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An Efficient Solution of the Perspective Problem via a Suitable Delay Riccati Equation

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Abstract— **In this paper it is proposed a new approach to the solution of the classical state estimation problem for a general time-varying perspective system. The solution is achieved by the definition of a virtual output which has the property to convert the original nonlinear output measurement function into a linear time-varying one. This allows the computation of an efficient observer gain matrix by the real-time solution of a suitable delay Riccati equation. Numerical results show high performances of the observer.**

I. Introduction

The estimation of the time-varying distance of an object from a camera along its optical axis is a classical problem in machine vision [9]. It has received a lot of attention because of its importance in several practical applications, such as autonomous vehicle navigation, aerial tracking, path planning, surveillance, etc. All these applications usually require to have at disposition the 3-D Euclidean coordinates of moving features or the position of a static object to be recovered from a 2-D image sequence provided by a charge-coupled device (CCD) camera. The overall class of practical problems can be generally represented by considering the relative motion between a perspective camera and an observed object.

In literature, there are many range identification techniques for perspective vision systems. Some of them utilize the extended Kalman filter (EKF) ([15], [19], [11], [4]). However, EKF involves linearization of the nonlinear vision model and requires *a priori* knowledge of the noise distribution. In order to overcome the shortcomings of the linear model, nonlinear system analysis and estimation tools are used to develop nonlinear observers able to identify the range when the motion parameters are known. Such an approach is introduced in [10]. The basic idea is developed in [16] where the perspective problem is treated as a particular case of implicit output system. Following works [3]and [8] suggest to build an observer by expressing the perspective system in terms of the nonlinear feature dynamic. A different approach is adopted in [1] and [6] where the estimation is carried out using methods from linear control theory. A reduced-order observer is suggested in [12] as an application of the more general observer technique introduced in [13]. Finally, interesting recent solutions are proposed in [14], [17] and [7].

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In this paper a new approach for the estimation of the 3-D position of a moving target is suggested. The main idea is the use of the measurements process as time-varying parameters affecting a suitable defined output matrix, derived by the introduction of a virtual output. This simple manipulation transforms the nonlinear stationary measurement map into a linear time-varying one. This allows to solve the problem avoiding any linearization procedure, via a linear time-varying observer whose gain matrix is computed through a delay Riccati equation (DRE).

The paper is organized as follows. In Section II the perspective problem is formalized. Section III introduces the new approach. Results are provided in Section IV. Finally, conclusions are summarized in Section V.

II. Problem Formulation

The motion of feature point on a rigid object relative to a calibrated pinhole camera can be described by the affine system [20], [3], [8], [12]

$$\dot{x}(t) = A(t)x(t) + b(t), \tag{1}$$

where $A(t) \in \mathbb{R}^{3\times 3}$, $b(t) \in \mathbb{R}^3$, and

$$x(t) = [x_1(t), x_2(t), x_3(t)]^T \ \in \ \mathbb{R}^3$$

contains the unmeasurable coordinates of the feature point in an inertial reference frame with x_3 being perpendicular with the camera image plane.

The dynamic matrix $A(t)$ and the drift term $b(t)$ are composed by motion parameters $a_{i,j}(t)$ and $b_i(t)$ which are possible time-varying and are assumed known. The measurable image-space coordinates, denoted by $y(t) \in \mathbb{R}^2$, are given as

$$y(t) = \left[\ y_1(t)\ y_2(t)\ \right]^T = \eta \left[\ \frac{x_1(t)}{x_3(t)} \quad \frac{x_2(t)}{x_3(t)}\ \right]^T, \tag{2}$$

where η is the focal length of the camera. Without loss of generality, it can be assumed that $\eta = 1$.

For the perspective system in (1) and (2), following assumptions are made [8].

Assumption 1: The motion parameters $a_{i,j}(t)$ and $b_i(t)$, $i, j = 1, 2, 3$ are bounded functions of time, i.e. $a_{i,j},\ b_i \in \mathcal{L}_\infty$.

Assumption 2: The image-space coordinates $y_1(t)$ and $y_2(t)$ are bounded functions of time, i.e. $y_1,\ y_2 \in \mathcal{L}_\infty$.

Assumption 3: The object feature motion avoids the degenerate case where the point feature intersect the image plane. That is $x_3(t) > \varepsilon_0$, where $\varepsilon_0 \in \mathbb{R}$ is an arbitrarily small positive constant. Moreover, $x_3 \in \mathcal{L}_\infty$.

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Remark 1: Assumptions 2 and 3 are standard hypothesis (see also [20], [3], [8], [12]) that are practically properties of the physical system rather than assumptions.

The problem objective is the reconstruction of the coordinates $x_1(t),\ x_2(t),\ x_3(t)$ from the measurements of the image-space coordinates $y_1(t)$ and $y_2(t)$.

III. The New Observer

A. The Virtual Measurement Map

Taking into account the first definition in (2), it follows that, for any t

$$x_1(t) - y_1(t)x_3(t) = 0, \tag{3}$$

$$x_2(t) - y_2(t)x_3(t) = 0. \tag{4}$$

It can be now defined the virtual output function which is identically equal to a zero vector, i.e.

$$y_v(t) := \left[\begin{array}{cc} 0 & 0 \end{array} \right]^T, \tag{5}$$

for which the following measurement map holds true:

$$y_v(t) = C(t)x(t), \tag{6}$$

with

$$C(t) = \left[\begin{array}{cc} I_2 & -y(t) \end{array} \right], \tag{7}$$

where I_n denotes the identity matrix in $\mathbb{R}^n$. The expression of the time-varying output matrix $C(t)$ in (7) directly follows from (3) and (4).

It is remarkable that the above manipulation transforms the nonlinear stationary measurement map (2) into a linear time-varying one (6). It can be said that *the definition of a virtual output converts nonlinearity into non-stationarity*. This naturally leads to a simpler and efficient solution to the proposed problem since no approximation (e.g. linearization) has to be performed. A similar approach was suggested by [5] for the planar tracking setting.

B. The Observer Equation

The reconstruction of coordinates $x(t)$ from the measurements of $y(t)$ can be obtained by the design of a linear time-varying observer for the affine system in (1) and (6). The standard approach to linear state observation, motivated partly on grounds of hindsight, is to generate an asymptotic estimate of the state by using another linear state equation that accepts as inputs the output $y(t)$ and input $b(t)$ signals. For the special case of this paper such a system state equation has the standard structure of a Luemberger time-varying observer [18]:

$$\begin{aligned} \dot{\hat{x}}(t) =& A(t)\hat{x}(t) + b(t) + K^*(t)\left(y_v(t) - C(t)\hat{x}(t)\right), \\ \hat{x}(t_0) =& \hat{x}_0, \end{aligned} \tag{8}$$

where $\hat{x}(t) \in \mathbb{R}^3$ is the estimated state, i.e. the estimated coordinates of the feature point, $\hat{x}_0 \in \mathbb{R}^3$ is the initial estimate, and $K^*(t) \in \mathbb{R}^{3\times 2}$ is the *observer gain matrix*. This matrix has the standard form

$$K^*(t) = P(t)C^T(t), \tag{9}$$

Algorithm 1 DRE Observer

given: $A|_{[t_0-\delta,T]}, b|_{[t_0,T]}, y|_{[t_0,T]}, \alpha, \delta > 0$;

initial conditions: $\hat{x}(t_0) = \hat{x}_0,\ P(t_0) > 0,$

$\Psi(t_0) = \Phi(t_0-\delta, t_0),\ y|_{[t_0-\delta,t_0)} = 0;$

instant time $t = t_0$, **start integrating:**

$$\begin{aligned}
&\dot{\hat{x}}(t) = \left(A(t) - P(t)C^T(t)C(t)\right)\hat{x}(t) + b(t);\\
&\dot{P}(t) = A(t)P(t) + P(t)A^T(t) + 4\alpha^2 P(t)\\
&\quad - 2P(t)C^T(t)C(t)P(t)\\
&\quad + 2e^{-4\alpha^2\delta}P(t)\Psi^T(t)C^T(t-\delta)C(t-\delta)\Psi(t)P(t);\\
&\dot{\Psi}(t) = A(t-\delta)\Psi(t) - \Psi(t)A(t);\\
&C(t) = \left[\begin{array}{cc} I_2 & -y(t) \end{array} \right].
\end{aligned}$$

where $P(t) \in \mathbb{R}^{3\times 3}$ is a symmetric matrix that has been chosen to satisfy the following matrix dynamical delay system which has a form that resembles the classical Riccati equation:

$$\begin{aligned} \dot{P}(t) =\ & A(t)P(t) + P(t)A^T(t) + 4\alpha^2 P(t)\\ & - 2P(t)C^T(t)C(t)P(t)\\ & + 2e^{-4\alpha^2\delta}P(t)\Psi^T(t)C^T(t-\delta)C(t-\delta)\Psi(t)P(t) \end{aligned} \tag{10}$$

with $P(t_0) > 0$ and where $\alpha, \delta \in \mathbb{R}$, $\delta > 0$, and $\Psi(t) \in \mathbb{R}^{3\times 3}$ is the solution of

$$\begin{aligned} \dot{\Psi}(t) &= A(t-\delta)\Psi(t) - \Psi(t)A(t),\\ \Psi(t_0) &= \Phi(t_0-\delta, t_0), \end{aligned} \tag{11}$$

$\Phi(t,\tau)$ denoting the state-transition matrix associated to $A(t)$. The role of parameters α and δ will be clarified in the sequel.

Taking into account (5) and (9), (8) can be rewritten as follows:

$$\dot{\hat{x}}(t) = \left(A(t) - P(t)C^T(t)C(t)\right)\hat{x}(t) + b(t). \tag{12}$$

Note that the delay Riccati equation (DRE) (10) is forced though the output matrix $C(\cdot)$ both by the actual value of measurements $y(t)$ and their delayed values $y(t-\delta)$. It is now known that using past values of the output signal helps observability [2].

In order to clarify the way the observer works, the proposed solution is summarized in Algorithm 1. Note that the values of measurements $y(t)$ before the initial time t_0 are zero valued. This is obviously one of the possible choices for real frameworks. A second possibility is to start the estimation at the delayed time $t_0 + \delta$ and use the output signal $y(t)$ which is actually available within the interval $[t_0, t_0 + \delta]$.

C. Proof of Convergence

Once introduced the observer equation, the relevant convergence is now proved. In order to quantify the performance of any observer, an estimation error, denoted by $e(t) \in \mathbb{R}^3$, is usually defined as

$$e(t) := x(t) - \hat{x}(t).$$

For observer forms such as (8) with a generic gain $K(t)$, the error time derivative is given by

$$\dot{e}(t) = (A(t) - K(t)C(t))\, e(t), \tag{13}$$

where $K(t)$ is the observer gain matrix.

The goal of any observer design is to make the observer-error system (13) asymptotically stable. In the sequel a sufficient condition for the uniform exponential stability of such a system is given in the particular case of observer (12).

In order to analyze the asymptotic behaviour of the observer-error system, a key result form [18] is recalled.

Let the reconstructibility Gramian for the plant in (1) and (6) be

$$N(t_0, t) = \int_{t_0}^{t} \Phi^T(\tau, t) C^T(\tau) C(\tau) \Phi(\tau, t) d\tau.$$

Note that $N(t_0, t)$ is a symmetric and positive semi-definite matrix. Furthermore, let

$$M(t_0, t) = \int_{t_0}^{t} 2e^{-4\alpha^2(\tau - t)} \Phi^T(\tau, t_0) C^T(\tau) C(\tau) \Phi(\tau, t_0) d\tau.$$

Theorem 1: Suppose for the linear state equations (1) and (6) there exist positive constants δ, ε_1, and ε_2 such that

$$\varepsilon_1 I_3 \leq N(t - \delta, t) \leq \varepsilon_2 I_3 \tag{14}$$

for all t. Then, given a constant $\alpha \in \mathbb{R}$, the observer gain matrix

$$K(t) = \left[\Phi^T(t - \delta, t) M(t - \delta, t) \Phi(t - \delta, t)\right]^{-1} C^T(t) \tag{15}$$

is such that the resulting observer-error state equation (13) is uniformly exponentially stable with rate α^2.
For the proof the reader is referred to [18].

Now, the main result of the paper is summarized in the following theorem.

Theorem 2: Suppose for the linear state equations (1) and (6) there exist positive constants δ and ε_1 such that

$$N(t - \delta, t) \geq \varepsilon_1 I_3 \tag{16}$$

for all t. Then, given a constant $\alpha \in \mathbb{R}$, the observer gain matrix $K^*(t)$ defined in (9), and satisfying (10) and (11), is such that the resulting observer-error state equation (13) is uniformly exponentially stable with rate α^2.

Proof: This proof consists of two parts. In the first part it will be proved that condition (16) implies condition (14) of Theorem 1. Then it will be demonstrate that the special form of the observer gain $K^*(t)$ (9) is identically equal to the generic form given by Theorem 1 in (15).

Condition (16) corresponds to the left-side inequality of condition (14). Since $N(t - \delta, t)$ is a symmetric matrix the following inequality holds true (Rayleigh-Ritz inequality):

$$N(t - \delta, t) \leq \lambda_{max}(N(t - \delta, t)) I_3,$$

for every t and δ, and where $\lambda_{max}(M)$ is the maximum eigenvalue of a given symmetric matrix M. As a consequence, the right-hand inequality of condition (14) is satisfied if there exists a positive finite constant ε_2 such that $\lambda_{max}(N(t - \delta, t)) \leq \varepsilon_2$. Recall that for a symmetric positive semi-definite matrix $Q \in \mathbb{R}^{n \times n}$ the following norm can be defined:

$$\| Q \| = \lambda_{max}(Q).$$

Therefore, (14) is proved if ε_2 is such that

$$\| N(t - \delta, t) \| \leq \varepsilon_2. \tag{17}$$

In order to demonstrate condition (17), note that from Assumptions 1 and 2 it follows that there exist two finite positive constants γ and σ such that, for any t:

$$\| A(t) \| \leq \gamma \qquad \| C^T(t) C(t) \| \leq \sigma. \tag{18}$$

Taking into account (18) and Lemma 3 it can be said that there exist a positive constant β such that, for every t and $\delta > 0$:

$$\begin{aligned}
\|N(t - \delta, t)\| &\leq \int_{t-\delta}^{t} \| \Phi^T(\tau, t) C^T(\tau) C(\tau) \Phi(\tau, t) \| d\tau \\
&\leq \int_{t-\delta}^{t} \| \Phi^T(\tau, t) \| \| C^T(\tau) C(\tau) \| \| \Phi(\tau, t) \| d\tau \\
&\leq \beta^2 \sigma \delta < +\infty,
\end{aligned}$$

which proves condition (17) with $\varepsilon_2 = \beta^2 \sigma \delta$.

Since (14) is satisfied, from Theorem 1 follows that the observer gain matrix in (15) guarantees the uniform exponential stability of the observer-error system (13). Therefore, it remains to prove that the solution of the DRE (10) satisfies, for any t,

$$P^{-1}(t) = \Phi^T(t - \delta, t) M(t - \delta, t) \Phi(t - \delta, t), \tag{19}$$

so that $K^*(t) = K(t)$, for every t.

In order to prove (19) the time derivative of $M(t - \delta, t)$ is needed. This can be computed utilizing properties in Lemma 4:

$$\begin{aligned}
\dot{M}(t - \delta, t) = &- 4\alpha^2 M(t - \delta, t) \\
&+ 2\Phi^T(t, t - \delta) C^T(t) C(t) \Phi(t, t - \delta) \\
&- 2e^{-4\alpha^2\delta} C^T(t - \delta) C(t - \delta) \\
&- A^T(t - \delta) M(t - \delta, t) \\
&- M(t - \delta, t) A(t - \delta).
\end{aligned} \tag{20}$$

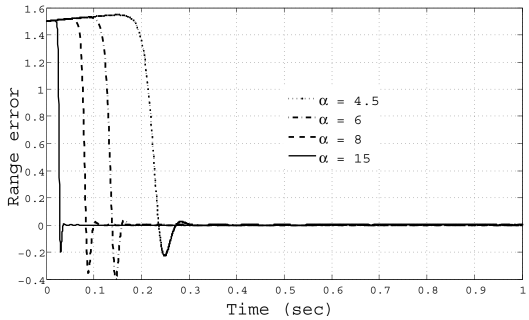

Fig. 1. Time history of the observation error for different values of α.

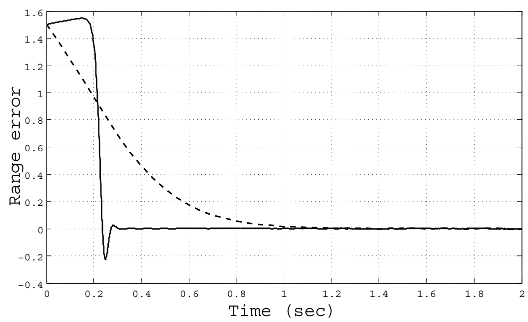

Fig. 2. State estimate error in x_3-coordinate. Solid curve is the new observer instead the dashed curve is the observer presented in [12].

The time derivative of both sides of (19) can be now obtained taking into account (20) and Lemma 4:

$$\begin{aligned}\dot{P}^{-1}(t) &= \dot{\Phi}^T(t-\delta,t)M(t-\delta,t)\Phi(t-\delta,t)\\ &\quad+\Phi^T(t-\delta,t)\dot{M}(t-\delta,t)\Phi(t-\delta,t)\\ &\quad+\Phi^T(t-\delta,t)M(t-\delta,t)\dot{\Phi}(t-\delta,t)\\ &=-A(t)^T P^{-1}(t)-P^{-1}(t)A(t)\\ &\quad-4\alpha^2 P^{-1}(t)+2C^T(t)C(t)\\ &\quad-2e^{-4\alpha^2\delta}\Phi^T(t-\delta,t)C^T(t-\delta)C(t-\delta)\Phi(t-\delta,t).\end{aligned} \tag{21}$$

By defining

$$\Psi(t) := \Phi(t-\delta,t),$$

which satisfies (11), as proved in Lemma 4, and using Lemma 1, the DRE (10) readily follows from (21). ■

IV. Simulation Results

In this section, a detailed simulation study is presented to evaluate the performance of the proposed estimation technique. Since the estimation is carried out through the the delayed measurement $y(t-\delta)$, these are set to zero for $t<\delta$.

Consider the example given in [3] and [7] of the perspective system:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix}=\begin{bmatrix}-0.2 & 0.4 & -0.6\\ 0.1 & -0.2 & 0.3\\ 0.3 & -0.4 & 0.4\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}+\begin{bmatrix}0.5\\ 0.25\\ 0.3\end{bmatrix},$$

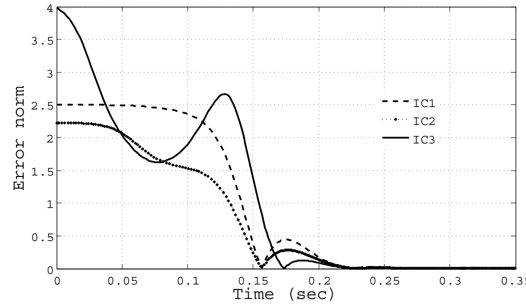

Fig. 3. Norm of the state estimate error.

with initial conditions

$$x(0) = [1\ 1.5\ 2.5]^T,$$
$$\hat{x}_0 = [0.4\ 0.6\ 1]^T.$$

The observer performance for different values of the rating parameter α, namely $\alpha = 4.5,\ 6,\ 8,\ 15$, is shown in Fig. 1, with $\delta = 0.01$. Note that the convergence rate can be arbitrarily chosen simply by setting parameter α.

In order to compare this observer with an existing approach, the observer in [12] is considered and the results are shown in Fig. 2 with $\alpha = 5$ and the constant design parameter of [12] $\lambda = 30$.

To demonstrate the robustness of the suggested tool, the system in [7] is taken into account:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix}=\begin{bmatrix}0 & -1 & 1\\ 1 & 0 & 1\\ -1 & -1 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}+\begin{bmatrix}1\\ 2\\ 1\end{bmatrix}.$$

Three set of initial condition (ICs) have been considered in the format $[x^T(0)\ \hat{x}_0^T]^T$, with $\delta = 0.1$:

$$\begin{aligned}IC1: &\quad [-1,2,2,1/6,1/3,1/3]^T,\\ IC2: &\quad [-1,2,1,-0.03,0.12,0.30]^T,\\ IC3: &\quad [-2,3,4,-0.4,2.4,0.4]^T.\end{aligned} \tag{22}$$

In Fig. 3 the norm of error $\parallel x(t)-\hat{x}(t)\parallel$ is graphically represented for the different ICs in (22).

V. Conclusions

As a concluding remark, it deserves to point out the novelty of the concept denoted as "virtual output measurement process", that in the opinion of the authors could be very promising for the treatment of important classes of output measurement functions. Moreover the gain matrix of the proposed observer, computed trough a special kind of Riccati equation, guarantees global convergence and high performances of this method.

Appendix

Lemma 1: Let $M:\mathbb{R}\to\mathbb{R}^{n\times n}$ be a continuous function. Suppose $M(t)$ to be invertible for all t, then

$$\dot{M}^{-1}(t) = -M^{-1}(t)\dot{M}(t)M^{-1}(t), \tag{23}$$

for every t.

Proof: Since $M(t)$ is invertible for all t, then

$$M(t)M^{-1}(t) = I_n \qquad \forall t.$$

From the time derivative of both sides of last equation, it follows that

$$\dot{M}(t)M^{-1}(t) + M(t)\dot{M}^{-1}(t) = 0$$

from which (23) can be trivially obtained. ■

Let $\Phi(t,\tau) \in \mathbb{R}^{n\times n}$ be the state-transition matrix associated with the linear system

$$\dot{\zeta}(t) = A(t)\zeta(t)$$

where $\zeta(t) \in \mathbb{R}^n$ and $A(t) \in \mathbb{R}^{n\times n}$. Following Lemmas give a few key properties of this matrix.

Lemma 2: The transition matrix for $A(t)$ is invertible for any $t, \tau \in \mathbb{R}$ and

$$\Phi^{-1}(t,\tau) = \Phi(\tau,t).$$

Lemma 3: If there exists a finite positive constant γ such that $\| A(t) \| \leq \gamma$, for all t, then given a finite $\delta > 0$ there exists a finite $\beta > 0$ such that $\| \Phi(t,\tau) \| \leq \beta$ for all $t, \tau \in \mathbb{R}$ such that $|t - \tau| < \delta$.

For the proof of Lemma 2 and 3 the reader is referred to [18].

Lemma 4: For every $t, \tau, \delta \in \mathbb{R}$, $\delta > 0$ it is:

$$\dot{\Phi}(t,\tau) = A(t)\Phi(t,\tau); \tag{24}$$

$$\frac{\partial \Phi(t,\tau)}{\partial \tau} = -\Phi(t,\tau)A(\tau); \tag{25}$$

$$\dot{\Phi}(t-\delta,t) = A(t-\delta)\Phi(t-\delta,t) - \Phi(t-\delta,t)A(t). \tag{26}$$

Proof: Equation (24) is a general property of the state-transition matrix.

From Lemma 1 and 2, and (24) the partial derivative of the $\Phi(t,\tau)$ with respect to the second argument τ can be computed as follows:

$$\begin{aligned}
\frac{\partial \Phi(t,\tau)}{\partial \tau} &= \frac{\partial \Phi^{-1}(\tau,t)}{\partial \tau} \\
&= -\Phi^{-1}(\tau,t)\frac{\partial \Phi(\tau,t)}{\partial \tau}\Phi^{-1}(\tau,t) \\
&= -\Phi^{-1}(\tau,t)A(\tau)\Phi(\tau,t)\Phi^{-1}(\tau,t) \\
&= -\Phi^{-1}(\tau,t)A(\tau) = -\Phi(t,\tau)A(\tau),
\end{aligned}$$

which proves (25).

Finally, (26) is trivially derived by applying properties (24) and (25). ■

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