2011 50th IEEE Conference on Decision and Control and
European Control Conference (CDC-ECC)
Orlando, FL, USA, December 12-15, 2011

Microstepping with Nonlinear Torque Modulation for Position Tracking Control in Permanent Magnet Stepper Motors

Wonhee Kim, Donghoon Shin and Chung Choo Chung†

Abstract— **A new approach to position control based on microstepping with nonlinear torque modulation in permanent magnet stepper motors is explored in this paper. The proposed method consists of nonlinear torque modulation, a commutation scheme, and a nonlinear current tracking controller. Previous microstepping control methods that guarantee the desired currents have relatively large position tracking errors due to their mechanical dynamics. A method of nonlinear torque modulation is designed to solve this problem. We propose a commutation scheme that is equivalent to microstepping in which the desired currents have time-varying amplitudes with $\frac{\pi}{2}$ electrical phase advance. The nonlinear controller in the electrical dynamics is developed to guarantee the desired current derived by the commutation scheme. The proposed control method is in the form of field oriented control (FOC) even though direct quadrature transformation so that the zero direct current is kept to maximize the torque. Thus, the efficiency of the energy consumption of the proposed method is better those that of previous microstepping control methods for guarantees of the desired currents. The performance of the proposed position control method was validated through simulations.**

I. Introduction

Microstepping is used to improve the resolution and increase the motion stability of permanent magnet stepper motors (PMSMs). Microstepping is a control method for two phase PMSMs in which the desired currents, two sinusoidal inputs that are shifted 90 degrees, are given to a PMSM for position tracking. Regulation of the desired currents is important in microstepping. Several feedback control methods have been studied in order to improve the current tracking performance of microstepping [1] - [4]. Generally, microstepping with current proportional and integral (PI) feedback loops is used to maintain desired currents in industrial applications [1]. However, the PI controllers cannot efficiently compensate for the effects of both back-emfs and the inductances. Microstepping with current PI and feedforward was proposed to improve the tracking performance of the desired currents [2]. Position feedback by resolvers or encoders built into PMSMs was previously used to improve microstepping in industrial applications [5]. Recently, Lyapunov-based controller was developed to guarantee the exponential convergence of the currents to the desired currents using full state feedback [3]. In [4], Lyapunov-based control with a nonlinear observer was implemented using only position feedback, and the stability of the closed-loop was presented. Since the electrical dynamics is much faster than the mechanical dynamics in PMSM, the position tracking error in the nonzero velocity appears due to the mechanical dynamics although the desired currents are guaranteed.

In order to improve the position tracking performance, various methods have been researched to control position of PMSM using direct quadrature (DQ) transformation, which makes mechanical dynamics linear [6] - [9]. A technique of exact feedback linearization using full state-feedback was proposed with extensions to the partial state-feedback in [6]. A field weakening control with state feedback was proposed for high speed operation [7]. A nonlinear adaptive controller for PMSMs with unknown parameters was presented to guarantee asymptotic tracking of a reference trajectory [8]. In [9], a conventional servo compensator-based controller was presented to improve transient performance. Although all of the methods improve the position tracking performance of PMSM, these method were not designed based on two phase frame.

We present a new approach to position control based on microstepping with nonlinear torque modulation in PMSMs. The proposed method includes nonlinear torque modulation, a commutation scheme, and a nonlinear current tracking controller. We study why the torque modulation is required to improve the position tracking performance of microstepping [4]. The latter is implemented to guarantee delivery of the desired current derived by the commutation scheme. The least lower bound of absolute steady-state position tracking error during a period of constant velocity is estimated when the desired current is regulated by conventional microstepping with feedback control. This steady-state position tracking error is the limitation of conventional microstepping with the feedback control appeared due to the mechanical dynamics of PMSM. Nonlinear torque modulation is proposed to overcome this problem. A commutation scheme implementing field oriented control (FOC) is developed to construct the desired current profile for torque generation even though DQ transformation is not used. FOC maintains zero direct current to maximize torque. Therefore, the energy efficiency of the proposed method is superior to those of previous microstepping control methods for the regulation of the desired currents. This commutation scheme is found to

†: Corresponding Author
This work was supported by Ministry of Knowledge Economy, Republic of Korea.
W. Kim and D. Shin are with the Department of Electrical Engineering, Hanyang University, Seoul, 133-791, Korea. alukard@hanyang.ac.kr, shin211@hanyang.ac.kr
C. C. Chung is with the Division of Electrical and Biomedical Engineering, Hanyang University, Seoul, 133-791, Korea cchung@hanyang.ac.kr Tel: +82-2-2220-1724, Fax: +82-2-2291-5307

be equivalent to microstepping control, in which the desired currents have time-varying amplitudes with $\frac{\pi}{2}$ electrical phase advance. We demonstrate that, in contexts without step-out, the required torque and quadrature are almost identical regardless of the type of control method used. The performance of the proposed position control method was validated through simulations.

II. Mathematical Model of Permanent Magnet Stepper Motor and Microstepping Performance Analysis

A. Mathematical Model

The dynamics of a PMSM can be represented in the state-space such that [7]

$$\begin{aligned}\dot{\theta} &= \omega \\ \dot{\omega} &= \frac{1}{J}[-K_m i_a \sin(N_r\theta) + K_m i_b \cos(N_r\theta) - B\omega - \tau_l] \\ \dot{i}_a &= \frac{1}{L}[v_a - Ri_a + K_m\omega\sin(N_r\theta)] \\ \dot{i}_b &= \frac{1}{L}[v_b - Ri_b - K_m\omega\cos(N_r\theta)]\end{aligned} \tag{1}$$

where v_a, v_b and i_a, i_b are the voltages [V] and currents [A] in phases A and B, respectively. ω is the rotor (angular) velocity [rad/s], θ is the rotor (angular) position [rad], B is the viscous friction coefficient [N·m·s/rad], J is the inertia of the motor [Kg·m^2], K_m is the motor torque constant [N·m/A], R is the phase winding resistance [Ω], L is the phase winding inductance [H], N_r is the number of rotor teeth, and τ_l is the load torque. Since detent torque in a PMSM does not significantly affect the torque produced by the motor, it can therefore be ignored [1]. In addition, the magnetic coupling between the phases is also ignored, as well as the variation in inductance due to magnetic saturation. Furthermore, an ideal sinusoidal flux distribution is assumed.

B. Microstepping Performance Analysis

In this subsection, the load torque perturbation, denoted by τ_L, is, for analysis purposes, assumed to be zero in this subsection. In [3], it was proven that if inputs $v^d_{a_{ms}}$, $v^d_{b_{ms}}$ of in conventional microstepping are given to the PMSM (1) as

$$v^d_{a_{ms}} = V_{\max}\cos(N_r\theta^d),\ v^d_{b_{ms}} = V_{\max}\sin(N_r\theta^d) \tag{2}$$

where θ^d is the static desired position, $V_{\max}$ is the amplitude of the microstepping input, then the states of the PMSM (1) locally asymptotically converge to equilibrium points $[\theta^d,\ 0,\ i^d_{a_{ms}},\ i^d_{b_{ms}}]$, i.e.

$$\lim_{t\to\infty}\theta(t) = \theta^d,\ \lim_{t\to\infty}\omega(t) = 0,\ \lim_{t\to\infty} i_a(t) = i^d_{a_{ms}},\ \lim_{t\to\infty} i_b(t) = i^d_{b_{ms}}. \tag{3}$$

where $i^d_{a_{ms}} = \frac{v^d_{a_{ms}}}{R}$, $i^d_{b_{ms}} = \frac{v^d_{b_{ms}}}{R}$ are the desired currents. Generally, various feedback controller in the current-loop is used to guarantee the desired current such as PI controller, PI and feedforward controller, nonlinear controller, etc [1] - [4]. In PMSM (1), the current dynamics is much faster than the mechanical dynamics. Therefore, a position tracking error appears although the feedback controller guarantees the desired currents $i^d_{a_{ms}}$, $i^d_{b_{ms}}$ during nonzero velocity period. That will be shown in next Proposition.

Proposition 1: Consider the PMSM (1). Suppose that the desired currents $i^d_{a_{ms}}$, $i^d_{b_{ms}}$ are guaranteed, and that there is no step-out. If the desired velocity is a constant, i.e. $\theta^d = \omega^d_{\max}t$ where $\omega^d_{\max}$ is the desired velocity, then the absolute steady-state position tracking error in the constant velocity period is greater than or equal to $\left|\frac{RB\omega^d_{\max}}{K_m V_{\max} N_r}\right|$. ◊

Proof: If $i_a = i^d_{a_{ms}}$, $i_b = i^d_{b_{ms}}$, PMSM (1) becomes

$$\begin{aligned}\dot{\theta} &= \omega \\ \dot{\omega} &= \frac{1}{J}[\frac{K_m V_{\max}}{R}\sin(N_r\theta^d - N_r\theta) - B\omega].\end{aligned} \tag{4}$$

Since the position θ is close to the desired position θ^d, PMSM (4) can be approximated as

$$\begin{aligned}\dot{\theta} &= \omega \\ \dot{\omega} &= \frac{1}{J}[\frac{K_m V_{\max}}{R}(N_r\theta^d - N_r\theta) - B\omega].\end{aligned} \tag{5}$$

From (5) we obtain the transfer functions from the desired position θ^d to the position θ as

$$\frac{\Theta(s)}{\Theta^d(s)} = \frac{G(s)}{1+G(s)} \tag{6}$$

where $G(s) = \frac{K_m V_{\max} N_r}{Rs(Js+B)}$. The constant velocity period in a desired trajectory, $\theta^d(t)$ can be regarded as a ramp input, $\omega^d_{\max} \times t$. From (6), the absolute steady-state position tracking error for the ramp input is obtained as

$$\begin{aligned}|\theta_e(\infty)| &= |\theta^d(\infty) - \theta(\infty)| \\ &= \lim_{s\to 0}\left|s\frac{1}{1+G(s)}\frac{\omega^d_{\max}}{s^2}\right| = \left|\frac{RB\omega^d_{\max}}{K_m V_{\max} N_r}\right|.\end{aligned} \tag{7}$$

The absolute steady-state position tracking error (7) is obtained under assumption that the currents are equal to the desired currents. Therefore, the actual absolute steady-state position tracking error during the constant velocity period is greater than or equal to $\left|\frac{RB\omega^d_{\max}}{K_m V_{\max} N_r}\right|$. ■

$\left|\frac{RB\omega^d_{\max}}{K_m V_{\max} N_r}\right|$ is the least lower bound of microstepping with feedback control of the electrical dynamics during the constant velocity period. This is a limitation of microstepping with feedback control for regulation of the desired current. Furthermore, the constant amplitude $V_{\max}$ in the microstepping inputs (2) results in degradation of energy efficiency.

III. Controller Design

The proposed method consists of three elements: i.e., nonlinear torque modulation, commutation scheme, and the current tracking controller. In this section, the design of the proposed method is presented, and the proposed method is analyzed in comparison to microstepping and FOC.

A. Nonlinear Torque Modulation

The mechanical dynamics of PMSM is

$$\begin{aligned}\dot{\theta} &= \omega \\ \dot{\omega} &= \frac{1}{J}[-K_m i_a \sin(N_r\theta) + K_m i_b \cos(N_r\theta) - B\omega - \tau_l].\end{aligned} \quad (8)$$

In the context of mechanical dynamics, the torque τ is the input as follows

$$\begin{aligned}\dot{\theta} &= \omega \\ \dot{\omega} &= \frac{1}{J}[\tau - B\omega - \tau_l]\end{aligned} \quad (9)$$

where $\tau = -K_m i_a \sin(N_r\theta) + K_m i_b \cos(N_r\theta)$. The tracking error of the mechanical dynamics is defined as

$$e_\theta = \theta^d - \theta, \text{ and } e_\omega = \omega^* - \omega \quad (10)$$

where where θ^d is the desired position profile and ω^* is yet to be defined. The tracking error dynamics of the mechanical dynamics is given by

$$\begin{aligned}\dot{e}_\theta &= \omega^d - \omega \\ \dot{e}_\omega &= \dot{\omega}^* - \frac{1}{J}[\tau - B\omega - \tau_l].\end{aligned} \quad (11)$$

Lemma 1: Consider the tracking error dynamics (11). If the nonlinear torque modulation is designed by

$$\begin{aligned}\omega^* &= \omega^d + k_1(\theta^d - \theta) \\ \tau &= k_2(\omega^* - \omega) + (\theta^d - \theta + B\omega + J\dot{\omega}^* + \tau_l)\end{aligned} \quad (12)$$

where k_1, k_2 are positive constant and $\omega^d = \dot{\theta}^d$ is the desired velocity, then the origin of the tracking error dynamics (11) is exponentially stable. ◊

Proof: For stability analysis, a Lyapunov candidate function V_1 is defined as

$$V_1 = \frac{1}{2}e_\theta^2 + \frac{J}{2}e_\omega^2. \quad (13)$$

The derivative of V_1 with respect to time is given by

$$\begin{aligned}\dot{V}_1 &= e_\theta \dot{e}_\theta + J e_\omega \dot{e}_\omega \\ &= e_\theta(\omega^d - \omega) + e_\omega(J\dot{\omega}^* - \tau + B\omega + \tau_l).\end{aligned} \quad (14)$$

With the control input (12), $\dot{V}_1$ becomes

$$\dot{V}_1 = -k_1 e_\theta^2 - k_2 e_\omega^2 < 0. \quad (15)$$

Therefore, the origin of the tracking error dynamics (11) is exponentially stable. ■

Remark 1: The physical meaning of ω^* in (12) is as follows: If $\theta^d > \theta$, then $\omega^* > \omega^d$ for θ to catch up with θ^d. And if $\theta^d < \theta$, then $\omega^* < \omega^d$ for θ to keep pace with θ^d. ◊

B. Commutation Scheme

In the mechanical dynamics (8), torque τ is generated by the current such that

$$\tau = -K_m i_a \sin(N_r\theta) + K_m i_b \cos(N_r\theta). \quad (16)$$

The nonlinear torque modulation (12) is designed under the assumption that the torque τ is the input in (8). The actual input of the mechanical dynamics (8), however, is not torque τ but represents the currents i_a, i_b. In order to generate the torque (12), we propose a new commutation scheme defined as

$$i_a^d = -\frac{\tau}{K_m}\sin(N_r\theta), \; i_b^d = \frac{\tau}{K_m}\cos(N_r\theta). \quad (17)$$

where i_a^d, i_b^d are the desired currents. The commutation scheme (17) is analyzed compared with the desired currents $i_{a_{ms}}^d$, $i_{b_{ms}}^d$ in microstepping (2). From the following relationships

$$\begin{aligned}i_a^d &= -\frac{\tau}{K_m}\sin(N_r\theta) = \frac{\tau}{K_m}\cos(N_r\theta + \frac{\pi}{2}) = \frac{\tau}{K_m}\cos(N_r\theta_{ms}^d), \\ i_b^d &= \frac{\tau}{K_m}\cos(N_r\theta) = \frac{\tau}{K_m}\sin(N_r\theta + \frac{\pi}{2}) = \frac{\tau}{K_m}\sin(N_r\theta_{ms}^d).\end{aligned} \quad (18)$$

$(N_r\theta + \frac{\pi}{2})$ can be regarded as the desired electrical position $N_r\theta_{ms}^d$ in the context of microstepping. Therefore, the proposed commutation scheme (17) is equivalent to the microstepping in which the desired currents have time-varying amplitudes with $\frac{\pi}{2}$ electrical phase advance. Note that the desired position θ_{ms}^d in the commutation scheme is not the same as desired position θ^d in nonlinear torque modulation (12). However θ_{ms}^d is the same as θ^d in microstepping (2). The desired currents generated by the commutation scheme (17) is illustrated in Fig. 1.

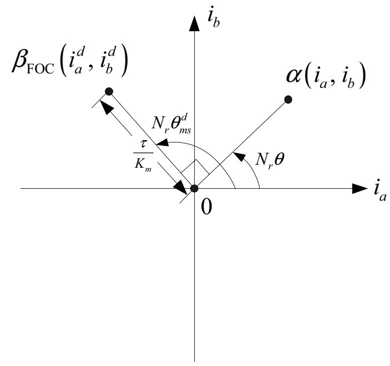


Fig. 1. Desired currents (17)

Remark 2: From (18), the commutation scheme (17) is equivalent to microstepping in which desired currents have time-varying amplitudes with $\frac{\pi}{2}$ electrical phase advance. Since the desired electrical position $N_r\theta_{ms}^d$ is always ahead of the present electrical position $N_r\theta$ since $\frac{\pi}{2}$, PMSM does not step out unless the inputs are saturated. ◊

C. Nonlinear Current Tracking Controller

The electrical dynamics in the PMSM (1) is

$$\begin{aligned} \dot{i}_a &= \frac{1}{L}[v_a - Ri_a + K_m\omega\sin(N_r\theta)] \\ \dot{i}_b &= \frac{1}{L}[v_b - Ri_b - K_m\omega\cos(N_r\theta)] \end{aligned} \tag{19}$$

Decreases of current and phase lags appear in the electrical dynamics due to the effects of back-emf and inductance. Therefore, our nonlinear controller is designed to guarantee the desired currents (17).

Let us define the tracking error of the electrical dynamics as

$$e_a = i_a^d - i_a, \text{ and } e_b = i_b^d - i_b. \tag{20}$$

The tracking error dynamics of the electrical dynamics is given by

$$\begin{aligned} \dot{e}_a =& \dot{i}_a^d - \frac{1}{L}[v_a - Ri_a + K_m\omega\sin(N_r\theta)] \\ \dot{e}_b =& \dot{i}_b^d - \frac{1}{L}[v_b - Ri_b - K_m\omega\cos(N_r\theta)]. \end{aligned} \tag{21}$$

Lemma 2: Consider the tracking error dynamics (21). If nonlinear current tracking controller is given to the electrical dynamics (19) as

$$\begin{aligned} v_a =& (Ri_a - K_m\omega\sin(N_r\theta)) + L(\dot{i}_a^d + k_3 e_a), \\ v_b =& (Ri_b + K_m\omega\cos(N_r\theta)) + L(\dot{i}_b^d + k_3 e_b), \end{aligned} \tag{22}$$

then the origin of the tracking error dynamics (21) is exponentially stable. ♢

Proof: A Lyapunov candidate function, V_2 is defined as

$$V_2 = \frac{1}{2}e_a^2 + \frac{1}{2}e_b^2. \tag{23}$$

Differentiating V_2 with respect to time yields

$$\begin{aligned} \dot{V}_2 =& e_a(\dot{i}_a^d - \dot{i}_a) + e_b(\dot{i}_b^d - \dot{i}_b) \\ =& e_a(\dot{i}_a^d - \frac{1}{L}(v_a - Ri_a + K_m\omega\sin(N_r\theta)) \\ &+ e_b(\dot{i}_b^d - \frac{1}{L}(v_b - Ri_b - K_m\omega\cos(N_r\theta)). \end{aligned} \tag{24}$$

Substituting the control law (22) in (24) gives us

$$\dot{V}_2 = -k_3(e_a^2 + e_b^2) < 0. \tag{25}$$

Therefore, the origin of the tracking error dynamics (21) is exponentially stable. ■

The nonlinear current tracking controller (22) guarantees the exponential convergences of the currents to the desired currents (17).

IV. Performance Analysis of the Proposed Method and Analysis of the Closed-Loop Stability

A. Performance Analysis of the Proposed Method

In this section, we study the relationship between microstepping and FOC. The DQ transformation [12] for the currents is defined as

$$\begin{bmatrix} i_d \\ i_q \end{bmatrix} = \begin{bmatrix} \cos(N_r\theta) & \sin(N_r\theta) \\ -\sin(N_r\theta) & \cos(N_r\theta) \end{bmatrix} \begin{bmatrix} i_a \\ i_b \end{bmatrix} \tag{26}$$

where i_d is the direct current and i_q is the quadrature current, respectively. Applying the DQ transformation to the desired currents in microstepping (2), the desired currents $i_{a_{ms}}^d$, $i_{b_{ms}}^d$ become

$$i_{d_{ms}}^d = \frac{V_{\max}}{R}\cos(N_r\theta^d - N_r\theta), \; i_{q_{ms}}^d = \frac{V_{\max}}{R}\sin(N_r\theta^d - N_r\theta). \tag{27}$$

where $i_{d_{ms}}^d$ and $i_{q_{ms}}^d$ are the desired direct and quadrature currents in microstepping (2). In PMSM, the torque $\tau = -K_m(i_a\sin(N_r\theta) + i_b\cos(N_r\theta))$ is equivalent to the quadrature current $K_m i_q$. Therefore, in order to improve energy efficiency, it is necessary to guarantee zero direct current.

Proposition 2: Consider the PMSM (1). Suppose that $i_a = i_a^d$ and $i_b = i_b^d$. Then the zero direct current is maintained by the proposed method. ♢

Proof: Applying the DQ transformation to the desired currents in (17), the desired currents become

$$i_d^d = 0, \; i_q^d = \frac{\tau}{K_m} \tag{28}$$

where i_d^d and i_q^d are the desired direct and quadrature currents in microstepping (2).

Therefore, if $i_a = i_a^d$ and $i_b = i_b^d$, then $i_d = 0$, $i_q = \frac{\tau}{K_m}$. ■

From Proposition 2, it is apparent that the proposed control is in the form of FOC, even though DQ transformation is not used.

Remark 3: From Remark 2 and Proposition 2, FOC is equivalent to microstepping, in which the desired currents have time-varying amplitudes with $\frac{\pi}{2}$ electrical phase advance in the proposed method. ♢

Now we study the desired quadrature current i_q^d for PMSM not to prevent step-out during the constant velocity period. For analysis, an Assumption 1 is made:

Assumption 1: e_θ and e_ω can be ignored since e_θ and e_ω are very small during the constant velocity period. ♢

Proposition 3: Consider the PMSM (1) under Assumption 1. Then in order to prevent step-out during the constant velocity period, the approximated required torque is $B\omega^d + \tau_L$ and the approximated desired quadrature current is $\frac{B\omega^d + \tau_L}{K_m}$. ♢

Proof: With Assumption 1, $\theta = \theta^d$, $\omega = \omega^d = \omega^*$. During constant velocity period, $\dot{\omega}$ should be almost zero in order to prevent step-out. Therefore, From (9), we obtain the approximated torque

$$\tau \approx B\omega^d + \tau_L. \tag{29}$$

Since $i_q^d = \frac{\tau}{K_m}$,

$$i_q^d \approx \frac{B\omega^d + \tau_L}{K_m}. \tag{30}$$

■

Remark 4: From Proposition 3, we conclude that the required torque and quadrature current are almost identical regardless of the type of control method used. ♢

B. Analysis of the Closed-loop Stability

Since the proposed method (12), (22) are separately designed, the stability of closed-loop should be proven. Let us define new errors as

$$e_m = \begin{bmatrix} e_\theta \\ e_\omega \end{bmatrix}, \quad e_e = \begin{bmatrix} e_a \\ e_b \end{bmatrix}. \tag{31}$$

The error dynamics of the closed-loop becomes

$$\dot{e}_m = A_m e_m + B_m(\theta) e_e \tag{32}$$

$$\dot{e}_e = A_e e_e \tag{33}$$

where $A_m = \begin{bmatrix} -k_1 & 1 \\ -1 & -k_2 \end{bmatrix}$, $B_m(\theta) = \begin{bmatrix} 0 & 0 \\ -\frac{K_m \sin(N_r\theta)}{J} & \frac{K_m \cos(N_r\theta)}{J} \end{bmatrix}$, $A_e = \begin{bmatrix} -k_3 & 0 \\ 0 & -k_3 \end{bmatrix}$.

Theorem 1: Consider the error dynamics of the closed-loop(32), (33). If k_1, k_2, and k_3 are positive constants, the origins of (32), (33) are asymptotically stable. ◊

Proof: If k_1, k_2, and k_3 are positive constants, A_m and A_e are Hurwitz. We define V_m as

$$V_m = \frac{1}{2} e_m^T e_m \tag{34}$$

$\dot{V}_m$ is given by

$$\dot{V}_m = -\frac{1}{2} e_m^T Q_m e_m + e_m^T B_m(\theta) e_e \tag{35}$$

where $Q_m = \begin{bmatrix} k_1 & 0 \\ 0 & k_2 \end{bmatrix}$. $B_m(\theta)$ is bounded. If we define $B_m(\theta)e_e$ as the input and e_m as the output in (32), (35) is

$$\underbrace{e_m^T}_{output} \underbrace{B_m(\theta) e_e}_{input} = \dot{V}_m + \underbrace{e_m^T Q_m e_m}_{>0}. \tag{36}$$

Equation (36) shows that the relationship between e_m and $B_m(\theta)e_e$ is strictly output passive [13]. And $\dot{e}_m = A_m e_m$ is zero-state observable since A_m is Hurwitz. Therefore, (32) is bounded input bounded output (BIBO) stable. By defining the Lyapunov candidate function, V_e, as

$$V_e = \frac{1}{2} e_e^T e_e, \tag{37}$$

we obtain

$$\dot{V}_e = -k_2 e_e^T Q_e e_e \tag{38}$$

where $Q_e = \begin{bmatrix} k_3 & 0 \\ 0 & k_3 \end{bmatrix}$. Thus the origin of (33) is exponentially stable and the origin of (32) is asymptotically stable. ■

V. Simulation Results

Simulations were performed to evaluate the performance of the proposed controller. The proposed method was compared with Lyapunov-based control in microstepping [4] such as

$$\begin{aligned} v_{a_{ms}}^d &= V_{\max} \cos(N_r \theta^d), \\ v_{b_{ms}}^d &= V_{\max} \sin(N_r \theta^d), \\ v_a &= Ri_a - K_m \omega \sin(N_r\theta) + L(\dot{v}_{a_{ms}}^d + \rho(v_{a_{ms}}^d - Ri_a)), \\ v_b &= Ri_b + K_m \omega \cos(N_r\theta) + L(\dot{v}_{b_{ms}}^d + \rho(v_{b_{ms}}^d - Ri_b)). \end{aligned} \tag{39}$$

TABLE I
Parameters of PMSM and controller gains

Parameter	Value	Parameter	Value
L	40 mH	R	14.8 Ω
J	8×10^{-5} kg·m^2	K_m	0.5 N·m /A
N_r	50	B	5×10^{-3} N·m·s/rad
τ_L	0.01	k_1	0.01
k_2	0.01	k_3	30000
$V_{\max}$	6.5	ρ	30000

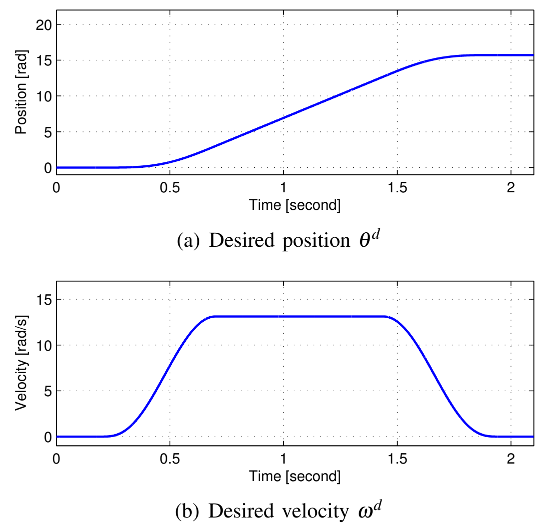


Fig. 2. Desired position and velocity

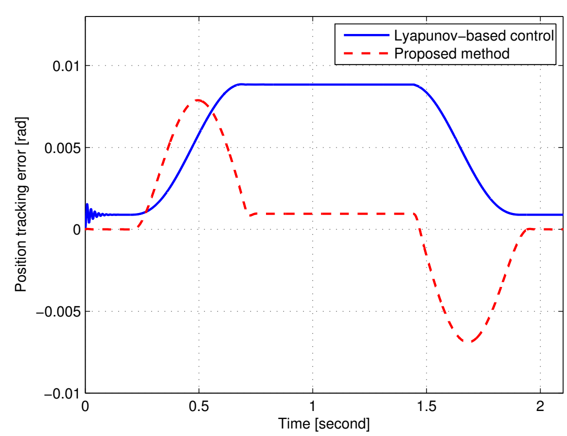


Fig. 3. Position tracking error e_θ

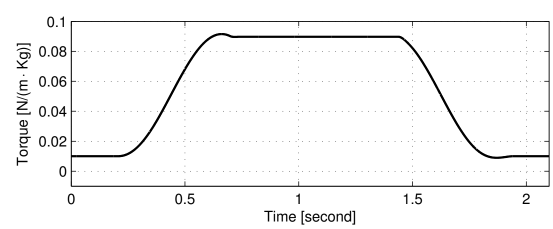
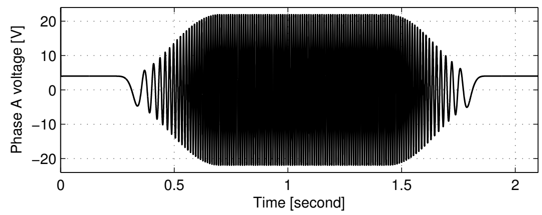
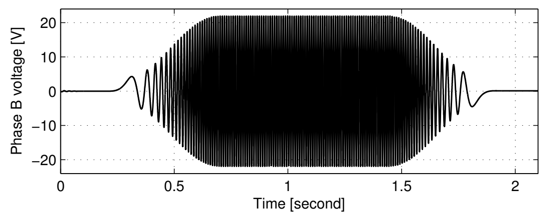


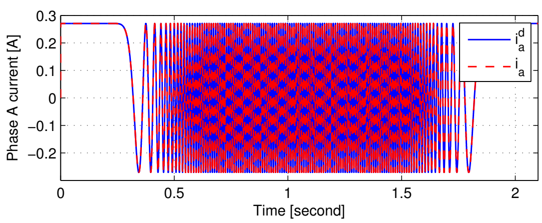
Fig. 4. Torque τ of the proposed method



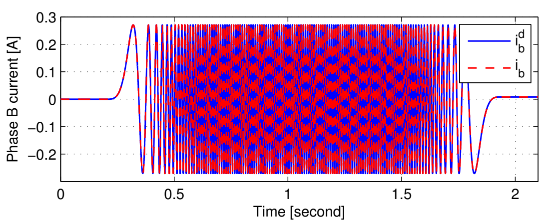
(a) Phase A voltage



(b) Phase B voltage

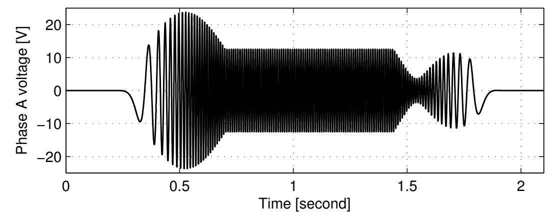


(c) Tracking performance of phase A current

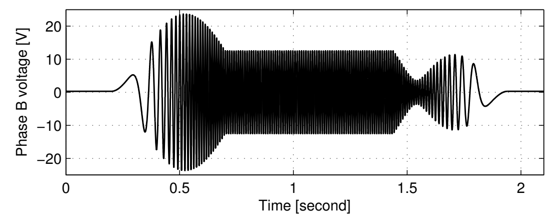


(d) Tracking performance of phase B current

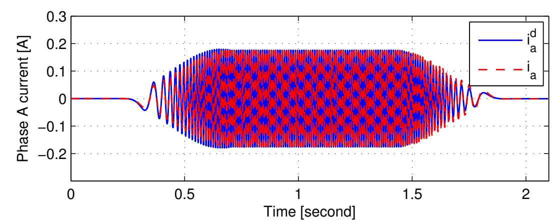
Fig. 5. Voltages and current tracking performances of Lyapunov-based control



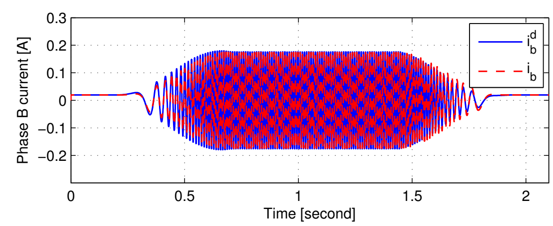
(a) Phase A voltage



(b) Phase B voltage



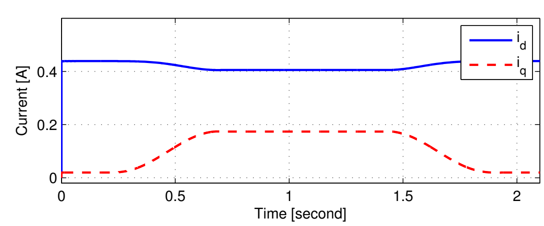
(c) Tracking performance of phase A current



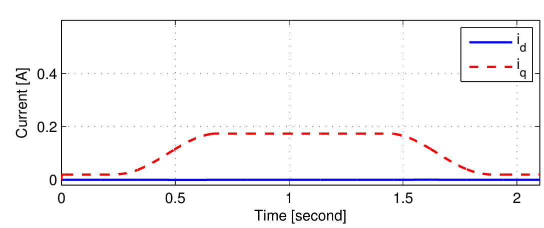
(d) Tracking performance of phase B current

Fig. 6. Voltages and current tracking performances of proposed method

The parameters of PK266-01B PMSM made by Oriental motor [15], the proposed method, and Lyapunov-based controller listed in Table I were used. Since the electrical dynamics is faster than the mechanical dynamics, k_3 should be bigger than k_1 and k_2. The desired position and velocity profiles shown in Figs. 2 were used. During the constant velocity period, $\omega^d = 13.13$ rad/s. Figure 3 indicates the position tracking errors of both the proposed method and Lyapunov-based control. The steady-state position tracking error of Lyapunov-based control during the constant velocity period was 0.0088 rad, and increased more than 0.0070 rad rad according to (7) due to the electrical dynamics and the load torque. DC offset position error was detected in the final position due to load torque. However, the steady-state position tracking error of the proposed method during the constant velocity period was only 0.00095 rad. The proposed method overcomes the limitations associated with microstepping. Since the nonlinear controller compensated



(a) Lyapunov based control



(b) Proposed method

Fig. 7. i_q and i_q

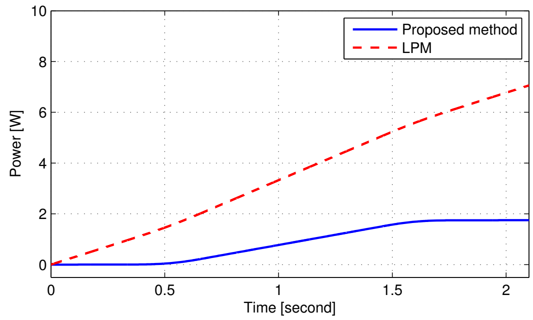


Fig. 8. Powers of both methods

for the load torque by torque modulation (12), the position error of the final position was zero. τ was derived by nonlinear torque modulation (12) and is shown in Fig. 4. During the constant velocity period, τ^d was 0.089, which is very close to the value of 0.088 obtained by (29). The torque increased during the acceleration period and decreased during the deceleration period. Although the acceleration is zero during the constant velocity period, the desired torque is not zero due to friction, load torque, and tracking errors of position and velocity. Figures 5 and 6 voltages and current tracking performances of both methods. In Lyapunov-based control, the voltages were increased to compensate for the effects of back-emfs and inductances as shown in Figs. 5 (a) and (b). Therefore, Figs. 5 (c) and (d) show that the desired current was maintained. When the proposed method was used, the voltages followed the torque τ as shown Figs. 5 (a), (b). During the acceleration period, the amplitudes of the voltages were increased to compensate for the effects of back-emfs and inductances in the proposed method. However, at the outset of the deceleration period the phase voltage inputs were decreased due to friction and back-emfs. The phase voltage inputs increased again so that the position would converge to the desired position. The current tracked the desired currents well, as shown in Figs. 6 (c) and (d). In addition, the currents used in the proposed method were reduced compared to those used in Lyapunov-based control. Since the magnitudes of the desired currents vary with time in the proposed method, i.e., the desired torque, the phase currents of the proposed method are unable to accurately track the desired phase currents during the acceleration and deceleration periods. Therefore, the absolute position tracking error increased during the acceleration and deceleration periods as shown in Fig. 3. Figure 7 shows the direct and quadrature currents of both methods. The quadrature currents of both methods were almost identical, as predicted by Remark 4. Furthermore, during the constant velocity period, the quadrature current was 0.0175, which is very close to the value of 0.0174 calculated by (30). In Lyapunov-based control, since $|N_r\theta^d - N_r\theta| < \frac{\pi}{2}$, i_d was always positive (27). On the other hand, i_d was maintained to be near zero by the proposed method. Consequently, the power of the proposed method was smaller that that of Lyapunov-based control, even though the position tracking performance of the proposed method was better that that of Lyapunov-based control as shown in Fig. 8.

VI. Conclusions

We present a new approach to achieve position control in PMSMs that is based on microstepping with torque modulation. Nonlinear torque modulation was proposed to achieve the desired torque. A commutation scheme was developed to determine the desired current profile for the desired torque. A nonlinear current tracking controller was designed to guarantee that the desired current would be derived by the commutation scheme. FOC is proposed even though DQ transformation is not used. FOC is equivalent to microstepping control if the desired currents have time-varying amplitudes with $\frac{\pi}{2}$ electrical phase advance. Our simulation results showed that the proposed method improves the position tracking performance of microstepping. FOC was achieved by the proposed method even though DQ transformation was not used. Furthermore, the energy efficiency of the proposed method is superior to that of previous microstepping control methods and guarantees the achievement of of the desired currents.

References

[1] A. Bellini, C. Concari, G. Franceschini, and A. Toscani, “Mixed-mode PWM for high-performance stepping motors,” *IEEE Trans. Ind. Electron.*, vol. 54, no. 6, pp. 3167-3177, Dec. 2007.

[2] W. Kim, I. Choi, and C. C. Chung, “Microstepping with PI feedback and feedforward for permanent magnet stepper motors,” in *Proc. ICCAS-SICE,* 2009, pp. 603-607.

[3] W. Kim, I. Choi, K. Bae, and C. C. Chung, “A Lyapunov method in microstepping control for permanent magnet stepper motors,” in *Proc. IEEE Int. Conf. Mechatron.,* 2009, pp. 1-5.

[4] W. Kim, I. Choi, and C. C. Chung, “Lyapunov-based control in microstepping with a nonlinear observer for permanent magnet stepper motors,” in *Proc. Amer. Contr. Conf.,* 2010, pp. 4313-4316.

[5] Orientalmotor “Basics of motion control.” [Online]. Available: http://www.orientalmotor.com/products/ac-dc-step-motors/alphastep-overview.html

[6] M. Zribi, J. Chiasson, “Position control of a PM stepper motor by exact linearization,” *IEEE Trans. Autom. Control,*, vol 36, no. 5, pp. 620-625, May 1991.

[7] M. Bodson, J. Chiasson, R. Novotnak, and R. Rekowski, “High-performance nonlinear feedback control of a permanent magnet stepper motor,” *IEEE Trans. Contr. Syst. Technol.*, vol. 1, no. 1, pp. 5-14, Mar. 1993.

[8] R. Marino, S. Peresada, and P. Tomei, “Nonlinear adaptive control of permanent magnet step motors,” *Automatica,* vol. 31, no. 11, pp. 1595-1604, 1995.

[9] S. Seshagiri, “Position control of permanent magnet stepper motors using conditional servocompensators,” *IET Contr. Theory Applications,* vol. 3, no. 9, pp. 1196-1208, 2008.

[10] F. Khorrami, P. Krishnamurthy, and H. Melkote, *Modeling and Adaptive Nonlinear Control of Electric Motors.* Heidelberg: Springer Verlag, 2003.

[11] J. Chiasson, *Modeling and High-Performance Control of Electric Machines.* Hoboken, NJ: Wiley-Interscience, 2005.

[12] R. H. Park, “Two-reaction theory of synchronous machines generalized method of analysis,“ *AIEE Trans.,* no. 48, pp. 716-727, Nov. 1929.

[13] H. Khalil, *Nonlinear Systems,* 3rd ed. Upper Saddle River, NJ: Prentice-Hall, 2002.

[14] S. Sastry, *Nonlinear systems: Analysis, Stability, and Control.* New York, Springer, 1999.

[15] Orientalmotor, ”PK Series Stepping Motors.” [Online]. Available: http://catalog.orientalmotor.com/viewitems/steppingmotors.1068/pk-series-stepping-motors