2011 50th IEEE Conference on Decision and Control and
European Control Conference (CDC-ECC)
Orlando, FL, USA, December 12-15, 2011

A stochastic reachability framework for autonomous surveillance with pan-tilt-zoom cameras

Nikolaos Kariotoglou, Davide M. Raimondo, Sean Summers, and John Lygeros

Abstract— **In this work a framework for camera-based autonomous surveillance is introduced based on the theory of stochastic reachability and random sets. We consider set-valued models of a single pan-tilt-zoom (PTZ) camera (pursuer) and multiple targets that need to be tracked (evaders). We define the stochastic pursuer process and the stochastic evader processes and consider the problem of maximizing the probability of satisfying safety (tracking), reachability (acquisition), and reach-avoid (tracking while acquiring) objectives. The solution of the safety, reachability, and reach-avoid tasks are computed via dynamic programming resulting in an optimal control policy for the PTZ camera. Experimental results are given for a single PTZ camera and multiple robotic evaders.**

I. INTRODUCTION

Intelligent surveillance systems deal with the real-time monitoring of persistent and transient objects within an environment [1]. The primary objectives of these systems are to automatically interpret scenes and to understand and predict the actions and interactions of the observed objects. Specific tasks of an intelligent surveillance system include moving object detection and recognition, patrolling, tracking, prediction, and target acquisition. The focus of this work is on the surveillance tasks of tracking and acquisition.

The use of PTZ cameras for tracking and acquisition objectives has recently been exploited in various works, e.g. [2]–[10]. In [8], the problem of optimally patrolling a one-dimensional perimeter with a network of cameras was considered resulting in a distributed control strategy based on local asynchronous communication. Optimal camera movement for the objective of minimizing the time necessary to monitor an environment was addressed in [7] and experimentally tested. In [3], a multi-camera tracking approach based on optimizing the capture of targets using a probabilistic objective function was considered. In [6], the authors introduced a tracking algorithm based on stochastic Model Predictive Control (MPC) coupled with an Extended Kalman Filter and Particle Filter for recursive target state estimation. In [4] a stochastic MPC approach to optimal patrolling was considered and a target tracking algorithm based on Min-Max and minimum time MPC was proposed.

In this work we consider surveillance tasks of target tracking and acquisition in the form of a probabilistic pursuit-evasion game [11]–[13] where the pursuer and the evaders are set-valued and governed by stochastic processes. In particular, we develop a single framework for autonomous surveillance tasks based on the theory of stochastic reachability (for discrete-time stochastic hybrid systems (DTSHS)) and random sets [14], [15]. Here the objective is to maximize the probability of satisfying safety, reachability, and reach-avoid objectives which are directly related tasks of tracking and target acquisition. It is shown that for each of these objectives this probability can be computed by dynamic programming, resulting in an optimal decision policy for the camera. In addition to the optimal decision policy, the solution of the dynamic program also provides a value for the maximum probability of successfully completing the considered surveillance objective. We argue that this knowledge can be useful when making high level decisions (e.g. track a single evader or two evaders), especially when considering a scenario involving multiple cameras where this information may be valuable to exchange between cooperating cameras.

An experimental test bed consisting of a single PTZ camera and multiple evaders was used to verify the feasibility and applicability of the developed framework. The accuracy of the dynamic programming solution was evaluated experimentally using repeated experimental runs. The positive results of the experimental analysis coupled with the consistency of the constructed framework for general surveillance objectives motivates further research in this area.

The rest of the work is arranged as follows. In Section II we briefly recall the theory of stochastic reachability for DTSHS. In Section III, we introduce a probabilistic reachability framework for autonomous surveillance. In Section IV, we introduce (stochastic) models for a single PTZ camera and multiple evaders. In Section V we provide experimental results for the autonomous surveillance system.

II. STOCHASTIC REACHABILITY AND RANDOM SETS

Here we recall the theory of stochastic reachability for DTSHS [14], [16] and stochastic reachability with random sets [15], [17] upon which the framework for autonomous surveillance is built. In particular, the results of this section can be found in detail in the work [15].

A DTSHS $\mathcal{H}$ can be described as a Markov control process with state space X, (compact) control space $\mathcal{A}$, and controlled transition probability function Q. Given a Markov

Nikolaos Kariotoglou, Sean Summers, and John Lygeros are with the Automatic Control Laboratory, Department of Information Technology and Electrical Engineering, ETH Zurich, Physikstrasse 3, 8092 Zurich, Switzerland. karioto,summers,lygeros@control.ee.ethz.ch

Davide M. Raimondo is with University of Pavia, Dipartimento di Informatica e Sistemistica, Via Ferrata, 1, 27100 Pavia, Italy. davide.raimondo@unipv.it. This work has been done while he was at the Automatic Control Laboratory, ETH Zurich.

The authors would like to thank S. Gasparella and D. Sturzenegger for their help in setting up the test bed.

This research was partially supported by the European Commission under the projects Feednetback FP7-ICT-223866 and MoVeS FP7-ICT-2009-257005.

The first three authors contributed equally.

control policy $\mu \in \mathcal{M}_m$ (where $\mathcal{M}_m$ denotes the set of all admissible Markov control policies) and initial state $x_0 \in X$, the execution $\{x_k, k = 0, ..., N\}$ is a time inhomogeneous stochastic process defined on the canonical sample space $\Omega = X^{N+1}$, endowed with its product $\sigma-$algebra $\mathcal{B}(\Omega)$ where $\mathcal{B}(\cdot)$ denotes the Borel $\sigma-$algebra. The probability measure $P^{\mu}_{x_0}$ is uniquely defined by the transition kernel Q, the Markov policy $\mu \in \mathcal{M}_m$, and the initial condition $x_0 \in X$ (see [18]).

For $k = 0, 1, 2, \dots, N$, let G_k be a Borel-measurable stochastic kernel on $\mathcal{Y}$ given $\mathcal{Y}$, $G_k : \mathcal{B}(\mathcal{Y}) \times \mathcal{Y} \to [0, 1]$, which assigns to each $\xi \in \mathcal{Y}$ a probability measure $G_k(\cdot|\xi)$ on the Borel space $(\mathcal{Y}, \mathcal{B}(\mathcal{Y}))$. That is, let G_k represent a collection of probability measures on $(\mathcal{Y}, \mathcal{B}(\mathcal{Y}))$ parameterized by the elements of $\mathcal{Y}$ and indexed by time k. A discrete-time time-inhomogeneous Markov process $\xi = (\xi_k)_{k\in\mathbb{N}_0}$ taking values in the Borel space $\mathcal{Y}$ is described by the stochastic kernel G_k.

Definition 1: A parameterization of a discrete-time set-valued stochastic process is a discrete-time Markov process $\xi = (\xi_k)_{k\in\mathbb{N}_0}$ with parameter space $\mathcal{Y}$ and transition probability function $G_k : \mathcal{B}(\mathcal{Y}) \times \mathcal{Y} \to [0, 1]$ together with a function $\gamma : \mathcal{Y} \to \mathcal{B}(X)$ representing a stochastic (Borel) set-valued evolution on the hybrid state space X (according to the process ξ). Consequently, it holds that there exists a Borel set $\bar{K} \in \mathcal{B}(X \times \mathcal{Y})$ defined

$$\bar{K} = \{(x, \xi) \in X \times \mathcal{Y} | x \in \gamma(\xi)\}.$$

In the spirit of the theory of random closed sets [19], [20], for all $x \in X$, $\xi_{k-1} \in \mathcal{Y}$, and $k \in \mathbb{N}$, we define the following covering function:

$$\begin{aligned} p_{\gamma(\xi_k)}(x) &= P_{\xi_{k-1}}\{x \in \gamma(\xi_k)\} = E_{\xi_{k-1}}[\mathbf{1}_{\gamma(\xi_k)}(x)] \\ &= \int_{\mathcal{Y}} \mathbf{1}_{\bar{K}}(x, \xi_k) G_k(d\xi_k|\xi_{k-1}). \end{aligned}$$

For all $x \in X$ and all $\xi_{k-1} \in \mathcal{Y}$, it follows that the covering function $p_{\gamma(\xi_k)}(x)$ is Borel measurable and bounded between 0 and 1. Now consider the set valued maps $\gamma_1 : \mathcal{Y} \to \mathcal{B}(X)$ and $\gamma_2 : \mathcal{Y} \to \mathcal{B}(X)$ where, for all $\xi \in \mathcal{Y}$, $\gamma_1(\xi) \subseteq \gamma_2(\xi)$. It follows that

$$p_{\gamma_2(\xi_k)\setminus\gamma_1(\xi_k)}(x) = p_{\gamma_2(\xi_k)}(x) - p_{\gamma_1(\xi_k)}(x).$$

A. Finite Horizon Reach-Avoid

Let $K_k, K'_k \in \mathcal{B}(X)$, with $K_k \subseteq K'_k$ for all $k = 0, 1, \dots, N$. Our goal is to evaluate the probability that the execution of the Markov control process associated with the Markov policy $\mu \in \mathcal{M}_m$ and the initial condition x_0 will hit K_k before hitting $X \setminus K'_k$ during the time horizon N. Let $\xi = (\xi_k)_{k\in\mathbb{N}_0}$ with stochastic kernel $G_k : \mathcal{B}(\mathcal{Y}) \times \mathcal{Y} \to [0, 1]$ together with the functions $\gamma_1 : \mathcal{Y} \to \mathcal{B}(X)$ and $\gamma_2 : \mathcal{Y} \to \mathcal{B}(X)$ be a parameterization of a discrete-time set-valued stochastic process. We assume that the initial set parameter state ξ_0 is known, hence $\gamma_1(\xi_0) = K_0$ and $\gamma_2(\xi_0) = K'_0$ is known, and $\gamma_1(\xi_k) = K_k$ and $\gamma_2(\xi_k) = K'_k$ for $k = 1, \dots, N$ is an execution of the stochastic set-valued process. The probability that the system initialized at $x_0 \in X$, with control policy $\mu \in \mathcal{M}_m$ and $\xi_0 \in \mathcal{Y}$, reaches K_k while avoiding $X \setminus K'_k$ for all $k = 0, 1, \dots, N$ is given by

$$\begin{aligned} r^{\mu}_{(x_0,\xi_0)} \quad := \quad & P^{\mu}_{(x_0,\xi_0)}\{\exists j \in [0, N] : x_j \in K_j \wedge \\ & \forall i \in [0, j-1] \;\; x_i \in K'_i \setminus K_i\}, \end{aligned}$$

where $\wedge$ denotes the logical AND, and we operate under the assumption that the requirement on i is automatically satisfied when $x_0 \in K_0$; subsequently we will use a similar convention for products, i.e. $\prod_{i=k}^{j}(\cdot) = 1$ if $k > j$. Note that while we assume knowledge of the initial state and initial set parameter, the consideration of a probabilistic initial condition for each is straightforward.

As in [14], [16], consider

$$\sum_{j=0}^{N} \left(\prod_{i=0}^{j-1} \mathbf{1}_{K'_i\setminus K_i}(x_i) \right) \mathbf{1}_{K_j}(x_j) = \begin{cases} 1, & \text{if } \exists j \in [0, N] : x_j \in K_j \wedge \\ & \quad \forall i \in [0, j-1] \;\; x_i \in K'_i \setminus K_i \\ 0, & \text{otherwise.} \end{cases}$$

Hence $r^{\mu}_{(x_0,\xi_0)}$ can be expressed as the expectation

$$r^{\mu}_{(x_0,\xi_0)} \quad = \quad E^{\mu}_{(x_0,\xi_0)} \left[\sum_{j=0}^{N} \left(\prod_{i=0}^{j-1} \mathbf{1}_{K'_i\setminus K_i}(x_i) \right) \mathbf{1}_{K_j}(x_j) \right].$$

In [15], [17], two classes of problems are considered for the analytical (and computational) evaluation of $r^{\mu}_{(x_0,\xi_0)}$. In the first class, the parameterized set-valued process is described (or can be fairly approximated) by a time-indexed independent distribution of stochastic parameters. In the second class, the parameters of the set-valued process are modeled as a Markov process. In this work we consider the former problem formulation. That is, we assume that the product measure of the parametric process is equal to (or well approximated by) the product measure of time-indexed independent stochastic kernels, i.e. for $N \in \mathbb{N}$

$$\prod_{j=0}^{N} G_j(d\xi_j|\xi_{j-1}) \approx \prod_{j=0}^{N} G_j(d\xi_j).$$

For a DTSHS with independent set-valued reach and safe sets ($\gamma_1(\xi_k) \subseteq \gamma_2(\xi_k)$ almost surely), it can be shown that

$$r^{\mu}_{(x_0,\xi_0)} \quad = \quad E^{\mu}_{x_0} \left[\sum_{j=0}^{N} \left(\prod_{i=0}^{j-1} p_{K'_i\setminus K_i}(x_i) \right) p_{K_j}(x_j) \right].$$

The covering functions are defined

$$\begin{aligned} p_{K_i}(x) &= E\left[\mathbf{1}_{\gamma_1(\xi_i)}(x)\right] = \int_{\mathcal{Y}} \mathbf{1}_{\gamma_1(\xi_i)}(x) G_i(d\xi_i), \\ p_{K'_i}(x) &= E\left[\mathbf{1}_{\gamma_2(\xi_i)}(x)\right] = \int_{\mathcal{Y}} \mathbf{1}_{\gamma_2(\xi_i)}(x) G_i(d\xi_i), \\ p_{K'_i\setminus K_i}(x) &= p_{K'_i}(x) - p_{K_i}(x). \end{aligned}$$

Let $\mathcal{F}$ denote the set of functions from X to $\mathbb{R}$ and define the operator $H : X \times \mathcal{A} \times \mathcal{F} \to \mathbb{R}$ as

$$H(x, a, Z) := \int_X Z(y) Q(dy|x, a). \qquad (2)$$

The following lemma shows that $r^{\mu}_{(x_0,\xi_0)}$ can be computed via a backwards recursion.

Lemma 3: Fix a Markov policy $\mu = (\mu_0, \mu_1, ...\mu_{N-1}) \in \mathcal{M}_m$. The functions $V_k^{\mu} : X \to [0, 1]$, $k = 0, 1, \ldots, N-1$ can be computed by the backward recursion:

$$V_k^{\mu}(x) = p_{K_k}(x) + p_{K'_k \setminus K_k}(x) H(x, \mu_k(x), V_{k+1}^{\mu}), \qquad (4)$$

initialized with $V_N^{\mu}(x) = p_{K_N}(x)$, $x \in X$.

Definition 5: Let $\mathcal{H}$ be a Markov control process, $\xi = (\xi_k)_{k\in\mathbb{N}_0}$ a parametric stochastic process, $K_k \in \mathcal{B}(X)$, $K'_k \in \mathcal{B}(X)$, with $K_k = \gamma_1(\xi_k)$, $K'_k = \gamma_2(\xi_k)$ and $K_k \subseteq K'_k$ almost surely, for all $k = 0, 1, 2, \ldots, N$. A Markov policy μ^* is a maximal reach-avoid policy if and only if $r^{\mu^*}_{(x_0,\xi_0)} = \sup_{\mu\in\mathcal{M}_m} r^{\mu}_{(x_0,\xi_0)}$, for all $x_0 \in X$.

Theorem 6: Define $V_k^* : X \to [0, 1]$, $k = 0, 1, ..., N$, by the backward recursion:

$$V_k^*(x) = \sup_{a\in\mathcal{A}} \{p_{K_k}(x) + p_{K'_k \setminus K_k}(x) H(x, a, V_{k+1}^*)\} \qquad (7)$$

$x \in X$, initialized with $V_N^*(x) = p_{K_N}(x)$, $x \in X$. Then, $V_0^*(x_0) = \sup_{\mu\in\mathcal{M}_m} r^{\mu}_{(x_0,\xi_0)}$, $x_0 \in X$ and $\xi_0 \in \mathcal{Y}$. If $\mu_k^* : X \to \mathcal{A}$, $k \in [0, N-1]$, is such that for all $x \in X$

$$\mu_k^*(x) = \arg\sup_{a\in\mathcal{A}} \{p_{K_k}(x) + p_{K'_k \setminus K_k}(x) H(x, a, V_{k+1}^*)\} \qquad (8)$$

then $\mu^* = (\mu_0^*, \mu_1^*, ..., \mu_{N-1}^*)$ is a maximal reach-avoid policy.

III. Autonoumous Surveillance Framework

Let n adversary objects be distributed in a general spatial frame $X_G \subset \mathbb{R}^3$ (e.g. an auditorium or a stadium). Each set-valued evader $\mathcal{O}^{(i)} \in \mathcal{B}(X_G)$, $i \in \{1, ..., n\}$, is parameterized by a set of parameters $x_e \in X_e$, where X_e is the state space of the adversary parameterization. It follows, by the definition of the kernels in Section II, that the evader set is constrained in X_G and the parametric representation constrained in X_e. We assume that the set-valued evaders are independent and can intersect.

We consider a parametric model for the camera with state x and state space X, $x \in X$. Naturally, there exists a mapping from the state of the camera parameterization x to the set-valued camera view in the spatial frame X_G (defined as the field of view (FOV)). In the general case, this function can be defined as a measureable mapping $\mathcal{L} : X \to \mathcal{B}(X_G)$.

In the spatial frame X_G, the set

$$S_1 = \{x_G \in X_G : x_G \in \cup_i \mathcal{O}^{(i)}\}$$

comprises all states that intersect with the set-valued region of one or more evaders (equivalently the union of the evader sets). S_1 is the coverage of the evaders in the spatial frame.

In the camera space X, the set

$$S_2 = \{x \in X : \mathcal{L}(x) \cap \mathcal{O}^{(i)} \neq \emptyset, \ \ \forall i \in \{1, ..., n\}\}$$

comprises all camera states for which every evader is in the field of view of the camera. Likewise, assuming $n_1 < n$,the set

$$S_3 = \{x \in X : \mathcal{L}(x) \cap \mathcal{O}^{(i)} \neq \emptyset, \ \ \forall i \in \{1, ..., n_1\}\}$$

comprises all camera states for which n_1 evaders are in the view of the camera. Lastly, the set

$$S_4 = \{x \in X : \exists i \in \{1, ..., n\}, \ \ \mathcal{L}(x) \cap \mathcal{O}^{(i)} = \emptyset\}$$

comprises all camera states for which one or more evaders is not visible by the camera.

Uncertainty plays a large role in the estimation and prediction of evader trajectories, dealing with this uncertainty is central to the success of an automated surveillance system. Consider, for instance, in the current example that the parametrization of each set-valued evader $i \in \{1, ..., n\}$ is distributed according to some probability distribution. Under this consideration, it is of interest to know the probability that the evader (or set of evaders) is visible to the camera when the camera is in state x?

It follows that S_1, S_2, S_3, and S_4 are random sets according to the random distribution of the evader parametrization considered above. Further, note that they are dependent random sets (they are parameterized by the random evader centers) and that $S_2 \subseteq S_3$ almost surely. Thus, the question above can be answered by computing the covering functions of the various sets. Specifically, the covering function $p_{S_1}(x_G)$ defines the probability that $x_G \in X_G$ will intersect with one or more evaders. Similarly, $p_{S_2}(x)$ and $p_{S_3}(x)$ represent the probability that the camera in state $x \in X$ will visually capture evaders $\{1, ..., n\}$ and $\{1, ..., n_1\}$ respectively.

Considering the probabilistic sets detailed above, it is possible to formulate surveillance tasks using the stochastic reachability framework of Section II.

Safety(tracking): Minimize the probability that the camera loses sight of one of the evaders at some point during the time horizon $k \in \{0, ..., N\}$. It follows that the safety problem can be formulated where X denotes the safe set and S_4 denotes the target set and the optimal control policy is obtained by solving the DP of Theorem 6 in the minimal case (replace the sup by inf).

Reach(acquisition): Maximize the probability that the camera can reach all n evaders at some point during the finite time horizon $k \in \{0, ..., N\}$. It follows that the reach problem can be formulated where X denotes the safe set and S_2 denotes the target set and the optimal control policy is obtained by solving the DP of Theorem 6.

Reach-Avoid: Maximize the probability that the camera can reach all n evaders at some point during the finite time horizon $k \in \{0, ..., N\}$ while avoiding losing a subset of evaders at each prior time point. It follows that the reach-avoid problem can be formulated where S_3 denotes the safe set and S_2 denotes the target set and the optimal control policy is obtained by solving the DP of Theorem 6.

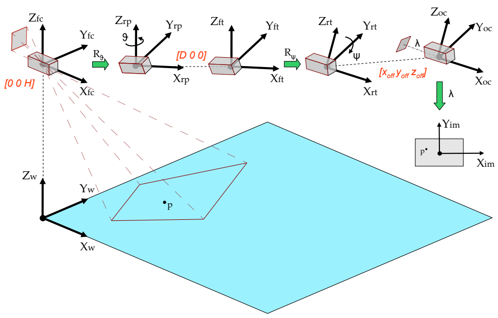

Fig. 1. Pinhole camera model

IV. Camera and Evader Models

The scenario we consider has a PTZ camera and multiple evaders moving on a planar ground plane, i.e. $X_G = \mathcal{B}(\mathbb{R}^2)$. The camera state is $x = [\theta, \psi, \zeta]^T$, where θ, ψ and ζ are respectively camera pan, tilt and zoom. By the ground plane assumption, given the position of a point p in the camera image, it is possible to obtain its position in X_G, see [6]. In the same way, it is possible to compute $\mathcal{L}(x)$. We first introduce the relation between optical center reference frame (oc) and world reference frame (w)

$$\begin{bmatrix} x_w \\ y_w \\ z_w \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ H \end{bmatrix} + R_\theta \left(\begin{bmatrix} D \\ 0 \\ 0 \end{bmatrix} + R_\psi \left(\begin{bmatrix} x_{off} \\ y_{off} \\ z_{off} \end{bmatrix} + \begin{bmatrix} x_{oc} \\ y_{oc} \\ z_{oc} \end{bmatrix} \right) \right)$$

where $p_w = [x_w, y_w, z_w]^T$ are the coordinates of a point p in X_G while $p_{oc} = [x_{oc}, y_{oc}, z_{oc}]^T$ are its coordinates in the optical center reference frame (see Figure 1). $H, D, x_{off}, y_{off}, z_{off}$ are parameters and R_θ, R_ψ the rotation matrices involved. Given a point in optical center reference frame, its position in the image view frame (im) is obtained as follows

$$x_{im} = \lambda(\zeta)\frac{y_{oc}}{x_{oc}} \qquad y_{im} = -\lambda(\zeta)\frac{z_{oc}}{x_{oc}}$$

where $\lambda(\zeta) = \lambda_1 \zeta$ is the focal length for a given level of zoom ζ, while λ_1 is λ value for $\zeta = 1$. In order to compute $\mathcal{L}(x)$ we calculate the coordinates of the optical center projected on X_G, i.e. $[\bar{x}_w, \bar{y}_w, \bar{z}_w]^T$.

Given $\tilde{x}^i_{im}$, $\tilde{y}^i_{im}$, $i = 1, \cdots, 4$, positions of the vertices of the FOV in the camera image, we can compute their projection on the ground plane. Note that the FOV is located at $-\lambda$ on X_{oc} in the optical center reference frame. The shift of $-\lambda$ is expressed in X_G coordinates. In order to find the projection of the FOV vertices on the ground plane, i.e. $\tilde{v}^{iP} = [\tilde{x}^{iP}_w, \tilde{y}^{iP}_w, \tilde{z}^{iP}_w]^T$, we intersect the line passing through $[\tilde{x}^i_w\ \tilde{y}^i_w, \tilde{z}^i_w]^T$ and $[\bar{x}_w, \bar{y}_w, \bar{z}_w]^T$ with the ground plane $Z_w = 0$.

The dynamics of the camera view over the time horizon $k \in \{0, ..., N\}$, $N \in \mathbb{N}$, are given by the stochastic difference equation

$$\begin{bmatrix} x_{1,k+1} \\ x_{2,k+1} \\ x_{3,k+1} \end{bmatrix} = \begin{bmatrix} x_{1,k} + u_{1,k} + w_1 \\ x_{2,k} + u_{2,k} + w_2 \\ x_{3,k} + u_{3,k} + w_3 \end{bmatrix} \qquad (9)$$

where, for all $k \in \{0, ..., N\}$, camera pan, tilt and zoom $[x_{1,k}, x_{2,k}, x_{3,k}]^T \in X \subseteq \mathbb{R}^3$, the camera inputs $[u_{1,k}, u_{2,k}, u_{3,k}]^T \in U \subseteq \mathbb{R}^3$, and the camera noise is i.i.d. according to $w_1 \sim \mathcal{N}(0, \nu_1)$, $w_2 \sim \mathcal{N}(0, \nu_2)$ and $w_3 \sim \mathcal{N}(0, \nu_3)$. This represents both process and measurement noise as a result of innacurate motor movements and disturbed image data.

Each set-valued evader $\mathcal{O}^{(i)} \in \mathcal{B}(X_G)$, $i \in \{1, ..., n\}$, is parameterized by its center and orientation $[x^{(i)}_e, y^{(i)}_e, \phi^{(i)}_e]^T \in X_e$, according to the relation

$$\mathcal{O}^{(i)} = \{x_G \in X_G : (x_{G,1} - x^{(i)}_e)^2 + (x_{G,2} - y^{(i)}_e)^2 \leq (r^{(i)}_e)^2\}$$

where $\mathcal{O}^{(i)}$ is a two-dimensional circle . The dynamics for the center of each evader $i \in \{1, ..., n\}$ over the time horizon $k = \{0, ..., N\}$, $N \in \mathbb{N}$, is modeled by the stochastic difference equation with sampling time Δ_t

$$\begin{bmatrix} x^{(i)}_{e,k+1} \\ y^{(i)}_{e,k+1} \\ \phi^{(i)}_{e,k+1} \end{bmatrix} = \begin{bmatrix} x^{(i)}_{e,k} + \bar{v}^{(i)}_{e,k} sin(\phi^{(i)}_{e,k})\Delta_t \\ y^{(i)}_{e,k} + \bar{v}^{(i)}_{e,k} cos(\phi^{(i)}_{e,k})\Delta_t \\ \phi^{(i)}_{e,k} + \Omega^{(i)}_{e,k}\Delta_t \end{bmatrix} \qquad (10)$$

where, for all $k \in \{0, ..., N\}$, the evader centers $[x^{(i)}_{e,k}, y^{(i)}_{e,k}]^T \in X_G$, the orientation $\phi^{(i)}_{e,k} \in [-\pi, \pi]$, the evader linear velocity $\bar{v}^{(i)}_{e,k} \in V \subseteq \mathbb{R}$. The evader angular velocity $\Omega^{(i)}_{e,k} \in W \subseteq \mathbb{R}$ is i.i.d according to $\mathcal{N}(0, w^{(i)}_\phi)$. This represents process noise associated with movement of the evaders and is completely independant from the camera noise.

Clearly the evader processes defined are Markov (i.e. the stochastic set-valued processes are driven by a parametric Markov process). Evaluating the problem for $n = 2$ evaders using the coupled approach of [15] would require an additional 6 states in the worst case (2 evaders with 3 states each) leading to computational intractability. We therefore make use of the following approximating assumption, to be able to use the decoupled approach [15] introduced in Section II. For each random parametric state ξ (where in this case ξ is the collection of parametric states of all n evaders), we approximate the product measure as

$$\prod_{k=0}^{N} G_k(d\xi_k|\xi_{k-1}) \approx \prod_{k=0}^{N} \hat{G}_k(d\xi_k)$$

where each stochastic kernel $\hat{G}_k(d\xi_k)$ is taken as the marginal (according to ξ_k) of the product measure, i.e.

$$\hat{G}_k(d\xi_k) = \int_{\mathcal{Y}^k} \prod_{i=0}^{k} G_i(d\xi_i|\xi_{i-1}).$$

This naturally leads to covering functions of the form

$$p_{\gamma_j(\xi_k)}(x) = E\left[\mathbf{1}_{\gamma_j(\xi_k)}(x)\right] = \int_{\mathcal{Y}} \mathbf{1}_{\gamma_j(\xi_k)}(x)\hat{G}_k(d\xi_k)$$

for $j \in \{1, 2\}$.

V. Experimental Results

Here we consider $n \in \{1, 2\}$ and a finite time horizon of $N = 9$. We evaluate four surveillance tasks as probabilistic reach-avoid problems with random sets. In each case, the stochastic reachability problem is solved via dynamic programming based on the computational methods of [21]. The result is an optimal control policy $\mu_k^*(x)$ for the DTSHS that can be applied in open loop (or receding horizon). The dynamic program also provides the optimal value function $V_k^*(x)$ which represents the probability of success of the considered surveillence tasks. Not only is this information valuable on its own, but it can also be used to make high level decisions. For example, if the probability of acquiring evader 2 while tracking evader 1 is significantly lower than only tracking evader 1, then the prudent thing for the camera may be to track evader 1.

The camera set evolves according to an *a priori* calculated optimal control policy while the evaders follow the differential robot dynamics introduced above. In the current experimental setup we apply the optimal policy over the time horizon in open loop in order to verify the accuracy of the dynamic programming results. An extension towards the application of the computed policies in receding horizon is currently being explored. The experimental setup follows.

With the support of Videotec S.p.A., ETH has set up a laboratory test bed comprising of a PTZ Ulisse compact camera capable of covering the whole evader space. For simplicity, we have assumed the zoom constant, an assumption that is beeing relaxed in current work.

The calibrated values of camera set-up have been calculated in an earlier study [6] and are: $H = 2.5$, $D = 0.124$, $x_{off} = 0.1485$ $y_{off} = -0.0275$, $z_{off} = 0.02$, $\lambda_1 = 0.0297$. The camera is positioned at $x_w = 0$ meters and $y_w = 4.55$ meters. The state space bounds are $x_{1,k} \in [-\pi/2, 0]$ and $x_{2,k} \in [\pi/5.5, \pi/2]$ where $x_{1,k}$ and $x_{2,k}$ are measured in radians. The control inputs are bounded according to $u_{1,k} \in [-0.0262, 0.0262]$ and $u_{2,k} \in [-0.0262, 0.0262]$ with units in radians and the variance of the noise is $\nu_1 = \nu_2 = \nu_3 = 10^{-6}$ (i.e. the camera movement is fairly accurate). The radius of the Khepera robots (i.e. the evaders) is $r_e^{(i)} = 0.06$ meters for all $i \in \{1, 2\}$ and the linear velocity of the robots is constant, $\bar{v}_{e,k}^{(i)} = 0.24$ meters per second. The angular velocity (radians per second), however, is assumed to be random and i.i.d. according to a bounded gaussian distribution, i.e. $\Omega_{e,k}^{(i)} \sim \mathcal{N}(0, \pi/8)$ with $|\Omega_{e,k}^{(i)}| \leq \pi$. The planar ground plane, is $X_G = \{[x_w, y_w] : x_w \in [0, 3.8], y_w \in [0, 4.5]\}$ (in meters). For computational purposes, the environment has been discretized with step 0.015 meters on both x_w and y_w and the camera state has been discretized by 0.0262 radians. The sampling time for both processes is $\Delta_t = 0.25$ seconds. The initial positions of the camera and robot evaders for each survailance objective are presented in Table I.

A. Acquiring a single evader

Consider the problem of maximizing the probability that the camera acquires evader 1 over the finite time horizon $k \in \{0, ..., N\}$. For each $k \in \{0, ..., N\}$, we define the sets

$$\begin{aligned} S_k &= S_k^{(1)} \times S_k^{(2)} \\ &= \{x_k \in X : \mathcal{L}(x_k) \cap \mathcal{O}_k^{(i)} \neq \emptyset, \ \forall i \in \{1\}\} \times X. \end{aligned}$$

The target set $K_k = S_k^{(1)}$ comprises all camera states where all evaders are visible to the camera at step k. The safe set, $K'_k = S_k^{(2)}$, comprises all camera states. Note that the camera has achieved the acquisition criteria if there exists a time $k \in \{0, ..., N\}$ such that $x_k \in S_k^{(1)}$. We pose this problem as a reachability problem where the objective is to maximize the probability of reaching $S_k^{(1)}$ at some point over the time horizon. The optimal value function indicates high probabilities of success for configurations defining FOVs close to the initial position of the evader. In Table II, the result of applying the same optimal policy on repeated experiments confirms the numerical results obtained by computer simulations.

B. Tracking a single evader

Consider the problem of maximizing the probability that the camera tracks evader 1 over the finite time horizon $k \in \{0, ..., N\}$. For each $k \in \{0, ..., N\}$, we define the sets

$$\begin{aligned} S_k &= S_k^{(1)} \times S_k^{(2)} \\ &= \{x_k \in X : \exists i \in \{1\}, \ \mathcal{L}(x_k) \cap \mathcal{O}_k^{(i)} = \emptyset\} \times X. \end{aligned}$$

The target set, $K_k = S_k^{(1)}$, comprises all camera states where one or more evaders is not visible to the camera at step k. The safe set, $K'_k = S_k^{(2)}$, comprises all camera states. Note that the camera has failed to meet the tracking criteria if there exists a time $k \in \{0, ..., N\}$ such that $x_k \in S_k^{(1)}$. Thus, it follows that maximizing the probability that the camera tracks the evaders (given as $V_k^*(x)$ for $k \in \{0, ..., N\}$) is equivalent to minimizing the probability that the camera loses view of any evader at some $k \in \{0, ..., N\}$ (given as $\hat{V}_k^-(x)$ for $k \in \{0, ..., N\}$) since $V_k^*(x) = 1 - \hat{V}_k^-(x)$ always [14], [16], [22]. As in [14], [16], [22], we pose this problem as a target hitting problem where the objective is to minimize the probability of attaining $S_k^{(1)}$ at some point over the time horizon. According to the solution, if the evader is covered at $k = 0$ the probability of success is very high and this was confirmed by the experiments (Table II).

C. Tracking a single evader while acquring a second evader

Here we consider the problem of maximizing the probability that the camera acquires all $n = 2$ evaders at some point during the finite time horizon $k \in \{0, ..., N\}$ while tracking evader 1 at each prior time point. For each $k \in \{0, ..., N\}$, we define the sets

$$\begin{aligned} S_k &= S_k^{(1)} \times S_k^{(2)} \\ &= \{x_k \in X : \mathcal{L}(x_k) \cap \mathcal{O}_k^{(i)} \neq \emptyset, \ \forall i \in \{1, 2\}\} \times \\ &\qquad \{x_k \in X : \mathcal{L}(x_k) \cap \mathcal{O}_k^{(i)} \neq \emptyset, \ \forall i \in \{1\}\}. \end{aligned}$$

The target set $K_k = S_k^{(1)}$ comprises all camera states where both evaders are visible to the camera at step k. Similarly,

	$p_0^{(1)}$	$p_0^{(2)}$	x_0
Acquire 1	[1.9, 2, 0.46]	-	[-60,46]
Track 1	[1.9, 2, 0.46]	-	[-52,40]
Track 1, Acquire 2	[1.9, 2, 0.46]	[2.5, 2.6, -2.35]	[-52,37]
Track 1,2	[1.9, 2, 0.46]	[2, 2.1, 0.77]	[-51,40]

TABLE I

INITIAL STATE VALUES FOR THE CAMERA AND THE MULTIPLE EVADERS

	$V_0^*(x_0)$	Success/Experiments
Acquire 1	0.967	23/25 = 0.92
Track 1	0.996	24/25 = 0.96
Track 1 and Acquire 2	0.787	19/25 = 0.76
Track 1,2	0.756	17/25 = 0.68

TABLE II

EXPERIMENTAL VALIDATION OF THE SURVEILLANCE FRAMEWORK

$K'_k = S_k^{(2)}$ comprises all camera states where evader 1 is visible to the camera at step k. Trivially, $S_k^{(1)} \subseteq S_k^{(2)}$.(Note that this holds always since if all 2 evaders are visible to the camera then evader $i \in \{1\}$ must also be visible to the camera.) Thus, the camera has achieved the target tracking and acquisition criteria if there exists a time $k \in \{0, ..., N\}$ such that $x_k \in S_k^{(1)}$ and $x_j \in S_j^{(2)}$ for all $j \in \{0, ..., k-1\}$. Experimental results of the tracking while acquiring problem are reported in Table II.

D. Tracking multiple evaders

Consider the problem of maximizing the probability that the camera tracks both evaders $i \in \{1, 2\}$ over the horizon $k \in \{0, ..., N\}$. For each $k \in \{0, ..., N\}$, we define the sets

$$\begin{aligned} S_k &= S_k^{(1)} \times S_k^{(2)} \\ &= \{x_k \in X : \exists i \in \{1, 2\},\ \mathcal{L}(x_k) \cap \mathcal{O}_k^{(i)} = \emptyset\} \times X. \end{aligned}$$

The target set, $K_k = S_k^{(1)}$, comprises all camera states where one or more evaders is not visible to the camera at step k. The safe set, $K'_k = S_k^{(2)}$, comprises all camera states. We again pose the tracking problem as a target hitting problem where the objective is to minimize the probability of attaining $S_k^{(1)}$ at some point over the time horizon. Experimental results of tracking both evaders are reported in Table II.

VI. CONCLUSION

In this work a framework for camera-based autonomous surveillance was introduced based on the theory stochastic reachability and random sets. The problem of maximizing the probability of satisfying safety (tracking), reachability (target acquisition), and reach-avoid (target tracking while acquiring a target) objectives was considered and solved via dynamic programming. Experimental results were provided for a single PTZ camera and multiple robotic evaders and compared to the theoretical solution.

REFERENCES

[1] M. Valera and S. Velastin, "Intelligent distributed surveillance systems: a review," *IEE Proceedings - Vision, Image, and Signal Processing*, vol. 152, no. 2, pp. 192–204, 2005.

[2] N. Bellotto, E. Sommerlade, B. Benfold, C. Bibby, I. Reid, D. Roth, C. Fernández, L. Van Gool, and J. Gonzalez, "A distributed camera system for multi-resolution surveillance," in *Third ACM/IEEE International Conference on Distributed Smart Cameras (ICDSC 2009)*, 2009.

[3] N. Krahnstoever, T. Yu, S. Lim, K. Patwardhan, and P. Tu, "Collaborative real-time control of active cameras in large scale surveillance systems," 2008.

[4] O. Avni, F. Borrelli, G. Katzir, E. Rivlin, and H. Rotstein, "Scanning and tracking with independent cameras–a biologically motivated approach based on model predictive control," *Autonomous Robots*, vol. 24, no. 3, pp. 285–302, 2008.

[5] I. Everts, N. Sebe, and G. Jones, "Cooperative object tracking with multiple ptz cameras," in *the 14th Intl Conf. on Image Analysis and Processing*, Citeseer, 2007.

[6] D. Raimondo, S. Gasparella, D. Sturzenegger, J. Lygeros, and M. Morari, "A tracking algorithm for PTZ cameras," in *2nd IFAC Workshop on Distributed Estimation and Control in Networked Systems (NecSys 10), Annecy, France*, 2010.

[7] G. Fiengo, D. Castiello, G. Grande, and M. Solla, "Optimal camera trajectory for video surveillance systems," in *American Control Conference, 2006*, p. 5, IEEE, 2006.

[8] M. Baseggio, A. Cenedese, P. Merlo, M. Pozzi, and L. Schenato, "Distributed perimeter patrolling and tracking for camera networks," in *Conference on Decision and Control (CDC10)*, 2010.

[9] B. Song, C. Soto, A. Roy-Chowdhury, and J. Farrell, "Decentralized camera network control using game theory," in *Distributed Smart Cameras, 2008. ICDSC 2008. Second ACM/IEEE International Conference on*, pp. 1–8, IEEE, 2008.

[10] D. Ilie, *On-Line Control of Active Camera Networks*. PhD thesis, University of North Carolina, 2010.

[11] J. Hespanha, H. J. Kim, and S. Sastry, "Multiple-agent probabilistic pursuit-evasion games," in *Decision and Control, 1999. Proceedings of the 38th IEEE Conference on*, 1999.

[12] R. Vidal, O. Shakernia, H. Kim, D. Shim, and S. Sastry, "Probabilistic pursuit-evasion games: theory, implementation, and experimental evaluation," *Robotics and Automation, IEEE Transactions on*, vol. 18, pp. 662 – 669, Oct. 2002.

[13] J. Hespanha, M. Prandini, and S. Sastry, "Probabilistic pursuit-evasion games: A one-step nash approach," in *Decision and Control, 2000. Proceedings of the 39th IEEE Conference on*, vol. 3, pp. 2272–2277, IEEE, 2002.

[14] S. Summers and J. Lygeros, "Verification of discrete time stochastic hybrid systems: A stochastic reach-avoid decision problem," *Automatica*, vol. 46, no. 12, pp. 1951 – 1961, 2010.

[15] S. Summers, M. Kamgarpour, C. Tomlin, and J. Lygeros, "A stochastic reach-avoid decision problem with random sets," tech. rep., ETH Zurich. vol. AUT11-13.

[16] S. Summers and J. Lygeros, "A Probabilistic Reach-Avoid Problem for Controlled Discrete Time Stochastic Hybrid Systems," in *IFAC Conference on Analysis and Design of Hybrid Systems, ADHS*, (Zaragoza, Spain), Sept. 2009.

[17] S. Summers, M. Kamgarpour, C. Tomlin, and J. Lygeros, "A Stochastic Reach-Avoid Problem with Random Obstacles," in *HSCC (Hybrid Systems: Computation and Control)*, (Chicago, USA), Apr. 2011.

[18] D. P. Bertsekas and S. E. Shreve, *Stochastic Optimal Control: The Discrete-Time Case*. Athena Scientific, February 2007.

[19] G. Matheron, *Random Sets and Integral Geometry*. New York: Wiley, 1975.

[20] I. Molchanov, *Theory of Random Sets*. New York: Springer, 2005.

[21] A. Abate, S. Amin, M. Prandini, J. Lygeros, and S. Sastry, "Computational approaches to reachability analysis of stochastic hybrid systems," in *Hybrid Systems: Computation and Control* (A. Bemporad, A. Bicchi, and G. C. Buttazzo, eds.), vol. 4416 of *Lecture Notes in Computer Science*, pp. 4–17, Springer, 2007.

[22] A. Abate, M. Prandini, J. Lygeros, and S. Sastry, "Probabilistic reachability and safety for controlled discrete time stochastic hybrid systems," *Automatica*, vol. 44, pp. 2724–2734, November 2008.