2011 50th IEEE Conference on Decision and Control and
European Control Conference (CDC-ECC)
Orlando, FL, USA, December 12-15, 2011

Derivative-Free Decentralized Adaptive Control of Large-Scale Interconnected Uncertain Systems

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Abstract— **This paper presents a derivative-free decentralized adaptive control architecture for large-scale interconnected systems with matched and unmatched time-varying uncertainties and interconnections. The assumption of unknown constant ideal weights is generalized to the existence of time-varying weights without assuming the existence of their derivatives in a time interval. As a result, the proposed approach is particularly well suited for disturbance rejection, and for adaptation in the presence of sudden change in each subsystem's uncertain dynamics, such as might be due to damage.**

I. Introduction

A fundamental assumption in most decentralized adaptive approaches is that an uncertainty is parameterized by unknown constants [1]–[7]. As a result, the class of uncertain systems that can be handled by adaptive control has been limited to that of systems with time-invariant uncertainties. In this paper, we show that the derivative-free approach developed in Refs. 8 and 9 can be extended to adaptive control of large-scale interconnected systems, and therefore both matched and unmatched time-varying uncertainties and interconnections can be handled in a decentralized adaptive control setting. A key element in this pursuit is that the assumption of unknown constants is generalized to the existence of time-varying variables, such that fast and possibly discontinuous variation in unknown parameters are allowed. This generalization adds a dimension in the tuning process such that the adaptive law uses the delayed weight estimates and the information contained in current known system states and errors.

Compared to the approach in Ref. 8, this paper shows how the state predictor developed in Ref. 11 can be employed in conjunction with the derivative-free adaptive law. In Ref. 11, it is shown that the state predictor adds a low-pass filtering effect to the weight update law and improves transient responses of adaptive systems. The state predictor resembles a reference model modified by an observer-like tracking error mismatch term such that the original reference model is recovered when the gain of the mismatch term is selected to be zero. When communication between subsystems is allowed, which renders the state of the state predictor available throughout all subsystems, it is shown that unknown matched, time-varying interconnections can seamlessly be handled by the proposed method. The proposed approach is particularly well suited for adaptation in the presence of sudden change in each subsystem's uncertain dynamics, such as might be due to damage. Boundedness of the error signals is shown by using a Lyapunov-Krasovskii functional without the need for modification terms in the adaptive law.

This research was supported in part by NASA Langley Research Center under Grant NNX08AC61A.

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The notation used in this paper is fairly standard. We write $\mathbb{R}^n$ for $n \times 1$ real column vectors, $\overline{\mathbb{R}}_+$ for the set of nonnegative real numbers, $\mathbb{R}^{n\times m}$ for the set of $n \times m$ real matrices, $(\cdot)^{\mathrm{T}}$ for transpose, $(\cdot)^{-1}$ for inverse, $\lambda_{\min}(A)$ (resp., $\lambda_{\max}(A)$) for the minimum (resp., maximum) eigenvalue of A, $|\cdot|$ for the Euclidian vector norm, $\|\cdot\|$ for the Frobenius matrix norm, $\mathrm{vec}(\cdot)$ for the column stacking operator, and $\mathrm{diag}[A, B]$ for a block diagonal matrix formed with matrices A and B on the diagonal.

II. Problem Formulation

We consider an uncertain system $\mathcal{G}$ consisting of N interconnected subsystems $\mathcal{G}_i$, $i = 1, 2, \ldots, N$. A subsystem $\mathcal{G}_i$ is described by

$$\dot{x}_i(t) = A_i x_i(t) + B_i\big[u_i(t) + \Delta_i(t, x_i)\big] + \delta_i(t, x(t)), \quad (1)$$

where $x_i(t) \in \mathbb{R}^{n_i}$ is the state of $\mathcal{G}_i$, $u_i(t) \in \mathbb{R}^{m_i}$ is the control input applied to $\mathcal{G}_i$, and $A_i \in \mathbb{R}^{n_i\times n_i}$ and $B_i \in \mathbb{R}^{n_i\times m_i}$ are known matrices. In addition, $\Delta_i : \overline{\mathbb{R}}_+ \times \mathbb{R}^{n_i} \to \mathbb{R}^{m_i}$ represents *matched* uncertainty and $\delta_i : \overline{\mathbb{R}}_+ \times \mathbb{R}^{n_1+n_2+\cdots+n_N} \to \mathbb{R}^{n_i}$ represents the possibly nonlinear and time-varying interactions with the other subsystems where $x(t) \triangleq \big[x_1^{\mathrm{T}}(t), x_2^{\mathrm{T}}(t), \ldots, x_N^{\mathrm{T}}(t)\big]^{\mathrm{T}}$. Notice that $\delta_i(t, x(t))$ is allowed to be *unmatched*. We assume that the pair (A_i, B_i) is controllable and $\mathcal{G}_i$ only has access to $x_i(t)$ and $u_i(t)$, where $u_i(t)$ is restricted to the class of admissible controls consisting of measurable functions. Hence, we consider the *strictly* decentralized control problem.

Assumption 1. The matched uncertainty in (1) can be linearly parameterized as

$$\Delta_i(t, x_i) = W_i^{\mathrm{T}}(t)\beta_i(x_i), \quad x_i \in \mathcal{D}_{xi}, \quad (2)$$

where $W_i(t) \in \mathbb{R}^{s_i\times m_i}$ is an unknown *time-varying* ideal weight matrix that satisfies $\|W_i(t)\| \le w_i^*$, $\beta_i : \mathbb{R}^{n_i} \to \mathbb{R}^{s_i}$ is a vector of known functions of the form $\beta_i(x_i) = \big[b_i,\ \beta_{1i}(x_i),\ \beta_{2i}(x_i), \ldots, \beta_{(s_i-1)i}(x_i)\big]^{\mathrm{T}} \in \mathbb{R}^{s_i}$ with a bias component $b_i > 0$, and $\mathcal{D}_{xi}$ is a sufficiently large compact set $\mathcal{D}_{xi} \in \mathbb{R}^{n_i}$.

Remark 1. Assumption 1 expands the class of uncertainties that can be represented by a given set of basis functions. That is, Assumption 1 encompasses broader classes of uncertainties than those parameterized by

$$\Delta_i(x_i) = W_i^{\mathrm{T}}\beta_i(x_i) + \varepsilon(x_i), \quad x_i \in \mathcal{D}_{xi}, \quad (3)$$

where W_i is an unknown *constant* ideal weight matrix and $\varepsilon(x_i)$ is the residual error, due to the fact that time-variation is allowed in the unknown ideal weight matrix. It also permits an explicit dependence of the uncertainty on time.

Remark 2. Assumption 1 does not place any restriction on the time derivative of the ideal weight matrix. However, the degree of time dependence will depend on how $\beta_i(x_i)$ is chosen.

Remark 3. In Assumption 1, we introduced a bias component $b_i > 0$ in $\beta_i(x_i)$. This captures the effect of external matched disturbances acting on subsystem $\mathcal{G}_i$.

Assumption 2. The function $\delta_i(t,x(t))$ in (1) satisfies

$$|\delta_i(t,x(t))| \;\le\; \alpha_i \sum_{j=1}^{N} |x_j(t)|, \quad \alpha_i > 0. \tag{4}$$

Remark 4. Assumption 2 is standard in the decentralized adaptive control literature (see, for example, [6], [7], [10]) which implies that the system interconnections satisfy a linear growth inequality.

III. Adaptive Control Architecture

Let the feedback control law for subsystem $\mathcal{G}_i$ be

$$u_i(t) = u_{\mathrm{n}i}(t) - u_{\mathrm{ad}i}(t), \tag{5}$$

where $u_{\mathrm{n}i}(t)$ is a nominal feedback control given by

$$u_{\mathrm{n}i}(t) = -K_{1i}x_i(t) + K_{2i}r_i(t), \tag{6}$$

where $K_{1i} \in \mathbb{R}^{m_i\times n_i}$ and $K_{2i} \in \mathbb{R}^{m_i\times r_i}$ are nominal control gains such that $A_{\mathrm{m}i} \triangleq A_i - B_iK_{1i}$ is Hurwitz, $B_{\mathrm{m}i} \triangleq B_iK_{2i}$, and $u_{\mathrm{ad}i}(t)$ is the adaptive feedback control given by

$$u_{\mathrm{ad}i}(t) = \hat{W}_i^{\mathrm{T}}(t)\beta_i(x_i(t)), \tag{7}$$

where $\hat{W}_i(t) \in \mathbb{R}^{s_i\times m_i}$ is an estimate of $W_i(t)$.

Consider the reference model for subsystem $\mathcal{G}_i$, characterizing the *desired* closed-loop behavior when $\Delta_i(t,x_i(t)) = 0$ and $\delta_i(t,x(t)) = 0$, given by

$$\dot{x}_{\mathrm{m}i}(t) = A_{\mathrm{m}i}x_{\mathrm{m}i}(t) + B_{\mathrm{m}i}r_i(t), \quad |r_i(t)| \le \bar{r}_i, \tag{8}$$

where $x_{\mathrm{m}i}(t) \in \mathbb{R}^{n_i}$, $r_i(t) \in \mathbb{R}^{r_i}$, $r_i \le m_i$, is a bounded piecewise continuous reference input. Since $r_i(t)$ is bounded, it follows that $x_{\mathrm{m}i}(t)$ is upper bounded by $\bar{x}_{\mathrm{m}i}(\bar{r}_i, \varepsilon_i)$ for all $x_{\mathrm{m}i}(0) \in \mathcal{B}_\varepsilon \triangleq \{x_i(t) \in \mathbb{R}^{n_i} : ||x(t)|| \le \varepsilon_i\}$.

Define the *state predictor* [11], as

$$\dot{\hat{x}}_i(t) = A_{\mathrm{m}i}\hat{x}_i(t) + B_{\mathrm{m}i}r_i(t) + L_i\big[x_i(t) - \hat{x}_i(t)\big], \tag{9}$$

where $L_i \in \mathbb{R}^{n_i\times n_i}$ is chosen such that $A_{\mathrm{e}i} \triangleq A_{\mathrm{m}i} - L_i$ is Hurwitz. Note that the state predictor serves as a reference model. Its dynamics are *approximately* same as the reference model in (8) when $\tilde{x}_i(t) \triangleq x_i(t) - \hat{x}_i(t)$ is sufficiently small.

Next, consider the *derivative-free* weight update law for a subsystem $\mathcal{G}_i$ given by

$$\hat{W}_i(t) = \Omega_{1i}\hat{W}_i(t-\tau_i) + \hat{\Omega}_{2i}(t), \tag{10}$$

where $\tau_i > 0$, and $\Omega_{1i} \in \mathbb{R}^{s_i\times s_i}$ and $\hat{\Omega}_{2i} : \mathbb{R}^{s_i} \times \mathbb{R}^{n_i} \to \mathbb{R}^{s_i\times m_i}$ satisfy

$$0 \le \Omega_{1i}^{\mathrm{T}}\Omega_{1i} < \kappa_{1i}I, \quad 0 \le \kappa_{1i} < 1, \tag{11}$$

$$\hat{\Omega}_{2i}(t) = \kappa_{2i}\beta_i(x_i(t))\tilde{x}_i^{\mathrm{T}}P_iB_i, \quad \kappa_{2i} > 0, \tag{12}$$

with $\tilde{x}_i(t) \triangleq x_i(t) - \hat{x}_i(t)$, and $P_i \in \mathbb{R}^{n_i\times n_i}$ satisfying the algebraic Riccati equation given by

$$0 = A_{\mathrm{e}i}^{\mathrm{T}}P_i + P_iA_{\mathrm{e}i} - b_i^2\kappa_{2i}P_iB_iB_i^{\mathrm{T}}P_i + Q_i, \tag{13}$$

for any symmetric matrix $Q_i > 0$.

Remark 5. We use $\tilde{x}_i(t)$ in (12) instead of using $e_i(t) \triangleq x_i(t) - x_{\mathrm{m}i}(t)$ to estimate $W_i(t)$. Note that for $L_i = 0$, (9) reduces to (8), hence, $\tilde{x}_i(t)$ becomes $e_i(t)$, if desired. However, using $\tilde{x}_i(t)$ in (12) instead of $e_i(t)$ adds a low-pass filtering effect to the weight update law. Choosing the state predictor dynamics faster than the reference model dynamics (choosing the eigenvalues of $A_{\mathrm{e}i}$ larger than the eigenvalues of $A_{\mathrm{m}i}$) aids in suppressing undesired transient behavior [11].

IV. Stability Analysis

This section presents a stability analysis for the derivative-free decentralized adaptive control architecture in Section III. Consider a parameter dependent Riccati equation [12], [13], given by

$$0 \;=\; A_{\mathrm{m}i}^{\mathrm{T}}P_{\mathrm{o}i} + P_{\mathrm{o}i}A_{\mathrm{m}i} + \bar{Q}_{\mathrm{o}i}, \tag{14}$$

$$\bar{Q}_{\mathrm{o}i} \;=\; Q_{\mathrm{o}i} + \mu_iP_{\mathrm{o}i}L_iL_i^{\mathrm{T}}P_{\mathrm{o}i}, \tag{15}$$

in which $Q_{\mathrm{o}i} > 0$ and $\mu_i > 0$.

Remark 6. Let $0 < \mu_i < \bar{\mu}_i$ define the largest set within which there is a positive-definite solution for $P_{\mathrm{o}i}$. Since $P_{\mathrm{o}i} > 0$ for $\mu_i = 0$ and $P_{\mathrm{o}i}$ depends continuously on μ_i, the existence of $P_{\mathrm{o}i}(\mu_i) > 0$ for $0 < \mu_i < \bar{\mu}_i$ is assured.

The next lemma shows that for $\mu_i < \bar{\mu}_i$, (14)–(15) can reliably be solved for $P_{\mathrm{o}i} > 0$ using the Potter approach given in Ref. 14. This also implies that $\bar{\mu}_i$ can be determined by searching for the boundary value that results in a failure of the algorithm to converge. We employ the notation $\mathrm{ric}(\cdot)$ and $\mathrm{dom}(\mathrm{ric})$ as defined in Ref. 15.

Lemma 1. Let $P_{\mathrm{o}i}$ satisfy the parameter dependent Riccati equation given by (14)–(15) and let the modified Hamiltonian be given by

$$H_i \equiv \begin{bmatrix} A_{\mathrm{m}i} & \mu_iL_iL_i^{\mathrm{T}} \\ -Q_{\mathrm{o}i} & -A_{\mathrm{m}i}^{\mathrm{T}} \end{bmatrix}. \tag{16}$$

Then, for all $0 < \mu_i < \bar{\mu}_i$, $H_i \in \mathrm{dom}(\mathrm{ric})$ and $P_{\mathrm{o}i} = \mathrm{ric}(H)$.

Proof. The proof follows from Lemma 1 and Lemma 2 of Ref. 15. □

Assumption 3. $d_{1i} \triangleq \lambda_{\min}(Q_i) - (3+N)\alpha_i\lambda_{\max}(P_i) - 1/\mu_i$ and $d_{2i} \triangleq \lambda_{\min}(Q_{\mathrm{o}i}) - \alpha_iN\lambda_{\max}(P_i)$ are positive by suitable selection of the design parameters.

Theorem 1. Consider the uncertain system $\mathcal{G}$ consisting of N interconnected subsystems $\mathcal{G}_i$ described by (1) subject to Assumptions 1, 2, and 3. Consider, in addition, the subsystem control laws given by (5), with the nominal controllers given by (6), and with the adaptive controllers given by (7) along with (9), (10) subject to the conditions in (11), (12). Then, $\tilde{x}_i(t)$, $\hat{e}_i(t) \triangleq \hat{x}_i(t) - x_{\mathrm{m}i}(t)$, and $\tilde{W}_i(t) \triangleq W_i(t) - \hat{W}_i(t)$ are uniformly ultimately bounded (UUB) for all $i = 1, 2, \ldots, N$.

Proof. We can write dynamics of $\hat{e}_i$ using (8) and (9) as

$$\dot{\hat{e}}_i(t) = A_{\mathrm{m}i}\hat{e}_i(t) + L_i\tilde{x}_i(t). \tag{17}$$

Consider

$$\mathcal{V}_{1i}(\hat{e}_i(t)) = \hat{e}_i^{\mathrm{T}}(t)P_{\mathrm{o}i}\hat{e}_i(t), \tag{18}$$

where $P_{\mathrm{o}i} > 0$ satisfies the parameter dependent Riccati equation (14)–(15) with $\mu_i < \bar{\mu}_i$. The time derivative of (18) along dynamics of $\hat{e}_i$ (17) is given by

$$\dot{\mathcal{V}}_{1i}(\cdot) \;=\; -\hat{e}_i^{\mathrm{T}}(t)\bar{Q}_{\mathrm{o}i}\hat{e}_i(t) + 2\hat{e}_i^{\mathrm{T}}(t)P_{\mathrm{o}i}L_i\tilde{x}_i(t). \tag{19}$$

Consider $|a^{\mathrm{T}}b| \le \gamma a^{\mathrm{T}}a + b^{\mathrm{T}}b/4\gamma$, $\gamma > 0$, that follows from Young's inequality [16] extended to the vector case for any vectors a and b. Applying this to the last term in (19) produces

$$2|\hat{e}_i^{\mathrm{T}}P_{\mathrm{o}i}L_i\tilde{x}_i| \;\le\; \mu_i\hat{e}_i^{\mathrm{T}}P_{\mathrm{o}i}L_iL_i^{\mathrm{T}}P_{\mathrm{o}i}\hat{e}_i + \tilde{x}_i^{\mathrm{T}}\tilde{x}_i/\mu_i. \tag{20}$$

Using (15) and (20) in (19) results in

$$\dot{\mathcal{V}}_{1i}(\cdot) \;=\; -\hat{e}_i^{\mathrm{T}}(t)Q_{\mathrm{o}i}\hat{e}_i(t) + \tilde{x}_i^{\mathrm{T}}(t)\tilde{x}_i(t)/\mu_i. \tag{21}$$

Next, we can write dynamics of $\tilde{x}_i$ using (1) and (9) as

$$\dot{\tilde{x}}_i(t) = A_{\mathrm{e}i}\tilde{x}_i(t) + B_i\tilde{W}_i^{\mathrm{T}}(t)\beta_i(x_i(t)) + \delta_i(t,x(t)). \tag{22}$$

Using (10) and defining

$$\Omega_{2i}(t) \triangleq W_i(t) - \Omega_{1i} W_i(t-\tau_i), \quad (23)$$

where $\|\Omega_{2i}(t)\| \le \Omega_i^*$, $\Omega_i^* = w_i^*(1+\|\Omega_{1i}(t)\|)$, the dynamics of $\tilde{W}_i$ can be written as

$$\tilde{W}_i(t) = \Omega_{1i}\tilde{W}_i(t-\tau_i) + \Omega_{2i}(t) - \hat{\Omega}_{2i}(t). \quad (24)$$

Using (24) in (22) under Assumption 1 we obtain

$$\begin{aligned}\dot{\tilde{x}}_i(t) &= A_{\mathrm{e}i}\tilde{x}_i(t) + B_i\big[\Omega_{1i}\tilde{W}_i(t-\tau_i) + \Omega_{2i}(t) \\ &\quad -\hat{\Omega}_{2i}(t)\big]^{\mathrm{T}}\beta_i(x_i(t)) + \delta_i(t,x(t)). \quad (25)\end{aligned}$$

Consider

$$\begin{aligned}\mathcal{V}_{2i}(\tilde{x}_i(t), \tilde{W}_{ti}) &= \tilde{x}_i^{\mathrm{T}}(t)P_i\tilde{x}_i(t) \\ &\quad +\rho_i \ \mathrm{tr}\Big[\int_{t-\tau_i}^{t} \tilde{W}_i^{\mathrm{T}}(s)\tilde{W}_i(s)\mathrm{d}s\Big], \quad (26)\end{aligned}$$

where $\rho_i > 0$, and $\tilde{W}_{ti}$ represents $\tilde{W}_i(t)$ over the time interval $t-\tau_i$ to t. The time derivative of (26) along the trajectories of (25) and (24) is given by

$$\begin{aligned}\dot{\mathcal{V}}_{2i}(\cdot) &= -\tilde{x}_i^{\mathrm{T}}(t)\bar{Q}_i\tilde{x}_i(t) + 2\tilde{x}_i(t)P_iB_i[\Omega_{1i}\tilde{W}_i(t-\tau_i)]^{\mathrm{T}} \\ &\quad \times\beta_i(x_i(t)) - 2\tilde{x}_i^{\mathrm{T}}(t)P_iB_i\hat{\Omega}_{2i}^{\mathrm{T}}(t)\beta_i(x_i(t)) \\ &\quad +2\tilde{x}_i^{\mathrm{T}}(t)P_iB_i\Omega_{2i}^{\mathrm{T}}(t)\beta_i(x_i(t)) + \rho_i \ \mathrm{tr}\big[-\xi_i\tilde{W}_i^{\mathrm{T}}(t) \\ &\quad \times\tilde{W}_i(t) + \eta_i\tilde{W}_i^{\mathrm{T}}(t)\tilde{W}_i(t) - \tilde{W}_i^{\mathrm{T}}(t-\tau_i) \\ &\quad \times\tilde{W}_i(t-\tau_i)\big] + 2\tilde{x}_i^{\mathrm{T}}P_i\delta_i(t,x(t)), \quad (27)\end{aligned}$$

where $\eta_i = 1+\xi_i$, $\xi_i > 0$, and

$$\bar{Q}_i \triangleq Q_i - b_i^2\kappa_{2i}P_iB_iB_i^{\mathrm{T}}P_i. \quad (28)$$

Using (24) to expand $\mathrm{tr}\big[\eta_i\tilde{W}_i^{\mathrm{T}}(t)\tilde{W}_i(t)\big]$ in (27) produces

$$\begin{aligned}\dot{\mathcal{V}}_{2i}(\cdot) &= -\tilde{x}_i^{\mathrm{T}}(t)\bar{Q}_i\tilde{x}_i(t) + 2\tilde{x}_i(t)P_iB_i[\Omega_{1i}\tilde{W}_i(t-\tau_i)]^{\mathrm{T}} \\ &\quad \times\beta_i(x_i(t)) - 2\tilde{x}_i^{\mathrm{T}}(t)P_iB_i\hat{\Omega}_{2i}^{\mathrm{T}}(t)\beta_i(x_i(t)) \\ &\quad +2\tilde{x}_i^{\mathrm{T}}(t)P_iB_i\Omega_{2i}^{\mathrm{T}}(t)\beta_i(x_i(t)) + \rho_i \ \mathrm{tr}\big[-\xi_i\tilde{W}_i^{\mathrm{T}}(t) \\ &\quad \times\tilde{W}_i(t) - \tilde{W}_i^{\mathrm{T}}(t-\tau_i)\tilde{W}_i(t-\tau_i) \\ &\quad +\eta_i\tilde{W}_i^{\mathrm{T}}(t-\tau_i)\Omega_{1i}^{\mathrm{T}}\Omega_{1i}\tilde{W}_i(t-\tau_i) + \eta_i\hat{\Omega}_{2i}^{\mathrm{T}}(t) \\ &\quad \times\hat{\Omega}_{2i}(t) + \eta_i\Omega_{2i}^{\mathrm{T}}(t)\Omega_{2i}(t) - 2\eta_i\hat{\Omega}_{2i}^{\mathrm{T}}(t)\Omega_{1i} \\ &\quad \times\tilde{W}_i(t-\tau_i) + 2\eta_i\tilde{W}_i^{\mathrm{T}}(t-\tau_i)\Omega_{1i}^{\mathrm{T}}\Omega_{2i}(t) \\ &\quad -2\eta_i\hat{\Omega}_{2i}^{\mathrm{T}}(t)\Omega_{2i}(t)\big] + 2\tilde{x}_i^{\mathrm{T}}P_i\delta_i(t,x(t)). \quad (29)\end{aligned}$$

Young's inequality can be generalized to matrices as $\mathrm{tr}\big[A^{\mathrm{T}}B\big] = \mathrm{vec}(A)^{\mathrm{T}}\mathrm{vec}(B) \le \gamma\mathrm{vec}(A)^{\mathrm{T}}\mathrm{vec}(A) + \mathrm{vec}(B)^{\mathrm{T}}\mathrm{vec}(B)/4\gamma = \gamma\mathrm{tr}\big[A^{\mathrm{T}}A\big] + \mathrm{tr}\big[B^{\mathrm{T}}B\big]/4\gamma$, $\gamma > 0$, for any matrices A and B having appropriate dimensions. Using this, we can write

$$\begin{aligned}&\mathrm{tr}\ \big[2\eta_i\tilde{W}_i^{\mathrm{T}}(t-\tau_i)\Omega_{1i}^{\mathrm{T}}\Omega_{2i}(t)\big] \\ &\quad \le \mathrm{tr}\ \big[\gamma_i\tilde{W}_i^{\mathrm{T}}(t-\tau_i)\Omega_{1i}^{\mathrm{T}}\Omega_{1i}\tilde{W}_i(t-\tau_i)\big] \\ &\quad\quad +\mathrm{tr}\ \big[\eta_i^2\Omega_{2i}^{\mathrm{T}}(t)\Omega_{2i}(t)/\gamma_i\big],\ \gamma_i > 0. \quad (30)\end{aligned}$$

Using (12) with $\kappa_{2i} \triangleq 1/\rho_i\eta_i > 0$ for $\hat{\Omega}_{2i}(t)$, and substituting (30) in (29), it follows that

$$\begin{aligned}\dot{\mathcal{V}}_{2i}(\cdot) &\le -\tilde{x}_i^{\mathrm{T}}(t)\bar{Q}_i\tilde{x}_i(t) - \kappa_{2i}\tilde{x}_i^{\mathrm{T}}(t)P_iB_iB_i^{\mathrm{T}}P_i\tilde{x}_i(t) \\ &\quad \times\beta_i^{\mathrm{T}}(x_i(t))\beta_i(x_i(t)) - \rho_i\xi_i\mathrm{tr}\ \big[\tilde{W}_i^{\mathrm{T}}(t)\tilde{W}_i(t)\big] \\ &\quad -\rho_i\mathrm{tr}\ \Big[\tilde{W}_i^{\mathrm{T}}(t-\tau_i)\big[I-(\eta_i+\gamma_i)\Omega_{1i}^{\mathrm{T}}\Omega_{1i}\big] \\ &\quad \times\tilde{W}_i(t-\tau_i)\Big] + \rho_i(\eta_i+\eta_i^2/\gamma_i)\mathrm{tr}\ \big[\Omega_{2i}^{\mathrm{T}}(t)\Omega 2i(t)\big] \\ &\quad +2\tilde{x}_i^{\mathrm{T}}P_i\delta_i(t,x(t)). \quad (31)\end{aligned}$$

Since $\beta_i^{\mathrm{T}}(x_i(t))\beta_i(x_i(t)) \ge b_i^2$, using (28) in (31) results in

$$\begin{aligned}\dot{\mathcal{V}}_{2i}(\cdot) &\le -\tilde{x}_i^{\mathrm{T}}(t)Q_i\tilde{x}_i(t) - \rho_i\xi_i\mathrm{tr}\ \big[\tilde{W}_i^{\mathrm{T}}(t)\tilde{W}_i(t)\big] \\ &\quad -\rho_i\mathrm{tr}\ \Big[\tilde{W}_i^{\mathrm{T}}(t-\tau_i)\big[I-(\eta_i+\gamma_i)\Omega_{1i}^{\mathrm{T}}\Omega_{1i}\big] \\ &\quad \times\tilde{W}_i(t-\tau_i)\Big] + \rho_i(\eta_i+\eta_i^2/\gamma_i)\mathrm{tr}\ \big[\Omega_{2i}^{\mathrm{T}}(t)\Omega_{2i}(t)\big] \\ &\quad +2\tilde{x}_i^{\mathrm{T}}P_i\delta_i(t,x(t)). \quad (32)\end{aligned}$$

Using (11) with $\kappa_{1i} \triangleq 1/(\eta_i+\gamma_i) < 1$ for Ω_{1i} yields

$$\begin{aligned}\dot{\mathcal{V}}_{2i}(\cdot) &\le -c_{1i}|\tilde{x}_i|^2 - c_{2i}\|\tilde{W}_i(t)\|^2 - c_{3i}\|\tilde{W}_i(t-\tau_i)\|^2 \\ &\quad +c_{4i} + 2\tilde{x}_i^{\mathrm{T}}P_i\delta_i(t,x(t)), \quad (33)\end{aligned}$$

where $c_{1i} \triangleq \lambda_{\min}(Q_i)$, $c_{2i} \triangleq \rho_i\xi_i$, $c_{3i} \triangleq \rho_i\lambda_{\min}(I - \kappa_{1i}^{-1}\Omega_{1i}^{\mathrm{T}}\Omega_{1i})$, and $c_{4i} \triangleq \rho_i(\eta_i+\eta_i^2/\gamma_i)\Omega_i^*$. Since $x_j(t) = \tilde{x}_j(t) + \hat{x}_j(t) = \tilde{x}_j(t) + \hat{e}_j(t) + x_{\mathrm{m}j}(t)$, it follows from Assumption 2 that

$$|\delta_i(t,x(t))| \le \sum_{j=1}^{N}\alpha_i\big[|\tilde{x}_j(t)| + |\hat{e}_j(t)| + \bar{x}_{\mathrm{m}j}\big]. \quad (34)$$

Furthermore, using (34) in the last term of (23) and applying Young's inequality results in

$$\begin{aligned}|2\tilde{x}_i^{\mathrm{T}}P_i\delta_i(t,x(t))| &\le \alpha_i\lambda_{\max}(P_i)\sum_{j=1}^{N}\big[3|\tilde{x}_i(t)|^2 + |\tilde{x}_j(t)|^2 \\ &\quad +|\hat{e}_j(t)|^2 + \bar{x}_{\mathrm{m}j}^2\big]. \quad (35)\end{aligned}$$

Hence, (33) becomes

$$\begin{aligned}\dot{\mathcal{V}}_{2i}(\cdot) &\le -\big[c_{1i} - 3\alpha_i\lambda_{\max}(P_i)\big]|\tilde{x}_i|^2 - c_{2i}\|\tilde{W}_i(t)\|^2 \\ &\quad -c_{3i}\|\tilde{W}_i(t-\tau_i)\|^2 + \alpha_i\lambda_{\max}(P_i)\sum_{j=1}^{N}|\tilde{x}_j|^2 \\ &\quad +\alpha_i\lambda_{\max}(P_i)\sum_{j=1}^{N}|\hat{e}_j|^2 + \varphi_i, \quad (36)\end{aligned}$$

where $\varphi_i \triangleq c_{4i} + \alpha_i\lambda_{\max}(P_i)\sum_{j=1}^{N}\bar{x}_{\mathrm{m}j}^2$.

Next, define the candidate Lyapunov-Krasovskii functional for the i-th subsystem

$$\mathcal{V}_i(\hat{e}_i(t), \tilde{x}_i(t), \tilde{W}_{ti}) \triangleq \mathcal{V}_{1i}(\hat{e}_i(t)) + \mathcal{V}_{2i}(\tilde{x}_i(t), \tilde{W}_{ti}). \quad (37)$$

The time derivative of (37) is written directly from (21) and (36) as

$$\begin{aligned}\dot{\mathcal{V}}_i(\cdot) &\le -\big[c_{1i} - 3\alpha_i\lambda_{\max}(P_i) - 1/\mu_i\big]|\tilde{x}_i|^2 \\ &\quad -\lambda_{\min}(Q_{\mathrm{o}i})|\hat{e}_i(t)|^2 - c_{2i}\|\tilde{W}_i(t)\|^2 \\ &\quad -c_{3i}\|\tilde{W}_i(t-\tau_i)\|^2 + \alpha_i\lambda_{\max}(P_i)\sum_{j=1}^{N}|\tilde{x}_j|^2 \\ &\quad +\alpha_i\lambda_{\max}(P_i)\sum_{j=1}^{N}|\hat{e}_j|^2 + \varphi_i. \quad (38)\end{aligned}$$

Now introducing

$$\mathcal{V}(\cdot) = \sum_{i=1}^{N}\mathcal{V}_i(\hat{e}_i(t), \tilde{x}_i(t), \tilde{W}_{ti}), \quad (39)$$

for the whole system $\mathcal{G}$ results in

$$\dot{\mathcal{V}}(\cdot) \le \sum_{i=1}^{N}\Big[-d_{1i}|\tilde{x}(t)|^2 - d_{2i}|\hat{e}_i(t)|^2 - c_{2i}\|\tilde{W}_i(t)\|^2$$

$$-c_{3i}\|\tilde{W}_i(t-\tau_i)\|^2+\varphi_i\Big], \quad (40)$$

where $d_{1i}>0$ and $d_{2i}>0$ are given in Assumption 3 (see Remark 9). Either $|\tilde{x}_i(t)|>\Psi_{1i}$ or $|\hat{e}_i(t)|>\Psi_{2i}$ or $\|\tilde{W}_i(t)\|>\Psi_{3i}$ or $\|\tilde{W}_i(t-\tau_i)\|>\Psi_{4i}$ renders $\dot{\mathcal{V}}(\cdot)<0$, where $\Psi_{1i}\triangleq\sqrt{\varphi_i/d_{1i}}$, $\Psi_{2i}\triangleq\sqrt{\varphi_i/d_{2i}}$, $\Psi_{3i}\triangleq\sqrt{\varphi_i/c_{2i}}$, and $\Psi_{4i}\triangleq\sqrt{\varphi_i/c_{3i}}$. Hence, $\tilde{x}_i(t)$, $\hat{e}_i(t)$, and $\tilde{W}_i(t)$ are UUB for all $i=1,2,\ldots,N$. □

Corollary 1. Under the conditions of Theorem 1, $e_i(t)$ is bounded for all $i=1,2,\ldots,N$.

Proof. It follows from

$$\begin{aligned}|e_i(t)| &= |x_i(t)-x_{\mathrm{m}i}(t)|\\ &\leq |x_i(t)-\hat{x}_i(t)|+|\hat{x}_i(t)-x_{\mathrm{m}i}(t)|\\ &= |\tilde{x}_i(t)|+|\hat{e}_i(t)|, \quad (41)\end{aligned}$$

and the boundedness of $\tilde{x}_i(t)$ and $\hat{e}_i(t)$ $i=1,2,\ldots,N$ by Theorem 1 that $e_i(t)$ is bounded for all $i=1,2,\ldots,N$. □

Remark 7. The derivative-free weight update law given by (10) does not require a modification term to prove the error dynamics, including the weight errors, are UUB.

Remark 8. Derivative-free adaptive control in (10) is not simply the equivalent of an Euler integration of a conventional adaptive control law. This point is discussed in Remark 3.1 of Ref. 8.

Remark 9. In the proof of Theorem 1, we require that Assumption 3 holds by suitable selections of the design parameters. This can be achieved by ensuring that $\lambda_{\max}(P_i)$ is sufficiently small. One way this can be done is to make the state predictor dynamics in (9) fast by choosing L_i to place eigenvalues of $A_{\mathrm{e}i}$ sufficiently far from the origin in the left half of the complex plane. Similar conditions are required to be satisfied in the decentralized adaptive control literature. See, for example, Assumption 6.1 of [6], (26) of [3], (19) of [4], and (17) of [5].

Remark 10. Derivative-free adaptive control does not employ an integrator in its weight update law. This is advantageous from the perspective of augmenting a nominal controller that employs integral action to ensure that the regulated output variables track $r_i(t)$ for constant disturbances, regardless of how these disturbances may enter the system. An example illustrating this advantage is provided in Section V of Ref. 8.

Define

$$q_i(t)\triangleq[\hat{e}_i^{\mathrm{T}}(t),\tilde{x}_i^{\mathrm{T}}(t),\tilde{v}_i(t,\tau_i)]^{\mathrm{T}}, \quad (42)$$

where $\tilde{v}_i^2(t,\tau_i)\triangleq\mathrm{tr}\big[\int_{t-\tau_i}^t\tilde{W}_i^{\mathrm{T}}(s)\tilde{W}_i(s)\mathrm{d}s\big]$, and let $\mathcal{B}_{ri}=\{q_i(t):|q_i(t)|<r_i\}$, such that $\mathcal{B}_{ri}\subset\mathcal{D}_{qi}$ for a sufficiently large compact set $\mathcal{D}_{qi}$. Then, we have the following corollary.

Corollary 2. Under the conditions of Theorem 1, an estimate for the ultimate bound for q_i is given by

$$r_i=\sqrt{\frac{\lambda_{\max}(P_i)\Psi_{1i}^2+\lambda_{\max}(P_{\mathrm{o}i})\Psi_{2i}^2+\rho_i\tau_i\Psi_{3i}^2}{\lambda_{\min}(\tilde{P}_i)}}, \quad (43)$$

for each subsystem $\mathcal{G}_i$, $i=1,2,\ldots,N$, where $\tilde{P}_i=\mathrm{diag}[P_i,P_{\mathrm{o}i},\rho_i]$.

Proof. Denote $\Omega_{\alpha i}=\{q_i(t)\in\mathcal{B}_{ri}:q_i^{\mathrm{T}}(t)\tilde{P}_iq_i(t)\leq\hat{\alpha}_i\}$, $\hat{\alpha}_i=\min_{||q_i(t)||=r_i}q_i^{\mathrm{T}}(t)\tilde{P}_iq_i(t)=r_i^2\underline{\lambda}(\tilde{P}_i)$. Since

$$\mathcal{V}_i(\hat{e}_i(t),\tilde{x}_i(t),\tilde{W}_{ti})=q_i^{\mathrm{T}}(t)\tilde{P}_iq_i(t), \quad (44)$$

it follows that $\Omega_{\alpha i}$ is an invariant set if and only if

$$\hat{\alpha}_i\geq\lambda_{\max}(P_i)\Psi_{1i}^2+\lambda_{\max}(P_{\mathrm{o}i})\Psi_{2i}^2+\rho_i\tau_i\Psi_{3i}^2. \quad (45)$$

Thus, the minimum size of $\mathcal{B}_{ri}$ that ensures this condition has radius given by (43). □

Remark 11. The proofs of Theorem 1 and Corollary 2 assume that the sets $\mathcal{D}_{xi}$ and $\mathcal{D}_{qi}$ for each subsystem are sufficiently large. If we define $\bar{\mathcal{B}}_{ri}$ as the largest ball contained in $\mathcal{D}_{qi}$, and assume that the initial conditions are such that $q_i(0)\subset\bar{\mathcal{B}}_{ri}$, then we have added the condition that $r_i<\bar{r}_i$, which implies a lower bound on ρ_i. It can be shown that in this case the lower bound must be such that $\lambda_{\min}(\tilde{P}_i)=\rho_i$. With r_i defined by (43) and $\lambda_{\min}(\tilde{P}_i)=\rho_i$, the condition $r_i<\bar{r}_i$ implies

$$\rho_i > \frac{\lambda_{\max}(P_i)\Psi_{1i}^2+\lambda_{\max}(P_{\mathrm{o}i})\Psi_{2i}^2}{\bar{r}^2-\tau_i\Psi_{3i}^2}. \quad (46)$$

Since $\kappa_{2i}=1/\rho_i\eta_i$, $\eta_i>1$, it follows from (46) that $\bar{r}_i$ should ensure that

$$\kappa_{2i} < \frac{\bar{r}^2-\tau_i\Psi_{3i}^2}{\lambda_{\max}(P_i)\Psi_{1i}^2+\lambda_{\max}(P_{\mathrm{o}i})\Psi_{2i}^2}. \quad (47)$$

Therefore, the meaning of $\mathcal{D}_{qi}$ *sufficiently large* in Corollary 2 is that $\bar{r}_i>\sqrt{\kappa_2\big(\lambda_{\max}(P_i)\Psi_{1i}^2+\lambda_{\max}(P_{\mathrm{o}i})\Psi_{2i}^2\big)+\tau_i\Psi_{3i}^2}$ and $q_i(0)\subset\mathcal{D}_{\bar{r}_i}$. The meaning of $\mathcal{D}_{xi}$ *sufficiently large* is difficult to characterize precisely since $x_i(t)$ depends on both $r_i(t)$ and $x_i(0)$. Nevertheless it can be seen that increasing κ_{2i} implies increasing the require size of the set $\mathcal{D}_{xi}$.

V. A Special Case

This section considers a special case in which the system interconnections are matched and the predictor states of each subsystems can be communicated across all other subsystems, so that

$$\delta_i(t,x(t))=B_if_i(t,\theta(t)), \quad (48)$$

where $f_i:\bar{\mathbb{R}}_+\times\mathbb{R}^{n_1+\cdots+n_{i-1}+n_{i+1}+\cdots+n_N}\to\mathbb{R}^{n_i}$,

$$\theta(t)\triangleq\big[x_1^{\mathrm{T}}(t),\ldots,x_{i-1}^{\mathrm{T}}(t),x_{i+1}^{\mathrm{T}}(t),\ldots,x_N^{\mathrm{T}}(t)\big]^{\mathrm{T}}, \quad (49)$$

and Assumption 2 is replaced by the following assumption.

Assumption 4. The matched interconnections in (48) can be linearly parameterized as

$$f_i(t,\theta(t))=V_i^{\mathrm{T}}(t)\sigma_i(\theta(t)),\quad \theta\in\mathcal{D}_\theta, \quad (50)$$

where $V_i(t)\in\mathbb{R}^{h_i\times m_i}$ is an unknown *time-varying* ideal weight matrix that satisfies $\|V_i(t)\|\leq v_i^*$, $\sigma_i:\mathbb{R}^{n_1+\cdots+n_{i-1}+n_{i+1}+\cdots+n_N}\to\mathbb{R}^{h_i}$ is a vector of known Lipschitz continuous functions of the form $\sigma_i(\theta)=\big[\varsigma_i,\ \sigma_{1i}(\theta),\ \sigma_{2i}(\theta),\ldots,\sigma_{(h_i-1)i}(\theta)\big]^{\mathrm{T}}\in\mathbb{R}^{h_i}$ with a bias component $\varsigma_i\geq0$, and $\mathcal{D}_\theta$ is a sufficiently large compact set $\mathcal{D}_\theta\in\mathbb{R}^{n_1+\cdots+n_{i-1}+n_{i+1}+\cdots+n_N}$.

We modify the adaptive feedback control given by (7) by using the predictor states of the other subsystems

$$u_{\mathrm{ad}i}(t)=\hat{W}_i^{\mathrm{T}}(t)\beta_i(x_i(t))+\hat{V}_i(t)\sigma_i(\hat{\theta}(t)), \quad (51)$$

where

$$\hat{\theta}(t)\triangleq\big[\hat{x}_1^{\mathrm{T}}(t),\ldots,\hat{x}_{i-1}^{\mathrm{T}}(t),\hat{x}_{i+1}^{\mathrm{T}}(t),\ldots,\hat{x}_N^{\mathrm{T}}(t)\big]^{\mathrm{T}} \quad (52)$$

$\hat{V}_i(t)\in\mathbb{R}^{h_i\times m_i}$ is an estimate of $V_i(t)$ obtained from the *derivative-free* weight update law given by

$$\hat{V}_i(t)=\Omega_{3i}\hat{V}_i(t-\bar{\tau}_i)+\hat{\Omega}_{4i}(t), \quad (53)$$

where $\bar{\tau}_i>0$, and $\Omega_{3i}\in\mathbb{R}^{h_i\times h_i}$ and $\hat{\Omega}_{4i}:\mathbb{R}^{h_i}\times$

$\mathbb{R}^{n_1+\cdots+n_{i-1}+n_{i+1}+\cdots+n_N} \to \mathbb{R}^{h_i\times m_i}$ satisfy

$$0 \le \Omega_{3i}^{\mathrm{T}}\Omega_{3i} < \kappa_{3i}I, \quad 0 \le \kappa_{3i} < 1, \tag{54}$$

$$\hat{\Omega}_{4i}(t) = \kappa_{4i}\sigma_i(\hat{\theta})\tilde{x}_i^{\mathrm{T}}P_iB_i, \quad \kappa_{4i} > 0. \tag{55}$$

Assumption 5. $\bar{d}_{1i} \triangleq \lambda_{\min}(Q_i) - Nv_i^* L_{\sigma i}\|P_iB_i\| > 0$ by suitable selection of the design parameters, where $L_{\sigma i}$ is the Lipschitz constant corresponding to $\sigma_i(\theta(t))$

Theorem 2. Consider the uncertain system $\mathcal{G}$ consisting of N interconnected subsystems $\mathcal{G}_i$ described by (1) subject to Assumptions 1, 4, and 5. Consider, in addition, the subsystem control laws given by (5), with the nominal controllers given by (6), and with the adaptive controllers given by (51) along with (9), (10), (53) subject to the conditions in (11), (12), (54), (55). Then, $\tilde{x}_i(t)$, $\hat{e}_i(t)$, $\tilde{W}_i(t)$, and $\tilde{V}_i(t) \triangleq V_i(t) - \hat{V}_i(t)$ are uniformly ultimately bounded (UUB) for all $i = 1, 2, \ldots, N$.

Proof. We can write dynamics of $\tilde{x}_i$ using (1) and (9) as

$$\begin{aligned}\dot{\tilde{x}}_i(t) &= A_{\mathrm{e}i}\tilde{x}_i(t) + B_i\tilde{W}_i^{\mathrm{T}}(t)\beta_i(x_i(t)) \\ &\quad + B_i\tilde{V}_i^{\mathrm{T}}(t)\sigma_i(\hat{\theta}(t)) + B_ig_i(\theta(t),\hat{\theta}(t)),\end{aligned} \tag{56}$$

where $g_i(\theta(t),\hat{\theta}(t)) = V_i^{\mathrm{T}}(t)\big[\sigma_i(\theta(t)) - \sigma_i(\hat{\theta}(t))\big]$ with

$$\begin{aligned}|g_i(\theta(t),\hat{\theta}(t))| &\le v_i^* L_{\sigma i}|\tilde{\theta}(t)| \\ &\le v_i^* L_{\sigma i}\sum_{j=1,\ j\ne i}^{N}|\tilde{x}_j(t)|,\end{aligned} \tag{57}$$

where $\tilde{\theta}(t) \triangleq \theta(t) - \hat{\theta}(t)$. Using (52) and defining

$$\Omega_{4i}(t) \triangleq V_i(t) - \Omega_{3i}V_i(t-\bar{\tau}_i), \tag{58}$$

where $\|\Omega_{4i}(t)\| \le \bar{\Omega}_i^*$, $\bar{\Omega}_i^* = v_i^*(1+\|\Omega_{3i}(t)\|)$, the dynamics of $\tilde{V}_i$ can be written as

$$\tilde{V}_i(t) = \Omega_{3i}\tilde{V}_i(t-\bar{\tau}_i) + \Omega_{4i}(t) - \hat{\Omega}_{4i}(t). \tag{59}$$

Using (24) and (59), dynamics of $\tilde{x}_i$ in (56) under Assumptions 1 and 4 becomes

$$\begin{aligned}\dot{\tilde{x}}_i(t) &= A_{\mathrm{e}i}\tilde{x}_i(t) + B_i\big[\Omega_{1i}\tilde{W}_i(t-\tau_i) + \Omega_{2i}(t) \\ &\quad -\hat{\Omega}_{2i}(t)\big]^{\mathrm{T}}\beta_i(x_i(t)) + B_i\big[\Omega_{3i}\tilde{V}_i(t-\bar{\tau}_i) \\ &\quad +\Omega_{4i}(t) - \hat{\Omega}_{4i}(t)\big]^{\mathrm{T}}\sigma_i(\hat{\theta}(t)) \\ &\quad +B_ig_i(\theta(t),\hat{\theta}(t)).\end{aligned} \tag{60}$$

Consider the Lyapunov-Krasovskii functional given by

$$\begin{aligned}\mathcal{V}_i(\tilde{x}_i(t),\tilde{W}_{ti},\tilde{V}_{ti}) &= \tilde{x}_i^{\mathrm{T}}(t)P_i\tilde{x}_i(t) \\ &\quad +\rho_i\ \mathrm{tr}\Big[\int_{t-\tau_i}^{t}\tilde{W}_i^{\mathrm{T}}(s)\tilde{W}_i(s)\mathrm{d}s\Big] \\ &\quad +\bar{\rho}_i\ \mathrm{tr}\Big[\int_{t-\bar{\tau}_i}^{t}\tilde{V}_i^{\mathrm{T}}(s)\tilde{V}_i(s)\mathrm{d}s\Big],\end{aligned} \tag{61}$$

where $\rho_i > 0$, $\bar{\rho}_i > 0$, $\tilde{W}_{ti}$ represents $\tilde{W}_i(t)$ over the time interval $t-\tau_i$ to t, and $\tilde{V}_{ti}$ represents $\tilde{V}_i(t)$ over the time interval $t-\bar{\tau}_i$ to t. The time derivative of (61) along the trajectories of (60), (24), and (59), using similar arguments as in the proof of Theorem 1, is given by

$$\begin{aligned}\dot{\mathcal{V}}_i(\cdot) &\le -c_{1i}|\tilde{x}_i(t)|^2 - c_{2i}\|\tilde{W}_i(t)\|^2 - c_{3i}\|\tilde{W}_i(t-\tau_i)\|^2 \\ &\quad -\bar{c}_{2i}\|\tilde{V}_i(t)\|^2 - \bar{c}_{3i}\|\tilde{V}_i(t-\bar{\tau}_i)\|^2 + \bar{c}_{4i} \\ &\quad +2\tilde{x}_i^{\mathrm{T}}(t)P_iB_ig_i(\theta(t),\hat{\theta}(t)),\end{aligned} \tag{62}$$

where $c_{1i} \triangleq \lambda_{\min}(Q_i)$, $c_{2i} \triangleq \rho_i\xi_i$, $\bar{c}_{2i} \triangleq \bar{\rho}_i\bar{\xi}_i$, $\bar{c}_{3i} \triangleq \bar{\rho}_i\lambda_{\min}(I - \bar{\kappa}_{3i}^{-1}\Omega_{3i}^{\mathrm{T}}\Omega_{3i})$, $\bar{c}_{4i} \triangleq \rho_i(\eta_i + \eta_i^2/\gamma_i)\Omega_i^* + \bar{\rho}_i(\bar{\eta}_i + \bar{\eta}_i^2/\bar{\gamma}_i)\bar{\Omega}_i^*$, $\eta_i = 1+\xi_i$, $\xi_i > 0$, $\bar{\eta}_i = 1+\bar{\xi}_i$, $\bar{\xi}_i > 0$, $\kappa_{1i} \triangleq 1/(\eta_i+\gamma_i) < 1$, $\kappa_{2i} \triangleq 1/\rho_i\eta_i > 0$, $\kappa_{3i} \triangleq 1/(\bar{\eta}_i+\bar{\gamma}_i) < 1$, and $\kappa_{4i} \triangleq 1/\bar{\rho}_i\bar{\eta}_i > 0$. Using (57) in the last term of (62) yields

$$\begin{aligned}&2|\tilde{x}_i^{\mathrm{T}}P_iB_ig_i(\theta(t),\hat{\theta}(t))| \\ &\quad \le 2|\tilde{x}_i(t)|\sum_{j=1,\ j\ne i}^{N}|\tilde{x}_j(t)| \\ &\quad \le v_i^* L_{\sigma i}\|P_iB_i\|\sum_{j=1,\ j\ne i}^{N}\big[|\tilde{x}_i(t)|^2 + |\tilde{x}_j(t)|^2\big],\end{aligned} \tag{63}$$

where we again used Young's inequality for the term $2|\tilde{x}_i(t)||\tilde{x}_j(t)|$ in the second line of (63). Hence, (61) becomes

$$\begin{aligned}\dot{\mathcal{V}}_i(\cdot) &\le -\big[c_{1i} - v_i^* L_{\sigma i}\|P_iB_i\|\big]|\tilde{x}_i(t)|^2 - c_{2i}\|\tilde{W}_i(t)\|^2 \\ &\quad -c_{3i}\|\tilde{W}_i(t-\tau_i)\|^2 - \bar{c}_{2i}\|\tilde{V}_i(t)\|^2 \\ &\quad -\bar{c}_{3i}\|\tilde{V}_i(t-\bar{\tau}_i)\|^2 + \bar{c}_{4i} + v_i^* L_{\sigma i}\|P_iB_i\| \\ &\quad \times\sum_{j=1,\ j\ne i}^{N}|\tilde{x}_j(t)|^2.\end{aligned} \tag{64}$$

Now introducing

$$\mathcal{V}(\cdot) = \sum_{i=1}^{N}\mathcal{V}_{4i}(\tilde{x}_i(t),\tilde{W}_{ti},\tilde{V}_{ti}), \tag{65}$$

for the whole system $\mathcal{G}$ results in

$$\begin{aligned}\dot{\mathcal{V}}(\cdot) &\le \sum_{i=1}^{N}\Big[-\bar{d}_{1i}|\tilde{x}_i(t)|^2 - c_{2i}\|\tilde{W}_i(t)\|^2 \\ &\quad -c_{3i}\|\tilde{W}_i(t-\tau_i)\|^2 - \bar{c}_{2i}\|\tilde{V}_i(t)\|^2 \\ &\quad -\bar{c}_{3i}\|\tilde{V}_i(t-\bar{\tau}_i)\|^2 + \bar{c}_{4i}\Big],\end{aligned} \tag{66}$$

where $\bar{d}_{1i} > 0$ is given in Assumption 5 (see Remark 13). Either $|\tilde{x}_i(t)| > \bar{\Psi}_{1i}$ or $|\tilde{W}_i(t)| > \bar{\Psi}_{2i}$ or $\|\tilde{W}_i(t-\tau_i)\| > \bar{\Psi}_{3i}$ or $|\tilde{V}_i(t)| > \bar{\Psi}_{4i}$ or $\|\tilde{V}_i(t-\bar{\tau}_i)\| > \bar{\Psi}_{5i}$ renders $\dot{\mathcal{V}}(\cdot) < 0$, where $\bar{\Psi}_{1i} \triangleq \sqrt{\bar{c}_{4i}/\bar{d}_{1i}}$, $\bar{\Psi}_{2i} \triangleq \sqrt{\bar{c}_{4i}/c_{2i}}$, $\bar{\Psi}_{3i} \triangleq \sqrt{\bar{c}_{4i}/c_{3i}}$, $\bar{\Psi}_{4i} \triangleq \sqrt{\bar{c}_{4i}/\bar{c}_{2i}}$, and $\bar{\Psi}_{5i} \triangleq \sqrt{\bar{c}_{4i}/\bar{c}_{3i}}$. Hence, $\tilde{x}_i(t)$, $\tilde{W}_i(t)$, and $\tilde{V}_i(t)$ are UUB for all $i = 1, 2, \ldots, N$. The boundedness of $\hat{e}_i(t)$ for all $i = 1, 2, \ldots, N$ follows directly from the boundedness of $\tilde{x}_i(t)$ for all $i = 1, 2, \ldots, N$. □

Corollary 3. Under the conditions of Theorem 2, $e_i(t)$ is bounded for all $i = 1, 2, \ldots, N$.

Proof. The proof is a direct consequence of the proof of Corollary 1. □

Remark 12. For the case when the system uncertainties and interconnections are matched, the parameter dependent Riccati equation given by (14) and (15) reduces to a Lyapunov equation given by

$$0 = A_{\mathrm{m}i}^{\mathrm{T}}P_{\mathrm{o}i} + P_{\mathrm{o}i}A_{\mathrm{m}i} + Q_{\mathrm{o}i}, \quad Q_{\mathrm{o}i} > 0. \tag{67}$$

Remark 13. In the special case addressed by Theorem 2, we still require via Assumption 5 that $\bar{d}_{1i} > 0$, and therefore

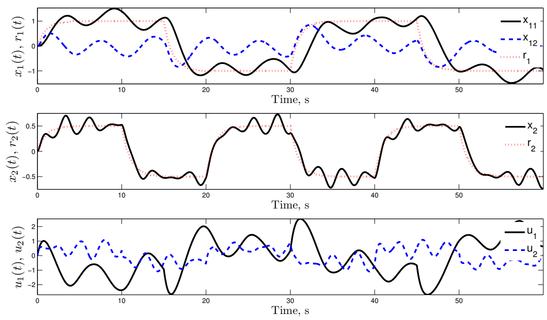

Fig. 1. Responses of reference input, state vector, reference model state vector, and control input of subsystems 1 and 2 with nominal controller.

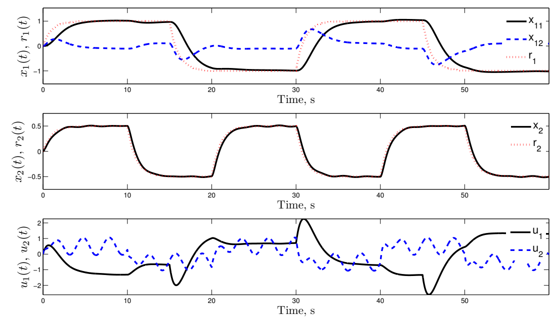

Fig. 2. Responses of reference input, state vector, reference model state vector, and control input of subsystems 1 and 2 with adaptive controller.

Remark 9 is still relevant.

Remark 14. Under the conditions of Theorem 2, an estimate for the ultimate bound can be expressed in a form similar to Corollary 2.

VI. Illustrative Example

In this section, we apply the derivative-free decentralized adaptive control architecture of Theorem 1 to an uncertain system consisting of 2 interconnected subsystems with time varying uncertainties, interconnections, and disturbances [10] given by

$$\dot{x}_1(t) = \underbrace{\begin{bmatrix}0 & 1\\ 1 & -2\end{bmatrix}}_{A_1} x_1(t) + \underbrace{\begin{bmatrix}0\\ 1\end{bmatrix}}_{B_1} u_1(t) + \begin{bmatrix}\vartheta_1\\ 0\end{bmatrix} x_2(t) + \begin{bmatrix}0\\ \vartheta_2\end{bmatrix} x_2(t)\sin(x_{11}^2(t)) + \begin{bmatrix}0\\ 1\end{bmatrix} \upsilon_1(t), \quad (68)$$

$$\dot{x}_2(t) = -\underbrace{[1]}_{A_2} x_2(t) + \underbrace{[1]}_{B_2} u_2(t) + \vartheta_3 x_{12}\cos(x_2^2) + \vartheta_4 x_2(t)\cos(2t) + \upsilon_2(t), \quad (69)$$

where $\vartheta_1 = \vartheta_2 = 0.2$, $\vartheta_3 = 0.02$, $\vartheta_4 = 1.1$, $\upsilon_1(t) = \sin(t)$, $\upsilon_2(t) = 0.5\sin(2.5t)$, and $x_1(t) = [x_{11}(t),\ x_{12}(t)]^{\mathrm{T}}$. It is assumed that the nominal part of each subsystem represented by A_1, B_1, A_2, and B_2 are known. The control objective of each subsystem is to track a given filtered square wave reference input. The nominal controller gains are $K_{11} = [3, 1]$, $K_{21} = 2$, $K_{12} = 4$, and $K_{22} = 5$, which correspond to the reference systems in (6.3) of Ref. 10. Responses using the nominal controller for these subsystems are shown in Fig. 1.

We set $L_1 = \begin{bmatrix}0 & 0\\ 2 & 3\end{bmatrix}$, $L_2 = 2$, $Q_1 = 2I_2$, $Q_2 = 2$, $\kappa_{11} = \kappa_{12} = 0.9$, $\kappa_{21} = \kappa_{22} = 100$, and $\tau_1 = \tau_2 = 0.05$ seconds. Furthermore, we chose $\beta_1(x_1) = [1, \mathrm{sigm}(x_{11}), \mathrm{sigm}(x_{12})]^{\mathrm{T}}$ and $\beta_2(x_2) = [1, \mathrm{sigm}(x_2)]^{\mathrm{T}}$, where $\mathrm{sigm}(x) \triangleq (1 - e^{-x})/(1 + e^{-x})$ represents a sigmoidal function. Fig. 2 shows that we were able to obtain a satisfactory system performance in terms of tracking the filtered square wave reference input.

VII. Conclusion

This paper extends a previously developed derivative-free adaptive control law to a decentralized form for controlling large-scale interconnected systems with matched and unmatched nonlinear time-varying system uncertainties and interconnections. The proposed controller is particularly useful for those applications in which uncertain parameters are time-varying, or for situations in which external disturbances are difficult to characterize.

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