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Adaptive Receding Horizon Control For A Class Of Nonlinear Differential Difference Systems

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***Abstract*— An adaptive receding horizon control (RHC) algorithm for a class of nonlinear differential difference systems with multiple time delays is proposed. This work extends previous research dealing with RHC of nonlinear delayed systems in several ways: first, a rigorous proof is given for that the RHC guarantees global, uniformly asymptotic stability of the nonlinear delayed systems. Second, the nonlinear system with multiple time delays is considered. Third, the procedure of an adaptive RHC controller is presented by combining the on-line delay value estimation. Through a simulation example, it is demonstrated that the proposed adaptive RHC has the guaranteed closed-loop stability for a special type of nonlinear differential difference systems.**

I. INTRODUCTION

Receding horizon control (RHC), also known as model predictive control (MPC), has been in the limelight of significant advanced control techniques because of its unparalleled advantages. These include: computational feasibility, ability to handle hard constraints on the controls as well as the system states, and good tracking performance. While the body of work concerning delay-free systems is now extensive, see [10], [13] for a comprehensive survey of previous contributions, much fewer results pertain to delayed systems which are common in process industry. Since time delays may significantly degrade the system performance, further research in receding horizon control of delayed systems is well motivated. Recent contributions in this area include [7],[8],[12] for linear delayed systems and [6], [9] for nonlinear delayed systems. Nevertheless, the methods pertain above only to the cases when the exact values of the delays are known a priori. Hence, any attempt to identify the real time delays, in an on-line mode, will radically reduce the conservativeness of delay uncertainties.

Several novel theoretically adaptive RHCs are introduced in [1], [2], [4], [5]. However, non of them investigate adaptive RHC in nonlinear delayed systems. To fill the gap, in this paper, an extension has been made to the research in [6] and [9] in three ways:

1) It is proved rigorously that the receding horizon strategy guarantees *global, uniformly asymptotic* stabilization.
2) The differential difference systems with an arbitrary number of state delays are considered.
3) To cope with model uncertainties with respect to state delays, an adaptive receding horizon strategy which combines feedback control with on-line identification of state delays is proposed.

In this paper, the control law is obtained by minimizing a finite horizon cost and its closed-loop stability is guaranteed by satisfying an inequality condition on the terminal functional. The delay estimation algorithm and RHC is related through the optimal cost functional. In other words, the estimation algorithm tries to generate a better delay estimate and RHC computes the perturbation control. Then a special type of nonlinear differential difference systems is introduced for constructing a systematic method to find a terminal weighting functional satisfying the proposed inequality condition. The closed-loop stability of the proposed RHC is shown through a simulation example.

This paper is organized as follows: the problem and notation are presented in Section II. The design of an adaptive RHC and the monotonicity of the optimal cost are stated in Section III. In Section IV, the stability of the RHC is investigated. In Section V, a systematic method to find a terminal weighting functional for a special type of time delayed systems is introduced. Finally, the efficiency of the resulting methodology is further demonstrated in a numerical example.

II. PROBLEM STATEMENT AND NOTATION

Receding horizon control is based on optimal control. In order to obtain a receding horizon control, we first focus on the optimal control problem listed below. The class of nonlinear differential difference systems with multiple unknown delays considered here is described by

$$\frac{d}{dt}x(t) = f(x(t), x(t-\tau_1), ..., x(t-\tau_k), u(t)) \qquad (1)$$

where $x(t) \in \mathbb{R}^n$ is the state, $u(t) \in \mathbb{R}^m$ is a continuous and uniformly bounded function that represents an exogenous input, and $0 < \tau_1 < \tau_2 < ... < \tau_k$ are the time delays which are unknown. The function $f(\cdot)$ is assumed to be a continuously differentiable function of its arguments and the initial condition is stated as

$$x(s) = \phi(s), \qquad -\tau_k \le s \le 0 \qquad (2)$$

The cost functional, to be minimized, is written in the form

$$J(x_{t_0}, u, t_0, t_f) \triangleq \int_{t_0}^{t_f} [q(x(s)) + r(u(s))]ds + q_0(x(t_f)) + \sum_{i=1}^{k} \int_{t_f-\tau_i}^{t_f} q_i(x(s))ds \qquad (3)$$

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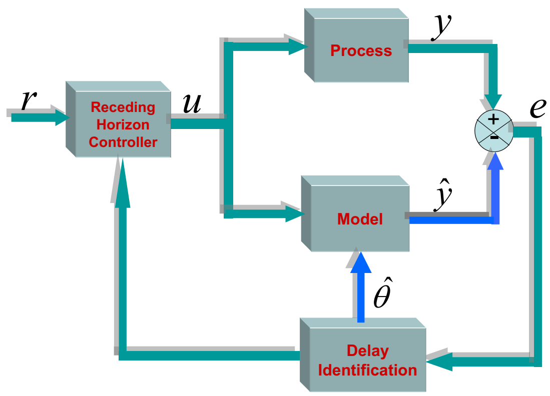

Fig. 1: Block diagram of the adaptive RHC scheme

where $t_0 > 0$ is an initial time, t_f is a final time, q and r are state and input cost functions, and q_i, $i = 0, ..., k$ are functions needed in the terminal weighting functional and x_t denotes $x_t(\theta) = x(t+\theta), \theta \in [-\tau_k, 0]$. Assume that $\alpha_L(\|x\|) \leq q(x) \leq \alpha_H(\|x\|)$, $\beta_L(\|u\|) \leq r(u) \leq \beta_H(\|u\|)$, and $\gamma_L^i\|x\| \leq q_i(x) \leq \gamma_H^i(\|x\|)$, $i = 0, ..., k$, where α_L, α_H, β_L, β_H, γ_L^i, and γ_H^i are continuous, positive-definite, strictly increasing functions satisfying $\alpha_L(0) = \alpha_H(0) = \beta_L(0) = \beta_H(0) = \gamma_L^i(0) = \gamma_H^i(0) = 0$, $i = 0, ..., k$.

With the system model (1), let the real system be represented by

$$\frac{d}{dt}\hat{x}(t) = f(\hat{x}(t), \hat{x}(t-\hat{\tau}_1), ..., \hat{x}(t-\hat{\tau}_k), u(t)) \quad (4)$$

with $0 < \hat{\tau}_1 < \hat{\tau}_2 < ... < \hat{\tau}_k$, and be equipped with the same initial condition

$$\hat{x}(s) = \phi(s), \quad -\hat{\tau}_k \leq s \leq 0 \quad (5)$$

Assume that the optimal control trajectory which minimizes $J(x_{t_0}, u, t_0, t_f)$ is given by

$$u^*(s) = u^*(s; x_0, t_f), \quad t_0 \leq s \leq t_f \quad (6)$$

In the following section, an adaptive RHC scheme is designed by combining on-line delay identifier and an inequality condition on the terminal weighting functional is presented for the proof of stability of adaptive RHC.

III. Design of Adaptive Receding Horizon Controller

In this section, the construction of an adaptive RHC algorithm for a class of nonlinear differential difference systems is described. The core concept is to combine the proposed RHC method with an adaptive delay identifier developed in [11]. The block diagram of the adaptive receding horizon control is depicted in Figure 1. A fundamental RHC problem with on-line identification is the adaptation of the identified model to the changes in the actual process dynamics. The predictive model will change at each sampling instant time t. The system model provides an estimate of the system output at the current instant of time using the current estimate of the delays. In the predictive controller, the estimated model is used to formulate the predictive model at t and also to derive the control law.

A. Delay Identifier

For any given initial and input functions ϕ and $Bu(t)$, let $H : \tau \mapsto x(\cdot)$ be the operator that maps the delay parameter vector $\tau = [\tau_1, ..., \tau_k]$ into the trajectory $x(t), t \in [0, T]$ of system (1). Then the identification problem is to find the solution of the following nonlinear operator equation, which assumes that $\hat{x}$ is given as the measured trajectory:

$$F(\tau, \hat{x}) \triangleq H(\tau) - \hat{x} = 0$$
$$\text{where } \hat{x} \in L^2([0,T], \mathbb{R}^n), \quad \tau \triangleq [\tau_1, ..., \tau_k] \quad (7)$$

The solution of the equation (7) is approached by introducing the cost functional to be minimized with respect to the unknown variable τ as follows:

$$\Psi(\tau, \hat{x}) \triangleq 0.5 < F(\tau, \hat{x}) \mid F(\tau, \hat{x}) >_2 = 0.5 \parallel H(\tau) - \hat{x} \parallel_2^2,$$
$$\text{for} \quad \hat{x} \in L^2([0,T], \mathbb{R}^n), \tau \in \mathbb{R}^k \quad (8)$$

Suppose that the Fréchet derivative $Y \triangleq \frac{\partial}{\partial \tau} F = \frac{\partial}{\partial \tau} H$. The above leads to a delay identification iteration:

$$\tau^{n+1} = \tau^n - \alpha(\tau^n) Y(\tau^n)^\dagger F(\tau^n, \hat{x}) \quad (9)$$

where τ^n is the approximation to the delay parameter vector in iteration step n. Here, $\alpha : \mathbb{R}^k \to (0, +\infty)$ is the step size function, $\mathrm{Y}(\tau)^*$ is the Hilbert adjoint of the operator $Y(\tau)$, and

$$Y(\tau^n)^\dagger \triangleq [Y(\tau^n)^* Y(\tau^n)]^{-1} Y(\tau^n)^* \quad (10)$$

Having obtained the delay estimates, the identified model can be used to formulate the RHC at t.

B. RHC with on-line delay identification

Before the discussion of the stability of the proposed adaptive RHC , the procedure to implement adaptive receding horizon control is presented as follows:

- Step1: **Initialization:** Set up initial conditions of system, prediction horizon N, measurement frequency, step size of numerical integration, step size α.
- Step2: **Model prediction:** Based on the known values up to t (past inputs and outputs), the delays are updated by using the delay identifier algorithm.
- Step3: **Prediction correction:** For an adopted horizon N, the predicted outputs $y(t+k|t)$, $k = 1, ..., N$, and future control signals $u(t+k|t)$, $k = 0, ..., N-1$ are obtained by optimizing the given criterion.
- Step4: **RHC control:** Only the control $u(t|t)$ is applied to the process and model over the interval $[t, t+1]$.
- Step5: **Repeat:** Go to Step 2.

C. Monotonicity of the optimal value function

As a standard approach in showing the stabilizing property of the receding horizon control law, the "monotonicity property" for the receding horizon optimal value function is shown first. In this section, an inequality condition on the terminal weighting functional under which the optimal value function has the non-increasing monotonicity is presented. Before directly discussion the receding horizon control, an open-loop optimal control problem with the cost functional in (3) will be considered first.

The open-loop optimal control problem is to find an optimal control that minimizes a Lipschitz continuous optimal value function $J(x_{t_0}, u, t_0, t_f)$ in (3). Let us denote the value of the optimal cost by $J^*(x_{t_0}, u^*, t_0, t_f)$. The following theorem is an extension of the result in the work of Kwon *et al.* ([6][9]) and shows that the optimal cost has the monotonicity property provided that an inequality condition on the terminal weighting functional is satisfied.

Theorem 1. *If there exist positive-definite functions* $q_0(\cdot)$, $q_1(\cdot)$, ... , $q_i(\cdot)$ *in (3) satisfying the following inequality for all* x_σ*:*

$$q(\bar{x}(\sigma)) + r(\bar{u}(\sigma)) + \sum_{i=1}^{k}[q_i(\bar{x}(\sigma)) - q_i(\bar{x}(\sigma - \tau_i))] + (\frac{\partial q_0}{\partial \bar{x}})^T f(\bar{x}(\sigma), \bar{x}(\sigma - \tau_1), ..., \bar{x}(\sigma - \tau_k), \bar{u}(\sigma)) \leq 0 \quad (11)$$

then the optimal cost $J^*(\bar{x}_\eta, \bar{u}^*, \eta, \sigma)$ *satisfies the following relation:*

$$D^+ J(\bar{x}_\eta, \bar{u}^*_{[\eta,\sigma]}, \eta, \sigma) \leq 0 \quad (12)$$

where $\bar{u}(t)^*$ *is the optimal control obtained from system (1) with estimated delays at time instant* t*,* $\bar{x}(t)$ *is the state trajectory derived by applying* $\bar{u}(t)$ *to the real system (4), and the right-sided Dini derivative is defined as*

$$\begin{aligned} &D^+ J(\bar{x}_\eta, \bar{u}^*_{[\eta,\sigma]}, \eta, \sigma) \\ &\triangleq \limsup_{\triangle \to 0+} \triangle^{-1} [J(\bar{x}_\eta, \bar{u}^*_{[\eta,\sigma+\triangle]}, \eta, \sigma+\triangle) \\ &\qquad - J(\bar{x}_\eta, \bar{u}^*_{[\eta,\sigma]}, \eta, \sigma)] \end{aligned} \quad (13)$$

Remark III.1. *In this approach, the delay estimation algorithm and RHC is related through the optimal cost functional* $J(\bar{x}_\eta, \bar{u}^*_{[\eta,\sigma]}, \eta, \sigma)$*. In other words, the estimation algorithm tries to generate a better delay estimate and RHC computes the perturbation control* $\bar{u}(t)$*.*

Proof: By definition of the cost function,

$$\begin{aligned} &J(\bar{x}_\eta, \bar{u}^*_{[\eta,\sigma+\triangle]}, \eta, \sigma+\triangle) \\ &\triangleq \int_\eta^{\sigma+\triangle} [q(\bar{x}^*(s)) + r(\bar{u}^*(s))]ds + q_0(\bar{x}^*(\sigma+\triangle)) \\ &\quad + \sum_{i=1}^{k} \int_{\sigma+\triangle-\tau_i}^{\sigma+\triangle} q_i(\bar{x}^*(s))ds \end{aligned} \quad (14)$$

where, $\bar{x}^*$ stands for $\bar{x}^*_{[\eta,\sigma+\triangle]}$, to simplify notation. Consider a sub-optimal control for $J(x_\eta, u, \eta, \sigma+\triangle)$ on $[\eta, \sigma+\triangle]$, obtained while employing the following sub-optimal control.

$$u_{sub}(v) \triangleq \begin{cases} \bar{u}^*_{[\eta,\sigma]}(v) & v \in [\eta, \sigma] \\ u_{cl}(v) & v \in [\sigma, \sigma+\triangle] \end{cases} \quad (15)$$

It is clear that the corresponding system trajectory $\bar{x}_{sub}(v), v \in [\eta, \sigma+\triangle]$, satisfies $\bar{x}_{sub}(v) = \bar{x}^*_{[\eta,\sigma]}$, for $v \in [\eta, \sigma]$ and

$$\begin{aligned} &J(\bar{x}_\eta, \bar{u}^*_{[\eta,\sigma+\triangle]}, \eta, \sigma+\triangle) \\ &\leq J(\bar{x}_\eta, u_{sub}, \eta, \sigma+\triangle) \\ &= \int_\eta^{\sigma+\triangle} [q(\bar{x}_{sub}(v)) + r(u_{sub}(v))]dv + q_0(\bar{x}_{sub}(\sigma+\triangle)) \\ &\qquad + \sum_{i=1}^{k} \int_{\sigma+\triangle-\tau_i}^{\sigma+\triangle} q_i(\bar{x}_{sub}(v))ds \\ &= \int_\eta^{\sigma} [q(\bar{x}^*(v)) + r(\bar{u}^*(v))]dv + \int_\sigma^{\sigma+\triangle} [q(\bar{x}_{sub}(v)) \\ &\qquad + r(u_{cl}(v))]dv + q_0(\bar{x}_{sub}(\sigma+\triangle)) \\ &\qquad + \sum_{i=1}^{k} \int_{\sigma+\triangle-\tau_i}^{\sigma+\triangle} q_i(\bar{x}_{sub}(v))ds \\ &= \int_\eta^{\sigma} [q(\bar{x}^*(v)) + r(u^*(v))]dv + \{q_0(\bar{x}^*(\sigma)) \\ &\quad + \sum_{i=1}^{k} \int_{\sigma-\tau_i}^{\sigma} q_i(\bar{x}^*(v))ds - q_0(\bar{x}^*(\sigma)) \\ &\quad - \sum_{i=1}^{k} \int_{\sigma-\tau_i}^{\sigma} q_i(\bar{x}^*(v))ds\} \\ &\quad + \int_\sigma^{\sigma+\triangle} [q(\bar{x}_{sub}(v)) + r(u_{cl}(v))]dv + q_0(\bar{x}_{sub}(\sigma+\triangle)) \\ &\quad + \sum_{i=1}^{k} \int_{\sigma+\triangle-\tau_i}^{\sigma+\triangle} q_i(\bar{x}_{sub}(v))ds \\ &= J(\bar{x}_\eta, u^*_{[\eta,\sigma]}, \eta, \sigma) - q_0(\bar{x}_{sub}(\sigma)) \\ &\quad - \sum_{i=1}^{k} \int_{\sigma-\tau_i}^{\sigma} q_i(\bar{x}_{sub}(v))ds \\ &\quad + \int_\sigma^{\sigma+\triangle} [q(\bar{x}_{sub}(v)) + r(u_{cl}(v))]dv \\ &\quad + q_0(\bar{x}_{sub}(\sigma+\triangle)) + \sum_{i=1}^{k} \int_{\sigma+\triangle-\tau_i}^{\sigma+\triangle} q_i(\bar{x}_{sub}(v))ds \end{aligned}$$

Rearranging the above and proceeding to the limit as $\triangle \to 0+$, yields

$$\begin{aligned}
&D^+J(\bar{x}_\eta, \bar{u}^*_{[\eta,\sigma]}, \eta, \sigma) \\
&= \limsup_{\triangle\to 0+} \triangle^{-1} [J(\bar{x}_\eta, \bar{u}^*_{[\eta,\sigma+\triangle]}, \eta, \sigma+\triangle) \\
&\qquad -J(\bar{x}_\eta, \bar{u}^*_{[\eta,\sigma]}, \eta, \sigma)] \\
&\leq \limsup_{\triangle\to 0+} \triangle^{-1} [\int_\sigma^{\sigma+\triangle} [q(\bar{x}_{sub}(v)) + r(u_{sub}(v))]dv \\
&\quad -q_0(\bar{x}_{sub}(\sigma)) - \sum_{i=1}^k \int_{\sigma-\tau_i}^{\sigma} q_i(\bar{x}_{sub}(v))ds \\
&\quad +q_0(\bar{x}_{sub}(\sigma+\triangle)) + \sum_{i=1}^k \int_{\sigma+\triangle-\tau_i}^{\sigma+\triangle} q_i(\bar{x}_{sub}(v))ds] \\
&= q(\bar{x}(\sigma)) + r(\bar{u}(\sigma)) + \sum_{i=1}^k [q_i(\bar{x}(\sigma)) - q_i(\bar{x}(\sigma-\tau_i))] \\
&\quad +(\frac{\partial q_0}{\partial x})^T f(\bar{x}(\sigma), \bar{x}(\sigma-\tau_1), ..., \bar{x}(\sigma-\tau_k), u(\sigma)) \\
&\leq 0
\end{aligned}$$

Hence, (12) is valid as required.

The Theorem delivers immediately the desired monotonicity of the optimal cost for the differential difference system with unknown delays (1).

IV. Stability Analysis

The RHC can be obtained by replacing t_0 by t, t_f by $t+T$, and x_{t_0} by x_t in (6) for $0 < T < \infty$, where T denotes the horizon length. Hence, the RHC is given by

$$u^*(t) = u^*(t; x_t, t+T) \tag{16}$$

The stability of the RHC hinges on the use of the optimal value function as the Lyapunov function for system with the receding horizon control law. It is first noted that the optimal value function has the following properties.

Proposition IV.1. *There exist continuous, nondecreasing functions $\tilde{u} : \mathbb{R}^+ \to \mathbb{R}^+$, $\tilde{v} : \mathbb{R}^+ \to \mathbb{R}^+$ with the properties that $\tilde{u}(0) = \tilde{v}(0) = 0$ and $\tilde{u}(s) > 0$, $\tilde{v}(s) > 0$ for $s > 0$, such that the optimal value function $J(x_t, u^*_{[t,t+T]}, t, t+T)$ satisfies*

$$\begin{aligned}
\tilde{u}(\| x(t) \|) \leq J(x_t, u^*_{[t,t+T]}, t, t+T) \leq \tilde{v}(\| x(t) \|) \\
\text{for all} \quad t \geq 0, x_t \in C([t-\tau_k, t], \mathbb{R}^n)
\end{aligned} \tag{17}$$

Additionally, there exists a continuous, nondecreasing function $\tilde{w} : \mathbb{R}^+ \to \mathbb{R}^+$, with the property that $\tilde{w}(s) > 0$ for $s > 0$, such that the right-sided derivative of the optimal value function along the system trajectory with the receding horizon control law satisfies

$$\begin{aligned}
D_t^+ J(x_t, u^*_{[t,t+T]}, t, t+T) \leq -w(\| x(t) \|) \\
\text{for all } t > 0,\ x_t \in C([t-\tau_k, t], \mathbb{R}^n)
\end{aligned} \tag{18}$$

Proof: First, we note that $\alpha_L(\|x\|) \leq q(x) \leq \alpha_H(\|x\|)$, $\beta_L(\|u\|) \leq r(u) \leq \beta_H(\|u\|)$, and $\gamma_L^i\|x\| \leq q_i(x) \leq \gamma_H^i(\|x\|)$, $i = 0, ..., k$, where α_L, α_H, β_L, β_H, γ_L^i, and γ_H^i are continuous, positive-definitive, strictly increasing function satisfying $\alpha_L(0) = \alpha_H(0) = \beta_L(0) = 0\beta_H(0) = 0\gamma_L^i(0) = \gamma_H^i(0) = 0$, $i = 0, ..., k$. Then

$$\begin{aligned}
&J(x_t, u^*_{[t,t+T]}, t, t+T) \\
&= \int_t^T [q(x(s)) + r(u(s))]ds + q_0(x(T)) \\
&\quad + \sum_{i=1}^k \int_{T-\tau_i}^T q_i(x(s))ds \\
&\leq \int_t^T [\alpha_H(\|x(s)\|) + \beta_H(\|u(s)\|)]ds + \gamma_H^0(\|x(t)\|) \\
&\quad + \sum_{i=1}^k \int_{T-\tau_i}^T \gamma_H^i(\|x(s)\|)ds
\end{aligned} \tag{19}$$

for all $t > 0$, and all $x_t \in C_{[t-\tau_k, t]}$, as required. On the other hand,

$$\begin{aligned}
&J(x_t, u^*_{[t,t+T]}, t, t+T) \\
&= \int_t^T [q(x(s)) + r(u(s))]ds + q_0(x(T)) \\
&\quad + \sum_{i=1}^k \int_{T-\tau_i}^T q_i(x(s))ds \\
&\geq \int_t^T [\alpha_L(\|x(s)\|) + \beta_L(\|u(s)\|)]ds + \gamma_L^0(\|x(t)\|) \\
&\quad + \sum_{i=1}^k \int_{T-\tau_i}^T \gamma_L^i(\|x(s)\|)ds
\end{aligned} \tag{20}$$

that delivers the desired function $\tilde{u}$.

The right-sided Dini derivative of the optimal value function along the trajectory of the RHC system is defined by

$$\begin{aligned}
&D_t^+ J(x_t, u^*_{[t,t+T]}, t, t+T) \\
&\triangleq \lim_{\sigma\to 0+} \sigma^{-1}[J(x_{t+\sigma}, u^*_{[t+\sigma,t+\sigma+T]}, t+\sigma, t+\sigma+T) \\
&\quad -J(x_t, u^*_{[t,t+T]}, t, t+T)]
\end{aligned} \tag{21}$$

Now, in view of the assumptions made and the result of Theorem 1, there exists a right-sided neighborhood of zero $\mathcal{N}(0) \triangleq \{\sigma \in \mathbb{R} \mid 0 \leq \sigma < \epsilon\}$ such that

$$\begin{aligned}
&J(x_t, u^*_{[t,t+T+\sigma]}, t, t+T+\sigma) \\
&\leq J(x_t, u^*_{[t,t+T]}, t, t+T)
\end{aligned} \tag{22}$$

for all $\sigma \in \mathcal{N}(0)$, all $t > 0$, and all $x_t \in C([t-\tau_k, t], \mathbb{R}^n)$. Hence, in particular, for any $\theta \in \mathcal{N}(0)$, employing Bellman's Principle of Optimality, one obtains

$$\begin{aligned}
&J(x_t, u^*_{[t,t+T]}, t, t+T) \\
&= \int_t^{t+\theta} [q(x^*(s)) + r(u^*_{[t,t+T]}(s)]ds \\
&\quad +J(x^*_{t+\theta}, u^*_{[t+\theta,t+T]}, t+\theta, t+T) \\
&\geq \int_t^{t+\theta} [q(x^*(s)) + r(u^*_{[t,t+T]}(s)]ds \\
&\quad +J(x^*_{t+\theta}, u^*_{[t+\theta,t+T+\theta]}, t+\theta, t+T+\theta)
\end{aligned} \tag{23}$$

where x^* denotes the trajectory corresponding to the optimal control $u^*_{[t,t+T]}$, and x^θ denotes the trajectory corresponding to the optimal control $u^*_{[t+\theta,t+T+\theta]}$. Rearranging the above and proceeding to the limit with $\theta \to 0+$ yields

$$\begin{aligned} & D_t^+ J(x_t, u^*_{[t,t+T]}, t, t+T) \\ & \le -[q(x^*(t)) + r(u^*_{[t,t+T]}(t)] \\ & \le -q(x^*(t)) \\ & \le -\alpha_L(\|x(t)\|) \end{aligned} \tag{24}$$

which holds for all $t > 0$, all $T > 0$, and all $x_t \in C([t-\tau_k, t], \mathbb{R}^n)$. It suffices to define

$$\tilde{w}(\| x(t) \|) = \alpha_L(\|x(t)\|) \tag{25}$$

which completes the proof.

The last proposition delivers immediately the desired stabilization result.

Theorem 2. *Assume that system (1) has the property specified in Theorem 1. Then, the receding horizon control law based on this cost functional is globally and uniformly stabilizing for system (1).*

Proof: The proof is immediate in view of the result of Proposition IV.1 and follows from a standard stability theorem for time delayed systems; see [3], p.132.

V. A PARTICULAR TYPE OF NONLINEAR DIFFERENTIAL DIFFERENCE SYSTEMS

A. Feasible Solution

In general, finding of the feasible $q_i(\cdot)$, $i = 0, ..., k$, in Theorem 1 for nonlinear differential difference systems (1) is difficult. In this section, we introduce an approach proposed in [9] for a particular type of nonlinear time delayed systems. In this approach, the feasible $q_i(\cdot)$, $i = 0, ..., k$, satisfying the inequality condition (11) can be easily obtained by solving an LMI problem.

The mentioned particular type of nonlinear differential difference system is presented as:

$$\begin{aligned} \dot{x}(t) &= f(x(t), x(t-\tau), Bu(t)) \\ &= Ax(t) + Hp(x(t)) + A_1 x(t-\tau) \\ &\quad + H_1 g(x(t-\tau)) + Bu(t) \end{aligned} \tag{26}$$

with initial condition:

$$x(s) = \phi(s), \quad s \in [-\tau, 0], \quad p(0) = 0, \quad g(0) = 0,$$

where H and H_1 are constant matrices with appropriate dimensions and the functions $f, p, g : \mathbb{R}^n \to \mathbb{R}^n$. There exist some constant matrices N, M, N_1, and M_1 for the functions p and g of (26) such that the following inequalities:

$$\|p(x(t)) - Nx(t)\|_2 \le \|Mx(t)\|_2, \tag{27}$$

$$\|g(x(t)) - N_1 x(t)\|_2 \le \|M_1 x(t)\|_2 \tag{28}$$

are valid. We assume that the cost penalties $q(\cdot)$ and $r(\cdot)$ have quadratic forms:

$$q(x(t)) = x^T(t) Q x(t), \quad r(u(t)) = u^T(t) R u(t) \tag{29}$$

where Q and R are symmetric, positive definite matrices. Then the following theorem for a terminal weighting functional provides a systematic method to obtain a receding horizon control law which stabilizes the nonlinear differential difference system (26):

Theorem 3. *([9]) If there exist a symmetric, positive definite matrix $X > 0$, as well as some matrices Y, Z, and scalars ε, δ such that*

$$\begin{bmatrix} W & VZ & X & X & Y^T & XM^T & 0 \\ ZV^T & -Z & 0 & 0 & 0 & 0 & ZM_1^T \\ X & 0 & -Z & 0 & 0 & 0 & 0 \\ X & 0 & 0 & -Q^{-1} & 0 & 0 & 0 \\ Y & 0 & 0 & 0 & -R^{-1} & 0 & 0 \\ MX & 0 & 0 & 0 & 0 & -\varepsilon I & 0 \\ 0 & M_1 Z & 0 & 0 & 0 & 0 & -\delta I \end{bmatrix} < 0 \tag{30}$$

where

$$\begin{aligned} W &\triangleq (AX + BY) + (AX + BY)^T + \varepsilon H H^T + \delta H_1 H_1^T \\ &\quad + HNX + XN^T H^T \\ V &\triangleq (A_1 + H_1 N_1) \end{aligned}$$

then the inequality condition (11) is satisfied with the control

$$u(x_t) \triangleq K(x(t)) \tag{31}$$

using $q_0(x(t)) = x^T(t) P x(t)$, $q_1(x(t)) = x^T(t) S x(t)$, where P and S are symmetric positive definite matrices, and P, S, and K can be obtained by letting

$$P = X^{-1}, \quad S = Z^{-1}, \quad K = Y X^{-1}$$

In next section, a simulation example is demonstrated by using the algorithm proposed in this section.

VI. NUMERICAL EXAMPLE

A. RHC for a special type of time delayed system

Consider the nonlinear time delayed system

$$\dot{x}(t) = x(t) \sin(x(t)) + x(t-1) + u(t), \quad 0 \le t \le 2$$

with initial condition

$$\phi(s) = 10, \quad -1 \le s \le 0$$

This system belongs to the class considered in Section V. Note that, for this system, we have

$$A = L_f = g = 0, \ \ H_f = M_f = A_1 = B = 1, \ \ f(x) = x \sin(x)$$

Applying Theorem 3 with $Q = 1$ and $R = 1$, we obtain

$$P = 7.17, \quad S = 2.944, \quad K = -7.5865$$

For RH implementation, state measurement is taken at the sample time of 0.05 and horizon length T is 1. Figure 2 compares the state trajectories for RHC with those for a constant state feedback $u(t) = Kx(t)$. With the above K value, Figure 3 compares the control trajectories. Integrated costs are given as follows:

$$J_{RHC} = 34.8201, \quad J_{KX} = 39.8288$$

where J_{RHC} is the cost for the RHC and J_{KX} is the cost for a constant state-feedback controller. Note that J_{RHC} is less than J_{KX} by about 15%. This result is obvious, since RHC has more degree of freedom than a constant state-feedback in minimizing the cost. From this numerical example, it is seen that the proposed RHC is stabilizing in closed loop.

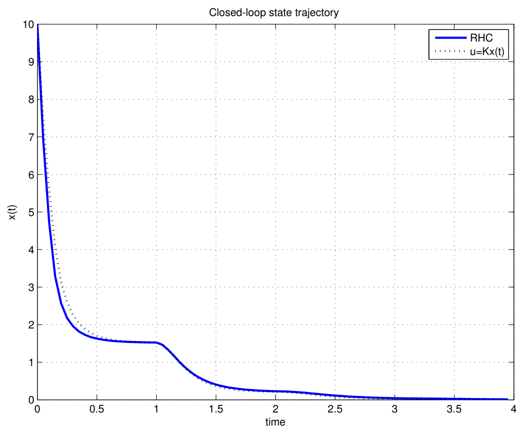

Fig. 2: State trajectories

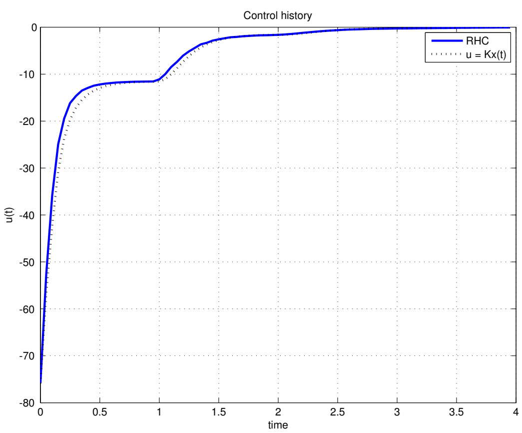

Fig. 3: Control trajectories

B. *Adaptive RHC of time delayed systems*

Here we assume the delay is unknown and the nonlinear time delayed system is addressed as

$$\dot{x}(t) = x(t)\sin(x(t)) + x(t-\tau) + u(t), \quad 0 \leq t \leq \; 2$$

with initial condition

$$\phi(s) = 10, \quad -\tau \leq s \leq 0$$

In this example, the actual delay is $\hat{\tau} = 1$ and the initial guess delay is taken to be $\tau = 0.1$. In simulation, with the on-line estimation of delay shown in Section III-B, the prediction horizon is $T = 1$, the state measurement is taken with a sample rate of 0.05. The RHC closed-loop state trajectories of the actual and estimated models are presented in Figure 4.

VII. Conclusions

This research has first presented a stablizing adaptive RHC for nonlinear differential difference systems with multiple time delays. An inequality condition on the terminal weighting functional was developed to insure the nonincreasing monotonicity of the optimal cost. The delay estimation algorithm and RHC is related through the optimal cost functional $J(\bar{x}_\eta, \bar{u}^*_{[\eta,\sigma]}, \eta, \sigma)$ which plays a vital role for the stability of the proposed adaptive RHC. The closed-loop stability of the proposed RHC is shown through a simulation example, and the effectiveness of the RHC with on-line delay estimation is confirmed.

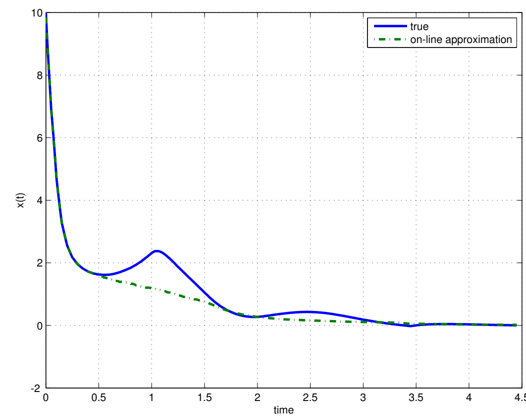

Fig. 4: RHC closed-loop state trajectories of actual and estimated models

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