters are uncertain the solution to Problem 1 could not be feasible anymore. In this section we apply some robust optimization techniques to the energy hub scheduling problem in order to take uncertainty into account. In particular we apply a well-known Robust Optimization (RO) technique [7], [9] to produce *robust* solutions which are in a sense 'immune' against bounded uncertainty. We assume that the efficiencies parameters of the coupling matrix $\mathbf{\Theta}$, θ_{ij}, are uncertain. We suppose that θ_{ij} are independent random variables taking values according to a symmetric distribution over the interval $[\bar{\theta}_{ij} - \tilde{\theta}_{ij}, \bar{\theta}_{ij} + \tilde{\theta}_{ij}]$.
We remark that the proposed technique can also be applied with uncertain data on the energy loads or on energy costs. In this case the hypothesis of the independence among random variables is less realistic. Despite this we believe that the adoption of the RO technique to the energy hub is still a new and promising technique.

A. Handling Equality Constraints

In deterministic optimization problems equality constraints need to be strictly satisfied to obtain a feasible solution. In robust problems, however, it could be impossible to satisfy equality constraint 'robustly', i.e for every possible determination of the uncertain parameter. If we suppose that Θ has uncertain parameters Problem 1 contains uncertain equality constraints; thus we adopt the approach outlined in [10] to prevent the possible infeasibility of a solution. Note that the authors in [10] replace the equality constraint by two inequalities that keep the original constraint satisfied to the maximum possible extent. Following this approach, for each time step k we convert each group of M equality constraints into inequality constraints:

$$-\mathbf{\Sigma}(k) \leq \mathbf{\Theta}\mathbf{P}(k) - \mathbf{Q}^{ch}(k) + \mathbf{Q}^{dis}(k) - \mathbf{L}(k) \leq \mathbf{\Sigma}(k),$$

where $\mathbf{\Sigma}(k) \in \mathbb{R}^M$ is a vector of auxiliary variables introduced to account for equality constraints violations. It has to be kept as small as possible.

V. Robust Formulation

The methodology proposed by Bertsimas and Sim consists in choosing among the set of uncertain coefficients those that are more likely to vary. The optimal robust solution will be robust only against the uncertainty of this subset of coefficients. The limit case where all uncertain parameters can deviate is known as the robust Soyster's model [11]. Recall that we denote by M the number of output power flows and by N the number of input power flows. Following the notation in [7], we denote by J_i the set of random coefficients $\theta_{ij} \neq 0$, $j \in J_i$, $i = 0, \dots, M$, and by Γ_i an integer parameter taking values in $[0, |J_i|]$. The aim of the proposed approach is to be insensitive against the deviation of at most Γ_i converter efficiencies. Then we can state a tractable formulation of the Problem 1 and solve a classic robust control problem. Consider the planning horizon $T \in \mathbb{N}_+$ and define:

$$\begin{aligned}
\mathcal{L} &= \left[\mathbf{L}_0^{'}, \dots, \mathbf{L}_{T-1}^{'}\right]^{'} \in \mathbb{R}^{TM} \\
\mathcal{P} &= \left[\mathbf{P}_0^{'}, \dots, \mathbf{P}_{T-1}^{'}\right]^{'} \in \mathbb{R}^{TN} \\
\mathcal{Q}^{ch} &= \left[\mathbf{Q^{ch}}_0^{'}, \dots, \mathbf{Q^{ch}}_{T-1}^{'}\right]^{'} \in \mathbb{R}^{TM} \\
\mathcal{Q}^{dis} &= \left[\mathbf{Q^{dis}}_0^{'}, \dots, \mathbf{Q^{dis}}_{T-1}^{'}\right]^{'} \in \mathbb{R}^{TM} \\
\mathcal{O} &= I_T \otimes \mathbf{\Theta},\ \bar{\mathcal{O}} = I_T \otimes \bar{\mathbf{\Theta}},\ \tilde{\mathcal{O}} = I_T \otimes \tilde{\mathbf{\Theta}}, \\
\tilde{\Sigma} &= \left[\mathbf{\Sigma}_0^{'}, \dots, \mathbf{\Sigma}_{T-1}^{'}\right]^{'} \in \mathbb{R}^{TM}
\end{aligned}$$

where $\otimes$ is the Kronecker product, I_T is the identity matrix $\in \mathbb{R}^{T\times T}$, $\bar{\mathbf{\Theta}}$ and $\tilde{\mathbf{\Theta}}$ are the coupling matrix where each uncertain element is replaced by respectively its mean value and its deviation. Then the equality constraints $\mathbf{\Theta}\mathbf{P}(k) - \mathbf{Q}^{ch}(k) + \mathbf{Q}^{dis}(k) - \mathbf{L}(k) = 0$, over the whole planning horizon, can be written as $\mathcal{O}\mathcal{P} - \mathcal{Q}^{ch} + \mathcal{Q}^{dis} - \mathcal{L} = 0$. Employing the Proposition 1 in [7] and the dualization theory, each equality constraint can be reformulated as a linear constraint and therefore it is tractable, although several auxiliary variables must be added. Precisely, the overall number of auxiliary variables that are introduced is $TM + TMN + TN$. The robust formulation of the Problem 1 can be stated as follows:

Problem 2 (Robust energy hub scheduling problem):

$$\min \sum_{k=0}^{T-1} \mathbf{c}(k)^{'} \mathbf{P}(k) + \rho \|\mathbf{\Sigma}(k)\|_2^2$$

$s.t.$

$$\begin{aligned}
&\mathbf{E}(k+1) = \mathbf{E}(k) + \mathbf{A}^{ch}\mathbf{Q}^{ch}(k) - \mathbf{A}^{dis}\mathbf{Q}^{dis}(k) - \mathbf{E} \\
&\mathbf{P}(k)^{min} \leq \mathbf{P}(k) \leq \mathbf{P}(k)^{max} \\
&0 \leq Q_l^{ch}(k) \leq \delta_l^{ch}(k) Q_l^{max}(k) \quad l = 1, \ldots, M \\
&0 \leq Q_l^{dis}(k) \leq \delta_l^{dis}(k) Q_l^{max}(k) \quad l = 1, \ldots, M \\
&\delta_l^{ch}(k) + \delta_l^{dis}(k) \leq 1 \qquad l = 1, \ldots, M \\
&\mathbf{E}(k)^{min} \leq \mathbf{E}(k) \leq \mathbf{E}(k)^{max} \\
&\mathbf{E}_0 = \mathbf{E}_T \\
&\sum_j \bar{\mathcal{O}}_{ij} \mathcal{P}_j - \mathcal{Q}_i^{ch} + \mathcal{Q}_i^{dis} - z_i \Gamma_i - \sum_j p_{ij} \geq \mathcal{L}_i - \tilde{\Sigma}_i \\
&z_i + p_{ij} \geq \tilde{\mathcal{O}}_{ij} y_j \\
&-y_j \leq \mathcal{P}_j \leq y_j
\end{aligned}$$

with ρ is a penalty weight, $i = 1, \ldots, TM$, $j = 1, \ldots, TN$ and $k = 0, \ldots, T-1$. All the auxiliary variables introduced because of the robust formulation, z_i, p_{ij}, y_j $\forall i, j$, are forced to be greater than or equal to zero. The term in the objective function is a feasibility penalty factor: this is introduced to account for the fact that it is not always possible to find a feasible solution for all data realizations and infeasibilities inevitably arise. The resulting RO problem is then quadratic. Note that, by varying $\Gamma_i = [0, |J_i|]$, the level of conservatism of the solution, and then the increase in cost, can be controlled. It is guaranteed that the computed solution is always feasible if less than the prescribed number of coefficients change.

VI. Simulation Results

We consider the hub shown in Figure 2 and we pose the optimization problem as in the previous section, exploiting the hub model proposed in Section II. Converter efficiencies are uncertain: for each converter the efficiency is supposed to vary within an interval of $\pm 10\%$ of its nominal value. Then we apply both the nominal and the RO approaches and we compare the purchase schedules in terms of costs and sensitivity to deviations of the efficiencies from their nominal values. The nominal schedule is computed by solving Problem 1. We consider a short-term planning horizon ($T = 24$). The demands of electricity, hydrogen and heat to be satisfied over the planning horizon are depicted in Figure 3 ($M = 3$). The variable prices over the planning horizon are depicted in Figure 4. The computed nominal schedule is shown in Figure 5: at the current point in time ($k = 0$), a plan of power flows to be purchased is formulated for the next 24 hours, based on the demand displayed in Figure 3, which is assumed to be known. We consider 4 different choices of the number of components subject to parameter uncertainty and then solve the robust scheduling problem 2. The choice is made acting on the parameters Γ_i of the RO model for each

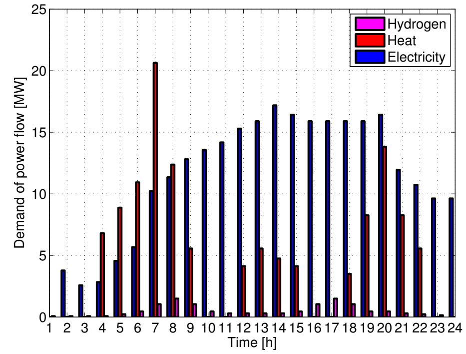

Fig. 3. Demand power flows over 24 hours.

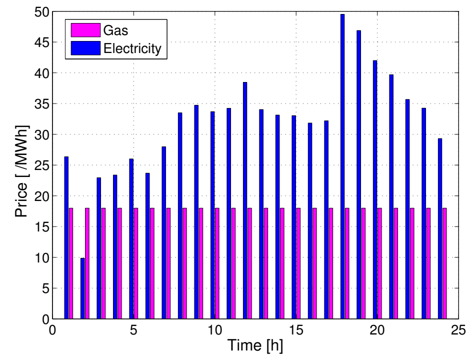

Fig. 4. Electricity and gas spot-prices.

i^{th} row of the matrix $\mathbf{\Theta}$: for example, if the efficiency of one component related to electricity output is known to be uncertain, we must set $\Gamma_1 = 1$ and $\Gamma_2 = \Gamma_3 = 0$. Note that each row of the matrix $\mathbf{\Theta}$ is related to a specific load: for example the electrical load is computed multiplying the first row of $\mathbf{\Theta}$ by the scheduled input power flows. The 4 cases are reported in the following:

1) no component is subject to parameter uncertainty (nominal case),
2) the efficiency of one component for each type of demand is uncertain,
3) the efficiencies of two components for electricity and one for heat are uncertain,
4) all components are subject to parameter uncertainty (Soyster's model),

For a robustness analysis, all efficiencies in the energy hub of Figure 2 are randomly changed within the prescribed range, considering only negative perturbations. If the optimal input power flow purchases at a time step k do not fulfill the deterministic load requirements at that time, we consider the demand at time k unmet. The robust optimization Problem 2 is solved $R = 1000$ times with different perturbations of the predefined number of uncertain efficiencies. We consider electrical and heat loads. Then the probability of electricity, hydrogen and heat demand satisfaction is calculate as $100 * \frac{V}{R}$, where V is the number of times for which the demand is unmet: these probabilities are denoted respec-

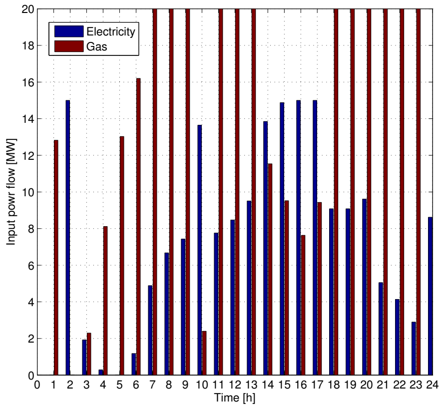

Fig. 5. Nominal schedule of input power flows over 24 hours.

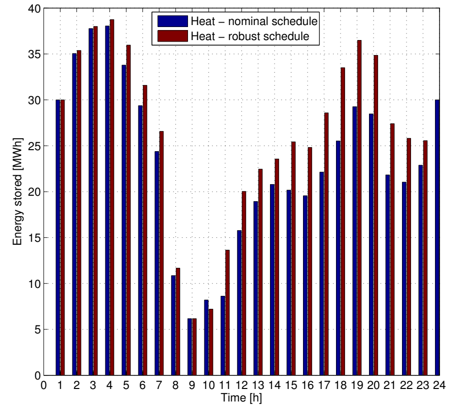

Fig. 6. Energy stored during the planning horizon in the heat storage element, both for the nominal and the robust schedules.

tively by p_{ue}, p_{uH} and p_{uh}. The above probabilities are a measure of how closely each RO formulation satisfies the required demands at the specified uncertainty level. The table I reports the result of the robustness analysis. For this simulation, the auxiliary variables introduced to

TABLE I

Robustness analysis.

Case	p_{ue}	p_{uH}	p_{uh}	% increase in cost
1	98.75	75.52	92.84	0
2	31.38	3.35	13.33	7.87
3	0	75.68	13.3	10.25
4	0	0	0	11.48

account for equality constraints violations are found not to be needed (i.e. they can be set to 0); then they are not reported here. Note that RO schedule is more costly but it is also less sensitive to efficiency uncertainty compared to the nominal schedule. The nominal schedule is the least expensive one but it doesn't provide a good protection against deviations of converter efficiencies from their nominal value. In presence of uncertainty the RO schedule outperforms the nominal solution in terms of demand satisfaction. Namely the robust schedule is more expensive than the nominal one, but the increase in cost can be controlled by a proper choice of Γ_i parameters. In the proposed example, with a small number of uncertain parameter for each load type, reasonable schedule performance can be obtained only guaranteeing a high protection against uncertainty (Case 3 and 4 of the Table I). Consider the robust schedule obtained in Case 3. The Figure 6 depicts the energy stored during the planning horizon of 24h in the heat energy storing device for both the robust and the nominal schedule. The energy stored in the heat storing device shows a meaningful difference since, as expected, the robust schedule requires storing a larger amount of heat than the nominal one to provide the desired robustness. All the simulations are run using CPLEX 12.0 and all computations are done on an Intel Core 2 Duo CPU, 2 GHz. Simulation times are less than 1 second.

VII. Conclusions and Future Research

In this paper we analyzed and solved the robust optimal scheduling problem of an energy hub. We introduce a generalized discrete time linear model of energy hub; this allows to give a mixed integer linear formulation of the operation scheduling problem. By applying a RO technique, we obtained robust schedules of input power flows that are significantly less sensitive to uncertain converter efficiencies than the nominal schedule. The tractability of the problem is preserved. More accurate models accounting for uncertainty in the energy prices and in the energy demand other than in the converter efficiencies, are under current studies.

References

[1] G. Chicco and P. Mancarella, "Distributed multi-generation: A comprehensive view," *Renewable and Sustainable Energy Reviews*, vol. 13, pp. 535–555, 2009.

[2] L. Carradore and F. Bignucolo, "Distributed multi-generation and application of the energy hub concept in future networks," in *43rd International Universities Power Engineering Conference*, 2008.

[3] M. Geidl, G. Koeppel, P. Favre-Perrod, B. Klöckl, G. Andersson, and K. Fröhlich, "Energy hubs for the future," *IEEE Power & Energy Magazine*, vol. 5, pp. 24–30, 2007.

[4] M. Geidl and G. Andersson, "Optimal coupling of energy infrastructures," in *IEEE Power Tech 2007*, 2007.

[5] M. Arnold, R. Negenborn, and B. D. Schutter, *Distributed Predictive Control for Energy Hub Coordination in Coupled Electricity and Gas Networks*. (Negenborn R. R., Lukszo Z., Hellendoorn H. Editors) Intelligent Infrastructures, Springer Verlag, London, 2009.

[6] F. Kienzle and G. Andersson, "Valuing investments in multi-energy generation plants under uncertainty: A real options analysis," in *IAEE European Conference, Vienna, Austria*, 2009.

[7] D. Bertsimas and M. Sim, "The price of robustness," *Operations Research*, vol. 52, no. 1, pp. 35–53, 2004.

[8] A. Hajimiragha, C. Canizares, M. Fowler, M. Geidl, and G. Andersson, "Optimal energy flow of integrated energy systems with hydrogen economy considerations," in *Bulk Power System Dynamics and Control - VII. Revitalizing Operational Reliability, 2007 iREP Symposium*, 2007, pp. 1–11.

[9] A. Ben-Tal and A. Nemirovski, "Robust solutions of uncertain linear programs," *Operations Research*, vol. 25, pp. 1–13, 1999.

[10] A. Messac and A. Ismail-Yahaya, "Multiobjective robust design using physical programming," *Structural and Multidisciplinary Optimization*, vol. 23, no. 5, pp. 357–371, 2002.

[11] A. Soyster, "Convex programming with set-inclusive constraints and applications to inexact linear programming," *Operations Research*, vol. 21, pp. 1154–1157, 1973.