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On Optimal Input Design in System Identification for Model Predictive Control

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Abstract— **This paper considers a method for optimal input design in system identification for model predictive control. The objective is to provide the user with a model that guarantees, with high probability, that a specified control performance is achieved. We see that, even though the system is nonlinear, using linear theory in the input design can reduce the experimental effort. The method is illustrated in a minimum power input signal design in identification of a water tank system.**

I. Introduction

Model predictive control (MPC) is a widely used model based control strategy in industry [1]. As its name entails, MPC predicts future states of the controlled process based on a model of the system. Given these predictions, MPC constructs the optimal control strategy. The control input at the first time step of the strategy is applied to the process. The procedure is then iterated at each time instant.

The performance of the controller is highly dependent on the quality of the model it is based on. Due to process–model mismatch, the control performance may not be up to standard. Thus, it is desirable to have an efficient and accurate method of identifying models in an MPC context. Reducing the cost of the experiment is also often of importance and optimal input design has been shown to give significant reduction of the experimental effort [2].

In this contribution we build on the optimal input design for models used in MPC developed in [3] and [4]. We present a method of performing optimal input design on a process controlled by MPC. The approach is illustrated on a water tank system. Given a model structure and a measurement of the control performance degradation, the method provides the user with the optimal input signal to be used in the identification experiment. There are other ideas for identification for MPC available, see e.g. [5], [6].

In Section II we outline the ideas of optimal input design for control presented in [12]. The specifics for MPC are outlined in Section III. Section III-C presents the design procedure for MPC which is illustrated in Section V-D.

II. Problem formulation

We consider identification of models on the form

$$\mathscr{M}(\theta): \qquad \begin{aligned} x_{t+1} &= F(\theta)x_t + G(\theta)u_t + v_t \\ y_t &= H(\theta)x_t + e_t \end{aligned} \tag{1}$$

where x_t is the state vector, $\{u_t\}_{t=1}^N$ is a known input sequence, v_t and e_t are zero-mean, Gaussian processes with covariance matrices R_v and R_e respectively and θ is an unknown parameter vector. We assume that there exists a parameter vector, say θ_o, such that the model (1) describes the true system, denoted $\mathscr{S}$.

The process is assumed to be controlled using MPC with the quadratic cost function

$$J(t) = \sum_{i=0}^{M-1} \left(\|\hat{y}_{t+i+1|t} - r_{t+i+1}\|_Q^2 + \|\Delta u_{t+i|t}\|_{R_1}^2 + \|u_{t+i|t} - u_{t+i}^d\|_{R_2}^2 \right), \tag{2}$$

where $\hat{y}_{t+i|t}$, $u_{t+i|t}$ and $\Delta u_{t+i|t} = u_{t+i} - u_{t+i-1}$ are i-step predictions of the output, input and input update of the system, respectively. The known reference trajectory is denoted r_t and u_t^d are target input values. The matrices Q, R_1 and R_2 are tunable weights. The norm $\|x\|_A$ is equal to $\sqrt{x^T A x}$. The cost function is minimized with respect to the input updates and the update Δu_{t+1} is applied to the process. The optimization is performed in each timestep in accordance with the receding horizon control philosophy. The cost (2) is used by the MPC toolbox in Matlab [7].

A major advantage of MPC is the ability to handle signal and state constraints on the process in the controller. However, there is no explicit solution to the optimization problem in the controller [8]. We will see that this is a limiting factor in the experiment design and requires numerical calculations.

To find the predicted output used in (2), a model of the process is needed. The more accurate the model, the better the MPC performance. The degradation in performance due to an inaccurate model is formalized in the next section.

A. Application cost

The performance of a controller designed based on the process model, is directly related to the quality of the model. If θ_o are available for the design, the performance specifications are met. However, for estimates different from θ_o, the performance degrades. Application cost relates model parameters to performance degradation and is denoted $V_{app}(\theta)$.

We choose the cost function such that its minimal value is zero and occurs when the true parameter vector θ_o is used, i.e., $V_{app}(\theta_o) = 0$, $V'_{app}(\theta_o) = 0$ and $V''_{app}(\theta_o) \succeq 0$[1]. A maximal allowed performance degradation gives

$$V_{app}(\theta) \leq 1/\gamma, \tag{3}$$

for some real-valued positive constant γ. The parameters corresponding to acceptable performance degradation belong

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[1] $A \succeq B$ means that $A - B$ is a positive semi-definite matrix.

to the set

$$\Theta = \{\theta \;|V_{app}(\theta) \leq 1/\gamma\}, \tag{4}$$

which we call the application set. Thus, the objective of system identification should be to deliver parameter estimates that belong to the application set.

We can make a convex approximation of Θ using a Taylor expansion. Hence, inequality (3) can be approximated by

$$[\theta - \theta_o]^T V''_{app}(\theta_o)[\theta - \theta_o] \leq 2/\gamma. \tag{5}$$

For sufficiently large γ, the set of acceptable parameters (4) can thus be approximated by the ellipsoidal set

$$\mathscr{E}_{app} = \{\theta \;|[\theta - \theta_o]^T V''_{app}(\theta_o)[\theta - \theta_o] \leq 2/\gamma\}. \tag{6}$$

We call this the application ellipsoid.

In [4] the scenario approach [9], [10], is presented as another possible approximation of the application set. The idea is to randomly select parameters that satisfy (3), called scenarios. If enough scenarios are used, the performance degradation can be guaranteed with high probability.

B. System identification

Let $\hat{\theta}$ be the estimated parameter vector of the model based on N input–output observations using the prediction error method (PEM). A result from PEM for open-loop identification is the asymptotic (in sample size) Gaussian distribution of the estimates $\hat{\theta}$ [11], i.e.,

$$\sqrt{N}(\hat{\theta} - \theta_o) \sim \mathscr{N}(0, \mathbf{P}), \tag{7}$$

$$\mathbf{P}^{-1} = \mathrm{E}\left\{ \frac{d}{d\theta}\hat{y}(t,\theta)\bigg|_{\theta=\theta_o} R_e^{-1} \frac{d}{d\theta}\hat{y}^H(t,\theta)\bigg|_{\theta=\theta_o} \right\}. \tag{8}$$

The confidence ellipsoids for the estimates are given by

$$\hat{\theta}_N \in \mathscr{E}_{SI} = \{\theta \;|[\theta - \theta_o]^T \mathbf{P}^{-1}[\theta - \theta_o] \leq \kappa/N\}, \text{ w. p. } \alpha. \tag{9}$$

The positive constant κ depends on the number of parameters to be estimated and the probability α. Its value is obtained from the χ^2-distribution. This means that the estimate lies inside the system identification set (9) with probability α.

C. Input design

We want the estimated parameters $\hat{\theta}$ to be acceptable with respect to control performance. Since the estimates are random variables, this is hard to guarantee. Therefore, we relax the condition and require only that the estimated parameters satisfy the control performance with some (high) probability. That is, we require the system identification set (9) to be contained in the application set (4), i.e.,

$$\mathscr{E}_{SI} \subseteq \Theta. \tag{10}$$

If we use approximation (6), both sets are ellipsoids and (10) is equivalent to the linear matrix inequality (LMI)

$$\frac{N}{\kappa}\mathbf{P}^{-1} \succeq \frac{\gamma}{2}\, V''_{app}(\theta_o). \tag{11}$$

LMI (11) together with (9) imply that the estimated parameters lie in the application ellipsoid, i.e., $\hat{\theta}_N \in \mathscr{E}_{app}$, with at least probability α. This idea is presented in [12].

If we use the scenario approach, we replace (10) with

$$[\theta_k - \theta_o]^T \frac{N}{\kappa}\mathbf{P}^{-1}[\theta_k - \theta_o] \geq \gamma V_{app}(\theta_k),\; k = 1,\ldots,K, \tag{12}$$

where $\theta_k \in \Theta$ are samples taken from a uniform distribution on Θ. For sufficiently large values of K (12) approximates the original constraint well. For more details see [10] and [4].

A natural objective of the input design is to minimize some experiment cost while guaranteeing that (10) holds. Experiment cost can, for instance, be experiment time, input power or input energy. The key is that the inverse covariance matrix $\mathbf{P}^{-1}$ can be expressed in the frequency domain as an affine function of the input spectrum. Hence, a linear parameterization, and any convex objective function of the spectrum will lead to input design problems that are semi-definite programs. This has been extensively discussed in the literature, see e.g., [13].

The application set gives the directions of high performance degradation with respect to model parameters. Thus, we can determine which linear combinations of elements of θ are important to estimate with high accuracy. The application set is linked to the identification through the input design. When the optimal input is applied to the system, the most sensitive parameter directions are excited while unimportant dynamics are not.

III. Identification for MPC

In this section we present a scheme for optimal input design in an MPC context. There are two major challenges with the implementation of the method. The first is that optimal input design relies on knowledge of the true system parameters. These are, of course, not known. The two proposed ways around this are to design inputs that are robust to parameter variations, e.g., [14], or to use an initial parameter estimate instead of the true parameters in the optimal input design. The latter approach is considered here. The second challenge conserns time domain constraints in MPC. There is, as of yet, no good way of including such constraints in the input design formulation considered here. The solution here is to include them in the calculation of the application cost but not to consider them in the identification part of the method. It may be possible to enforce some time domain constraints when the optimal signal is generated, e.g., [15].

A. Application cost

A reasonable application cost for the MPC case is the difference between the output of the process controlled by MPC based on a model using $\theta \neq \theta_o$ and one based on θ_o, denoted $y_t(\theta)$ and $y_t(\theta_o)$, respectively. Therefore, we choose

$$V_{app}(\theta) = \frac{1}{N}\sum_{t=1}^{N} \|y_t(\theta_o) - y_t(\theta)\|, \tag{13}$$

which has the desired properties mentioned in Section II-A.

In an application, it is unlikely that one can evaluate (13) using outputs from the real process. Since this requires controlling the process based on models with more or less

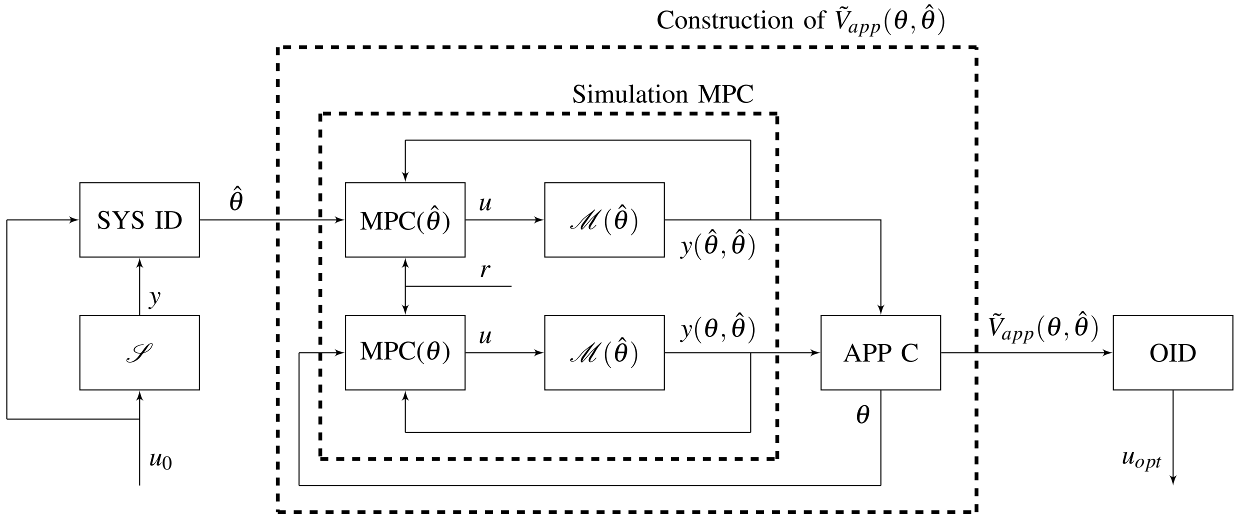

Fig. 1. The method of optimal input design for MPC. The true system ($\mathscr{S}$) is excited by a white noise sequence (u_0). Initial parameter estimates ($\hat{\theta}$) are obtained from system identification (SYS ID). The control performance and how it is effected by different values of the parameters (θ) is examined. This is done by simulating a model of the system ($\mathscr{M}(\hat{\theta})$) controlled with an MPC using $\hat{\theta}$ and another MPC using θ (MPC($\hat{\theta}$) and MPC(θ) respectively). Based on this, the approximate application cost ($\tilde{V}_{app}(\theta,\hat{\theta})$) is calculated (APP C). The application cost and initial estimate are then used in the optimal input design (OID) and the optimal input signal is obtained. The input is optimal for $\mathscr{M}(\hat{\theta})$ but might not be optimal for the system $\mathscr{S}$.

arbitrary parameter values. Instead, we introduce an approximation of V_{app} where the true system is replaced with the linear model using estimated parameter values. This gives

$$\tilde{V}_{app}(\theta,\hat{\theta}) = \frac{1}{N}\sum_{t=1}^{N} \|y_t(\hat{\theta},\hat{\theta}) - y_t(\theta,\hat{\theta})\|. \tag{14}$$

The first argument is the parameter used by the MPC and the second argument the parameter used in the linear model replacing the system, *cf. Simulation MPC* block in Figure 1.

The choice of acceptable performance degradation is highly application dependent. We consider the reference tracking capability of the MPC using a model with θ_o,

$$V(\theta_o) = \frac{1}{N}\sum_{t=1}^{N} \|y_t(\theta_o) - r_t\|^2, \tag{15}$$

and allow for a certain level of degradation, e.g., a 1 % degradation of the performance corresponds to

$$\gamma = 100/V(\theta_o). \tag{16}$$

In equations (15) and (16), we use an initial estimate of the parameters instead of θ_o in the implementation.

B. Input design

Optimal input design minimizes experimental cost while guaranteeing performance. Quantifying experiment cost is not obvious, however we choose to focus on input power. To formalize, we can write the full input design problem as

$$\begin{aligned} &\underset{\phi_{u(\omega)}}{\text{minimize}} && \text{trace}\left(\frac{1}{2\pi}\int_{-\pi}^{\pi}\phi_u(\omega)d\omega\right) && \text{(17a)}\\ &\text{subject to} && \mathscr{E}_{SI} \subseteq \Theta && \text{(17b)}\\ & && \phi_u(\omega) \geq 0 \quad \forall\omega && \text{(17c)}\end{aligned}$$

where $\phi_u(\omega)$ is the input spectrum. Depending on if we choose the ellipsoidal approximation of Θ or the scenario approach, (17b) is replaced by (11) or (12), respectively. With linear parameterization of $\phi_u(\omega)$, problem (17) can be written as a semi-definite program, see e.g., [13].

C. Identification algorithm

We construct an optimal input design and identification method to estimate models to be used in MPC. The true parameters in the expressions are replaced with estimates thereof. The proposed method is described by the following algorithm and further illustrated in Figure 1.

Algorithm

Step 0 Find an initial estimate of the parameters using white noise as input in the identification experiment.

Step 1 Find the application cost based on simulations of the model with the parameter estimates.

Step 2 Design the optimal input signal based on the application cost and parameter estimates.

Step 3 Find a new estimate of the parameters using the optimal input signal in the identification experiment.

Note: If a good initial guess of the parameters are available, e.g., through physical insight of the process, this guess can replace the initial estimation in Step 0.

The algorithm can be iterated so that the estimate from Step 3 is used in Step 1 and 2 to calculate a new input design. As more data is used in the identification step and if there exist parameters θ_o such that $\mathscr{S} = \mathscr{M}(\theta_o)$, the estimates will converge to their true values. Therefore, one can expect the input design to converge to what would be obtained had θ_o been known. A discussion on this and a formal proof for the case with ARX systems are found in [16].

IV. Water Tank Process

We have implemented the method of system identification for MPC described in Section III on the water tank process presented in [17]. It consists of four interconnected water tanks. The layout of the process is shown in Figure 2. The control objective is to regulate the water levels of the two lower tanks, according to a reference trajectory, using MPC. The process is nonlinear, however, a linearized and discretized model of the process is used in the MPC. We want to estimate the parameters of the linearized model.

A. Process description

The four water tanks are connected to two pumps that deliver water into the tanks. Two valves are used to control the amount of water pumped into the upper and lower tanks respectively. The input signals are the voltages of the two pumps and the outputs are the water levels in the two lower tanks. There are physical constraints on the process, such as input voltages to the pumps and water levels in the tanks.

We derive a nonlinear model of the process from Torricelli's principle,

$$\begin{aligned}
\frac{dx_1}{dt} &= -\frac{a_1}{A_1}\sqrt{2gx_1} + \frac{a_3}{A_1}\sqrt{2gx_3} + \frac{\gamma_1 k_1}{A_1}u_1,\\
\frac{dx_2}{dt} &= -\frac{a_2}{A_2}\sqrt{2gx_2} + \frac{a_4}{A_2}\sqrt{2gx_4} + \frac{\gamma_2 k_2}{A_2}u_2,\\
\frac{dx_3}{dt} &= -\frac{a_3}{A_3}\sqrt{2gx_3} + \frac{(1-\gamma_2)k_2}{A_3}u_2,\\
\frac{dx_4}{dt} &= -\frac{a_4}{A_4}\sqrt{2gx_4} + \frac{(1-\gamma_1)k_1}{A_4}u_1,
\end{aligned}$$

where x_i is the water level in centimeters of tank i and u_j is the voltage in volt of pump j. The parameters of the process and their nominal values are found in Table I.

TABLE I

PHYSICAL PARAMETERS OF THE FOUR TANK PROCESS.

Parameter	Nominal	Description
a_i	$\{0.17\ 0.15\ 0.11\ 0.08\}$ cm^2	area of outlet of tank i
A_i	15.5 cm^2	area of tank i
γ_j	0.625	parameter of valve j
k_j	4.14 cm^3/(sV)	parameter of pump j

B. Linear Model

We derive a linear and time discrete model of the process that will be used in MPC. The nonlinear model is linearized around its equlibrium points, x^0 and u^0, giving

$$\frac{d\bar{x}}{dt} = \begin{bmatrix} \frac{-1}{\tau_1} & 0 & \frac{A_3}{A_1\tau_3} & 0\\ 0 & \frac{-1}{\tau_2} & 0 & \frac{A_4}{A_2\tau_4}\\ 0 & 0 & \frac{-1}{\tau_3} & 0\\ 0 & 0 & 0 & \frac{-1}{\tau_3}\end{bmatrix}\bar{x}_t + \begin{bmatrix} \frac{\gamma_1 k_1}{A_1} & 0\\ 0 & \frac{\gamma_2 k_2}{A_2}\\ 0 & \frac{(1-\gamma_2)k_2}{A_3}\\ \frac{(1-\gamma_1)k_1}{A_4} & 0\end{bmatrix}\bar{u}_t,$$

$$y = \begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\end{bmatrix}\bar{x}_t + e_t,$$

where $\bar{x} = x - x^0$, $\bar{u} = u - u^0$ and $\tau_i = \frac{A_i}{a_i}\sqrt{\frac{2x_i^0}{g}}$. The measurement noise e_t is assumed to be zero-mean Gaussian with covariance matrix R_e. The equilibrium points of our process are $x^0 = [15\ \ 15\ \ 3\ \ 12]^T$ cm and $u^0 = [7.8\ \ 5.25]^T$ V. The linear model is then discretized assuming zero-order hold sampling at a sampling rate of $T_s = 1$ Hz. The parameters to be estimated in the identification experiment are the physical parameters presented in Table I. The equilibrium points and gravity are considered known and hence the factor $\sqrt{2x_i^0/g}$ in τ_i, $i = 1, \ldots, 4$, is also known.

C. Control Strategy

The objective of the controller is to perform reference tracking of the water levels in the two lower tanks. The MPC implemented is provided by the MPC Toolbox in Matlab. The MPC constructs an optimal control strategy by minimizing the cost function defined by (2), with the deviation variables used instead of $\hat{y}$, r, Δu and u, subject to the constraints of the process. These constraints are listed in Table II.

TABLE II

PHYSICAL CONSTRAINTS OF THE FOUR TANK PROCESS.

Parameter	Limit	Description
$x_{i,max}$	25 cm	maximum water level of tank i
$x_{i,min}$	0 cm	minimum water level of tank i
$u_{j,max}$	15 V	maximum voltage of pump j
$u_{j,min}$	0 V	minimum voltage of pump j

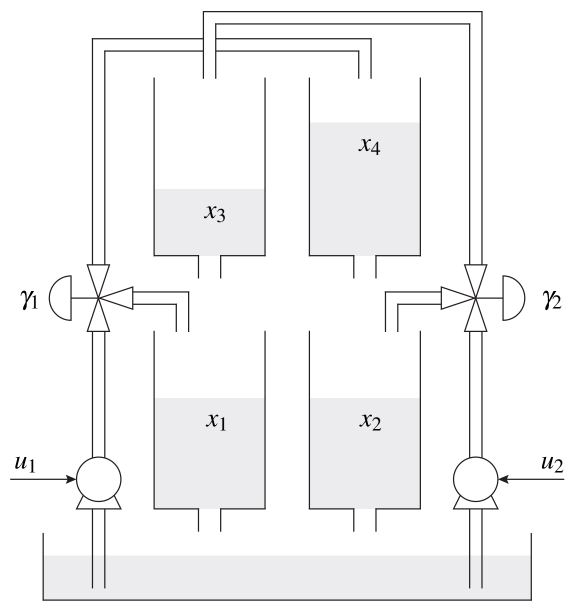

Fig. 2. The water process. Water is pumped from the reservoir into the four tanks. The voltages to the pumps are input signals and the levels in the two lower tanks, x_1 and x_2, are measured outputs. The setting of the two valves regulate how much water is pumped into the upper and lower tank.

V. OPTIMAL INPUT DESIGN EXAMPLE

In this section we implement the method on simulations of the water tank process presented in Section IV. The optimal input design is found and identified models are evaluated.

A. Simulation Setup

The MPC prediction and control horizons are 10 time steps. There are no constraints on the input rate in the MPC control problem and the covariance matrix R_e is set to zero for the calculations of the scenarios in the input design. For all other settings we use the default values provided by the MPC Toolbox in Matlab. The optimal input design problem is (17) using the scenario approach, i.e., constraint (12). The problem is solved with $R_e = 10^{-3} I_{\{2\times 2\}}$, $N = 400$ and κ from the $\chi^2(10)$-distribution with $\alpha = 0.95$. The number of scenarios used is 3,000.

B. Input Design

We compare the optimal design obtained when using θ_o and one where the design is based on an initial estimate of the parameters. These estimates were obtained using a zero mean white Gaussian input with variance 0.01. The optimal input spectra for 1 % performance degradation, i.e., γ given by (15) is shown in Figure 3. The optimization problem is implemented in CVX and solved using SDPT3 [18], [19].

We see that the optimal spectrum is temporally colored but almost spatially white. The spectrum has high energy at low frequencies, indicating that the static gain of the system is important. This is expected since the application cost relates to reference tracking and therefore emphasizes the static gain. The design obtained from the initial estimate is very close to the optimal.

We constructed the Hessian of the application cost. It gives that it is most important to estimate γ_1 with high accuracy. This seems reasonable since the pump corresponding to γ_1 is supplying water to tank 1 and tank 4, which have the largest outflows of the process. Thus, an error in γ_1 would highly effect the control performance. We also see that it is important to estimate the parameters related to tanks 2 and 4 of the process, i.e., a_2, a_4 and γ_2, with high accuracy.

C. Control Performance Comparison

As a motivation for optimal input design, we estimate the water tank process using an optimal input with minimum power and a white noise input with the same power. We then compare the performance of the MPC controllers based on these estimates. The simulation is performed with the setting specified in Section V-A and γ is defined by (16). The resulting output trajectories are shown in Figure 4.

We see that the optimal input outperforms white noise in terms of satisfying the specification on control performance. In total 91 % of the models estimated using the optimal input satisfy requirement (3) compared to only 15 % of the models estimated with white noise.

If we instead use a white noise input with variance equal to the optimal, eight times more samples are needed to achieve the same control performance as for the optimal input.

The reason that we do not reach the goal of 95 % acceptable models is, to some extent, due to that the identified system is nonlinear and no θ_o exists. If a linear system is estimated, using the same signal realizations, 94 % of the models are acceptable, which is closer to the specifications.

D. Reducing Performance Degradation

We look at the effects of decreasing the upper limit on the performance degradation when a linear model is used for a nonlinear process. To decrease the application cost, we want

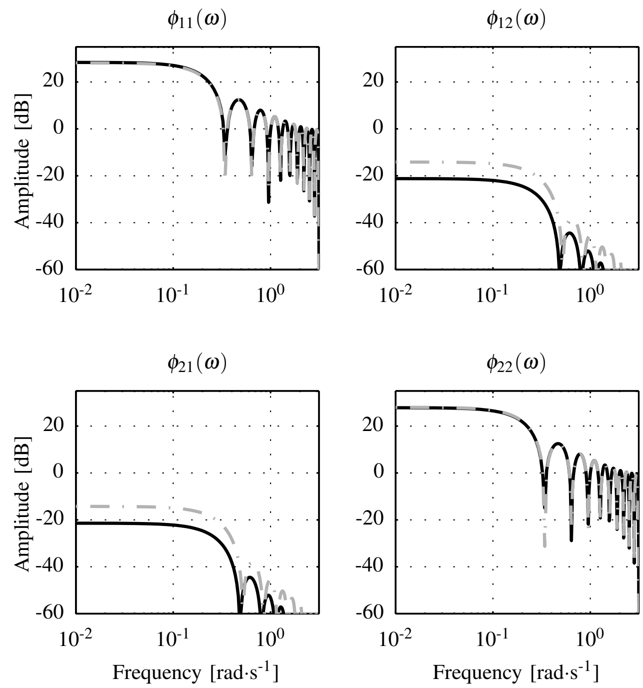

Fig. 3. The input spectra obtained using optimal input design based on θ_o (solid) and an initial estimate of the parameters (dashed). $\phi_{ij}(\omega)$ is the cross spectrum between u_i and u_j.

estimates with lower variance. Had the process been linear, increasing input power, $\mathrm{var}(u)$, or the experiment length, N, would both reduce the estimate variance. We can always trade power for experiment time or vice versa. However, when the process is nonlinear, increasing input power might drive the process too far from the linearization point for the model to be accurate. Therefore, one might have to increase experiment time to reduce the variance.

To investigate this for the tank process, we conduct two experiments. In the first we use $N = 100$ in the input design which gives a high input power solution. In the second we use $N = 10,000$ which gives a low power solution. Note that both designs use the same input energy. We allow for 0.01 % performance degradation, i.e., $\gamma = 10,000/V(\theta_o)$. Figure 5 shows the resulting trajectories, when the estimated models from the two experiments are used in the MPC.

We see that increasing power can degrade the quality of the estimates when the identified plant is nonlinear. If experiment time is increased, higher quality estimates are obtained. In total 85 % of the models from the low input power identification satisfy the requirements while none of the estimates from the high input power identification do.

VI. Conclusion

We presented a method for optimal input design for MPC. The identified model is guaranteed, with high probability, to give a prescribed control performance. The method thereby links system identification and intended use of the model. Optimal input design requires knowledge of the true parameters. These are not known. The proposed solution is to use an

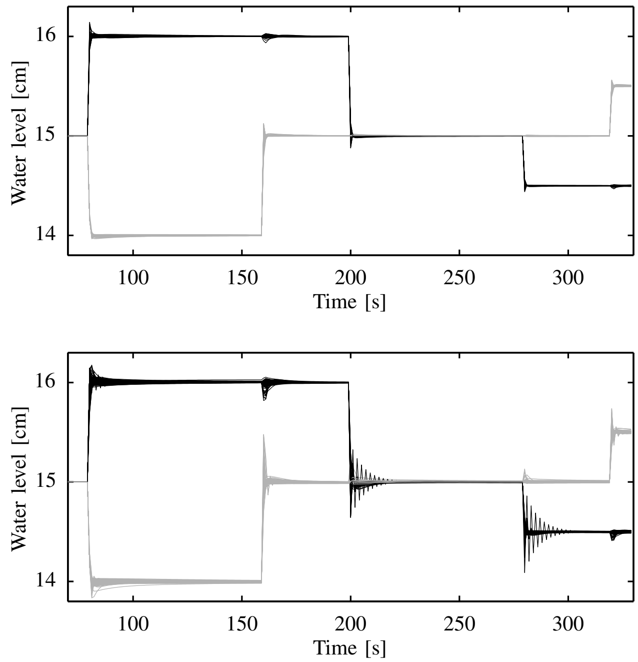

Fig. 4. The trajectories of the process controlled by MPC with models based on estimates from 100 identification experiments. In the upper plot optimal input has been used whereas in the lower plot a white input has been used. For the upper plot 91 % of the trajectories satisfy the performance degradation requirements while only 15 % of the trajectories in the lower plot satisfy the requirements.

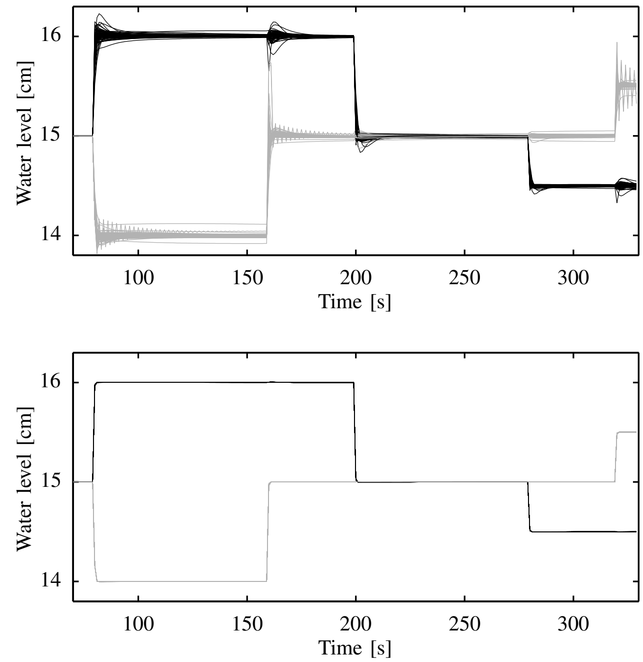

Fig. 5. The trajectories of the process controlled by MPC with models based on estimates from 100 different identification experiments. In the upper plot $N = 100$ samples are used in the identification whereas in the lower plot $N = 10,000$ samples are used. The higher input power in the first case drives the system from the linearization point, giving estimates with high variance. Both experiments have the same total input energy.

initial estimate instead. It can be obtained in an identification experiment or through knowledge of the system. The method requires an evaluation of the control performance degradation with respect to the model. The evaluation may effect the behavior of the system, preventing it from being performed on-line. We propose that it is based on simulations of the system. We used a linear model to approximate the process, even though the true system was nonlinear. We saw that one have to be careful when trading power of the input signal with number of observations in the identification experiment. This is because we use a linear model for input design but identify a nonlinear process. A high variance of the input signal may drive the process state from its linearization point and thus the model is no longer accurate.

REFERENCES

[1] Y. Zhu, "System identification for process control: recent experiment and outlook," in *Proceedings of the 14th IFAC Symposium on System Identification*, Newcastle, Australia, 2006, plenary presentation.

[2] M. Barenthin, H. Jansson, and H. Hjalmarsson, "Applications of mixed $\mathscr{H}_\infty$ and $\mathscr{H}_2$ input design in identification," in *16th World Congress on Automatic Control*. Prague, Czech Republik: IFAC, 2005, paper Tu-A13-TO/1.

[3] B. Wahlberg, H. Hjalmarsson, and M. Annergren, "On Optimal Input Design in System Identification for Control," *kth.se*, 2010.

[4] C. A. Larsson, C. R. Rojas, and H. Hjalmarsson, "MPC oriented experiment design," in *Proceedings of the 18th IFAC World Congress*, no. 1986, Milano, Italy, 2011.

[5] J. Jørgensen and S. Jørgensen, "MPC-Relevant Prediction-Error Identification," in *American Control Conference, 2007. ACC '07*, july 2007, pp. 128 –133.

[6] J. W. MacArthur and C. Zhan, "A practical global multi-stage method for fully automated closed-loop identification of industrial processes," *Journal of Process Control*, vol. 17, no. 10, pp. 770 – 786, 2007.

[7] A. Bemporad, M. Morari, and N. L. Ricker, *Model Predictive Control Toolbox: User's Guide*, The MathWorks, Inc., Natick, MA, 2011.

[8] J. M. Maciejowski, *Predictive Control with Constraints*. Edinburgh Gate, Harlow, Essex, England: Prentice Hall, 2002.

[9] G. Calafiore and M. Campi, "The Scenario Approach to Robust Control Design," *IEEE Transactions on Automatic Control*, no. 5, pp. 742–753, May.

[10] M. Campi and S. Garatti, "The exact feasibility of randomized solutions of robust convex programs," *Optimization online (www. optimization-online. org)*, pp. 1–28, 2007.

[11] L. Ljung, *System Identification: Theory for the User*, 2nd ed. Englewood Cliffs, NJ: Prentice-Hall, 1999.

[12] H. Hjalmarsson, "System identification of complex and structured systems," in *European Control Conference*, Budapest, Hugary, 2009, pp. 3424–3452, plenary address.

[13] H. Jansson, "Experiment design with applications in identification for control," Doctoral thesis, KTH, Stockholm, Sweden, 2004.

[14] C. R. Rojas, "Robust experiment design," Ph.D. dissertation, The University of Newcastle, June 2008.

[15] C. R. Rojas, J. S. Welsh, and G. C. Goodwin, "A receding horizon algorithm to generate binary signals with a prescribed autocovariance," in *Proceedings of the 2007 American Control Conference (ACC)*, New York, July 2007, pp. 122–127.

[16] L. Gerencsér, H. Hjalmarsson, and J. Mårtensson, "Identification of ARX systems with non-stationary inputs - asymptotic analysis with application to adaptive input design," *Automatica*, vol. 45, no. 3, pp. 623–633, March 2009.

[17] K. Johansson, a. Horch, O. Wijk, and a. Hansson, "Teaching multivariable control using the quadruple-tank process," *Proceedings of the 38th IEEE Conference on Decision and Control (Cat. No.99CH36304)*, no. December, pp. 807–812.

[18] M. Grant and S. Boyd, "CVX: Matlab software for disciplined convex programming, version 1.21," http://cvxr.com/cvx, Sept. 2010.

[19] K. C. Toh, M. J. Todd, and R. H. Tutuncu, "SDPT3 – a MATLAB software package for semidefinite programming," *Optimization Methods and Software*, vol. 11, pp. 545–581, 1999.