2011 50th IEEE Conference on Decision and Control and
European Control Conference (CDC-ECC)
Orlando, FL, USA, December 12-15, 2011

Periodic H_2 Synthesis for Spacecraft Attitude Control with Magnetorquers and Reaction Wheels

Jean-François Trégouët, Denis Arzelier, Dimitri Peaucelle,
Yoshio Ebihara, Christelle Pittet and Alexandre Falcoz

***Abstract*— Particularly attractive for small satellites, the use of magnetic torquers for attitude control is still a difficult problem. Indeed, equations are naturally time-varying and suffers from controllability issues. In this paper, a generic model, taking different kinds of pointing and different kinds of actuators into account, is proposed, linearized and then discretized. Recent studies demonstrate how combining magnetorquers and reaction wheels is attractive. Following this line, latest LMI synthesis techniques for static periodic controller are applied in this paper to the attitude control problem of a spacecraft equipped with both actuation systems. Simulation results are provided, showing the performance of the obtained control law.**

I. INTRODUCTION

The use of magnetic torquers, also called magnetorquers, for spacecraft attitude control of satellites has been a challenging issue for the scientific community for decades ([11] - [15]) and constitutes more than ever, an active area of research. Complexity of the associated control problem and recent advent of small satellites missions which frequently resort to this kind of actuators, for cheapness, efficiency, reliability and weight reasons, explain this renewed interest.

These actuators generate control torques which lie in the orthogonal plane of the local geomagnetic field vector. As this field is not uniform, due to the rotation of the satellite around the earth, the synthesis models are naturally periodic at the orbital frequency. Even if it has been proved recently in [9] that using these actuators solely, stabilization can be achieved for inertial pointing of a spacecraft, the time-varying nature of the problem as well as controllability issues will always lead to closed-loop performance limitations which are not acceptable when operating in mission mode. Therefore, modern spacecraft are usually endowed with magnetorquers and some type of mechanical actuator, such as reaction wheels. Typically, the latter are used for fine attitude control while the magnetic torquers are responsible for momentum dumping of reaction wheels. Thus, recent studies propose control laws intending to make both actuators working together in concert, see e.g. [3] and [7].

This work is supported by CNES and EADS-Astrium

Jean-François Trégouët, Denis Arzelier and Dimitri Peaucelle are with CNRS; LAAS; 7, avenue du Colonel Roche, F-31077 Toulouse, France, and Université de Toulouse; UPS , INSA , INP, ISAE ; LAAS ; F-31077 Toulouse, France tregouet@laas.fr

Yoshio Ebihara is with Department of Electrical Engineering, Kyoto University, Kyotodaigaku-Katsura, Nishikyo-ku, Kyoto 615-8510, Japan

Christelle Pittet is with CNES, 18 avenue Edouard Belin, F-31401 TOULOUSE

Alexandre Falcoz is with ASTRIUM, 31 Rue Cosmonautes, F-31400 Toulouse, France

The set of models available in the literature and taking magnetorquers into account is very heterogeneous depending on the kind of pointing or actuators considered. Furthermore, derivation of linear discrete-time model is very briefly documented, see e.g. [14] and [18]. Therefore, this paper contributes to the modelling effort by proposing a unifying model including, on the first hand, different choices of pointing, inertial or geocentric, traditionally associated with star and Earth observation missions, and, on the other hand, different choices of actuation, magnetorquers and/or reaction wheels. Moreover, challenging issues of the linearization stage are discussed.

In the literature, the problem of attitude control by means of magnetorquers has been tackled in the field of non-linear ([7] - [9]) and linear control ([10] - [17]). In the latter, among techniques specifically designed for periodic time-varying models, two major trends emerge depending on the kind of tools used : Periodic Riccati equations, see [10], and Linear Matrix Inequalities (LMI). Relying on efficient numerical algorithms and offering more flexibility in taking constraints of robustness and performance into account, the latter has become a matter of primary interest.

The main contribution of this study is to apply the latest LMI synthesis techniques for static periodic controller on the attitude control problem of a spacecraft equipped with magnetorquers and reaction wheels. As in [18], the obtained control law aims to minimize an H_2 criterion. Here, closed loop performance is improved by adding reaction wheels whereas in [18], magnetorquers solely are considered. Moreover, the paper initiates a more ambitious work which aims at dealing with uncertainties by designing a robust periodic control law.

Notations : The transpose of a matrix A is denoted A^T. $\mathbf{1}$ stands for identity matrix and $\mathbf{0}$ for the zero matrix with appropriate dimensions. For a real square matrix A, we define $\mathrm{He}\{A\} = A + A^T$. The convex hull of the collection of N elements $A_1, \cdots, A_N$ is denoted by $\mathrm{co}\{A_1, \cdots, A_N\}$. $\mathbb{S}^n$ denotes the set of symmetric matrices of $\mathbb{R}^{n\times n}$.

II. DYNAMIC AND KINEMATIC NON-LINEAR EQUATIONS

A. *Frames definition depending on the pointing case*

In order to derive dynamic and kinematic equations, the following frames are defined (see Fig 1) :

- F_I : Earth-Centered Inertial (ECI) frame, located at the earth center ;
- F_B : Body-fixed frame located at the center of mass of the satellite and along its principal axis ;

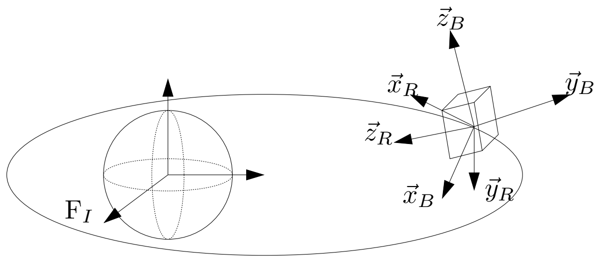

Fig. 1. Inertial, body-fixed and reference frames

– F_R : Reference frame located at the center of mass of the satellite and independent of its attitude.

Attitude regulation has to be understood here as the control problem which consists in aligning the frame F_B with F_R by means of actuators. The frame F_R is usually chosen to be the pointing target. Thus, it may correspond to F_I in the case of star-pointing control system ([7], [9]) and to a local orbital frame for geocentric pointing ([1],[15]). The angular velocity of F_R with respect to F_I is denoted $\omega_{R/I}$ and considered to be a given constant vector.

B. Dynamic and kinematic equations

Let us consider the case of a rigid satellite equipped with magnetorquers creating a torque T_m and reaction wheels whose axes (expressed in the body-fixed frame F_B as all quantities of (1) and (2)) are the columns of the matrix X_W. Classical dynamic equations are derived by applying the principle of angular momentum conservation :

$$\dot{h}_s = T_w \quad (1)$$

$$J\dot{\omega}_{B/I} + \omega_{B/I}^{\times}\left(J\omega_{B/I} + X_W h_s\right) = T_{ext}^{[B]} - X_W T_w + T_m \quad (2)$$

where T_w and h_s refer respectively to the torques applied by the wheels and their angular momentum, $\omega_{B/I}$, the angular velocity of the satellite with respect to the inertial frame, and $T_{ext}^{[B]}$, the external torques. Finally, J stands for the inertia matrix. Furthermore, we define for $g = [g_x, g_y, g_z]^T$

$$g^{\times} = \begin{bmatrix} 0 & -g_z & g_y \\ g_z & 0 & -g_x \\ -g_y & g_x & 0 \end{bmatrix} \quad (3)$$

such that $\left(g^{\times}\right)^T = -g^{\times}$ and $g^{\times}g = 0$.

Attitude of the satellite w.r.t. the reference frame F_R is characterized by the quaternion $q_{B/R} = [\epsilon^T \eta]_{B/R}^T \in \mathbb{R}^4$ subject to the unit norm constraint $q^T q = 1$. In [8], the following relationship is stated :

$$\dot{q}_{B/R} = \frac{1}{2}\begin{bmatrix} -\left(\omega_{B/R}\right)^{\times} & \omega_{B/R} \\ -\left(\omega_{B/R}\right)^{T} & 0 \end{bmatrix} \cdot q_{B/R} \quad (4)$$

which represents the attitude kinematic.

C. Magnetorquers

Use of magnetorquers, generating a magnetic moment M considered as a control variable, creates a torque T_m, expressed in F_B as the following :

$$T_m = \left(M^{[B]}\right)^{\times} B^{[B]} = -\left(B^{[B]}\right)^{\times} M^{[B]} \quad (5)$$

where $B^{[B]}$ refers to the geomagnetic field viewed by the satellite. Hence, at a given time, T_m lies in the orthogonal plane of the local geomagnetic field vector, image of the matrix $\left(B^{[B]}\right)^{\times}$. Nevertheless, over a whole orbit, the direction of $B(t)$ varies and the reachable torque space becomes $\mathbb{R}^3$ [15].

Using $C_{B/R}$, which stands for the rotation matrix of F_B w.r.t. F_R, $B^{[B]}$ can be decomposed as :

$$B^{[B]}(t) = C_{B/R} \cdot B^{[R]}(t) \quad (6)$$

This relationship brings in light the dual dependence of $B^{[B]}$ with respect to attitude, via $C_{B/R}$, and to its position along the orbit, via $B^{[R]}(t)$. The former induces a coupling between kinematics and dynamics while the latter makes the model pseudo-periodic at the orbital period.

$B^{[R]}(t)$ is assumed to be given, because it may be estimated on board or computed using the International Geomagnetic Reference Field (IGRF) which gives access to the local magnetic field at any point in space [16].

D. Disturbances

As emphasis is put on attitude control, orbital perturbations are neglected and the orbit is assumed to be circular of period ω_0.

Predominant for low orbits, where magnitude of the geomagnetic field is sufficient to use magnetorquers, disturbance torques such as gravity gradient, aerodynamic drag and magnetic torques disturbance, due to the interaction of current loops with the magnetic field, can be modeled as periodic at one or two times the orbital frequency ω_0 (see [2]) :

$$T_{ext}^{[I]}(t) = \begin{bmatrix} T_{0x} + T_{1x}\sin(\omega_0 t + \varphi_{1x}) + T_{2x}\sin(2\omega_0 t + \varphi_{2x}) \\ T_{0y} + T_{1y}\sin(\omega_0 t + \varphi_{1y}) + T_{2y}\sin(2\omega_0 t + \varphi_{2y}) \\ T_{0z} + T_{1z}\sin(\omega_0 t + \varphi_{1z}) + T_{2z}\sin(2\omega_0 t + \varphi_{2z}) \end{bmatrix} \quad (7)$$

For simplicity, these torques are expressed in F_R using the given angular velocity $\omega_{R/I}$.

E. State space model representation

State space equations corresponding to the vector $X = \begin{bmatrix} \omega_{B/I}^T & q_{B/R}^T & h_s^T \end{bmatrix}^T$ are pseudo-periodic at the orbital period :

$$\begin{aligned} \dot{\omega}_{B/I} = J^{-1}\Big(&-\omega_{B/I}^{\times}\left(J\omega_{B/I} + X_W h_s\right) + C_{B/R}T_{ext}^{[R]}(t) \\ &-X_W T_w - C_{B/R}\left(B^{[R]}(t)\right)^{\times} C_{B/R}^T M^{[B]}\Big) \end{aligned} \quad (8)$$

$$\dot{q}_{B/R} = \begin{bmatrix} -\omega_{B/I}^{\times} + C_{B/R}\omega_{R/I}^{\times}C_{B/R}^T & \omega_{B/I} - C_{B/R}\omega_{R/I} \\ -\omega_{B/I}^T + \omega_{R/I}^T C_{B/R}^T & 0 \end{bmatrix} \cdot \frac{1}{2} q_{B/R} \quad (9)$$

$$\dot{h}_s = T_w \quad (10)$$

where $C_{B/R}$ is a function of $q_{B/R}$ according to the general relationship provided by [8]

$$C(q) = \left(\eta^2 - \epsilon^T \epsilon\right) \underline{1} + 2\epsilon\epsilon^T - 2\eta\epsilon^{\times} \tag{11}$$

Inputs are composed of T_w and $M^{[B]}$, for control, and $T_{ext}^{[R]}$, for external disturbances.

III. Control Design

A. Control objectives and comparisons with existing studies

The control law aims at ensuring the best achievable attitude performance despite disturbances and modeling uncertainties, while controlling the angular momentum of the wheels which tends to diverge because of the DC component of the disturbing torque. Consequently, the equilibrium sought is such that the axis of the spacecraft are aligned with the reference frame and the angular momentum of the wheels remains around an average value chosen to avoid critical cases, e.g. saturation, non linear dynamics when wheels stop, frequencies exciting flexible modes, etc. Hence, the desired trajectory, denoted tr, is defined by

$$\left\{ \begin{array}{l} \left(\omega_{B/R}\right)_{tr} = \mathbf{0} \\ \left(q_{B/R}\right)_{tr} = \left[\begin{array}{cccc} 0 & 0 & 0 & 1 \end{array} \right]^T \\ \left(h_s\right)_{tr} = \left(h_s\right)_{eq} = \left[\left(h_{1s}\right)_{eq} \left(h_{2s}\right)_{eq} \left(h_{3s}\right)_{eq} \right]^T \end{array} \right. \tag{12}$$

where $\left(h_s\right)_{eq}$ corresponds to half of the maximum of h_s.

In [5], the problem of robust stabilization of linear discrete-time systems via a periodic state-feedback control law is tackled along with the minimization of the worst-case H_2 norm of the uncertain closed-loop system. Hence, when applied to the problem of this paper, in addition to control the attitude, the obtained control law guarantees that h_s not only remains bounded, since it is part of the state vector, but also is kept as close as possible to a predefined trajectory. Furthermore, unlike in [18] where another periodic H_2 synthesis theorem is proposed, uncertainties are explicitly taken into account in the design procedure.

Both [7] and [13] also deals with spacecraft endowed with magnetorquers and mechanical actuators which are considered under a general formulation. The retained control strategy consist in solving a conventional attitude control problem first and then allocating the desired torque by giving priority to the magnetic actuators. However, this approach has not been retained in this paper for two reasons : first, when applied to reactions wheels, it can not be ensure that divergence of h_s is avoided. Second, the knowledge about the time variation of the model is only taken into account when torques are distributed but not in the design process.

B. Synthesis model

In order to use the design theorem proposed in [5], the model is first linearized and then discretized. Furthermore, for control purpose, it is assumed that $B^{[R]}(t)$ is periodic.

a) Linear state space: Under the assumption of small angles between F_B and F_R, the model is linearized around tr on which states of the obtained model are

$$(X)_{tr} = \left[\omega_{B/I}^T \; \epsilon_{B/R}^T \; h_s^T \right]_{tr}^T = \left[\omega_{R/I}^T \; \mathbf{0} \; (h_s)_{eq}^T \right]^T \tag{13}$$

where, dynamics of $\eta_{B/R}$ has been omitted, as it is usually done in the literature (see e.g. [1]). Indeed, when dealing with small angles, there is no risk of singularities and therefore η can always be recovered from the knowledge of ϵ using the norm constraint $q^T \cdot q = 1$.

Inputs expression on tr is established such that $(\dot{X})_{tr} = \mathbf{0}$. Then, using (1) and (8), one gets :

$$\left\{ \begin{array}{l} (T_s)_{tr} = \mathbf{0} \\ \left(B^{[R]}(t)\right)^{\times} (M^{[B]})_{tr} = T_{ext}^{[R]} - \omega_{R/I}^{\times} \left(J\omega_{R/I} + X_W \left(h_s\right)_{tr}\right) \end{array} \right. \tag{14}$$

Unfortunately, the condition on $\left(M^{[B]}\right)_{tr}$ is not verified in any case since the matrix $\left(B^{[R]}(t)\right)^{\times}$ is singular. It means that there is no explicit expression of $(M^{[B]})_{tr}$ and $M^{[B]} = \delta M + (M^{[B]})_{tr}$ cannot be computed. One way to overcome this problem is to add artificially a constant external torque, T_a, to the non-linear model, in order that $(M^{[B]})_{tr} = \mathbf{0}$ and that tr becomes an equilibrium. From (14), expression of T_a in F_B is obviously $T_{ext}^{[R]} - \omega_{R/I}^{\times} \left(J\omega_{R/I} + X_W h_{seq}\right)$. Addition of T_a, makes the linearization trajectory tr an equilibrium for zero inputs but does not, by itself, solve the control problem since a controller is still necessary to reach this trajectory. Unacceptable from the practical point of view, this artificial torque T_a will be treated later on as a virtual constant perturbation.

By means of (11), derivation of (8), (9) and (10) leads to the following linear state space model :

$$\delta\dot{X} = A(t)\delta X + E(t) \left[\begin{array}{c} \delta T_w \\ \delta M \end{array} \right] \tag{15}$$

with

$$A(t) = \left[\begin{array}{ccc} J^{-1}R & 2J^{-1}\left(T_{ext}^{[R]}(t)\right)^{\times} & -J^{-1}\omega_{R/I}^{\times}X_W \\ \frac{1}{2}\cdot\mathbf{1} & -\omega_{R/I}^{\times} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{array} \right] \tag{16}$$

$$R = \left(J\cdot\omega_{R/I} + X_W\cdot h_{seq}\right)^{\times} - \omega_{R/I}^{\times}\cdot J \tag{17}$$

$$E(t) = \left[\begin{array}{cc} -J^{-1}X_W & -J^{-1}\left(B^{[R]}(t)\right)^{\times} \\ \mathbf{0} & \mathbf{0} \\ \mathbf{1} & \mathbf{0} \end{array} \right] \tag{18}$$

This formulation encompasses those proposed in [10], [14] and [17] established in the particular case of a geocentric pointing taking into account the gravity gradient torque.

b) Model discretization: Adding a zero-order hold, of period T_s, at the input of a continuous-time periodic model $\{A_c(t), E_c(t), C_c(t), F_c(t)\}$ of period $T = NT_s$, leads to the following discrete-time model :

$$\left\{ \begin{array}{l} x(t_k + T_s) = A_k x(t_k) + E_k u(t_k) \\ y(t_k) = C_k x(t_k) + F_k u(t_k) \end{array} \right. \tag{19}$$

where state-space matrices are N-periodic (i.e. $(A, E, C, F)_{k+N} = (A, E, C, F)_k$). The relationship between these two models stems from analytical integration of continuous-time state-space considering $u(t)$ to be frozen for a duration T_s :

$$\begin{array}{cc} A_k = \Phi\left(t_k + T_s, t_k\right) & B_k = \int_{t_k}^{t_k+T_s} \Phi\left(t_k + T_s, \tau\right) B_c(\tau)\mathrm{d}\tau \\ C_k = C_c(t_k) & D_k = D_c(t_k) \end{array} \tag{20}$$

where $\Phi\left(t, t_0\right)$ refers to the state-transition matrix, solution of $\dot{\Phi}\left(t, t_0\right) = A(t)\Phi\left(t, t_0\right)$.

Based on Nyquist-Shannon sampling theorem the choice of the sampling period for time-invariant models discretized with zero-order hold is a well understood problem. However, the case of time-varying models is more tricky. Indeed, the reliability of this choice depends on the quality of the solving of (20) on the first hand, and, as for every linearized model, on how close stays the resulting linear model to the linearization trajectory, on the other hand. This requirement is particularly critical because the expression of the field $B^{[R]}(t)$ has been calculated analytically *for this trajectory* tr. For the closed-loop system, divergence from tr can be due, not only to the effect of disturbances but also to the fact that, $u(t)$ remains constant between t_k and t_{k+1}, which is the assumption leading to (20). The choice of T_s is discussed in section IV.

C. Controller synthesis

Let the linear uncertain discrete-time time-varying system $\Sigma(\lambda)$ defined by the following state-space realization :

$$\left[\begin{array}{c} x_{k+1} \\ z_k \end{array}\right] = \underbrace{\left[\begin{array}{ccc} A_k(\lambda) & B_k(\lambda) & E_k(\lambda) \\ C_k(\lambda) & D_k(\lambda) & F_k(\lambda) \end{array}\right]}_{M_k(\lambda)} \left[\begin{array}{c} x_k \\ w_k \\ u_k \end{array}\right] \tag{21}$$

where $x_k \in \mathbb{R}^n$ is the state vector, $u_k \in \mathbb{R}^m$ is the control vector, $w_k \in \mathbb{R}^{m_w}$ is the disturbance vector, $z_k \in \mathbb{R}^{p_z}$ is the controlled output vector and $\lambda \in \Lambda$ is a vector of parametric uncertainties. For each k, the parameter-dependent system matrix $M_k(\lambda)$ belongs to the convex polytope $\mathcal{M}_k = \mathrm{co}\left\{M_k^{[1]}, \cdots, M_k^{[L]}\right\}$ where

$$M_k^{[i]} = \left[\begin{array}{ccc} A_k^{[i]} & B_k^{[i]} & E_k^{[i]} \\ C_k^{[i]} & D_k^{[i]} & F_k^{[i]} \end{array}\right] \tag{22}$$

The sequence of polytopes $\{\mathcal{M}_k\}_{k\in\mathbb{N}}$ is assumed to be N-periodic.

In [5], the subsequent Theorem 3.1 is established. It offers a way to compute a periodic state-feedback control law which robustly stabilizes $\Sigma(\lambda)$ and minimizes the worst-case H_2 norm of the uncertain closed-loop system.

Theorem 3.1: Let Υ be the arbitrary couple of sequences $(\left\{A_k^0\right\}_{k\in\mathbb{N}}, \left\{C_k^0\right\}_{k\in\mathbb{N}})$ whose elements belong to $\mathbb{R}^{n\times n}$ and $\mathbb{R}^{p_z\times n}$ respectively, and define the following optimization problem :

$$\chi_{sub}(\Upsilon) = \min_{X_k^{[i]}\in\mathbb{S}^n, Z_k\in\mathbb{S}^{p_z}, G_k\in\mathbb{R}^{n\times n}, Y_k\in\mathbb{R}^{m\times n}} \left(\frac{1}{N}\mathrm{Trace}\sum_{k=1}^{N} Z_k\right) \tag{23}$$

constrained by the Υ dependent LMIs ($k \in \{1\cdots N\}$, $i \in \{1\cdots L\}$)

$$\begin{array}{l} \left[\begin{array}{cc} -X_{k+1}^{[i]} + B_k^{[i]} B_k^{[i]\,T} & \mathbf{0} \\ \mathbf{0} & X_k^{[i]} \end{array}\right] \\ +\mathrm{He}\left\{\left[\begin{array}{c} -A_k^{[i]}G_k - E_k^{[i]}Y_k \\ G_k \end{array}\right]\left[\begin{array}{cc} A_k^{0\,T} & -\mathbf{1} \end{array}\right]\right\} \prec 0 \end{array} \tag{24}$$

$$\begin{array}{l} \left[\begin{array}{cc} -Z_k + D_k^{[i]} D_k^{[i]\,T} & \mathbf{0} \\ \mathbf{0} & X_k^{[i]} \end{array}\right] \\ +\mathrm{He}\left\{\left[\begin{array}{c} -C_k^{[i]}G_k - F_k^{[i]}Y_k \\ G_k \end{array}\right]\left[\begin{array}{cc} C_k^{0\,T} & -\mathbf{1} \end{array}\right]\right\} \prec 0 \end{array} \tag{25}$$

$$X_k^{[i]} \succ 0 \ , \ X_{N+1}^{[i]} = X_1^{[i]} \tag{26}$$

then the N-periodic controller $u_k = K_k(\Upsilon)x_k$ defined by $K_k(\Upsilon) = Y_k G_k^{-1}$ for $k \in \{1\cdots N\}$ is such that $\chi_{sub}(\Upsilon)$ is a squared guaranteed H_2 cost for the uncertain closed-loop system $\Sigma_{cl}(\lambda)$, i.e.

$$\chi_{sub}(\Upsilon) \geq \max_{\lambda\in\Lambda} \|\Sigma_{cl}(\lambda)\|_2^2 \tag{27}$$

It has been demonstrated in [5] that performance in the robust case is substantially improved by taking advantage of the degrees of freedom offered by the choice of Υ.

Note that in the nominal case, there is no need to deal neither with this sequence nor with the additional variables G_k. The best achievable result is provided in [6] (based on Theorem 1 in [5]) which is similar to the design theorem established in [18].

IV. Simulation results

A. Simulation set-up

Numerical data correspond to the micro-satellite Demeter designed by CNES, the French space agency [12]. The inertia matrix is composed of the following diagonal terms : 37.9, 23.1 and 28.7 (kg.m^2). The three reaction wheels are such that $X_W = \mathbf{1}$. In this example, we will assume that on the trajectory tr, their velocity is nominal and equal to 1400 (rpm). Finally, the circular orbit is characterized by an altitude of 660 (km), an inclination of 98.23° and a local time of ascending node of 22h15.

The controller is designed in the case of geocentric pointing and, as a first approach, without uncertainties. $T_{ext}^{[I]}(t)$, depicted by Fig. 2, is defined by (7) and $T_{0x} = T_{0y} = T_{0z} = 1.0\cdot 10^{-7}$ (N.m), $T_{1x} = T_{1y} = T_{1z} = 2.1\cdot 10^{-5}$ (N.m), $T_{2x} = T_{2y} = T_{2z} = 2.1\cdot 10^{-5}$ (N.m), $\varphi_{T1x} = -\varphi_{T2x} = \pi/4$, $\varphi_{T1y} = -\varphi_{T2y} = -\pi/4$, $\varphi_{T1z} = 0$ and $\varphi_{T2z} = \pi/2$.

Using the IGRF model, a time history of the geomagnetic field, expressed in the LVLH, along three orbits is shown in Fig. 2. If B_x and B_z have an almost periodic behavior, B_y is not as regular. Indeed, the x and z axis of the LVLH lie in the orbital plane while y axis is normal to it. Therefore, noticing that the orbit is almost polar, variations of B_x and B_z are due to orbital motion whereas B_y behavior is affected by Earth's spinning which is much slower.

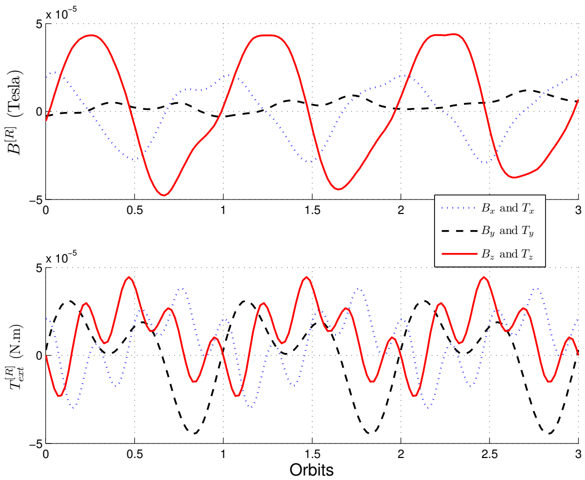

Fig. 2. Geomagnetic field and external disturbances in LVLH

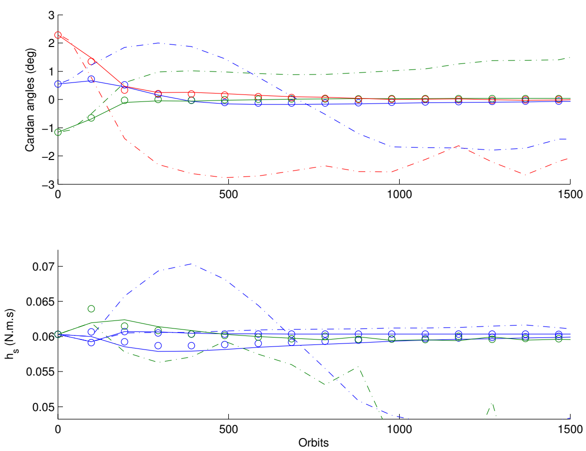

Fig. 3. Non-linear and linear models with periodic PD

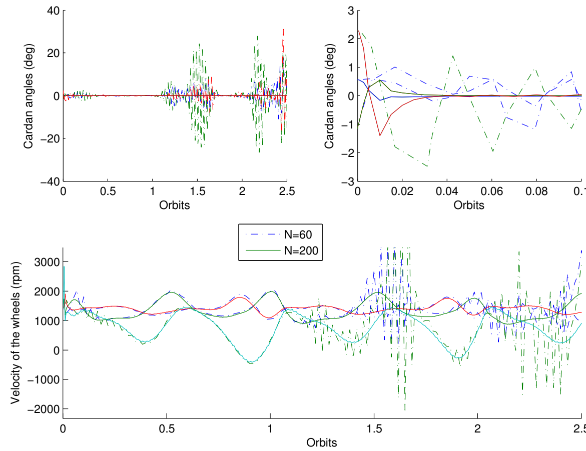

Fig. 4. Non-linear and linear models with periodic PID

B. Validation of the linear discrete-time model

As in every spacecraft attitude control problem, the open-loop model is not stable and, hence, simulations have to be performed on the closed-loop model. To this end, a first periodic controller is designed in the nominal case using theorem 3.1. With this controller, discrete-time linear and nonlinear models simulation results, displayed by Fig. 3, are almost similar. This validates the linearization and discretization stages. However, as previously explained, to have a fair comparison, the constant external torque T_a is artificially added when simulating the non-linear model.

C. Implementation of the control strategy

Unacceptable from the practical point of view, this artificial torque T_a can also be viewed as a virtual constant perturbation. Hence, as usually done to deal with such an issue, a digital integrator is added in the feedback loop after the measurement of $\epsilon_{B/R}$.

As already pointed out in [18], use of LMI techniques for H_2 synthesis with periodic models gives rise to heavy computations. Consequently, and as a first approach, only the nominal case is treated such that the number of variables can be lowered down to 6481 for $N = 60$ and to 21601 for $N = 200$. When considering uncertainties in the model, even if only 2 vertices are considered, if $N = 60$ (respectively $N = 200$) LMI solvers have to manipulate 27001 (90001) variables constrained by 2 set of 60 (200) LMIs of size 24×24 and 27×27. Nevertheless, exploitation of sparsity patterns and structure in those LMIs should allow to derive interesting results. This issue is under current investigation.

D. Simulation of the non linear closed-loop model

Fig. 4 depicts Euler angles of the closed-loop nonlinear model without T_a for $N = 60$ and $N = 200$. As previously mentioned, N appears to be an important tuning parameter since the model for $N = 200$ converges when the one for $N = 60$ does not. On this figure, checkered patterns are due to the zero-order hold.

As expected, comparing with [18] where magnetorquers solely are considered, adding reaction wheels improves significantly the performance of the control law. Moreover, Fig. 4 shows that velocity of the reaction wheels remains between -500 (rpm) and 2000 (rpm). Therefore, fine attitude regulation is achieved while at the same time avoiding divergence of the momentum of reaction wheels. This was the main requirement of the design and it would not have been possible without the use of both types of actuators. Thus, Fig. 5 shows that the control effort is shared between both actuators.

Whatever is the control law, as soon as the desirable steady-state attitude regulation is reached, control torques exactly compensate the perturbation torques. This is indeed what can be seen in Fig. 5 which compares the addition of every control torques, $T_m + T_w$, with the sum of the main disturbances, namely the external torques T_{ext} and gyroscopic torques due to reaction wheels spinning $T_g = \omega_{B/I}^{\times} X_W h_s$.

Fig. 6 gives information about the resultant control gains. It highlights how controllability issue is handled by the

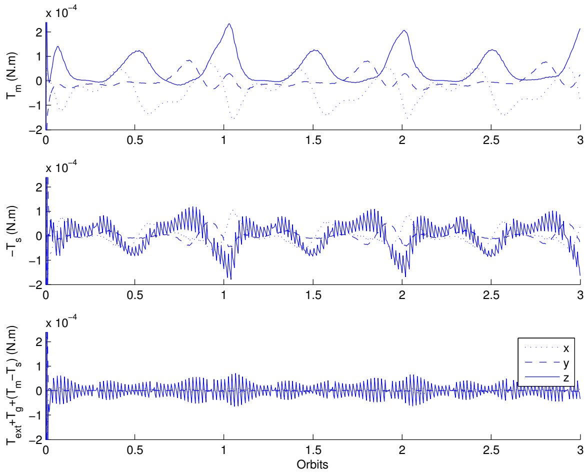

Fig. 5. Control input torques

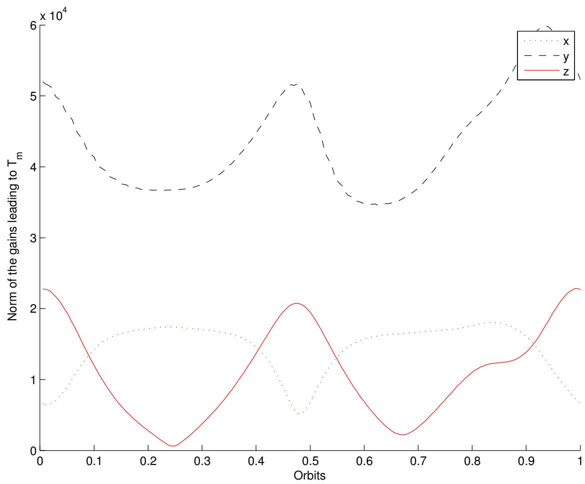

Fig. 6. Norm of coefficient gains submatrix leading to T_m

synthesis procedure. Indeed, norm of the submatrix of the controller leading to magnetic torque T_m increases when magnitude of magnetic field decreases. For instance, around the polar zones where B_x is almost zero, norm of the gains devoted to the computation of $(T_m)_x$ reaches its highest value.

V. Conclusions and Future Works

If attractive results have been recently proposed in the literature about joint use of magnetorquers and mechanical actuators, this paper demonstrates that in the particular case of reaction wheels, controlling both attitude and angular momentum of these actuators remains an open issue. As far as the modelling part is concerned, difficulties raised at the linearization and discretization stages are discussed. Mainly, it has been shown that the linearization trajectory tr is not an equilibrium by itself and the use of a zero-order hold may move the model away from this trajectory.

On the other hand, this paper constitutes the first step of the application of latest LMI synthesis techniques for robust static periodic controller on a complex example coming from the space industry. Indeed, promising simulation results have been obtained on the nominal case. Future works will be dedicated to the synthesis of attitude control law robust to parametric uncertainties, e.g. errors on the estimation of the geomagnetic field and perturbation torques. Considering results obtained in [5] on an academic example, significant improvements are expected as robust requirements are explicitly taken into account in the design theorem.

Furthermore, a new promising research axis has been initiated in [4] by proposing a design procedure leading to a special kind of dynamical state-feedback controller. Using this result on this specific demanding applicative set-up is currently under investigation.

References

[1] M. Corno and M. Lovera. Spacecraft attitude dynamics and control in the presence of large magnetic residuals. *Control Engineering Practice*, 17(4) :456 – 68, 2009.

[2] Michel Courtois. *Space Technology Course - spacecraft techniques and technology - Volume 3 : Platforms, Module XII : Attitude control and pointing*. 2005.

[3] Christopher J. Damaren. Hybrid magnetic attitude control gain selection. volume 2, pages 1340 – 1349, Hilton Head, SC, United states, 2007.

[4] Yoshio Ebihara, Dimitri Peaucelle, and Denis Arzelier. Periodically time-varying dynamical controller synthesis for polytopic-type uncertain discrete-time linear systems. pages 5438 – 5443, Cancun, Mexico, 2008.

[5] C. Farges, D. Peaucelle, D. Arzerlier, and J. Daafouz. Robust H_2 performance analysis and synthesis of linear polytopic discrete-time periodic systems via LMIs. *Systems & Control Letters*, 56(2) :159–166, February 2007.

[6] Christophe Farges. *Méthodes d'analyse et de synthèse robustes pour les systèmes linéaires périodiques*. PhD thesis, LAAS-CNRS and Université Paul Sabatier, 2006.

[7] James Richard Forbes and Christopher John Damaren. Geometric approach to spacecraft attitude control using magnetic and mechanical actuation. *Journal of Guidance, Control, and Dynamics*, 33(2) :590 – 595, 2010.

[8] Peter C. Hugues. *Spacecraft attitude dynamics*. Dover Publications, 1986.

[9] M. Lovera and A. Astolfi. Spacecraft attitude control using magnetic actuators. *Automatica*, 40(8) :1405 – 1414, 2004.

[10] Marco Lovera, Eliana De Marchi, and Sergio Bittanti. Periodic attitude control techniques for small satellites with magnetic actuators. *IEEE Transactions on Control Systems Technology*, 10(1) :90 – 95, 2002.

[11] Robert J. McElvain. Satellite angular momentum removal utilizing the earth's magnetic field. In *Torques and attitude sensing in Earth satellites*, pages 137–158. S.F. Singer, New York, 1964.

[12] C. Pittet and D. Arzelier. Demeter : a benchmark for robust analysis and control of the attitude of flexible microsatellites. In *ROCOND*, Toulouse, France, 2006.

[13] Tiziano Pulecchi and Marco Lovera. Attitude control of spacecraft with partially magnetic actuation. In *17th IFAC Symposium on Automatic Control in Aerospace*. Toulouse, France, 2007.

[14] Tiziano Pulecchi, Marco Lovera, and Andras Varga. Optimal discrete-time design of three-axis magnetic attitude control laws. *IEEE Transactions on Control Systems Technology*, 18(3) :714 – 722, 2010.

[15] Enrico Silani and Marco Lovera. Magnetic spacecraft attitude control : A survey and some new results. *Control Engineering Practice*, 13(3) :357 – 371, 2005.

[16] J.R. Wertz. *Spacecraft attitude determination and control*. Kluwer Academc Publishers, 1978.

[17] Rafael Wisniewski and Mogens Blanke. Three-axis attitude control based on magnetic torquing. *Automatica*, 35(7) :1201–1214, 1999.

[18] Rafael Wisniewski and Jakob Stoustrupy. Periodic H_2 synthesis for spacecraft attitude control with magnetometers. *Journal of Guidance, Control, and Dynamics*, 27(5) :874–881, 2004.