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Distributed Mobility and Power Control for Noncooperative Robotic Ad Hoc and Sensor Networks

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***Abstract*— In this paper we propose novel algorithms for noncooperative power and position control in mobile ad hoc networks. The algorithms are distributed and adaptive, *i.e.*, they are able to deal with the agents' lack of knowledge about the environmental conditions and about the actions, positions and properties of the other agents, which is the essential challenge in these networks. The agents' cost functions consist of a term proportional to the achievable rate of communication with the neighbors, explicitly depending on the interference from the other agents, and a pricing term penalizing excessive power (for the power control scheme) or deviation from predefined positions (for the position control scheme). We formulate conditions for the existence and uniqueness of the Nash equilibrium and prove that the algorithms converge to it almost surely, based only on local measurements and local signaling between the neighbors. The position control algorithm can be adopted to specific motion dynamics of the networked mobile robots. We illustrate the main properties of the algorithms through simulations.**

I. INTRODUCTION

Wireless ad hoc networks, including robotic sensor networks, have recently attracted much attention among researchers and practitioners (see, *e.g.*, [1]–[10]). In general, the main challenge in these multi-agent networked systems is how to deal with the lack of infrastructure, or a central/fusion node, since the information that agents have about the environment as well as about the actions/properties of the other agents is limited only to certain local measurements and local communications.

There has been a large surge of interest in treating these systems in the game theoretic framework, which is a natural approach to *cooperative control* problems (*e.g.*, [6], [11], [12]), and has been shown to be effective for dealing with *resource allocation* problems in networking (*e.g.*, [13], [14]). Specifically, the effectiveness of game theoretic approach to *power control* in wireless networks has been shown in the existing literature (*e.g.*, [14]–[18] and references therein). However, none of the existing approaches are able to cope with the problems specific to ad hoc networks originating from the lack of detailed/global information.

There are only very few results so far in exploiting possibility of *controlling mobility* of the agents in the network in order to enhance the overall communication capabilities. In [19] it has been demonstrated how a mobile robot, communicating with a base station, can exploit the effects of multi-path fading by performing small deviations from its predefined path. In [20] and references therein, some attempts are presented to include possibility of nodes' mobility and reconfigurability in order to improve energy efficiency of MAC protocol. *Connectivity control* in mobile sensor networks has been analyzed intensively in the existing literature from different points of view (*e.g.*, [10] and references therein). However, the existing approaches treat the peer-to-peer connectivity in a very simplistic way (proximity based) without explicitly taking into account limited communication resources.

In this paper we first propose a novel algorithm for *noncooperative power control* in ad hoc networks. The cost functions of the agents have a similar structure as in, *e.g.*, [15], consisting of a pricing function (penalizing excessive power levels) and a term proportional to the achievable rate (capacity) of the link. However, in our algorithm the agents do not need to have a detailed knowledge about the parameters of the cost functions (for calculating the local gradients) which makes it applicable to ad hoc networks. We formulate conditions for the existence and uniqueness of the Nash equilibrium and prove that the power levels converge to the equilibrium. The proposed algorithm is based on the recently proposed scheme for distributed seeking of Nash equilibria [6], [11].

Based on a similar idea, we then propose a new algorithm for *noncooperative position control* in robotic (mobile) sensor networks. Each cost function consists of the achievable rates of communications with the neighbors (explicitly depending on the positions of all the other agents due to interference), and a pricing function, now penalizing deviation from some predefined positions. The agents are able to locally control their mobility in order to reach positions minimizing their local costs. The most important property of the proposed algorithm is that it is based only on *locally available information* (measurements of the local cost functions), which is of essential importance due to the agents' lack of knowledge about the structural parameters of the costs as well as about their absolute or relative positions. Another important property of the algorithm is that it can be adopted to specific *motion dynamics* of the robots. We formulate conditions on the cost function parameters under which we prove existence and uniqueness of the Nash equilibrium. Under the same assumptions we prove that by using the proposed algorithm the agents converge to the Nash equilibrium positions almost surely. Simulation results are given which illustrate the main properties of the schemes.

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II. Noncooperative Power Control Algorithm

Consider an ad hoc wireless network which consists of N links between nodes/agents. Each agent communicates only with a subset of the other agents, called neighbors, so that there is no central node or base station which could communicate to all the agents. We assume that the physical layer communication is performed using the CDMA technology, so that for each link we can model the received signal to noise+interference ratio γ_i, $i \in \{1, ..., N\}$, in the following way [21]

$$\gamma_i(p) = \frac{L h_{ii} p_i}{\sigma^2 + \sum_{j \neq i} h_{ji} p_j}, \tag{1}$$

where L is the spreading gain, h_{ii} is the gain of the link i, h_{ji} is the gain of the link j which is interfering to link i and σ^2 is the noise power level, which for simplicity, we assume is constant for all links. The objective of each agent is to online adjust the transmit power level $p_i \in [p^i_{min}, p^i_{max}]$, where p^i_{min} is the minimal power needed for the link i to be established and p^i_{max} is the maximal power level allowed, such that the following local cost is minimized:

$$J_i(p) = P_i(p_i) - u_i \log(1 + \gamma_i(p)), \tag{2}$$

where $p = [p_1, ..., p_N]^T$, u_i is a positive parameter and $P_i(p_i)$ is a pricing function of link i. Thus, the second term can be interpreted as being proportional to the capacity of the wireless channel. As the cost function of each link depends on the power levels of all the other links we are dealing with a noncooperative static game, where the optimality can be characterized by the Nash equilibrium [22]. We assume that the channel gains h_{ij} are slowly changing in time (compared to the proposed power control algorithm's convergence rate) so that we can assume they are constant. The randomness of these parameters (due to, *e.g.*, fast fading effects) can be modeled as additive noise in the cost function measurements as discussed later. The pricing function $P_i(p_i)$ is a nondecreasing function, and it reflects the cost of transmitting with the power level p_i for the transmitter i. It can be interpreted as the cost of the battery usage for the link i. We have assumed that the links have already been established so that each agent have found the minimal power level p^i_{min} needed at least for exchanging signaling information. Obviously, the local pricing functions should be designed such that they do not penalize the agents when they are using minimal power levels.

In general, depending on the parameters of the cost functions and the chosen pricing functions the formulated game may admit multiple, unique or no Nash equilibria. Therefore, in order to ensure existence and uniqueness of the *inner* Nash equilibrium $p^* = [p^*_1, ..., p^*_N]^T$, $p^*_i \in (p^i_{min}, p^i_{max})$, we first need to introduce the following assumptions on the cost functions. For the existence of the Nash equilibrium the following is sufficient:

(A.1) The pricing functions $P_i(p_i)$ are smooth and convex in p_i, *i.e.* $\frac{\partial^2 P_i}{\partial p_i^2}(p_i) \geq 0$, for all p_i and for every $i = 1, ..., N$.

To ensure uniqueness of the Nash equilibrium, we will use the following assumption, easily satisfied for large enough spreading gain L:

(A.2) $L h_{ii} > \sum_{j \neq i} h_{ji}$, for all $i \in \{1, ..., N\}$.

Finally, we want to ensure that the Nash equilibrium is inner, *i.e.*, that $p^*_i \in (p^i_{min}, p^i_{max})$ for all i, which can be guaranteed with the following assumption (see also [18] where a similar assumption is introduced):

(A.3) The parameters of the cost function J_i are chosen such that $\frac{\partial J_i}{\partial p_i}(p) < 0$ for all such p that $p_i = p^i_{min}$, and $\frac{\partial J_i}{\partial p_i}(p) > 0$ for all such p that $p_i = p^i_{max}$.

We can now prove the following theorem:

Theorem 1: Let the Assumptions (A.1)–(A.3) be satisfied. Then the formulated power control game with the cost functions given in (2) admits a unique inner Nash equilibrium $p^* = [p^*_1, ..., p^*_N]^T$, $p^*_i \in (p^i_{min}, p^i_{max})$.

Proof: For partial derivatives of the cost functions (2) with respect to local actions p_i we obtain:

$$\frac{\partial J_i}{\partial p_i}(p) = \frac{\partial P_i}{\partial p_i}(p_i) - \frac{u_i L h_{ii}}{\sigma^2 + \sum_{j \neq i} h_{ji} p_j + L h_{ii} p_i}, \tag{3}$$

and for the second derivatives we obtain:

$$\frac{\partial^2 J_i}{\partial p_i^2}(p) = \frac{\partial^2 P_i}{\partial p_i^2}(p_i) + \frac{u_i L^2 h_{ii}^2}{(\sigma^2 + \sum_{j \neq i} h_{ji} p_j + L h_{ii} p_i)^2}, \tag{4}$$

$$\frac{\partial^2 J_i}{\partial p_i \partial p_j}(p) = \frac{u_i L h_{ii} h_{ji}}{(\sigma^2 + \sum_{k \neq i} h_{ki} p_k + L h_{ii} p_i)^2}. \tag{5}$$

Because of (A.1) we have that $\frac{\partial^2 J_i}{\partial p_i^2}(p) > 0$ for all p so that the cost functions J_i are strictly convex with respect to local actions p_i. Since the overall action space is closed, bounded and convex ($p_i \in [p^i_{min}, p^i_{max}]$ for all i) it follows that a Nash equilibrium p^* exists (see, *e.g.*, Theorem 4.3 in [22]). According to (A.3) it has to be inner.

To prove uniqueness, consider the Jacobian matrix of the vector $g(p) = [\frac{\partial J_1}{\partial p_1}(p), ..., \frac{\partial J_N}{\partial p_N}(p)]^T$ which is given by $G(p) = [\frac{\partial^2 J_i}{\partial p_i \partial p_j}]_{ij}$, $i, j = 1, ..., N$. From (4), (5), (A.1) and (A.3) it clearly follows that $G(p)$ is diagonally dominant with positive diagonal elements. Therefore, $G(p)$ is positive definite. According to the definition of the Nash equilibrium and (A.3), we have that $g(p^*) = 0$. According to the mean value theorem for vector functions, for every $p, h \in \mathcal{R}^N$ we have $g(p + h) - g(p) = M(p, h)h$, where $M(p, h) = \int_0^1 G(p + th)dt > 0$ (because $G(p + th)$ is positive definite for all t). By letting $p = p^*$, we have that for every $h \neq 0$, $g(p^* + h) = M(p^*, h)h$. Therefore, $g(p^* + h) \neq 0$ for all $h \neq 0$ which means that there is no point except p^* for which $g(p) = 0$. This proves the theorem. ■

From (3), it is clear that the assumptions (A.1) and (A.3) can be guaranteed with the simple quadratic pricing functions $P_i(p_i) = b_i(p_i - p^i_{min})^2$, with b_i large enough, depending on p^i_{max} (see also Example 1 below). Assumption (A.2) imposes that the spreading gain should be large enough compared to the number of links in the network (since typically $h_{ii} \geq h_{ji}$, $j \neq i$), which is typically satisfied in real life ad hoc networks [21].

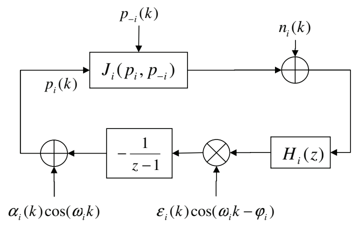

Fig. 1. Noncoopearative power control algorithm

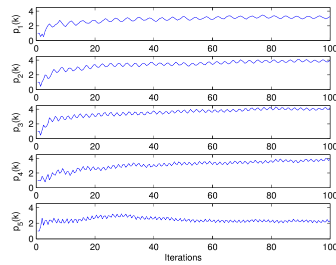

Fig. 2. Power levels of all 5 links

Under the formulated conditions, we want to find an algorithm which is based only on locally available information, that will drive the transmit power levels of all the agents to the Nash equilibrium. It is reasonable to assume that each agent has information about its current achievable rates of transmissions, or current signal to noise ratios (1) (using low bandwidth signaling feedback from the receiving node). Therefore, the agents can only obtain the current values of their local costs (2). Having in mind this information structure, we are going to apply the recently proposed algorithm in [6], [11] for distributed seeking of Nash equilibria. It can be directly applied to our problem as illustrated in Fig. 1. Each agent implements a local discrete-time extremum seeking algorithm using sinusoidal perturbations with vanishing amplitudes [5]. The "measurements" of the cost functions are corrupted with noise $n_i(k)$ which account for the uncertainties in the currently obtained rate due to, *e.g.*, fast fading effects and other unreliabilities. The parameters of the scheme should be chosen in the following way [11]:

(A.4) $\varepsilon_i(k) = e_i k^{-m_\varepsilon}$, $\alpha_i(k) = a_i k^{-m_\alpha}$ where $0.5 < m_\varepsilon < 1$, $0 < m_\alpha < 0.5$, $m_\varepsilon + m_\alpha \leq 1$, $e_i, a_i > 0$.

(A.5) $\omega_i \in (0, \pi)$ and $\omega_i \neq \omega_j$ for all $i, j = 1, ..., N$.

(A.6) $\varphi_i + \mathrm{Arg}\left\{H_i(e^{j\omega_i})\right\} = 0$ for all $i = 1, ..., N$.

We can now formulate a convergence theorem:

Theorem 2: Consider the noncooperative power control algorithm shown in Fig. 1. Let the Assumptions (A.1)–(A.5) be satisfied. Then the power levels $p(k) = [p_1(k), \ldots,\ p_N(k)]^T$ of all the links converge to the Nash equilibrium level p^* almost surely.

Proof: We have already proved the existence and uniqueness of the Nash equilibrium (Theorem 1). Therefore, the only condition from Theorem 1 in [11] that is left to prove is the stability condition (A.12 in [11]), *i.e.*, we need to show that there exists a Lyapunov function $V(p)$ such that $V(p^*) = 0$ and $\phi(k) = g^T(p)K^T\nabla_p V(p) > 0$, for all $p \neq p^*$, where $g(p) = [\frac{\partial J_1}{\partial p_1}(p), ..., \frac{\partial J_N}{\partial p_N}(p)]^T$, $K = \mathrm{diag}\{k_1, ..., k_N\}$, $k_i = e_i a_i H_i(1) > 0$, and $\nabla_p V(p)$ denotes the gradient of $V(\cdot)$ in p.

Consider the following quadratic Lyapunov function candidate $V(p) = \frac{1}{2}(p-p^*)^T(p-p^*)$. Obviously $V(p) = 0$ if and only if $p = p^*$. From the proof of Theorem 1, the Jacobian of $g(p)$ is positive definite and diagonally dominant for all p, so that the Jacobian of $g_1(p) = Kg(p)$ is also positive definite (since K is diagonal and positive definite). By using the mean value theorem for vector functions we have that for every $p \neq p^*$ it holds that $g_1(p) = M_1(p^*, p)(p - p^*)$ where $M_1(p^*, p) > 0$. Therefore, $\phi(k) = g_1(p)^T\nabla_p V(p) = (p-p^*)^T M(p^*, p)(p-p^*) > 0$ for every $p \neq p^*$ which proves the theorem.

■

Example 1: We illustrate the above power control algorithm in a simulation of a network of 5 links. Each agent controls the power for only one link; agents 1 and 2 transmit to each other, agent 3 to agent 2, agent 4 to agent 5, and agent 5 to agent 2. The channel gains h_{ij} are assumed to be inversely proportional to the distance between agents i and j, and we assume that the agents are stationary and positioned at the following locations: $r_1 = (-4, 0)$, $r_2 = (-3, 1)$, $r_3 = (-3, -1)$, $r_4 = (-1, 1)$ and $r_5 = (0, -1)$, where r_i is the position of agent i. For the rest of the parameters of the cost functions (2) we choose $L = 16$, $\sigma^2 = 1$, $P_i(p_i) = b_i(p_i - p^i_{min})^2$, $b_i = 1$, $p^i_{min} = 0.5$, $u_i = 40$ for all $i = 1, ..., 5$. We choose the following parameters of the proposed algorithm: $\varphi_i = \pi/8$, $H_i(z) = \frac{z-1}{z+0.07}$ (washout filters), $\varepsilon_i(k) = 0.1k^{-0.6} + 0.01$, $\alpha_i(k) = 0.6k^{-0.25} + 0.01$, for $i = 1, ..., 5$ (in order to improve convergence rate, we have set small lower bounds 0.01 for the gains $\alpha_i(k)$ and $\varepsilon_i(k)$), $\omega_1 = 0.26\pi$, $\omega_2 = 0.36\pi$, $\omega_3 = 0.48\pi$, $\omega_4 = 0.58\pi$, $\omega_5 = 0.7\pi$ and the "measurement" noise variance is $\mathrm{var}\{n_i(k)\} = 2$, for all i and k. It easy to check that all the assumptions (A.1)-(A.6) are satisfied. We choose the same initial conditions for all the agents, $p_i(1) = 1$, $i = 1, ..., 5$. The power levels for all the agents are shown in Fig. 2, as functions of the number of iterations k. Since all the agents have the same pricing function, it can be seen that the agent 5, which has the highest interference, will actually transmit at the lowest power level at the equilibrium, thus achieving the worst signal to interference+noise ratio. This is because the other links have taken more "resources" so that it is too costly for him to transmit with higher power. However, if we decrease the slope of its pricing function or increase u_i this agent will transmit at the higher power level, and, hence, achieve better signal to noise ratio.

III. Noncooperative Position Control Algorithm

We consider a CDMA wireless network as in the previous section, but now the players/agents are mobile robots that can *control their positions* such that their local costs are noncooperatively optimized. By T_i we denote the subset of agents to which the agent i is transmitting, and by R_i the subset of agents from which the agent i is receiving messages. In this case, the position of an agent influences not only the quality of the transmitted communications (by that agent), but also the receiving ones (unlike the power control scenario). Therefore, we define the local cost functions in the following way:

$$\begin{aligned} J_i(p) &= P_i(x_i, y_i) - U_i(x_i, y_i, x_{-i}, y_{-i}) = \\ &= P_i(x_i, y_i) - \sum_{j \in T_i} u^t_{ij} \log(1 + \gamma_{ij}(p)) \\ &\quad - \sum_{j \in R_i} u^r_{ij} \log(1 + \gamma_{ji}(p)), \end{aligned} \tag{6}$$

where

$$\gamma_{ij}(p) = \frac{L h_{ij}(x_i, y_i, x_j, y_j) p_{ij}}{\sigma^2 + \sum_{k \neq i} h_{kj} p_k}, \tag{7}$$

is the signal to interference+noise ratio of ij link, L is the spreading gain, $[x_i, y_i]^T = r_i$ is the position of agent i, x_{-i}, y_{-i} are the coordinates of all the other agents, h_{ij} is the link gain from agent i to agent j, $i \neq j$, p_{ij} is the transmission power from agent i to agent j, $p_k = \sum_{i \in T_k} p_{ik}$ is the overall transmission power of agent k, and σ^2 is the noise power level. Unlike the power control game, in this case, the agents' actions are their positions in the plain and the action spaces are unbounded and two dimensional. The pricing functions $P_i(x_i, y_i)$ can be interpreted as the costs of moving away from some predefined point which can be related to some primary mission of the network. In the case of sensor networks, it could penalize the agent if it moves away from the optimal sensing point (see [2], [5]–[7] where the problems of optimal positioning of mobile sensors were treated in details). For robotic ad hoc networks the pricing function can characterize the cost for the battery consumption for actuating the robot (for moving away from initial condition), or it can characterize the approximate regions that each robot should cover in order to ensure overall network connectivity (see Example 2 below).

First, let us formulate sufficient conditions for the existence and uniqueness of the Nash equilibrium for the formulated game. For clarity of presentation, we assume that $u^r_{ij} = 0$ for all (i, j), *i.e.*, that the agents are locally interested only in improving transmitted communication. Similar conditions can be obtained for general gains u^r_{ij}. For the existence of a Nash equilibrium we introduce the following assumptions:

$$\text{(B.1)} \ \frac{\partial^2 P_i}{\partial \chi_i^2} > 2 \sum_{j \in T_i} \frac{u^t_{ij}}{h_{ij}(\cdot)} \left(\frac{\partial^2 h_{ij}}{\partial \chi_i^2} - \frac{1}{(\lambda + 1) h_{ij}(\cdot)} (\frac{\partial h_{ij}}{\partial \chi_i})^2 \right) \tag{8}$$

for all $r = [r_1^T, ..., r_N^T]^T \in \mathcal{R}^{2N}$, where χ_i is either x_i or y_i and $\lambda = 1/\gamma_{min}$ is the maximal interference+noise to signal ratio (γ_{min} is the minimal signal to interference+noise ratio) in the whole network for all the links. Normally, this parameter is small, certainly less than 1.

(B.2) The Hessian of $P_i(r_i)$ is diagonal.

We assume that the dependence of a link gain h_{ij} on the distance between the nodes i and j is given by the following form

$$h_{ij}(r_i, r_j) = \frac{h^0_{ij}}{((x_i - x_j)^2 + (y_i - y_j)^2)^{n_{ij}/2} + h^0_{ij}}, \tag{9}$$

where n_{ij} depends on the environment in which the radio waves propagate (in open air we have $n_{ij} = 2$), and where we have normalized the function such that it is equal to 1 for $r_i = r_j$. For this function it can be shown that there is a region in $(x_i - x_j, y_i - y_j)$ plane in which the sum terms on the right hand side of (B.1) can be positive (in this region $h_{ij}(r_i, r_j)$ is in the medium range). The value of these terms in this region is obviously larger for smaller γ_{min} (or larger λ) or larger u_{ij}-s. Therefore, besides being strictly convex, the pricing function $P_i(r_i)$ needs to have large enough second derivative in this region so that the inequality (B.1) holds. This can easily be satisfied with simple quadratic function $P_i(x_i, y_i) = b^x_i (x_i - x^0_i) + b^y_i (y_i - x^0_y)$ with large enough b^x_i and b^y_i, depending on x^0_i and y^0_i.

Due to highly nonlinear dependence given in (9), for clarity of presentation, we introduce a general condition ensuring uniqueness of the Nash equilibrium:

(B.3) The Jacobian of the vector

$$g(r) = [\nabla_1 J_1(r)^T, ..., \nabla_N J_N(r)^T]^T, \tag{10}$$

where ∇_i denotes the gradient with respect to r_i, is diagonally dominant.

It will be evident after the proof of the next theorem that condition (B.3) can easily be satisfied if the number of "interfering" agents is not large compared to the spreading gain L (similarly as in (A.2)).

Theorem 3: Let the Assumptions (B.1)–(B.3) be satisfied. Then the formulated position control game with the cost functions given in (6) admits a unique Nash equilibrium $r^* = [r_1^{*T}, ..., r_N^{*T}]^T$.

Proof: The proof will be based on similar arguments as in the proof of Theorem 1. For the gradient of a cost function with respect to local position we obtain

$$\nabla_i J_i(r) = [\frac{\partial J_i}{\partial x_i}(r), \frac{\partial J_i}{\partial y_i}(r)]^T, \tag{11}$$

where the partial derivatives are given by

$$\frac{\partial J_i}{\partial x_i}(r) = \frac{\partial P_i}{\partial x_i}(r) - \sum_{j \in T_i} \frac{u^t_{ij} L p_{ij}}{\sigma^2 + \sum_{k \neq i} h_{kj} p_k + L h_{ij} p_{ij}} \frac{\partial h_{ij}}{\partial x_i}(r)$$

and we have a similar form for the partial derivative with respect to y_i. Furthermore, for the diagonal second partial derivatives we obtain

$$\frac{\partial^2 J_i}{\partial x_i^2}(r) =$$
$$\frac{\partial^2 P_i}{\partial x_i^2}(r) + \sum_{j\in T_i} u_{ij}^t [\frac{Lp_{ij}}{\sigma^2 + \sum_{k\neq i} h_{kj}p_k + Lh_{ij}p_{ij}} \frac{\partial^2 h_{ij}}{\partial x_i^2}(r)$$
$$- \frac{L^2 p_{ij}^2}{(\sigma^2 + \sum_{k\neq i} h_{kj}p_k + Lh_{ij}p_{ij})^2} \left(\frac{\partial h_{ij}}{\partial x_i}(r)\right)^2], \qquad (12)$$

with a similar formula for $\frac{\partial^2 J_i}{\partial y_i^2}$. From (12) and (B.1) it is evident that $\frac{\partial^2 J_i}{\partial x_i^2}(r) > 0$ and $\frac{\partial^2 J_i}{\partial y_i^2}(r) > 0$. By doing similar calculations, it can be shown that $\left|\frac{\partial^2 J_i}{\partial x_i \partial y_i}(r)\right| \leq \left|\frac{\partial^2 J_i}{\partial x_i^2}(r)\right|$ so that by using (B.1) and (B.2) we conclude that the Jacobian of the local gradients (11) is positive definite. Therefore, the cost functions J_i are strictly convex with respect to the local actions r_i. Furthermore, from (B.1), (B.2) and the logarithmic dependence of U_i in the cost function (6), we conclude that the cost functions are radially unbounded in local decisions. Therefore, we can use standard results in game theory (*e.g.*, Corollary 4.2 in [22]) and conclude that there exists a Nash equilibrium of the underlying game.

Uniqueness follows from (B.3) using similar arguments as in the proof of Theorem 1. ■

To show that Assumption (B.3) is not restrictive, for $k \notin T_i$, we have:

$$\frac{\partial^2 J_i}{\partial x_i \partial x_k}(r) =$$
$$\sum_{j\in T_i} \frac{u_{ij}^t L p_{ij} p_k}{(\sigma^2 + \sum_{l\neq i} h_{lj}p_l + Lh_{ij}p_{ij})^2} \frac{\partial h_{ij}}{\partial x_i} \frac{\partial h_{kj}}{\partial x_k}, \qquad (13)$$

which is small for large L compared to the parameters u_{ij}^t, so that (B.3) is easily satisfied if the number of "interfering" agents is reasonably small compared to the spreading gain L.

Let us now consider the problem of the agents' positioning to the equilibrium point. Similarly as in the power control game, it is impossible for the agents to know the exact values of all the parameters in the cost functions as well as the relative or absolute positions, which would enable them to use a gradient decent or best response strategies [22]. However, the agents are able to access the values of their local costs (6) at their current positions, since they can obtain current achievable rates, or signal to noise ratios using feedback from the receiver. Therefore, we again propose to use the Nash equilibrium seeking scheme developed in [11], adapted to given particular motion dynamics of the mobile robots. Assuming single integrator (velocity actuated) dynamics, we propose the algorithm depicted in Fig. 4 (see [6], [7], [11] for similar schemes involving double integrator or unicycle robots' dynamics). Since now we have two dimensional action spaces (unlike the power control game) the agents implement orthogonal sinusoidal perturbations with vanishing gains $\hat{s}_i^x(k) = \alpha_i(k)\cos(\omega_i k) - \alpha_i(k-1)\cos(\omega_i(k-1))$ and $\hat{s}_i^y(k) = \alpha_i(k)\sin(\omega_i k) - \alpha_i(k-1)\sin(\omega_i(k-1))$ which are differentiated since they are moved in front of the integrators

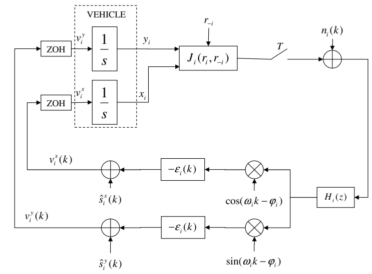

Fig. 3. Noncoopearative position control algorithm for velocity actuated vehicles

appearing in the vehicles' dynamics in this case. Since we are implementing the algorithm in discrete time and the vehicle dynamics are in continuous time, we introduced sampling with the period T (large enough so that the value of the cost can be obtained) with the zero-order-hold (ZOH) blocks at the input.The parameters of the scheme should be chosen as specified in (A.4)–(A.6) [11].

The conditions (B.1)–(B.3) are sufficient, together with (A.4)–(A.6) for the stability of our algorithm. Note that when distance between the agents goes to infinity condition (B.1) is not needed, so that we shouldn't expect instabilities even if $\frac{\partial^2 P_i}{\partial \chi_i^2}(r)$ is not large enough to ensure (B.1) for all r. However, if there is a region in which (B.1) is not satisfied, the Nash equilibrium might not exist, and we may have cyclic behavior of our algorithm in this region.

Now we can state the main convergence theorem:

Theorem 4: Consider the scheme in Fig. 4 where the cost functions $J_i(r_i, r_{-i})$ are given in (6). Let Assumptions (B.1)–(B.3) and (A.4)–(A.6) be satisfied. Then the positions of the agents converge to the Nash equilibrium positions almost surely.

Proof: Similarly as in the proof of Theorem 2, according to [11] and having in mind (B.1)–(B.3) and (A.4)–(A.6), all we need to show is that there exists a Lyapunov function $V(r)$ such that $V(r^*) = 0$ and $\psi(r) = g^T(r)K^T\nabla_r V(r) > 0$, for all $r \neq r^*$, where $g(r) = [\nabla_{r_1} J_1(r)^T, ..., \nabla_{r_N} J_N(r)^T]^T$, $K = I_2 \otimes \mathrm{diag}\{k_1, ..., k_N\}$, $k_i = e_i a_i H_i(1) > 0$. Because of the diagonal dominance and positive definiteness of the Jacobian of $g(r)$ (see proof of Theorem 3) we can choose quadratic Lyapunov function $V(r) = \frac{1}{2}(r - r^*)^T(r - r^*)$ and similarly as in Theorem 2 show that $\psi(r) > 0$, for all $r \neq r^*$ proving the theorem. ■

Remark 1: In both the power and the position control schemes we have adopted cost functions proportional to the channel capacity and with the additive pricing terms $P_i(\cdot)$. However, it is possible to introduce pricing effects in different, nonadditive manners (such as in, *e.g.*, [16] for power control), which could perhaps lead to simpler stability conditions, especially for the position control scheme because of the highly nonlinear functions $h_{ij}(\cdot)$.

Example 2: In this example we show simulation results for the position control algorithm in Fig. 4, for a network of 5 velocity actuated mobile robots having the cost functions

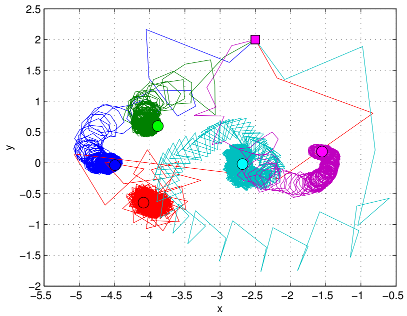

Fig. 4. Trajectories of the vehicles

(6) with $h_{ij}(\cdot)$-s given in (9) with the following parameters: $P_i(x_i, y_i) = b_i^x(x_i - x_i^0) + b_i^y(y_i - x_y^0)$, $b_i^x = b_i^y = 5$ for all $i \in \{1, ..., 5\}$, $r_1^0 = (x_1^0, y_1^0) = (-5, 0)$, $r_2^0 = (-4, 1)$, $r_3^0 = (-4, -1)$, $r_4^0 = (-3, 0)$, $r_5^0 = (-1, 0)$, $T_1 = R_1 = \{2, 3\}$, $T_2 = R_2 = \{1, 4\}$, $T_3 = R_3 = \{1, 4\}$, $T_4 = R_4 = \{5\}$, $T_5 = R_5 = \{4\}$, $p_{ij} = 1$, $h_{ij}^0 = 1$ for all $i, j \in \{1, ..., 5\}$, $L = 32$ and $\sigma^2 = 1$. Similarly to Example 1, for the parameters of the algorithm we set: $\varphi_i = \pi/8$, $T = 1$, $H_i(z) = \frac{z-1}{z+0.07}$ (washout filters), $\varepsilon_i(k) = 0.15k^{-0.6} + 0.002$, $\alpha_i(k) = 0.5k^{-0.25} + 0.03$ for $i = 1, ..., 5$, $\omega_1 = 0.26\pi$, $\omega_2 = 0.36\pi$, $\omega_3 = 0.58\pi$, $\omega_4 = 0.66\pi$, $\omega_5 = 0.16\pi$ and the "measurement" noise variance is $\text{var}\{n_i(k)\} = 1$, for all i and k. It easy to check that all the assumptions (B.1)–(B.3) and (A.4)–(A.6) are satisfied. In the context of robotic ad hoc networks, these cost functions can be interpreted as a way of establishing a communication bridge between the agents 1 and 5 (or between the regions having the centers in the minima of their pricing functions). In the sensor network context, pricing functions $P_i(r_i)$ may represent criteria describing the quality of local sensing which is the best at its minimum.

To illustrate the dependence among the agents through interference, we have set $T_4 = R_4 = \{5\}$, $T_5 = R_5 = \{4\}$ so that the cost functions of agents 4 and 5 depend only on their mutual communication rates, and the other agents can influence their positions only by interfering with their communication. First we set $u_{ij}^t = u_{ij}^r = 3$ for all the neighboring pairs (i, j) so that the importance of the communication rates in the agents' costs are not significant. All the agents are initially at the same point $(-2.5, 2)$. In Fig. 5 trajectories of the vehicles are shown where it can be seen that, at the Nash equilibrium, the deviations from the goals characterized by the pricing functions P_i are not significant, but the agents still tend to get closer to each other. The final destinations (Nash equilibrium) are marked with the circles. Next, if we set $u_{12}^r = u_{12}^t = u_{13}^r = u_{13}^r = u_{24}^t = u_{24}^r = u_{34}^t = u_{34}^r = 8$ and keep the other parameters the same, so that the importance of the achievable communication rates for agents 1,2 and 3 are higher, we obtain that the agents 1,2 and 3 move much closer to each other and to agent 4, deviating more from "local goals". However, the agents 4 and 5 would move away from each other due to the higher interference of the other agents in their mutual communications. Now it is more costly for them to move closer to each other than in the previous case. This is the expected effect because of the noncooperative nature of the algorithm.

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