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Stability and stabilization of positive Takagi-Sugeno fuzzy continuous systems with delay

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Abstract— **This paper deals with the problem of stability and stabilization of Takagi-Sugeno (T-S) fuzzy systems with a fixed delay by linear programming (LP) while imposing positivity in closed-loop. The stabilization conditions are derived using the single Lyapunov-Krasovskii Functional (LKF). An example of a real plant is studied to show the advantages of the design procedure.**

Key-words: T-S fuzzy systems, positive systems, Lyapunov-Krasovskii functional, stabilization, Linear programming.

I. Introduction

The problem concerns a special class of nonlinear systems called Takagi-Sugeno models (T-S) [7]. From the history of the approach, this class can be interpreted as a collection of linear models interconnected by nonlinear functions, called membership functions, which are dependent variables. The most delicate problem is the choice of premise variables that partition the space [6], [8].

Positive systems have been of great interest to researchers in recent years [9], [1], [4], [5] and [10]. The class of positive T-S fuzzy systems was considered for the first time in [2]. The obtained results were presented using LMIs.

In this paper, the conditions of stability and stabilization of such systems are studied by using linear programming (LP). An application on the model of a real process is considered. A comparison of the obtained results with those of [3] is proposed. The rest of this paper is organized as follows: In section 2, the description of T-S fuzzy models with fixed state delay and fuzzy control law based on PDC structure is given. New delay independent stabilization conditions are established for positive systems in section 3. In section 4, an example of a real plant is given to show the need for such controllers. Some conclusions are given in section 5.

Notation:

- M^T denotes the transpose of a real matrix M.
- F is called a positive matrix denoted by $F \succ 0$ if all its elements are positive and there is a strictly positive element $(f_{ij} \geq 0, \forall (i,j), \exists (i,j) : f_{ij} \succ 0)$.
- A matrix $A \in \Re^{n\times n}$ is called a Metzler matrix if its off-diagonal elements are nonnegative. That is, if $A = \{a_{ij}\}_{i,j=1}^{n}$, A is Metzler if $a_{ij} \geq 0$ whenever $i \neq j$.

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II. Problem formulation and preliminary results

Specifically, the Takagi-Sugeno fuzzy system is described by fuzzy IF-THEN rules, which locally represent linear input-output relations of a system. The fuzzy system is of the following form:

Rule i: IF $z_1(t)$ is F_i^1 and $\cdots$ and $z_p(t)$ is F_i^p Then:

$$\begin{aligned} \dot{x}(t) &= A_i\, x(t) + A_{i1}\, x(t-\tau) + B_i\, u(t) \qquad (1)\\ x(t) &= \Psi(t) \succ 0, t \in [-\tau, 0] \qquad (2) \end{aligned}$$

where $x(t) \in \mathbb{R}^n$ is the state, $u(t) \in \mathbb{R}^m$ is the control input, τ is a fixed delay, with $i = 1,2,...,r$, r is the number of IF-THEN rules, $z_1(t)\cdots z_p(t)$ and F_i^j are respectively the premise variable and the fuzzy sets.

The control law is chosen to be a state feedback one given by:

$$u(t) = K_i\, x(t), \qquad (3)$$

Systems (1) will be represented by T-S fuzzy models described by:

$$\dot{x}(t) = \sum_{i=1}^{r} h_i(z(t))(A_i\, x(t) + A_{i1}\, x(t-\tau) + B_i\, u(t)) \qquad (4)$$

The control used in this work is the so called PDC control:

$$u(t) = \sum_{i=1}^{r} h_i(z(t))K_i\, x(t), \qquad (5)$$

where $h_i(z(t)) = \frac{w_i(z(t))}{\sum_{i=1}^{r} w_i(z(t))}$; $w_i(t) = \prod_{j=1}^{p} F_i^j(z(t))$,

with $h_i(z(t)) \geq 0;\ \ \forall t \geq 0;\ \sum_{i=1}^{r} h_i(z(t)) = 1$,

$i = 1,2,...,r$ and $j = 1,2,...,p$.

By using (5), the closed-loop system (4) is then written as:

$$\begin{aligned} \dot{x}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(t))h_j(z(t))\,[(A_i \\ + B_iK_j\,)\,x(t) + A_{i1}\,x(t-\tau)] \qquad (6)\\ x(t) = \Psi(t) \succ 0, t \in [-\tau, 0]. \end{aligned}$$

The aim of this work is to present new sufficient conditions of existence of state feedback controllers allowing the state to be always nonnegative for continuous-time fuzzy systems with fixed delay.

Definition 1: The T-S fuzzy system (4) is said to be controlled positive if, given any nonnegative initial state and any input function $u(t) \geq 0$, the corresponding trajectory remains in the positive orthant for all t: $x(t) \in R_+^n$.

Lemma 1: [4] The autonomous delayed system (4) is positive if and only if A_i is a Metzler matrix and A_{i1} is a nonnegative matrix for $i = 1,...,r$.

Now, the conditions of stability and stabilization of T-S fuzzy system (4), using LMI method as presented in [3], are recalled.

Theorem 1: [3] For positive matrices A_{i1} and Metzler matrices A_i, the autonomous system (4) is asymptotically stable, if there exist a diagonal matrix $P = P^T \succ 0$ and a matrix $R = R^T \succ 0$ satisfying the following LMIs: $M_i = \begin{pmatrix} A_i^T P + PA_i + R & PA_{i1} \\ * & -R \end{pmatrix} \prec 0$; $i = 1,2,...,r$.

Theorem 2: [3] For positive matrices A_{i1}, if there exist a diagonal matrix $X = X^T \succ 0$, matrices Y_j; $j = 1,2,...,r$ and Z satisfying the following LMIs:

$$\begin{cases} M_{ij} + M_{ji} \prec 0 \\ A_iX + B_iY_j \; is\; Metzler \end{cases} ;i = 1,2,...,r;\; i \leq j,$$

where

$$M_{ij}=\begin{pmatrix} XA_i^T + Y_j^T B_i^T + A_iX + B_iY_j + Z & A_{i1}X \\ * & -Z \end{pmatrix}.$$

Then system (6) with $P = X^{-1}$; $K_j = Y_jX^{-1}$ and $R = X^{-1}ZX^{-1}$ is asymptotically stable and controlled positive.

To establish these conditions, the following Lyapunov-Krasovskii functional was used:

$$V(x(t)) = x(t)^T Px(t) + \int_{t-\tau}^{t} x(\upsilon)^T Rx(\upsilon)d\upsilon \tag{7}$$

Note that these results are a particular case of the ones given by [3].

III. Main results

This section concerns the study of the conditions of stability and stabilization of the fuzzy system (4) using a linear program (LP) method.

Remark: Knowing that the dual system (4) is asymptotically stable, if and only if the system (4) is asymptotically stable, then we simply demonstrate the stability of the dual system.

Theorem 3: For positive matrices A_{i1} and Metzler matrices A_i , the autonomous system (4) is asymptotically stable for all $\tau \succ 0$ if there exists a vector $\lambda \in R^n$; satisfying the following LPs:

$$\begin{cases} (A_i + A_{i1})\lambda \prec 0\, ;\; i = 1,...,r, \\ \lambda \succ 0. \end{cases}$$

Proof 1: The choice of the Lyapunov-Krasovskii functional in this case will be:

$V(x(t)) = x^T(t)\lambda + \sum_{i=1}^{r}\int_{t-\tau}^{t} x^T(s)A_{i1}\lambda ds\, ;\; \lambda \succ 0.$

As noted above, we can deal with the stability of the autonomous dual system of (4) given by:

$$\dot{x}(t) = \sum_{i=1}^{r} h_i(z(t))(A_i^T\, x(t) + A_{i1}^T\, x(t-\tau)). \tag{8}$$

The time derivative of the Lyapunov-Krasovskii functional is:

$$\dot{V}(x(t)) = \dot{x}^T(t)\lambda + x^T(t)\sum_{i=1}^{r} A_{i1}\lambda - x^T(t-\tau)\sum_{i=1}^{r} A_{i1}\lambda. \tag{9}$$

Replace the $\dot{x}^T(t)$ by the expression of the autonomous dual system (8), then the derivative of the functional will be of the form:

$$\dot{V}(x(t)) = \sum_{i=1}^{r} h_i(z(t))\left[x^T(t)A_i + x^T(t-\tau)A_{i1}\right]\lambda + \sum_{i=1}^{r}\left[x^T(t)A_{i1} - x^T(t-\tau)A_{i1}\right]\lambda.$$

As $0 \preceq h_i(z(t)) \preceq 1$, $A_{i1} \succ 0$ and $x(t-\tau) \succeq 0$, it follows that:

$$\sum_{i=1}^{r} h_i(z(t))\left[x^T(t)A_i + x^T(t-\tau)A_{i1}\right]\lambda \prec, \quad \sum_{i=1}^{r}\left[h_i(z(t))x^T(t)A_i + x^T(t-\tau)A_{i1}\right]\lambda. \tag{10}$$

Thus, $\dot{V}(x(t)) \leq \sum_{i=1}^{r}\left[h_i(z(t))x^T(t)A_i + x^T(t-\tau)A_{i1}\right]\lambda + \sum_{i=1}^{r}\left[x^T(t)A_{i1} - x^T(t-\tau)A_{i1}\right]\lambda$
$\leq \sum_{i=1}^{r} h_i(z(t))x^T(t)\left[A_i + A_{i1}\right]\lambda + \sum_{i=1}^{r}(1 - h_i(z(t)))x^T(t)A_{i1}\lambda.$

It is then obvious that $(A_i + A_{i1})\lambda \prec 0$ $, i = 1,...,r$ implies $\dot{V}(x(t)) \prec 0$. This result can be easily extended to design controllers ensuring asymptotic stability while imposing positivity in closed-loop. □

Theorem 4: For positive matrices A_{i1}, system (6) is asymptotically stable and controlled positive if there exist a vector $\lambda = [\lambda_1 \ldots \lambda_n]^T \in R^n$ and vectors $y_1^j,....,y_n^j \in R^m$ / $j = 1,...,r$; satisfying the following LPs:

$$\begin{cases} (A_i + A_{i1})\lambda + B_i\sum_{s=1}^{n} y_s^j \prec 0, i,j \in \{1,2,...,r\}, \\ a_{ls}^i\lambda_l + b_l^i y_s^j \succeq 0, l \neq s = 1,\ldots,n; i,j \in \{1,2,...,r\}, \\ \lambda \succ 0, \end{cases}$$

with $K_j = [\frac{y_1^j}{\lambda_1}, \frac{y_2^j}{\lambda_2} \ldots, \frac{y_n^j}{\lambda_n}]$; $j = 1,...,r$; and

$$A_i = (a^i)_{ls}, l,s = 1,\ldots,n; B_i = \begin{bmatrix} b_1^i \\ b_2^i \\ \ldots \\ b_n^i \end{bmatrix}. \tag{11}$$

Proof 2: Following the same reasoning and replacing the $\dot{x}^T(t)$ in equation (9) by the formula of the dual system of (6), which is as follows:

$$\dot{x}(t) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(t))h_j(z(t))\left[(A_i + B_iK_j)^T\, x(t) + A_{i1}^T\, x(t-\tau)\right].$$

The expression of the derivative of the functional (9) becomes:

$$\dot{V}(x(t)) = \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(t))h_j(z(t))\left[x^T(t)(A_i+B_iK_j)\right.$$

$$\left.+x^T(t-\tau)A_{i1}\right]\lambda + \sum_{i=1}^{r}\left[x^T(t)A_{i1} - x^T(t-\tau)A_{i1}\right]\lambda$$

$$\leq \sum_{i=1}^{r}\sum_{j=1}^{r}\left[h_i(z(t))h_j(z(t))x^T(t)(A_i+B_iK_j)\right.$$

$$\left.+x^T(t-\tau)A_{i1}\right]\lambda + \sum_{i=1}^{r}\left[x^T(t)A_{i1} - x^T(t-\tau)A_{i1}\right]\lambda$$

$$\leq \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(t))h_j(z(t))x^T(t)\left[A_i + A_{i1} + B_iK_j\right]\lambda +$$

$$\sum_{i=1}^{r}\sum_{j=1}^{r}\left[1 - h_i(z(t))h_j(z(t))\right]x^T(t)A_{i1}\lambda.$$

Finally,

$$(A_i + A_{i1})\lambda + B_iK_j\lambda \prec 0\,;\; i,j \in \{1,...,r\} \tag{12}$$

implies $\dot{V}(x(t)) \prec 0$. To ensure that the trajectory remains in the positive orthant, matrices $A_i + B_iK_j$ must be Metzler. By using (11), the off-diagonal elements of matrices in closed-loop are given by: $(A_i+B_iK_j)_{ls} = a^i_{ls} + b^i_l \frac{y^j_s}{\lambda_l}, l \neq s = 1,\ldots,n; i,j \in \{1,2,...,r\}$, which are nonnegative if and only if $a^i_{ls}\lambda_l + b^i_l y^j_s \succeq 0, l \neq s$, λ_l being positive. Now, by letting $K_j = [K^j_1\ K^j_2 \ldots\ K^j_n]$ where K^j_s are vectors in R^m, one has $K_j\lambda = \sum_{s=1}^{n} K^j_s\lambda_s = \sum_{s=1}^{n} y^j_s$, with $K^j_s\lambda_s = y^j_s$. Consequently, inequality (12) can be written as

$$(A_i + A_{i1})\lambda + B_i \sum_{s=1}^{n} y^j_s \prec 0,$$

and

$$K_j = \left[\frac{y^j_1}{\lambda_1}, \frac{y^j_2}{\lambda_2} \ldots, \frac{y^j_n}{\lambda_n}\right]; j = 1,...,r.$$

□

This result can be extended to positive T-S systems, that is systems with matrices A_i Metzler and positive matrices A_{i1} and B_i. In this case, the control has to be positive, which is guaranteed by imposing $y^j_s \geq 0$.

Corollary 1: For positive matrices A_{i1} and B_i and matrices A_i Metzler, system (6) is asymptotically stable and positive if there exist a vector λ, vectors $y^j_1, ..., y^j_r \in R^m$ / $j = 1,...,r$; satisfying the following LPs:

$$\begin{cases} (A_i + A_{i1})\lambda + B_i \sum_{s=1}^{n} y^j_s \prec 0; i,j \in \{1,2,...,r\} \\ y^j_s \succeq 0, \\ \lambda \succ 0, \end{cases}$$

with

$K_j = \left[\frac{y^j_1}{\lambda_1}, \frac{y^j_2}{\lambda_2} \ldots, \frac{y^j_n}{\lambda_n}\right]; j = 1,...,r.$

It is worth noting that the conditions of stability and stabilization of the T-S fuzzy system without delay can be obtained as a particular case of the studied system with delay (6).

IV. Application to a real plant model

Consider the process composed of two linked tanks of 22 liter capacity each. This system can be described by:

$$\dot{x}_1(t) = u_1(t) - q_{12}(t) - q_1(t)$$
$$\dot{x}_2(t) = u_2(t) - q_{12}(t) - q_2(t),$$

where x_i holds for the level in of the tank in liters, u_j represents the flow in liters/mn of pump j, q_{12} is the variation of the flow between the two tanks and q_i the loss flow of each tank. Applying the Torricelli law, one obtains:
$q_1 = \gamma_1\sigma_1\sqrt{2gx_1} = R_1\sqrt{x_1}$
$q_2 = \gamma_1\sigma_2\sqrt{2gx_2} = R_2\sqrt{x_2}$
$q_{12} = \gamma_{12}\sigma_1\sqrt{2g|x_1 - x_2|}sign(x_1 - x_2) = R_{12}\sqrt{|x_1 - x_2|}sign(x_1 - x_2)$,

where γ_i and γ_{ij} are physical constants, σ_i is the tank section and g the gravity acceleration. The process model is then as follows:

$$\dot{x}_1(t) = u_1 - R_1\sqrt{x_1} - R_{12}\sqrt{|x_1 - x_2|}sign(x_1 - x_2)$$
$$\dot{x}_2(t) = u_2 - R_2\sqrt{x_2} - R_{12}\sqrt{|x_1 - x_2|}sign(x_1 - x_2).$$

The obtained model is then nonlinear. To obtain a T-S fuzzy representation for this nonlinear system, the classical transformation: $\sqrt{x_i} = \frac{x_i}{\sqrt{x_i}} = x_iz_i$ with $z_i = \frac{1}{\sqrt{x_i}}; \frac{1}{\sqrt{|x_1-x_2}} = \frac{z_1z_2}{\sqrt{|z_2^2-z_1^2|}}$ is used.

The corresponding model is then given by: $\begin{cases}\dot{x}(t) = A(z_1,z_2)x(t) + Bu(t) \\ y(t) = Cx(t)\end{cases}$ where matrix $A(z_1,z_2)$ has the general following form: $A(z_1,z_2) = \begin{pmatrix} -R_1z_1 - \frac{R_{12}}{\sqrt{|z_1^2-z_2^2}} & \frac{R_{12}z_1z_2}{\sqrt{|z_1^2-z_2^2}} \\ \frac{R_{12}z_1z_2}{\sqrt{|z_1^2-z_2^2}} & -R_2z_2 - \frac{R_{12}}{\sqrt{|z_1^2-z_2^2}} \end{pmatrix}$ $B = I_2$; $C = I_2$.

The delayed model can be written as:

$$\begin{cases} \dot{x}(t) = (1-\varepsilon)A(z_1,z_2)x(t) + \varepsilon|A(z_1,z_2)|x(t-\tau) \\ +Bu(t) \\ y(t) = Cx(t), \end{cases}$$

with $\varepsilon \in [0,1]$ and τ: fixed delay.

The objective is that the output y tracks a given reference y_r. The following control is used: $u(t) = K(\theta)x(t) + L(\theta)y_r$, where controller gain $K(\theta)$ ensures the asymptotic stability together with the positivity in closed-loop, while the controller gain $L(\theta)$ achieves the tracking objective, one obtains: $X(s) = (sI - \hat{A}(\theta) - A_\tau(\theta)e^{-s\tau})^{-1}BL(\theta)Y_r(s)$; so: $Y(s) = \frac{(sI-\hat{A}(\theta)-A_\tau(\theta)e^{-s\tau})^{-1}BL(\theta)Y_r(s)}{s}$. Using the final value theorem, one can deduce: $y(\infty) = -C[\hat{A}(\theta) + A_\tau(\theta)]^{-1}L(\theta)y_r$ with $\hat{A}(\theta) = (1-\varepsilon)A(\theta) + BK(\theta); A_\tau(\theta) = \varepsilon|A(\theta)|$. If one chooses $L_i = -\hat{A}_i - A_{i1} = -(1-\varepsilon)A_i - \varepsilon|A_i|; i = 1,..,4$, the tracking objective will be reached with $y(\infty) = y_r$. Present this system as the T-S fuzzy model:
by considering that $z_i \in [a_i;b_i]; i = 1,2,$ the

four following rules are taken into account:
$$\begin{cases} If\ z_1\ is\ a_1\ and\ z_2\ is\ a_2\ Then: A(z_1,z_2)=A_1 \\ If\ z_1\ is\ a_1\ and\ z_2\ is\ b_2\ Then: A(z_1,z_2)=A_2 \\ If\ z_1\ is\ b_1\ and\ z_2\ is\ a_2\ Then: A(z_1,z_2)=A_3 \\ If\ z_1\ is\ b_1\ and\ z_2\ is\ b_2\ Then: A(z_1,z_2)=A_4 \end{cases}$$

The membership functions are given by:
$h_1(t)=f_{11}(t)f_{21}(t)$; $h_2(t)=f_{11}(t)f_{22}(t)$;
$h_3(t)=f_{12}(t)f_{21}(t)$; $h_4(t)=f_{12}(t)f_{22}(t)$;
where $f_{i1}(t)=\frac{z_i(t)-b_i}{a_i-b_i}$ and $f_{i2}(t)=1-f_{i1}(t)=\frac{a_i-z_i(t)}{a_i-b_i}$; $i=1,2$.

The membership functions are finally as:
$h_1(t)=\frac{(z_1(t)-b_1)(z_2(t)-b_2)}{(a_1-b_1)(a_2-b_2)}$; $h_2(t)=\frac{(z_1(t)-b_1)(a_2(t)-z_2(t))}{(a_1-b_1)(a_2-b_2)}$;
$h_3(t)=\frac{(a_1-z_1(t))(z_2(t)-b_2)}{(a_1-b_1)(a_2-b_2)}$; $h_4(t)=\frac{(a_1-z_1(t))(a_2-z_2(t))}{(a_1-b_1)(a_2-b_2)}$;

The obtained matrices A_i of the subsystems are: $A_1=\begin{pmatrix} -R_1a_1-\frac{R_{12}a_1a_2}{\sqrt{|a_1^2-a_2^2|}} & \frac{R_{12}a_1a_2}{\sqrt{|a_1^2-a_2^2|}} \\ \frac{R_{12}a_1a_2}{\sqrt{|a_1^2-a_2^2|}} & -R_2a_2-\frac{R_{12}a_1a_2}{\sqrt{|a_1^2-a_2^2|}} \end{pmatrix}$;

$A_2=\begin{pmatrix} -R_1a_1-\frac{R_{12}a_1b_2}{\sqrt{|a_1^2-b_2^2|}} & \frac{R_{12}a_1b_2}{\sqrt{|a_1^2-b_2^2|}} \\ \frac{R_{12}a_1b_2}{\sqrt{|a_1^2-b_2^2|}} & -R_2b_2-\frac{R_{12}a_1b_2}{\sqrt{|a_1^2-b_2^2|}} \end{pmatrix}$;

$A_3=\begin{pmatrix} -R_1b_1-\frac{R_{12}b_1a_2}{\sqrt{|b_1^2-a_2^2|}} & \frac{R_{12}b_1a_2}{\sqrt{|b_1^2-a_2^2|}} \\ \frac{R_{12}b_1a_2}{\sqrt{|b_1^2-a_2^2|}} & -R_2a_2-\frac{R_{12}b_1a_2}{\sqrt{|b_1^2-a_2^2|}} \end{pmatrix}$;

$A_4=\begin{pmatrix} -R_1b_1-\frac{R_{12}b_1b_2}{\sqrt{|b_1^2-b_2^2|}} & \frac{R_{12}b_1b_2}{\sqrt{|b_1^2-b_2^2|}} \\ \frac{R_{12}b_1b_2}{\sqrt{|b_1^2-b_2^2|}} & -R_2b_2-\frac{R_{12}b_1b_2}{\sqrt{|b_1^2-b_2^2|}} \end{pmatrix}$;

One can notice that matrix B in this example is common, which reduces considerably the number of the LMIs to be solved. The obtained T-S fuzzy model without delay is given by:

$$\begin{cases} \dot{x}(t)=\sum_{i=1}^{4}h_i(z(t))(A_ix(t)+Bu(t)) \\ y(t)=\sum_{i=1}^{4}h_i(z(t))C_ix(t) \end{cases} \tag{13}$$

The corresponding T-S model with fixed delay can be given as follows [3]:

$$\begin{cases} \dot{x}(t)=\sum_{i=1}^{4}h_i(z(t))((1-\varepsilon)A_ix(t)+\varepsilon|A_i|x(t-\tau)+Bu(t)) \\ y(t)=\sum_{i=1}^{4}h_i(z(t))C_ix(t) \end{cases} \tag{14}$$

The objective is to design controllers ensuring stabilization of systems (14) associated to the real plant model, for which matrices A_i are Metzler and matrices A_{i1} and B are positive, using the conditions of Theorem 2 and Corollary 1.

A. Simulation results of the system without delay

The use of the LMI method without delay of Theorem 2 leads to the following results:
$P=\begin{pmatrix} 0.1069 & -0.0692 \\ -0.0692 & 0.1053 \end{pmatrix}$,
$K_1=\begin{pmatrix} -0.0328 & 0.2235 \\ 0.1957 & 0.0173 \end{pmatrix}$; $K_2=\begin{pmatrix} -0.1553 & 0.3709 \\ 0.2146 & 0.0927 \end{pmatrix}$
$K_3=\begin{pmatrix} 0.1470 & 0.1801 \\ 0.3458 & -0.0991 \end{pmatrix}$; $K_4=\begin{pmatrix} 0.4437 & -0.1231 \\ -0.0909 & 0.3979 \end{pmatrix}$
Matrices in closed-loop are obtained as:
$\hat{A}_1=\begin{pmatrix} 0.4777-0.4560 \\ -0.42820.4605 \end{pmatrix}$; $\hat{A}_2=\begin{pmatrix} 0.5067 & -0.5099 \\ -0.3535 & 0.4340 \end{pmatrix}$
$\hat{A}_3=\begin{pmatrix} 0.4423-0.3445 \\ -0.51030.5088 \end{pmatrix}$; $\hat{A}_4=\begin{pmatrix} 0.5009 & -0.3966 \\ -0.4288 & 0.5096 \end{pmatrix}$

The obtained solutions of the LP method are as follows:
$\lambda=\begin{pmatrix} 0.1011 \\ 0.1063 \end{pmatrix}$,
$K_1=\begin{pmatrix} 0.0655 & 0.0623 \\ 0.0878 & 0.0834 \end{pmatrix}$; $K_2=\begin{pmatrix} 0.0670 & 0.0637 \\ 0.1364 & 0.1296 \end{pmatrix}$
$K_3=\begin{pmatrix} 0.1389 & 0.1320 \\ 0.0867 & 0.0824 \end{pmatrix}$; $K_4=\begin{pmatrix} 0.1335 & 0.1269 \\ 0.1416 & 0.1346 \end{pmatrix}$
Matrices in closed-loop are obtained as:
$\hat{A}_1=\begin{pmatrix} 0.3794 & -0.2948 \\ -0.3203 & 0.3944 \end{pmatrix}$; $\hat{A}_2=\begin{pmatrix} 0.2844 & -0.2027 \\ -0.2754 & 0.3971 \end{pmatrix}$
$\hat{A}_3=\begin{pmatrix} 0.4504 & -0.2965 \\ -0.2512 & 0.3273 \end{pmatrix}$; $\hat{A}_4=\begin{pmatrix} 0.8110 & -0.6466 \\ -0.6613 & 0.7729 \end{pmatrix}$

The results of the simulation, with the following data: initial point $x_0=[6,7]^T$ and the trajectory reference $y_r=[15,15]^T$, are obtained as follows:

B. Simulation results of the system with fixed delay

The use of the LMI method with fixed delay of Theorem 2 leads to the following results:

$P=\begin{pmatrix} 0.0161 & 0 \\ 0 & 0.0160 \end{pmatrix}$; $R=\begin{pmatrix} 0.0058 & -0.0025 \\ -0.0025 & 0.0060 \end{pmatrix}$
$K_1=\begin{pmatrix} 0.0214 & 0.0213 \\ 0.0214 & 0.0237 \end{pmatrix}$; $K_2=\begin{pmatrix} 0.0322 & 0.0485 \\ 0.0488 & 0.0745 \end{pmatrix}$
$K_3=\begin{pmatrix} 0.0894 & 0.0543 \\ 0.0547 & 0.0412 \end{pmatrix}$; $K_4=\begin{pmatrix} 0.0272 & 0.0228 \\ 0.0230 & 0.0249 \end{pmatrix}$
Matrices in closed-loop are obtained as:
$\hat{A}_1=\begin{pmatrix} 0.3345 & -0.2538 \\ -0.2540 & 0.3586 \end{pmatrix}$; $\hat{A}_2=\begin{pmatrix} 0.2490 & -0.1875 \\ -0.1878 & 0.3469 \end{pmatrix}$
$\hat{A}_3=\begin{pmatrix} 0.3821 & -0.2187 \\ -0.2191 & 0.2866 \end{pmatrix}$; $\hat{A}_4=\begin{pmatrix} 0.7285 & -0.5425 \\ -0.5427 & 0.7011 \end{pmatrix}$

The use of the LP method with fixed delay of Theorem 4 leads to the following results:
$\lambda=\begin{pmatrix} 0.8967 \\ 0.9209 \end{pmatrix}$,
$K_1=\begin{pmatrix} 0.0396 & 0.0386 \\ 0.0523 & 0.0509 \end{pmatrix}$; $K_2=\begin{pmatrix} 0.0444 & 0.0432 \\ 0.0983 & 0.0957 \end{pmatrix}$
$K_3=\begin{pmatrix} 0.1014 & 0.0988 \\ 0.0555 & 0.0540 \end{pmatrix}$; $K_4=\begin{pmatrix} 0.0745 & 0.0725 \\ 0.0745 & 0.0725 \end{pmatrix}$
Matrices in closed-loop are obtained as:
$\hat{A}_1=\begin{pmatrix} 0.3608 & -0.2478 \\ -0.2616 & 0.3791 \end{pmatrix}$; $\hat{A}_2=\begin{pmatrix} 0.2719 & -0.1683 \\ -0.2233 & 0.3784 \end{pmatrix}$
$\hat{A}_3=\begin{pmatrix} 0.4289 & -0.2468 \\ -0.2035 & 0.3147 \end{pmatrix}$; $\hat{A}_4=\begin{pmatrix} 0.7756 & -0.5403 \\ -0.5422 & 0.7442 \end{pmatrix}$

The results of simulation with the following data: $\varepsilon=0.1$; initial points $\Psi(t)=[8,7]^T, t\in[-\tau,0]$ and the trajectory reference $y_r=[15,15]^T$ are obtained as:

C. Comparison between the LMI and LP methods:

In this section, a comparison between the feasibility of the results of Theorem 2 and the ones of Theorem 4 is presented based on the real plant model.

Based on the comparison of the two presented methods, the LMI and linear programming, we note that the domain of feasibility of conditions based on linear programming is much larger than the LMI based ones.

V. Conclusion

In this paper, we are concerned with the study of positive non linear systems. To obtain conditions of stability and stabilization of nonlinear systems, while imposing positivity in closed-loop, the T-S fuzzy techniques are used. The study is performed by using a linear programming method. Finally, an application to a real model of a process with two tanks was presented together with a comparison between our results and the ones of [3] obtained with LMIs.

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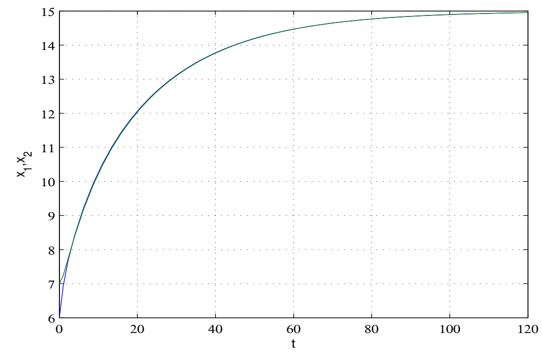

Fig. 1. This figure plots the evolution of the states x_1 and x_2(LMI)

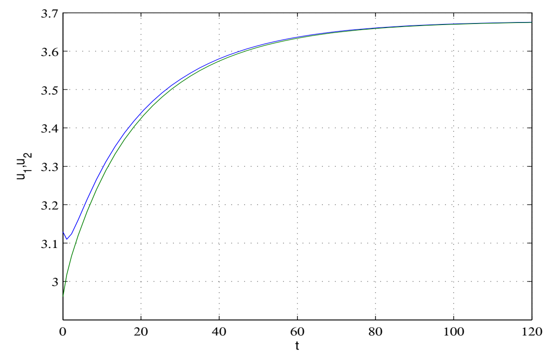

Fig. 2. This figure plots the evolution of the two pump flows(LMI)

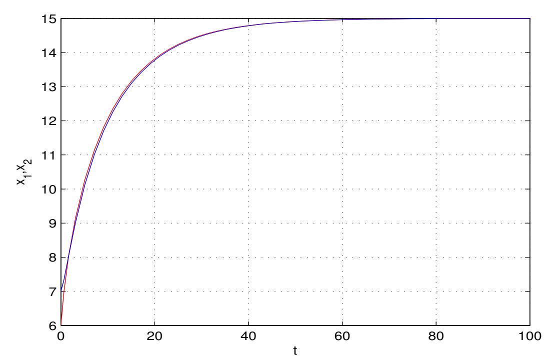

Fig. 3. This figure plots the evolution of the states x_1 and x_2(LP)

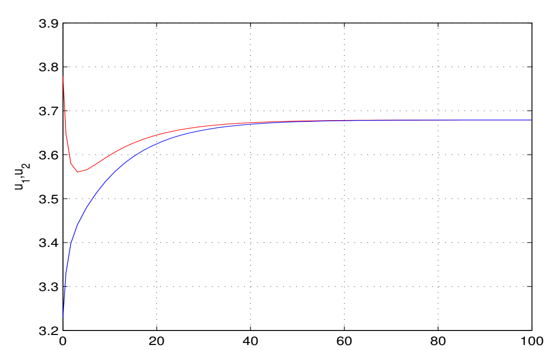

Fig. 4. This figure plots the evolution of the two pump flows(LP)

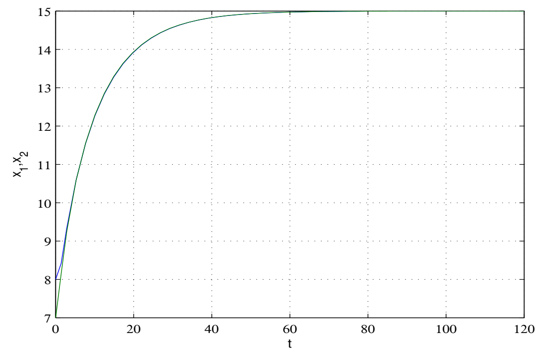

Fig. 5. This figure plots the evolution of the states x_1 and x_2(LMI)

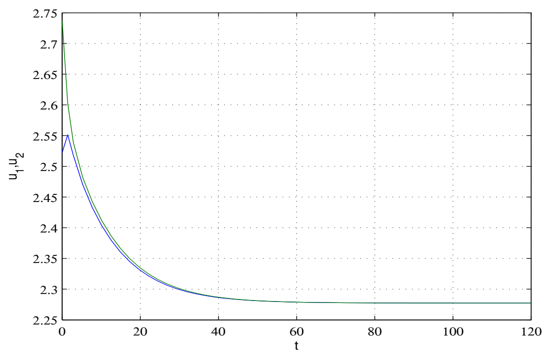

Fig. 6. This figure plots the evolution of the two pump flows(LMI)

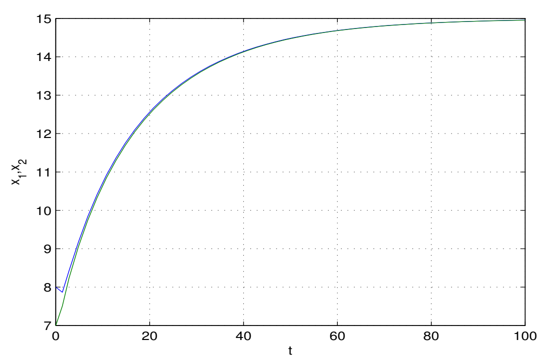

Fig. 7. This figure plots the evolution of the states x_1 and x_2(LP)

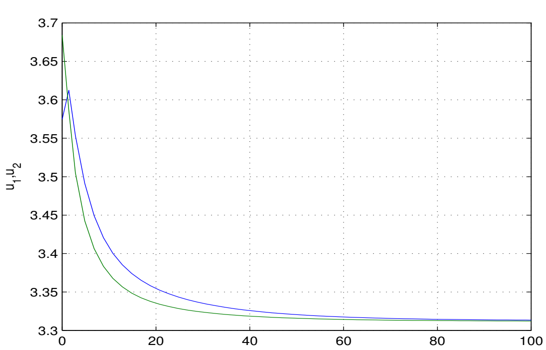

Fig. 8. This figure plots the evolution of the two pump flows(LP)

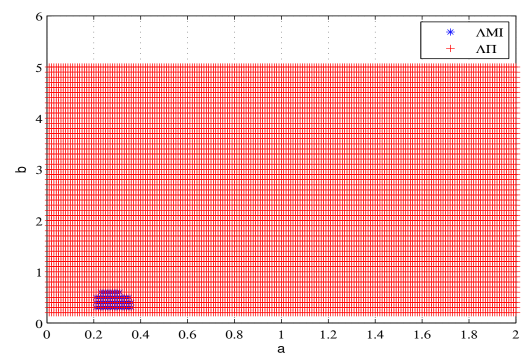

Fig. 9. Comparing the field feasibility of the LMI and LP