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A Multi-Stage Stochastic Optimization Approach to Optimal Bidding on Energy Markets

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Abstract— **One of the most challenging tasks for an energy producer is represented by the optimal bidding on energy markets. Each eligible plant has to submit bids for the spot market one day before the delivery time and bids for the ancillary services provision. Allocating the optimal amount of energy, jointly minimizing the risk and maximizing profits is not a trivial task, since one has to face several sources of stochasticity, such as the high volatility of energy prices and the uncertainty of the production, due to the deregulation and to the growing importance of renewable sources. In this paper the optimal bidding problem is formulated as a multi-stage optimization problem to be solved in a receding horizon fashion, where at each time step a risk measure is minimized in order to obtain optimal quantities to bid on the day ahead market, while reserving the remaining production to the ancillary market. Simulation results show the optimal bid profile for a trading day, based on stochastic models identified from historical data series from the Italian energy market.**

I. Introduction

During the last few years, energy-related issues emerged as a relevant application area of stochastic control, especially since electricity markets have been deregulated and have become very complex, leading to a wide range of new opportunities for power producers to optimize their productivity. The main tasks power producers have to face include production planning and optimal bidding. Scheduling operations and trading on the electricity market involve dealing with several sources of uncertainty, such as stochasticity of prices and of produced power in case of intermittent renewable sources. An overview on how the electricity markets work is presented in [1], while a macroeconomic description of power plant investments is given in [2].

A firm which produces and sells energy has many options to diversify its output; basically it will hedge against the risk associated with the aforementioned uncertainty sources by stipulating bilateral contracts, and it will try to increase its margins by trading on the day-ahead market or by presenting bids for ancillary services, such as primary or secondary capacity reserves (see [3] for a comparison between electricity contracts and the more common financial products such as options, futures and other derivatives).

Bilateral contracts are mainly traded on a long-term basis, as shown in [4], where the expiration date of such contracts is fixed at one year from the present date. Nonetheless, in a more and more *real-time* market, it is not excluded that bilateral contracts will be traded in a more short-term fashion.

Existing approaches focused on jointly providing optimal bidding strategies and unit commitment under stochasticity, as done in [5]–[9] and [10], which considered the case of a price-taker hydropower producer. The extension of the latter, in [11], aims at solving the unit commitment problem, once that the bidding auction has taken place but information about prices and weather has not been disclosed yet. A general overview on stochastic programming models applied to energy topics is presented in [12]. Stochastic models of electricity prices, which are affected by high volatility and jumps, are presented in [13] and [14]. The latter considers the dynamical evolution of volatility and introduces parameter-varying models such as GARCH, that will be also considered in this work.

This paper focuses on the optimal bidding problem. We do not consider unit commitment, which is assumed decoupled (this is the assumption of most producers that own several production sources), in a generalized formulation that can be extended and scaled to different kinds of power plants. For a simple power producer without marginal costs and from a price-taker point of view, the problem is to decide the following quantities:

- the optimal amount of installed capacity that should be traded on bilateral contracts,
- which amount should be traded on the day ahead;
- how much energy should be allocated and stored for the following ancillary services market (if the plant is eligible for this kind of production).

In other words, our goal is to split the production into three different output channels, each of which has a different level of risk and profit.

A control approach based on stochastic optimization is proposed to solve the optimal bidding problem, relying on the minimization of the Conditional Value at Risk (CVaR) risk measure [15], since the pure maximization of the expected profit would lead to excessively risky decisions. Simulations have been carried on the basis of past values of the Italian national cleared prices and on past offers submitted by producers, which are public and available online [16].

The paper is organized as follows. The considered energy market model is described in Section II. In Section III the optimal bidding problem is presented, under a risk-penalizing objective function, and the control algorithm is defined.

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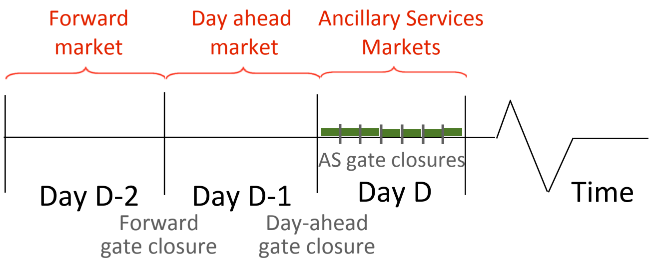

Fig. 1. Scheme of electricity markets

Simulation results are shown in Section IV, and conclusions are drawn in Section V.

II. MARKET MODEL DESCRIPTION

At noon of the day before the day of the delivery, referred to as day D, the spot pool auction is closed. By that moment, participants must have presented their bids (couples quantity/price) for each hour of the following day. Then, they still have the possibility to present bids for the ancillary services market, up to one hour and a half before the delivery time. Moreover, between the day-ahead and the ancillary services market the *intra-day* market takes place, where market participants are allowed to modify those parts of the program that cannot be satisfied. A scheme of the chronological order of the energy markets is shown in Figure 1.

The spot price (or cleared price) comes from a so called *clearing process*, aimed to find the national price for spot transactions, The *clearing process* works as follows. Purchasers and suppliers submit their bids (which are couples price-volume, expressed in €/MWh). Selling bids from all suppliers are aggregated and ordered by increasing prices while purchasing bids are sorted by decreasing prices. The matching point between demand and supply determines the cleared price (for further information on the clearing price mechanism, see [1]). Three cases are possible: *(i)* the bid is accepted, and paid at the price asked by the participant, *(ii)* the bid is completely rejected, or *(iii)* the bid is accepted but changed by the system, thus resulting in a different quantity. The last is the case when the bid is set exactly at the cleared price and the volume may not be entirely dispatched.

In this work we consider a simplified market scenario, which does not account for the intra-day market and partial fulfillment of bids in the day-ahead market. In this case either the total volume is supplied or the bid does not take place. Note that the price at which the transaction is settled is however the cleared price, that is the same for all the parties.

The ancillary services market works in a *price as bid* mechanism instead. This means that bids from market participants are dispatched at the proposed price according to an economic order (lower selling bids/higher purchasing bids are activated first), until the total demand is covered.

For a time slot $t \in \mathbb{N}$ (generally one hour) of a given day, the profit is given by:

$$P(t) = s_{BL}(t)u_{BL}(t) + s_{DA}(t)u_{DA}(t) + s_{AS}(t)'u_{AS}(t) \quad (1)$$

where $u_{BL} \in \mathbb{R}$, $u_{DA} \in \mathbb{R}$ and $u_{AS} \in \mathbb{R}^L$ are the energy quantities allocated to bilateral contracts, day-ahead and ancillary services markets, respectively, while s_{BL}, $s_{DA} \in \mathbb{R}$ and $s_{AS} \in \mathbb{R}^L$ are the corresponding prices and L is the number of bids presented.

Probabilistic models of the cleared price evolution and of the bid prices are required to generate relevant price scenarios, that will be exploited by the optimal control policy. Those models are introduced in the following.

A. Spot price modeling

As a result of deregulation electricity prices forecasting has assumed a crucial role in the recent years. Several models have been developed to catch the dynamics of electricity prices, such as ARMAX models, multiple regression, different specifications of multiple regression models, non linear Markov switching regression models and time-varying parameter regression models (see, e.g., [17], [18]). Energy can be assimilated to a particular traded stock, whose price is affected by high volatility and unpredictability.

Consider the following stochastic differential equations, where s is the spot price of an asset and y represents some internal variables such as variance

$$ds(\tau) = \mu^s(s(\tau), y(\tau))d\tau + \sigma^s(s(\tau), y(\tau))dz^s \quad (2a)$$

$$dy(\tau) = \mu^y(y(\tau))d\tau + \sigma^y(y(\tau))dz^y \quad (2b)$$

where $z^s(\tau)$, $z^y(\tau)$ are Wiener processes, namely dz^s, dz^y are correlated Gaussian variables with zero mean and variance $d\tau$. In (2) we assume $s \geq 0$, $\forall \tau \geq 0$. For control synthesis purposes, we discretize (2) to obtain the difference equations

$$s(t+1) = f(s(t), y(t), z^s(t)) \quad (3a)$$

$$y(t+1) = g(y(t), z^y(t)) \quad (3b)$$

where $[\cdot](t)$ denotes the value of $[\cdot]$ at time $\tau = t\Delta_T$, Δ_T is a constant trading interval, $z^s(t), z^y(t) \in \mathbb{R}^n$ are random Gaussian vectors with zero mean $E[z^s(t)] = 0$, $E[z^y(t)] = 0$, $\forall t \geq 0$, and covariance matrix $\Phi^s(t) = E[z^s(t)z^s(t)']$, $\Phi^y = E[z^y(t)z^y(t)']$, with $\Phi^s(t)$, $\Phi^y(t)$ positive semidefinite $\forall t \geq 0$. Model (2) is a general form that covers several popular models, including the time-varying parameter GARCH models, which allow for a specific dynamics of the volatility itself. For a price s_t, the GARCH(1,1) Gaussian model with normally distributed innovation is a simple constant mean model and is defined as follows:

$$s_t = C + \epsilon_t \quad (4a)$$

$$\sigma_t^2 = \kappa + G_1\sigma_{t-1}^2 + A_1\epsilon_{t-1}^2 \quad (4b)$$

$$\epsilon_t = \eta_t\sigma_t \quad (4c)$$

where ϵ_t is the innovation process with variance σ_t^2 and $\eta_t \sim \mathcal{N}(0,1)$ is a random variable drawn from a standard Gaussian distribution. The scalar parameters C, G_1, A_1, κ

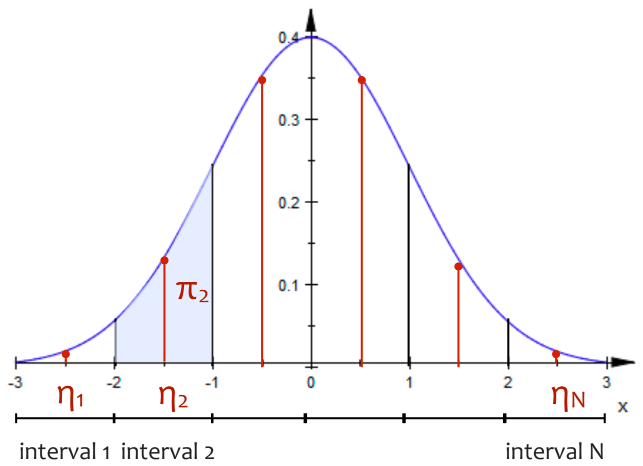

Fig. 2. Sampling the Gaussian distribution to obtain quantized spot prices

are identified form historical data series, considering past spot prices observed at the same hour of the day.

When dealing with optimization problems in the presence of stochastic data, the approximation of continuous uncertainty to a discrete domain is often used, and constructed in a way to preserve the main statistical properties of the underlying continuous process [19], [20].

For optimization purposes we quantize the number of future spot prices for the day-ahead market in N possible values. This is done by sampling the Gaussian standard distribution into N intervals, taking the central value of each interval, and substituting it into η_t in (4c). In this way we obtain a finite and discrete set of possible innovations that leads to quantized spot prices s^i_{DA} with discrete realization probabilities π_i, $i \in \{1, 2, \dots, N\}$. The sampling process is illustrated in Figure 2. Note that bids from other competitors are not taken into account for the generation of spot prices. This is done on the assumption that produced volumes are not relevant enough to affect the clearing price process, so that we can consider it as an external stochastic process, condensing all the outside information. In other words, the historical series of prices captures the past behavior of all generators in the system. This allows reducing the complexity of modeling the behavior of each competitor and purchaser.

B. Bid-price modeling

On the ancillary services market multiple bids are possible, that is, a power plant can submit more than one bid at different prices independently. In our framework we allow for such a possibility, fixing *a priori* the price for each of the L possible bids to present to the ancillary services market, and optimizing over the energy quantity to allocate per bid. To this end, given a spot price s^i_{DA}, $i \in \{1, 2, \dots, N\}$, we need to identify L representative price values s^{ij}_{AS}, $j \in \{1, 2, \dots, L\}$, such that the probability of acceptance π^{ij}_{AS} of a bid with price s^{ij}_{AS}, conditioned to s^i_{DA}, is sufficiently high.

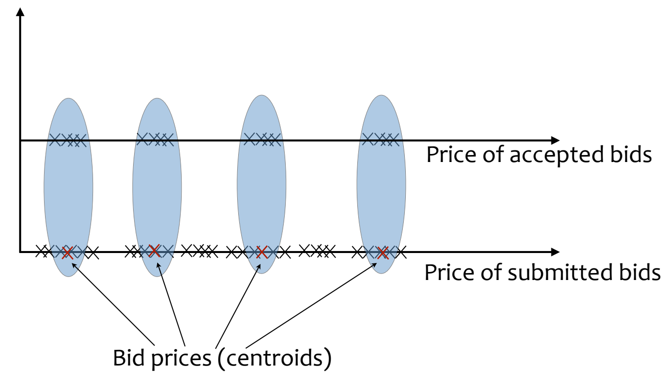

Fig. 3. Bid-price model

We present an empirical approach to derive such representative bid prices, based on the distribution of the past accepted bids on the Italian energy market, which are published on the GME (Gestore dei Mercati Energetici) website [16]. The Italian electricity market is divided into six physical zones, namely: North, North-Center, South-Center, South, Sicily and Sardinia. For each of these zones a different ancillary services market takes place. Since several kinds of ancillary services exist on the market (such as primary and secondary reserves, which differ in the time needed to activate them), the presented bids are filtered both by specific kind of service and by zone.

Let $\mathcal{P} = \{p_1, p_2, \dots, p_N\}$ be the set of the prices of the presented bids, where $p_i \leq p_j$, $\forall i \leq j$, and let $\mathcal{A} = \{a_1, a_2, \dots, a_M\}$ bet the set of the prices of the accepted bids, with $a_i \leq a_j$, $\forall i \leq j$, $\mathcal{A} \subseteq \mathcal{P}$ and $M \leq N$. Now, consider a partition $\mathcal{A}_1, \mathcal{A}_2, \dots, \mathcal{A}_L$, obtained by clustering $\mathcal{A}$ into L subsets. We take the representative bid price s^{ij}_{AS} as the the centroid of the cluster $\mathcal{A}_j$, for $j = 1, 2, \dots, L$. Let a^{min}_i and a^{max}_i be respectively the minimum and the maximum price in cluster $\mathcal{A}_i$, $i = 1, \dots, L$, and consider the subsets $\mathcal{P}_1, \mathcal{P}_2, \dots, \mathcal{P}_L$ of $\mathcal{P}$ computed such that $p_j \in \mathcal{P}_i$ if and only if $a^{min}_i \leq p_j \leq a^{max}_i$, for all $j = 1, 2, \dots, N$ and $i = 1, 2, \dots, L$. We have that $\mathcal{P}_i \cap \mathcal{P}_j = \emptyset$, $\forall i, j$, and $\bigcup_{i=1}^{L} \mathcal{P}_i \subseteq \mathcal{P}$. The acceptance probability in each cluster, that is the probability π^{ij}_{AS}, is then computed by

$$\pi^{ij}_{AS} = \frac{|\mathcal{A}_j|}{|\mathcal{P}_j|} \tag{5}$$

for all $j = 1, 2, \dots, L$, where $|\cdot|$ denotes the cardinality operator.

III. Control problem formulation

We tackle the optimal bidding problem as a multistage stochastic optimization problem with sampling time one hour, since hourly bids are presented in the Italian electricity market.

The problem formulation involves the setup of a scenario-based tree with two optimization stages, where the root node is represented by the current price of the bilateral contract $s_{BL}(t)$ and by the last observed cleared price $s_{DA}(t)$.

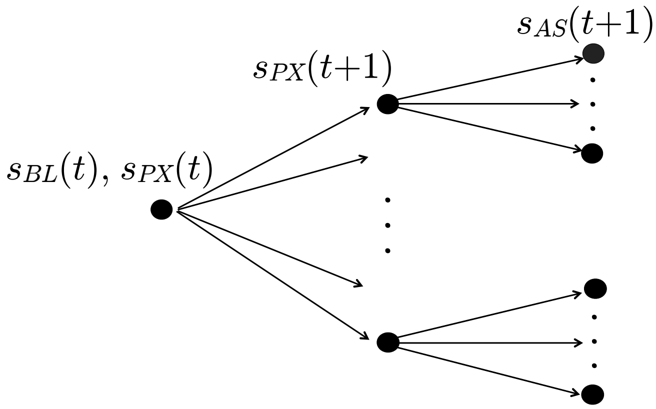

Fig. 4. Two-stage optimization tree for the optimal bidding problem

The first optimization stage at time t models the decision on the quantity of energy to allocate on the bilateral contracts, and on the day-ahead market. At $t{+}1$, the N branches are represented by the cleared prices at the same hour of the following day. Each one of these nodes further splits into L possibly accepted bid prices for the ancillary services, presented on the same operational day (see Figure 4).

A. Conditional Value at Risk

One of the most common approaches in optimal bidding problems is based on the maximization of the expected profit. The main drawback of this strategy is that it can lead to very risky decisions, since high risk associated to energy markets are not taken into account.

In order to compensate for this disadvantage and to find a tradeoff between risk and expected revenue, in this work a common risk measure is exploited, namely Conditional Value at Risk (CVaR). Let $f(u,s)$ be a loss function of the decision vector $u \in \mathbb{R}^n$ and of the random variable $s \in \mathbb{R}^m$, here the loss function is the opposite of the profit introduced in (1)

$$f(u,s) = -s_{BL}(t)u_{BL}(t) - s_{DA}(t)u_{DA}(t) - s_{AS}(t)' u_{AS}(t)$$

where

$$u = [u_{BL},\, u_{DA},\, u_{AS}']' \in \mathbb{R}^{2+L}$$
$$s = [s_{BL},\, s_{DA},\, s_{AS}']' \in \mathbb{R}^{2+L}.$$

Let $p(s)$ be the probability density function of the random vector s. With respect to a given probability β, $0 \le \beta \le 1$, the β-VaR (Value at Risk) is defined as the lowest value ℓ, such that the loss $f(u,s)$ will not exceed ℓ with probability β. A more accurate index is the β-CVaR, which is the conditional expectation of the loss function above ℓ, quantifying what the *average loss* is when one loses *more than* ℓ, with probability $1-\beta$ [15].

The probability of $f(u,s)$ not exceeding the threshold ℓ is

$$\psi(u,\ell) = \int_{f(u,s)\le \ell} p(s)ds. \tag{6}$$

The β-VaR and the β-CVaR are defined respectively as

$$\ell_\beta(u) = \min\{\ell \in \mathbb{R} :\ \psi(u,\ell) \ge \beta\} \tag{7a}$$
$$\phi_\beta(u) = (1-\beta)^{-1} \int_{f(u,s)\ge \ell_\beta(u)} f(u,s)p(s)ds. \tag{7b}$$

In a nutshell, CVaR is derived by taking a weighted average between VaR and losses exceeding VaR. In [15] the authors show that the β-CVaR of the loss associated with any u can be determined by the formula

$$\phi_\beta(u) = \min_{\ell \in \mathbb{R}} F_\beta(u,\ell) \tag{8}$$

where

$$F_\beta(u,\ell) = \ell + (1-\beta)^{-1} \int_{s\in\mathbb{R}^m} [f(u,s)-\ell]^+ p(s)ds \tag{9}$$

and $[\cdot]^+$ denotes the positive part of its argument,

$$[f]^+ \begin{cases} f & \text{if } f \ge 0 \\ 0 & \text{if } f < 0. \end{cases}$$

The integral in (9) can be approximated by sampling the distribution of s, according to the density function $p(s)$. If the sampling generates a collection of M vectors $s_1,\ \ldots,\ s_M$, each of which has probability π_i of occurring, then the corresponding approximation $\tilde{F}_\beta(u,\ell)$ is given by

$$\tilde{F}_\beta(u,\ell) = \ell + \frac{1}{(1-\beta)} \sum_{i=1}^{M} \pi_i [f(u,s_i)-\ell]^+. \tag{10}$$

We will use (10) as the objective function to minimize in the stochastic optimization problem, as detailed in the following.

B. Problem formulation

For a generic price-taker power producer (i.e., whose bids do not influence the outcome of the clearing process), and given a desired probability β, the optimal bidding problem at each hour of the operational day D can be cast as

$$\min_{u,\,\ell} \quad \ell + \frac{1}{1-\beta} \sum_{i=1}^{N} \pi_i v_i \tag{11a}$$
$$\text{s.t.} \quad u_{BL} \ge 0, \tag{11b}$$
$$u_{DA} \ge 0, \tag{11c}$$
$$u_{AS}^{ij} \ge 0, \tag{11d}$$
$$v_i \ge 0, \tag{11e}$$
$$u_{BL} + u_{DA} + \sum_{k=1}^{L} u_{AS}^{ik} \le 1, \tag{11f}$$
$$v_i \ge -s_{BL}u_{BL} - s_{DA}^{i}u_{DA} - \sum_{k=1}^{L} \pi_{AS}^{ik} s_{AS}^{ik} u_{AS}^{ik}, \tag{11g}$$
$$i = 1,2,\ldots,N,\ j = 1,2,\ldots,L.$$

Constraints (11b), (11c) and (11d) impose nonnegativity of the allocated quantities, and (11f) expresses a bound on the available energy. Constraints (11e) and (11g), together with the objective function (11a), model function (10) and allow the minimization of the β-CVaR.

Causality of the control action is ensured by the fact that a unique value u_{DA} is allowed for the day-ahead market, since the actual spot price is not known by the time the optimization problem is solved.

As the considered power plant is a *price-taker* participant, only quantities u are to be decided. Otherwise, if also spot and bid prices s_{DA}, s_{AS} were optimization variables, constraint (11g) would make the problem bilinear.

IV. Simulation Results

The algorithm has been tested on an instance of the problem, relative to the trading day of April 1st, 2010 in the Italian energy market. The cleared prices of the three months preceding the simulation day (January - March, 2010) have been considered to fit the GARCH model (4) and to forecast future values of the spot price $s_{DA}(t)$.

Following the procedure presented in Section II-A, the Gaussian distribution has been sampled into $N = 9$ values varying between -3σ and 3σ to generate predicted cleared prices (or spot prices). This value of N resulted as a good trade-off in order to include a sufficient amount of different scenarios without overloading the computational burden. Bilateral contracts are not considered in this example, since no consistent data is available online to estimate the price s_{BL}. This is due to the fact that in Italy future bilateral contracts are traded on a long term basis, differently from our market assumptions. For each predicted scenario of spot price, a representative number of $L = 10$ bid prices are fixed. Then, according to Section II-B, empirical probabilities are inferred from the presented bids relative to the same period of the cleared prices (January - March, 2010). Public bids on the ancillary services market were downloaded from the website of the Italian authority [16]. The time needed for formulating and solving an instance of the optimization problem (11) on 2.66 GHz processor was 540 ms in average (max 980 ms), running `linprog` on Matlab R2009b.

In Figure 5 the obtained optimal hourly allocation for the day April, 1st, 2010 is reported. Note that the algorithm allocates the whole production on the day-ahead market in some of the central hours of the day, when the spot price is higher. Table IV shows how the production splits into different bids, for three representative spot price scenarios. The highlighted results are relative to hour 20, when the last observed price is €65.90.

V. Conclusions

In this paper a decision algorithm based on multi-stage stochastic optimization is proposed to solve the optimal bidding problem for a price-taker power producer. The algorithm is aimed at finding the optimal allocation of production between bilateral contracts, day-ahead market and ancillary services market. A first step involves time series analysis and identification of the past spot energy prices, by means of linear parameter-varying regression models, in order to forecast a given number of scenarios for the future cleared price. The empirical probability of acceptance of bid prices is then inferred from past public bids submitted by the participants to the energy market. The optimization algorithm minimizes the conditional value at risk (CVaR), matching a trade-off between profit maximization and risk control. Simulations have shown how the proposed control scheme allocates a fraction of energy production on the ancillary services in those hours of the day when this market is attractive, otherwise it reserves the whole production for the day ahead market.

Future work involves extensions of the optimal bidding problem to consider a combined production unit, consisting of more than one generator, with different characteristics and feeding (coal, hydro, wind), thus leading to dynamical modeling of the plant behavior, subject to ramp-up/ramp-down constraints.

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References

[1] T. Nogueira, A. Vale, and Z. Vale, "An electricity day ahead market simulation model," *International Conference on Renewable Energy and Power Quality*, 2003.
[2] F. Olsina, F. Garces, and H. J. Haubrich, "Modeling long-term dynamics of electricity markets," *Energy Policy*, vol. 34, pp. 1411–1433, 2006.
[3] G. Unger, "Hedging strategy and electricity contract engineering," Ph.D. dissertation, Swiss Federal Institute of Technology, 2002.
[4] W. K. Klein Haneveld and M. H. van der Vlerk, "Optimizing electricity distribution using two-stage integer recourse models," *Stochastic Optimization: Algorithms and Applications*, pp. 137–154, 2001.
[5] V. P. Gountis and A. G. Bakirtzis, "Bidding strategies for electricity producers in a competitive electricity marketplace," *IEEE Transactions on Power Systems*, vol. 19, no. 1, pp. 356–366, 2004.
[6] A. J. Wood and B. F. Wollenberg, *Power Generation Operation and Control*. Wiley, 1996.
[7] R. Rajaraman and F. Alvarado, *Optimal Bidding Strategy in Electricity Markets Under Uncertain Energy and Reserve Prices*, Power Systems Engineering Research Center, 2003.
[8] M. P. Nowak, R. Nürnberg, W. Römisch, R. Schultz, and M. Westphalen, "Stochastic programming for power production and trading under uncertainty," 2000.
[9] M. Korpaas, A. T. Holen, and R. Hildrum, "Operation and sizing of energy storage for wind power plants in a market system," *Electrical Power and Energy Systems*, vol. 25, pp. 599–606, 2003.
[10] S. E. Fleten and T. K. Kristoffersen, "Stochastic programming for optimizing bidding strategies of a nordic hydropower producer," *European Journal of Operational Research*, vol. 181, pp. 916–928, 2007.
[11] ——, "Short-term hydropower production planning by stochastic programming," *Computers & Operations Research*, vol. 35, pp. 2656–2671, 2008.
[12] S. W. Wallace and S.-E. Fleten, *Stochastic Programming Models in Energy*. Ruszczyn, A. ski and Shapiro, A., 2003, vol. 10, ch. 10.
[13] D. Most and D. Keles, "A survey of stochastic modelling approaches for liberalized electricity markets," *European Journal of Operational Research*, vol. 207, pp. 543–556, 2010.
[14] P. Theodorou and D. Karyampas, "Modeling the return and volatility of the greek electricity marginal system price," *Energy Policy*, vol. 36, pp. 2601–2609, 2008.
[15] R. T. Rockafellar and S. Uryasev, "Optimization of conditional value at risk," *Journal of Risk*, 2000.
[16] [Online]. Available: www.mercatoelettrico.org
[17] J. Janczura and R. Weron, "An empirical comparison of alternate regime-switching models for electricity spot prices," *Energy Economics*, vol. 32, pp. 1059–1073, 2010.

TABLE I
OPTIMAL BID ALLOCATION FOR ANCILLARY SERVICES AT HOUR 20

scenario	bid no.	allocated production (%)	price (€)	prob. of acceptance (%)
scenario 1, spot = €37.3	1	37.65	12.13	55.07
scenario 5, spot = €66.02	1	3.11	29.07	26.56
	2	3.28	44.94	21.37
	3	12.09	500	21.05
scenario 6, spot = €76.15	1	9.10	40.34	76.51
	2	28.56	38.23	69.17
scenario 7, spot = 87.83 €	1	4.18	27.19	6.45
	2	4.18	37.88	73.35
	3	4.18	75.97	7.91
scenario 8, spot = 101.3 €	1	4.18	45.21	12.56
	2	4.18	75.40	13.44
	3	4.18	40.18	84.76

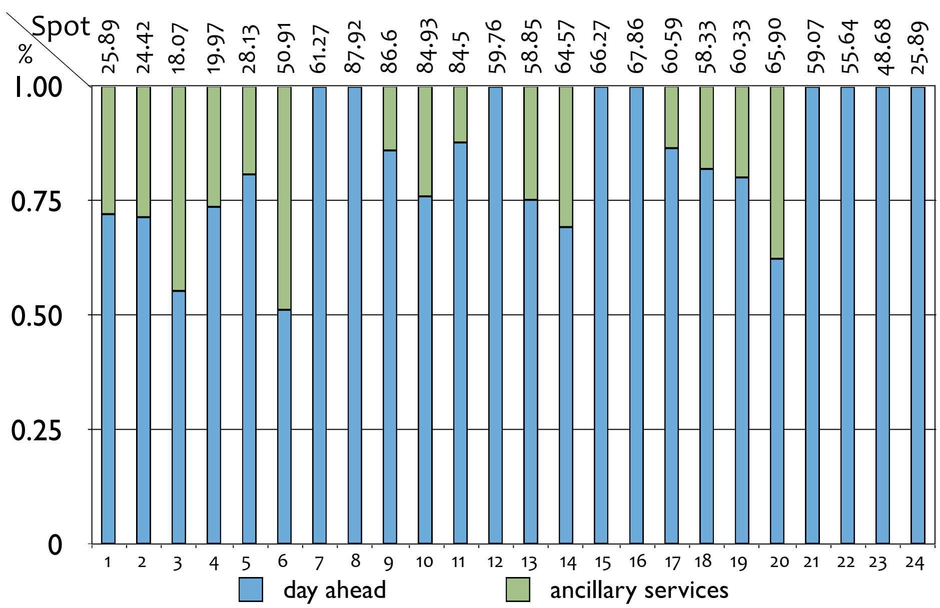

Fig. 5. Optimal hourly allocation for April 1st, 2010

[18] A. Misiorek, S. Trueck, and R. Weron, “Point and interval forecasting of spot electricity prices: Linear vs. non-linear time series models,” *Studies in Nonlinear Dynamics and Econometrics*, vol. 10, no. 3, 2006.

[19] K. Høyland and S. W. Wallace, “Generating scenario trees for multistage decision problems,” *Management Science*, vol. 47, no. 2, pp. 295–307, 2001.

[20] J. Dupačova, G. Consigli, and S. W. Wallace, “Scenarios for multistage stochastic programs,” *Annals of Operations Research*, vol. 100, pp. 25–53, 2004.