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Sensor fault diagnosis for bilinear systems using data-based residuals

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Abstract— **The proposed data-based FDI method has the advantage to require only available data: control signals and measured outputs. The unique information that we have about the system is its structure but the parameters values are supposed to be unknown. This data-based residual method was described, by the same authors in previous publication, for linear systems. In this paper we apply this method to bilinear structure models. A particular focus is made on computational complexity reduction. It will be shown that a part of the system dynamics may be neglected with the consequence to simplify the on-line residual computation. This method is illustrated on a simulated Activated Sludge Process.**

Index Terms— **Data driven methods, bilinear systems, activated sludge process.**

I. INTRODUCTION

Fault occurrence and propagation in industrial systems can cause human injuries, environmental impact and economic loss. Real time system supervision for early fault detection and isolation is a crucial task for systems safety and reliability. Indeed early fault detection and isolation (FDI) allow fault accommodation and avoid system breakdown. Several FDI approaches have been developed. In general, these approaches can be categorized into signal-based and model-based methods [2]. Model-based methods aim is to compare system behavior to normal one represented by a mathematical model. A residual which is a fault indicator signal is used. The residual is close to zero in a normal situation and differs from zero if a fault occurs. Three main classes of residual generation methods are proposed in the literature:

- observer-based methods [5], [6] and [7]
- analytical redundancy relation (ARR) based methods [1]
- parameters estimation methods [4], [8]

In some cases, an accurate system model is not available.

In many applications data-based methods [2], [12] are proposed for fault detection and isolation in these situations. Signal-based methods [11] aim at detecting faults by testing specific properties of measurement signals using spectral or statistical analysis. The proposed method here uses only the information about the system class and no model parameters knowledge is needed. Thus, the only information which is needed to apply this method, is the structure of the model equations, considered as bilinear in this paper. The residual is obtained using an input-output matrix relation and under the hypothesis that the system is stable. This input-output matrix relation is projected into the kernel of the input matrix.

The proposed method has the advantage of generating structured residuals for multiple sensors faults, which are consequently easily detected and isolated.

This data-based residual generation method was described in details in [10] for switching systems and in [9] linear systems. We propose here an extension for bilinear structure models.

Bilinear systems, as a special class of nonlinear systems have been extensively studied in recent years [3] for three main reasons. First, it has been shown that bilinear systems are mathematical models that correspond to many practical applications. Second, bilinear systems provide an approximation to a large class of nonlinear systems. Third, bilinear systems have rich geometric and algebraic structures whose manipulation is not trivial and consequently are interesting from theoretical point of view.

The objective of this paper is to extend the data-based residual generation method for bilinear structure models. The obtained residual expression is rather complex in this situation and leads to manipulate high dimensional input-output matrices. Consequently, the method could be difficult to implement for on-line diagnosis, where computing resources are often limited. With the aim to reduce the computations, we propose to neglect certain terms in the residual expression. A criterion is proposed for choosing the tuning parameters of the method (i.e. the window sizes) which maximizes the fault sensitivity and minimizes the neglected terms sensitivity. The proposed approach is applied to a simulated activated sludge process, where detection and isolation of sensors faults are successfully conducted.

The paper is organized as follows. In section 2, the treated problem addressed in this paper is presented. In section 3, the output expression is given. In section 4, data-based residual for sensor fault diagnosis is proposed. A criterion is proposed to choose the tuning parameters (i.e. window sizes) that allows certain dynamics to be neglected. In section 5, simulation results on an Activated Sludge Process are presented to show the effectiveness of our method. Finally, a conclusion is drawn in section 6.

II. PROBLEM FORMULATION

Consider N known inputs $u_k \in R^m$ and outputs $y_k \in R^\ell$ affected by colored white noise $w_k \in R^\ell$. These values are collected from the following discrete-time bilinear system

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expression given by:

$$\begin{cases} x_{k+1} = Ax_k + G(x_k \otimes u_k) + Bu_k \\ y_k = Cx_k + Du_k + f_k + w_k \end{cases} \quad (1)$$

where $\otimes$ represents the Kronecker product, and $f_k \in R^\ell$ is the sensor fault vector.

The target is to detect and to isolate sensor faults when supposing that the only available information is the system structure (bilinear) and input/output data. The system parameters ($A \in R^{n\times n}$, $B \in R^{n\times m}$, $C \in R^{\ell\times n}$, $D \in R^{\ell\times m}$, $G \in R^{n\times nm}$) are supposed to be unknown. The linear and bilinear parts are assumed to be stable i.e. $max(|eig(A)|) < 1$ and $max(|eig(G)|) < 1$.

We will detail our data-based residual generation method for bilinear systems in section III. Let us first give a general principle of this method. Under the stability conditions, it is possible to express the vector of outputs on a given time window as a function of the inputs and system parameters. The following expression is thus obtained.

$$Y = HU \quad (2)$$

where H depends only on system parameters, U and Y are matrices of inputs and outputs collected on a given time window. If the chosen time window is sufficiently large, we can then project equation (2) on the right kernel Π of U ($U\Pi = 0$) and we can derive the residual:

$$r = Y\Pi = 0 \quad (3)$$

This relation must be verified in absence of disturbances and faults.

When a fault occurs, r becomes different from zero and it can be used for FDI.

It is clear that no system parameter or state estimation is needed for residual computation since Π depends only on inputs, which makes residual expression (3) independent on model parameters.

III. Output expression y_k on a time-window of size i

The objective of this section is to show how to derive equation (2) from system (1). A general expression of the output y_k is first obtained. Then it is shown that the influence of the state may be neglected under the stability conditions, which leads to equation (2).

A. Exact expression of the output y_k

A general output expression y_k can be derived, which is given in the following proposition and proved by recurrence in the appendix.

Proposition 1: The general expression of the output y_k in function of the state x_{k-i}, the inputs and system parameters A, B, C, D, G is given by:

$$\forall i \geq 0 :$$
$$y_k = CA^i x_{k-i} + \overline{H}_i \overline{u}_{k,i} + \widetilde{H}_i \widetilde{u}_{k,i} + \overline{\overline{H}}_i \overline{u}_{k,i} + \widetilde{\widetilde{H}}_i \widetilde{u}_{k,i} + f_k + w_k \quad (4)$$

where $\widetilde{H}$, $\widetilde{\widetilde{H}}$, $\widetilde{u}$, $\overline{H}$, $\overline{\overline{H}}$ and $\overline{u}$ are detailed in what follows.

a) Expression of $\widetilde{u}_{k,i}$, $\widetilde{H}_i$ and $\widetilde{\widetilde{H}}_i$, which represent the bilinear part of equation (4) are provided in the following paragraph.

- $$\widetilde{u}_{k,i} = \begin{bmatrix} \vdots \\ (u_{k-s_1(j_2)} \otimes u_{k-s_2(j_2)} \otimes ... \otimes u_{k-s_d(j_2)}) \\ \vdots \end{bmatrix}$$

$\in R^{M(i)\times 1}$ is constructed with all possible combinations of inputs in a time window of size i (there is $K(i) = \sum_{\alpha=0}^{\alpha=i-2} C_i^{i-\alpha}$ possible combination[1]), where $M(i) = \sum_{\alpha=0}^{i-2} m^{i-\alpha} C_i^{i-\alpha}$, with the following constraint:

$$\forall j_2 \in \{1, ..., K(i)\}, \forall d \in \{2, ..., i\}, \forall s_p(j_2) \in \{1, ..., i-1\},$$

$$\forall p \in \{2, ..., d\}, \forall s_1(j_2) \in \{2, ..., i\} : s_1(j_2) > s_2(j_2) > ... > s_d(j_2).$$

Illustration: From the output expression (4) for $i = 3$, $\widetilde{u}_{k,3}$ is given by:

$$\widetilde{u}_{k,3} = \begin{bmatrix} u_{k-3} \otimes u_{k-2} \otimes u_{k-1} \\ u_{k-3} \otimes u_{k-2} \\ u_{k-3} \otimes u_{k-1} \\ u_{k-2} \otimes u_{k-1} \end{bmatrix}$$

where $s_1(1) = 3$, $s_2(1) = 2$ and $s_3(1) = 1$
$s_1(2) = 3$ and $s_2(2) = 2$
$s_1(3) = 3$ and $s_2(3) = 1$
$s_1(4) = 2$ and $s_2(4) = 1$

- $\widetilde{H}_i$ depends on system parameters, and it is given by:

$$\widetilde{H}_i = \left[C.\Gamma_{1,i}.(B \otimes I) |\, C.\Gamma_{2,i}.(B \otimes I) | \cdots |\, C.\Gamma_{K(i),i}.(B \otimes I) \right]$$

where $\Gamma_{j_2,i}$ is a product of the matrices A, $(A \otimes I)$, G and $(G \otimes I)$. For each $j_2 \in \{1, ..., K(i)\}$, these matrices are arranged differently, and they can appear several times with different powers, the sums of these powers is given by: $\rho_1(j_2)$ for A, $\rho_2(j_2)$ for $(A \otimes I)$, $\rho_3(j_2)$ for G and $\rho_4(j_2)$ for $(G \otimes I)$. We have the following property:

$$\forall j_2 \in \{1, ..., K(i)\} : \rho_1(j_2) + \rho_2(j_2) + \rho_3(j_2) + \rho_4(j_2) \leq i - 1 \quad (5)$$

Illustration: From the output expression (4) for $i = 3$, $\widetilde{H}_3$ is given by:

$$\widetilde{H}_3 = \left[CG.(G \otimes I_m).(B \otimes I_{m^2}) \,|CAG.(B \otimes I_m)| \right.$$
$$\left. CG.(A \otimes I_m).(B \otimes I_m) \,|CG.(B \otimes I_m) \right]$$

where $\rho_1(1) = \rho_2(1) = 0$ and $\rho_3(1) = \rho_4(1) = 1$
$\rho_2(2) = \rho_4(2) = 0$ and $\rho_1(2) = \rho_3(2) = 1$
$\rho_1(3) = \rho_4(3) = 0$ and $\rho_2(3) = \rho_3(3) = 1$
$\rho_1(4) = \rho_2(4) = \rho_4(4) = 0$ and $\rho_3(4) = 1$

[1] $C_n^p = \frac{n!}{(n-p)!p!}$

• Unlike $\widetilde{H}_i$, $\widetilde{\widetilde{H}}_i$ does not only depend on system parameters but also on state, it is defined by:

$$\widetilde{\widetilde{H}}_i = \big[C.\Psi_{1,i}.(x_{k-i} \otimes I)|\, C.\Psi_{2,i}.(x_{k-i} \otimes I)| \cdots$$
$$|C.\Psi_{K(i),i}.(x_{k-i} \otimes I)\big]$$

where $\Psi_{j_2,i}$ is a product of the matrices A, $(A \otimes I)$, G and $(G \otimes I)$. For each $j_2 \in \{1,...,K(i)\}$, these matrices are arranged differently, and they can appear several times with different powers, the sums of these powers is given by: $\lambda_1(j_2)$ for A, $\lambda_2(j_2)$ for $(A \otimes I)$, $\lambda_3(j_2)$ for G and $\lambda_4(j_2)$ for $(G \otimes I)$. We have the following property:

$$\forall j_2 \in \{1,...,K(i)\} : \lambda_1(j_2) + \lambda_2(j_2) + \lambda_3(j_2) + \lambda_4(j_2) = i \quad (6)$$

Illustration: From the output expression (4) for $i = 3$, $\widetilde{\widetilde{H}}_3$ is given by:

$$\widetilde{\widetilde{H}}_3 = \big[CG.(G \otimes I_m).(G \otimes I_{m^2}).(x_{k-3} \otimes I_{m^3})\;|$$
$$CAG.(G \otimes I_m).(x_{k-3} \otimes I_{m^2})\;|$$
$$CG.(A \otimes I_m).(G \otimes I_m).(x_{k-3} \otimes I_{m^2})\;|$$
$$CG.(G \otimes I_m).(A \otimes I_{m^2}).(x_{k-3} \otimes I_{m^2})\big]$$

where $\lambda_1(1) = \lambda_2(1) = 0$, $\lambda_3(1) = 1$ and $\lambda_4(1) = 2$
$\lambda_2(2) = 0$ and $\lambda_1(2) = \lambda_3(2) = \lambda_4(2) = 1$
$\lambda_1(3) = 0$ and $\lambda_2(3) = \lambda_3(3) = \lambda_4(3) = 1$
$\lambda_1(4) = 0$ and $\lambda_2(4) = \lambda_3(4) = \lambda_4(4) = 1$

b) Expression of $\overline{u}_{k,i}$, $\overline{H}_i$ and $\overline{\overline{H}}_i$, which represent the linear part of equation (4) are provided in the following paragraph.

• $\overline{u}_{k,i} = \big[u_{k-i}^T \cdots u_{k-1}^T\, u_k^T\big]^T \in R^{m.(i+1)\times 1}$

• $\overline{H}_i$ depends on system parameters, and it is given by:

$$\overline{H}_i = \big[CA^{i-1}B| \cdots |\, CB|\, D\big] \in R^{\ell \times m.(i+1)}.$$

• Unlike $\overline{H}_i$, $\overline{\overline{H}}_i$ does not depend only on system parameters but also on state, and it is given by:

$$\overline{\overline{H}}_i = \big[CA^{i-1}G.(x_{k-i} \otimes I_m)| \cdots |\, CAG.(A \otimes I_m)^{i-2}.(x_{k-i} \otimes I_m)$$
$$|CG.(A \otimes I_m)^{i-1}.(x_{k-i} \otimes I_m)|\, 0_{\ell,m}\big] \in R^{\ell \times m.(i+1)}$$

B. State-free expression of the output y_k

For i sufficiently large, under the stability conditions, the state influence may be neglected in expression (4). This leads to the following approximated expression of the output y_k, so one does not need to estimate the system state for fault diagnosis process.

$$y_k \cong \overline{H}_i \overline{u}_{k,i} + \widetilde{H}_i \widetilde{u}_{k,i} + f_k + w_k \quad (7)$$

The aforementioned simplification of the state terms is based on the stability of the linear and bilinear parts (which is not the global stability).

a) *First simplification*:
If the linear part of the system is stable, then A^i tends to zero for i sufficiently large (this implies that $(A \otimes I)^i$ also tends to zero).

Hence,

$$CA^i x_{k-i} \to 0 \quad (8)$$

b) *Second simplification*:
$\forall j_1 \in \{1,...,i\}$, using the sub-multiplicative norm, $0 < \left\|A^{i-j_1} G(A \otimes I_m)^{j_1-1}\right\|_2 < (max(\|A\|_2\,, \|G\|_2\,, \|A \otimes I_m\|_2))^i$
The power of the maximum is i, because it represents the sum of the powers of the matrices A, G and $(A \otimes I_m)$. If the linear and bilinear parts of the system are stable, then $(max(\|A\|_2\,, \|G\|_2\,, \|A \otimes I_m\|_2))^i \to 0$ for i sufficiently large. As a consequence

$\left\|A^{i-j_1} G(A \otimes I_m)^{j_1-1}\right\|_2 \to 0.$

Hence,

$$\overline{\overline{H}}\overline{u}_{k,i} \to 0 \quad (9)$$

c) *Third simplification*:

$\forall j_2 \in \{1,...,K(i)\}$, using the sub-multiplicative norm, $0 < \|\Psi_{j_2,i}\|_2 < (max(\|A\|_2\,, \|A \otimes I\|_2\,, \|G\|_2\,, \|G \otimes I\|_2))^i$. From equation (6), the power of the maximum is i.

If the linear and bilinear parts of the system are stable, then

$$(max(\|A\|_2\,, \|A \otimes I\|_2\,, \|G\|_2\,, \|G \otimes I\|_2))^i \to 0$$

for i sufficiently large. Consequently $\|\Psi_{j_2,i}\|_2 \to 0$.

Hence,

$$\widetilde{\widetilde{H}}\widetilde{u}_{k,i} \to 0 \quad (10)$$

IV. Data-based residual for sensor fault diagnosis in a bilinear system

In this section, a data-based residual ϵ_k is proposed for sensor fault detection and isolation.
Consider an integer L, which is chosen such that $L > m.(i + i!) + \ell$. Let us define the following short-hand matrices:
$Y_k = \big[y_{k-L+1} \cdots y_{k-1}\, y_k\big] \in R^{\ell \times L}$

$$U_k = \begin{bmatrix} \overline{u}_{k-L+1,i} \cdots \overline{u}_{k-1,i}\, \overline{u}_{k,i} \\ \widetilde{u}_{k-L+1,i} \cdots \widetilde{u}_{k-1,i}\, \widetilde{u}_{k,i} \end{bmatrix} \in R^{m(i+1)+M(i) \times L}$$

and $\Pi_k = I_L - U_k^T (U_k U_k^T)^{-1} U_k \in R^{L \times L}$ is the projection matrix, which defines the right kernel of the inputs matrix U_k. Consequently, we have:

$$U_k \Pi_k = 0. \quad (11)$$

Proposition 2: The proposed data-based residual is defined as follows:

$$\epsilon_k = Y_k \Pi_k Z \in R^\ell \quad (12)$$

with $Z = \big[0| \cdots |\, 0|\, 1\big]^T \in R^\ell$ (the last column of $Y_k \Pi_k$ is chosen since it represents the current information), It is proved that:(fault detection)

- $E[\epsilon_k] = 0$ when no fault occurs.
- $E[\epsilon_k] \neq 0$ when a fault occurs.

For fault isolation, it is clear to notice that the proposed residual is structured, where each row of ϵ_k is sensitive to the corresponding sensor fault.

Proof:

By stacking equation (7) on a time window of size L, it leads to the following expression:

$$Y_k = \left[\overline{H}_i | \widetilde{H}_i\right] U_k + \Delta_k^i + F_k + W_k \quad (13)$$

where the fault matrix F_k and the noise matrix W_k are constructed similarly as the output matrix Y_k using respectively f_k and w_k instead of y_k. $\Delta_k^i = \left[\delta_{k-L+1}^i | \cdots | \delta_{k-1}^i | \delta_k^i\right]$ represents the neglected part (8), (9) and (10) using the stability assumption on a time window of size L, where $\delta_k^i = CA^i x_{k-i} + \overline{\overline{H}}_i \overline{u}_{k,i} + \widetilde{\widetilde{H}}_i \widetilde{u}_{k,i}$.
By projecting the output matrix Y_k on Π_k which is the right-orthogonal matrix to the input matrix U_k, and by choosing the last column (using the vector Z) which represents the current information, equation (13) becomes:
$Y_k \Pi_k Z = \left[\overline{H}_i | \widetilde{H}_i\right] U_k \Pi_k Z + F_k \Pi_k Z + W_k \Pi_k Z + \Delta_k^i \Pi_k Z.$

From relation (11), it is easy to find that $\left[\overline{H}_i | \widetilde{H}_i\right] U_k \Pi_k Z = 0_{\ell \times 1}$. Therefore, for residual computation, one does not need to estimate analytically the system parameters involved in $\overline{H}_i$ and $\widetilde{H}_i$.

Hence, the evaluation form of the residual $\epsilon_k = Y_k \Pi_k Z$ is obtained as: $\epsilon_k = F_k \Pi_k Z + W_k \Pi_k Z + \Delta_k^i \Pi_k Z$.

The mathematical expectation of the residual ϵ_k is:

$$E[\epsilon_k] = E[F_k \Pi_k Z] + E[W_k \Pi_k Z] + E[\Delta_k^i \Pi_k Z] \quad (14)$$

Since noise is centered, we have $E[W_k \Pi_k Z] = 0$, and relation (14) becomes:$E[\epsilon_k] = E[F_k \Pi_k Z] + E[\Delta_k^i \Pi_k Z]$.
Since deterministic faults are considered, and Δ_k^i is also a deterministic matrix, we have: $E[\epsilon_k] = F_k \Pi_k Z + \Delta_k^i \Pi_k Z$, where $\Delta_k^i \Pi_k Z$ represents the contribution of the initial state vector, theoretically it is not equal to zero but it will be discussed in the next section. A threshold decision procedure has to be done for residual mean $E[\epsilon_k]$ evaluation, because residual mean depends on faults and the neglected dynamics.

Notice that the proposed data-based residual ϵ_k is not sensitive to sensor faults f_k if $span(F_k) \subset span(U_k)$.

■

V. Data-based residual computation

In the previous section, a data-based residual has been proposed for sensor fault detection and isolation in bilinear systems. Note that the computation form of the residual ϵ_k depends only on outputs Y_k and inputs through matrix Π_k (Π_k is the right kernel of U_k). However, the on-line computation of $\Pi_k \in R^{L \times L}$ needs the input matrix $U_k \in R^{m(i+1)+M(i) \times L}$ which is of high dimension. In this section, we will show that the computations may be reduced, because certain terms that appears in residual computation form may be neglected.

The effect of the residual neglected terms is minimized and the residual sensitivity to faults is maximized by proposing an appropriate criterion.

We introduce index i_1. Theoretically, we should have $i = i_1$, but i and i_1 are chosen different to overcome the heavy calculations.
If $i_1 < i$, expression of y_k given by equation (7) can be written differently as follows:

$$y_k = \overline{H}_i \overline{u}_{k,i} + \left[\widetilde{H}_{i_1} | \widetilde{H}_{i_1+1:i}\right] \begin{bmatrix} \widetilde{u}_{k,i_1} \\ \widetilde{u}_{k,i_1+1:i} \end{bmatrix} + f_k + w_k + \delta_k^i \quad (15)$$

where $\widetilde{u}_{k,i_1+1:i} = \begin{bmatrix} \vdots \\ (u_{k-g_1(j_5)} \otimes u_{k-g_2(j_5)} \otimes \ldots \otimes u_{k-g_d(j_5)}) \\ \vdots \end{bmatrix}$

$\in R^{(M(i)-M(i_1)) \times 1}$ is similar to $\widetilde{u}_{k,i}$ by removing combinations with $g_1 < i_1 + 1$, in other words, $\widetilde{u}_{k,i_1+1:i}$ is constructed with all possible combinations of inputs in a time window of size i with the following constraint:
$\forall j_5 \in \{1, ..., (K(i) - K(i_1))\}, \forall b \in \{2, ..., i\},$

$\forall g_q(j_5) \in \{1, ..., i-1\}, \forall q \in \{2, ..., d\}, \forall g_1(j_5) \in \{i_1 + 1, ..., i\}:$

$g_1(j_5) > g_2(j_5) > ... > g_d(j_5).$

Illustration: From the output expression (4) for $i = 3$, if $i_1 = 2$, the expression of $\widetilde{u}_{k,2+1:3}$ is given by:

$$\widetilde{u}_{k,2+1:3} = \begin{bmatrix} u_{k-3} \otimes u_{k-2} \otimes u_{k-1} \\ u_{k-3} \otimes u_{k-2} \\ u_{k-3} \otimes u_{k-1} \end{bmatrix}$$

and $\widetilde{H}_{i_1+1:i}$ contains the corresponding terms for $\widetilde{u}_{k,i_1+1:i}$.
Illustration: From the output expression (4) for $i = 3$, if $i_1 = 2$, $\widetilde{H}_{i_1+1:i}$ is given by: $\widetilde{H}_{i_1+1:i} = \left[CG.(G \otimes I_m).(B \otimes I_{m^2}) \,|CAG.(B \otimes I_m)| \; CG.(A \otimes I_m).(B \otimes I_m)\right]$.

The neglected part which can not be calculated is $\widetilde{H}_{i_1+1:i} \widetilde{u}_{k,i_1+1:i}$. Consequently, another output expression y_k is given by:

$$y_k = \overline{H}_i \overline{u}_{k,i} + \widetilde{H}_{i_1} \widetilde{u}_{k,i_1} + f_k + w_k + \delta_k^{i,i_1} \quad (16)$$

- i appears in terms with $\overline{u}_{k,i} \in R^{m(i+1) \times 1}$, where is chosen sufficiently large to be able to neglect the contribution of the initial state, this choice is involved with the inputs $\overline{u}$ because the inputs $\overline{u}$ are not big in size
- i_1 appears in terms with $\tilde{u}_{k,i_1} \in R^{M(i_1) \times 1}$, where i_1 is small enough to allow the residual computation.

By stacking equation (16) on a time window of size L, it leads to the following expression:

$$Y_k = \left[\overline{H}_i | \widetilde{H}_{i_1}\right] U_k + F_k + W_k + \Delta_k^{i,i_1} \quad (17)$$

Consequently, data-based residual is calculated similarly as in equation (12) but input matrix U_k becomes:

$$U_k = \begin{bmatrix} \overline{u}_{k-L+1,i} & \cdots & \overline{u}_{k-1,i} & \overline{u}_{k,i} \\ \widetilde{u}_{k-L+1,i_1} & \cdots & \widetilde{u}_{k-1,i_1} & \widetilde{u}_{k,i_1} \end{bmatrix} \in R^{m(i+1)+M(i_1) \times L}$$

where it is obvious that the size of U_k becomes lower than the previous one, with $\Delta_k^{i,i_1} =$

$\Delta_k^i + \widetilde{H}_{i_1+1:i}\left[\widetilde{u}_{k-L+1,i_1+1:i}\,|\cdots|\widetilde{u}_{k-1,i_1+1:i}\,|\widetilde{u}_{k,i_1+1:i}\right]$ represents the initial state contribution and $\widetilde{H}_{i_1+1:i}\widetilde{u}_{k,i_1+1:i}$. In order to find a good trade-off between the high sensitivity to sensor faults and low sensitivity to the neglected terms, the following criterion is proposed.

The choice of the two indexes i and i_1 corresponds to $\min(J(i,i_1))$, where $J(i,i_1)$ is the proposed criterion for a good trade-off between high sensitivity to sensor faults and low sensitivity to neglected part.

The criterion $J(i,i_1)$ is given by:

$$J(i,i_1) = \frac{J_1(i,i_1)}{J_2(i,i_1)} = \frac{\sum_{k=i_0+1}^{k=i+\varphi}\|Y_k\Pi_k\|}{\sum_{k=i_0+1}^{k=i+\varphi}\|F_k\Pi_k\|} = \frac{\sum_{k=i_0+1}^{k=i+\varphi}\left\|\Delta_k^{i,i_1}\Pi_k+W_k\Pi_k\right\|}{\sum_{k=i_0+1}^{k=i+\varphi}\|F_k\Pi_k\|}$$

Calculation is started from i_0, since matrices collection needs historical data in this paper.

$Y_k\Pi_k$, $F_k\Pi_k$ represent respectively the projection on the input kernel Π_k of the output matrix Y_k in a healthy case and the fault matrix F_k with a unique direction ($f_k = \begin{bmatrix}1\cdots 1\end{bmatrix}^T$ in this paper because it contains all the other possible faults directions).

Where φ is an integer to be selected by the designer, and $J_1(i,i_1)$, $J_2(i,i_1)$ are defined as follows:

$$J_1(i,i_1) = \frac{\sum_{k=i_0+1}^{k=i+\varphi}\|Y_k\Pi_k\|}{\sum_{k=i_0+1}^{k=i+\varphi}\|Y_k\|} = \frac{\sum_{k=i_0+1}^{k=i+\varphi}\left\|\Delta_k^{i,i_1}\Pi_k+W_k\Pi_k\right\|}{\sum_{k=i_0+1}^{k=i+\varphi}\|Y_k\|}$$

$$J_2(i,i_1) = \frac{\sum_{k=i_0+1}^{k=i+\varphi}\|F_k\Pi_k\|}{\sum_{k=i_0+1}^{k=i+\varphi}\|Y_k\|}$$

with i_0 is the starting time for calculation, since the proposed residual need the collection of the historical data.

Remark:

The denominator $\sum_{k=i_0+1}^{k=i+\varphi}\|Y_k\|$ of $J_1(i,i_1)$ and $J_2(i,i_1)$ is used to normalize $J_1(i,i_1)$ and $J_2(i,i_1)$. To prove the normalization process, we have the sub-multiplicative norm: $\|Y_k\Pi_k\| \leq \|Y_k\|\,\|\Pi_k\|$.

It is known that $\|\Pi_k\| = 1$, this leads to $\frac{\|Y_k\Pi_k\|}{\|Y_k\|} \leq 1$

VI. Example and results

A. The activated sludge process

The activated sludge process (ASP) is a biological method of wastewater treatment that is performed by a variable and mixed community of micro-organisms in an aerobic aquatic environment.

The overall goal of the activated-sludge process is to remove substances that have a demand for oxygen from the system.

The simulated process [13] is divided into three sub processes, denoted S_1, S_2 and S_3. The first sub process S_1 describes the rate of change of the ammonium concentration in the aerobic compartments. The inputs are the measurable d_1 and the control variable u_1, thus the process describes the nitrification process, which generally occurs when the time that the sludge stays in the system (called the mean cell residence time, or MCRT) is increased.

The second and the third sub processes are not detailed, because we did the application of our method only on the first process, which is a discrete-time bilinear system.

Numerical values of the bilinear model for the ASP are taken from [13]. These values are used for input/output generation but not used for residuals generation. System inputs and system outputs are plotted in figures (3) and (4).

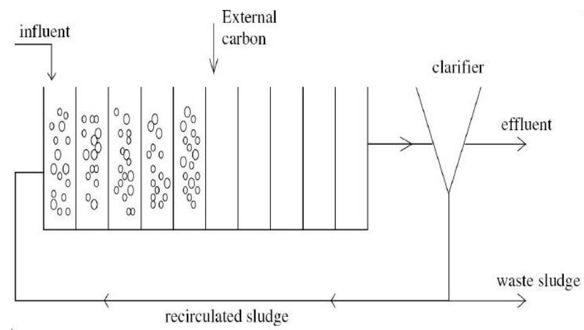

Fig. 1. Activated sludge process

B. Computation of the criterion J and the residual

We choose $\varphi = 100$, $L = 223$, $i_0 = 244$.

The criterion J is computed and plotted in figure (2). The optimal couple (i,i_1) corresponds to the minimum of $J(i,i_1)$ is $(i = 20, i_1 = 3)$.

The 3 data-based residuals are computed and represented in figure (5).

It is obvious that we can easily detect and isolate the three faults. Note that in a no fault situation, residuals are not exactly zero due to the contribution of noise and the neglected terms.

When the fault disappears, the corresponding residual stays strongly different from zero during one time window. This is due to the computation method of the residual that uses historical data on a time window.

VII. Conclusion

A data-based residual method is proposed in this paper for sensor fault diagnosis in a bilinear system without using system parameters values. The proposed method has the advantage of generating structured residuals for multiple sensors faults, but a drawback is that high dimensional input-output matrices must be manipulated. A method is proposed to reduce computational complexity, by neglecting certain terms in the residual evaluation, while maintaining the fault sensitivity. The residual generation method approach is illustrated on a simulated activated sludge process, where detection and isolation of sensors faults were successfully conducted.

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VIII. Appendix

Recurrence method is used to prove correctness of the general output expression (4), which can be written differently as follows:

$$\begin{aligned}
&y_k = CA^i x_{k-i} + \\
&C\sum_{j_1=1}^{i} A^{j_1-1}G.(A\otimes I_m)^{i-j_1}.(x_{k-i}\otimes I_m).u_{k-j_1} \\
&+C(Du_k + \sum_{j_1=1}^{i} A^{j_1-1}Bu_{k-j_1}) \\
&+C\sum_{j_2=1}^{K(i)} \Psi_{j_2,i}.(x_{k-i}\otimes I). \\
&(u_{k-s_1(j_2)}\otimes u_{k-s_2(j_2)}\otimes ...\otimes u_{k-s_d(j_2)}) \\
&+C\sum_{j_2=1}^{K(i)} \Gamma_{j_2,i}.(B\otimes I). \\
&(u_{k-s_1(j_2)}\otimes u_{k-s_2(j_2)}\otimes ...\otimes u_{k-s_d(j_2)}) + f_k + w_k
\end{aligned} \quad (18)$$

1) ***Writing the proposal that we are going to prove***

   The proposal to be demonstrated is given by equations (18), (6) and (5) for $\forall i \geq 0$.

2) ***Inductive step***

   Assuming that the proposal holds for i, let us prove that it holds for $i+1$ also:

$$\begin{aligned}
&y_k = CA^{i+1}x_{k-i-1} + \\
&C\sum_{j_3=1}^{i+1} A^{j_3-1}G.(A\otimes I_m)^{i+1-j_3}.(x_{k-i-1}\otimes I_m).u_{k-j_3} \\
&+C(Du_k + \sum_{j_3=1}^{i+1} A^{j_3-1}Bu_{k-j_3}) \\
&+C\sum_{j_4=1}^{K(i+1)} \Psi_{j_4,i+1}.(x_{k-i-1}\otimes I). \\
&(u_{k-s_1(j_4)}\otimes u_{k-s_2(j_4)}\otimes ...\otimes u_{k-s_d(j_4)}) \\
&+C\sum_{j_4=1}^{K(i+1)} \Gamma_{j_4,i+1}.(B\otimes I). \\
&(u_{k-s_1(j_4)}\otimes u_{k-s_2(j_4)}\otimes ...\otimes u_{k-s_d(j_4)}) + f_k + w_k
\end{aligned} \quad (19)$$

$$\vartheta_1(j_4) + \vartheta_2(j_4) + \vartheta_3(j_4) + \vartheta_4(j_4) = i+1 \quad (20)$$

$$\phi_1(j_4) + \phi_2(j_4) + \phi_3(j_4) + \phi_4(j_4) \leq i \quad (21)$$

where ϑ, ϕ are defined for $\Psi_{j_4,i+1}$ and $\Gamma_{j_4,i+1}$ similarly as λ, ρ are defined for $\Psi_{j_2,i}$ and $\Gamma_{j_2,i}$ respectively, with $\forall j_4 \in \{1,...,K(i+1)\}, \forall j_3 \in \{1,...,i+1\}, \forall d \in \{2,...,i+1\}$,

$\forall s_p(j_4) \in \{1,...,i\}, \forall p \in \{2,...,d\}$:

$s_1(j_4) \in \{2,...,i+1\} : s_1(j_4) > s_2(j_4) > ... > s_d(j_4)$.

Using equation (1), we replace $x_{k-i} = Ax_{k-i-1} + G.(x_{k-i-1}\otimes u_{k-i-1}) + Bu_{k-i-1}$ into the expression (18) of y_k at i , we derive the following expression:

$$\begin{aligned}
&y_k = CA^{i+1}x_{k-i-1} + CA^iG.(x_{k-i-1}\otimes I_m).u_{k-i-1} + \\
&CA^iBu_{k-i-1} + C(Du_k + \sum_{j_1=1}^{i} A^{j_1-1}Bu_{k-j_1}) + \\
&C\sum_{j_1=1}^{i} A^{j_1-1}G.(A\otimes I_m)^{i-j_1+1}.(x_{k-i-1}\otimes I_m).u_{k-j_1} \\
&+C\sum_{j_1=1}^{i} A^{j_1-1}G.(A\otimes I_m)^{i-j_1}.(G\otimes I_m).(x_{k-i-1}\otimes I_{m^2}). \\
&(u_{k-i-1}\otimes u_{k-j_1}) + \\
&C\sum_{j_1=1}^{i} A^{j_1-1}G.(A\otimes I_m)^{i-j_1}.(B\otimes I_m).(u_{k-i-1}\otimes u_{k-j_1}) + \\
&C\sum_{j_2=1}^{K(i)} \Gamma_{j_2,i}.(B\otimes I) \\
&.(u_{k-s_1(j_2)}\otimes u_{k-s_2(j_2)}\otimes ...\otimes u_{k-s_d(j_2)})
\end{aligned} \quad (22)$$

$$+C\sum_{j_2=1}^{K(i)}\Psi_{j_2,i}.(A\otimes I).(x_{k-i-1}\otimes I).$$

$$(u_{k-s_1(j_2)}\otimes u_{k-s_2(j_2)}\otimes ...\otimes u_{k-s_d(j_2)})$$

$$+C\sum_{j_2=1}^{K(i)}\Psi_{j_2,i}.(G\otimes I).(x_{k-i-1}\otimes I).$$

$$(u_{k-i-1}\otimes u_{k-s_1(j_2)}\otimes u_{k-s_2(j_2)}\otimes ...\otimes u_{k-s_d(j_2)})$$

$$+C\sum_{j_2=1}^{K(i)}\Psi_{j_2,i}.(B\otimes I).$$

$$(u_{k-i-1}\otimes u_{k-s_1(j_2)}\otimes u_{k-s_2(j_2)}\otimes ...\otimes u_{k-s_d(j_2)})+f_k+w_k$$

Let us present the used mathematical expressions for the previous equation simplification, which are easy to prove:

- $(Z_1\otimes Z_2)(Z_3\otimes Z_4)=(Z_1\ Z_3)\otimes(Z_2\ Z_4)$
- $(Z_1\otimes Z_2)=(Z_1\otimes I)Z_2$
- $i+2K(i)=K(i+1)$
- $C_{p+1}^{q+1}=C_p^{q+1}+C_p^q$

where Z_1, Z_2, Z_3, Z_4 are matrices with appropriate dimensions. By identifying expressions (19) and (22), one can find:

$$C\sum_{j_3=1}^{i+1}A^{j_3-1}G.(A\otimes I_m)^{i+1-j_3}.(x_{k-i-1}\otimes I_m).u_{k-j_3} = CA^iG.(x_{k-i-1}\otimes I_m).u_{k-i-1}+$$

$$C\sum_{j_1=1}^{i}A^{j_1-1}G.(A\otimes I_m)^{i-j_1+1}.(x_{k-i-1}\otimes I_m).u_{k-j_1}$$

$$C(Du_k+\sum_{j_3=1}^{i+1}A^{j_3-1}Bu_{k-j_3})=$$

$$CA^iBu_{k-i-1}+C(Du_k+\sum_{j_1=1}^{i}A^{j_1-1}Bu_{k-j_1})$$

$$\left[\Psi_{1,i+1}\,|\cdots|\Psi_{K(i+1)-1,i+1}\,|\Psi_{K(i+1),i+1}\right]=$$

$$\left[G.(A\otimes I_m)^{i-1}.(G\otimes I_m)\,|A^1G.(A\otimes I_m)^{i-2}.(G\otimes I_m)|\right.$$

$$\cdots|A^{i-1}G.(A\otimes I_m)^{i-i}.(G\otimes I_m)\,|\Psi_{1,i}.(A\otimes I_m)|$$

$$\Psi_{2,i}.(A\otimes I_m)\,|\cdots|\Psi_{K(i),i}.(A\otimes I_m)\,|\Psi_{1,i}.(G\otimes I_m)|$$

$$\left.\Psi_{2,i}.(G\otimes I_m)\,|\cdots|\Psi_{K(i),i}.(G\otimes I_m)\right]$$

$$\left[\Gamma_{1,i+1}\,|\cdots|\Gamma_{K(i+1)-1,i+1}\,|\Gamma_{K(i+1),i+1}\right]=$$

$$\left[G.(A\otimes I_m)^{i-1}\,|A^1G.(A\otimes I_m)^{i-2}\,|\cdots|\right.$$

$$A^{i-1}G.(A\otimes I_m)^{i-i}\,|\Gamma_{1,i}\,|\Gamma_{2,i}\,|\cdots|\Gamma_{K(i),i}|$$

$$\left.\Psi_{1,i}\,|\Psi_{2,i}\,|\cdots|\Psi_{K(i),i}\right]$$

It is straightforward to prove that the proposal for $i+1$ (equations (19), (20) and (21)) holds.

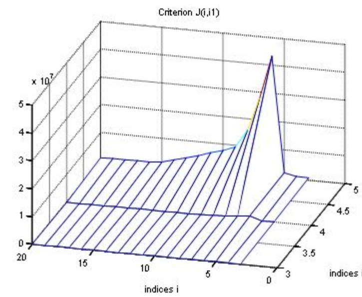

Fig. 2. Criterion to be minimized $J(i,i_1)$

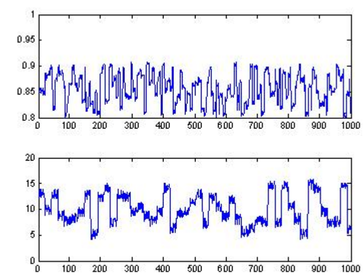

Fig. 3. System input

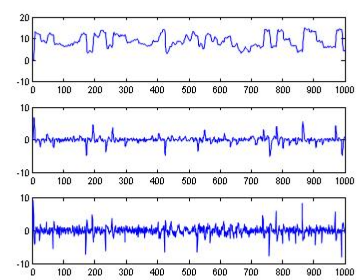

Fig. 4. System output

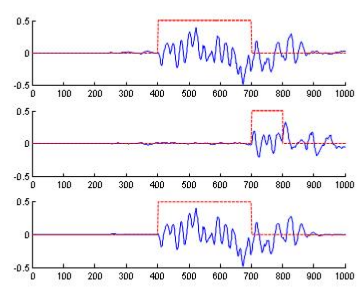

Fig. 5. blue: residuals expectation, red: faults occurrence