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Pantograph Catenary Control and Observation using the LMI Approach

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Abstract— **This paper deals with the control of the Pantograph-catenary interaction for high-speed-trains. This problem is directly related to the electric train current collection quality and efficiency. Our main contributions are the derivation of new mathematical description in terms of a multiple model as an alternative to existing time varying ones and further the proposition of an LMI control law to keep the contact force between the pantograph and the catenary close to a desired value in various operating conditions.**

I. INTRODUCTION

High-speed trains are well developping due to their numerous advantages. Indeed, they are safe, sustainable, energy efficient and very convenient for passengers. One of the main drawbacks comes from the current collection generally ensured by a pantograph in mechanical and electrical contact with the overhead equipement, the catenary.

As the speed increases, this contact becomes not permanent and many losses occur due to operational conditions (catenary structure variations, overhead contact line oscillation, train vibrations, etc) and to perturbations (aerodynamics, wind, ice, etc).

Therefore, it is very important to control the PAC (PAntograph-Catenay) contact force to a suitable value, generally about 100 N, to guarantee a satisfatory mechanical contact avoiding excessive wear and a good current/power collection (Figure 1).

The catenary constitutes the main difficulty since it is a complex structure which depends on time and space. In [1] to [4], one can find interesting developments on the modelling of the PAC system.

The pantograph is a suspension system which is generally mathematically described by a linear model of order 1 or 2. Reference [5] is a pioneering work on asymetric pantograph dynamics discussing lumped parameter pantograph models and addressing a general 3 degree of freedom model and corresponding laboratory experiments.

For control purposes, many simplifications have been considered. [6] and [7] presents basic fundamental and experimental results on the PAC system control considering a very simple model of the catenary. Many authors have dealt with the same problem by considered various simplifications on the catenary or pantograph models and by using different control techniques ([8]-[13]).

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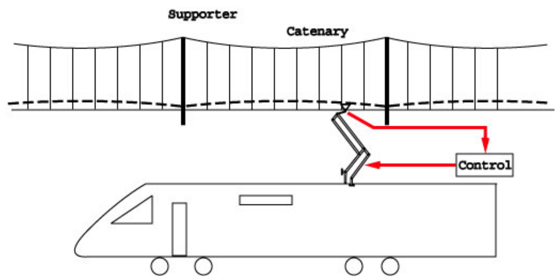

Figure 1: Active control of the PAC system

II. PROBLEM STATEMENT

The pantograph is shown in Figure 2 and it is represented by the 2 degree of freedom mechanical system in Figure 3. The catenary is generally represented by a time-variant stifness $k(t)$ as in [9].

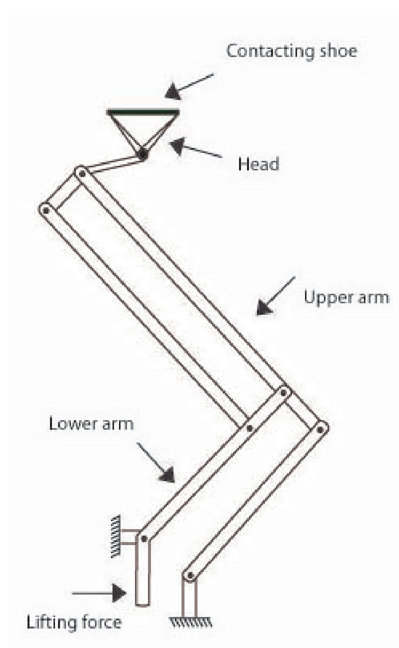

Figure 2: Pantograph mechanism

The state space representation of the PAC system is [7] :

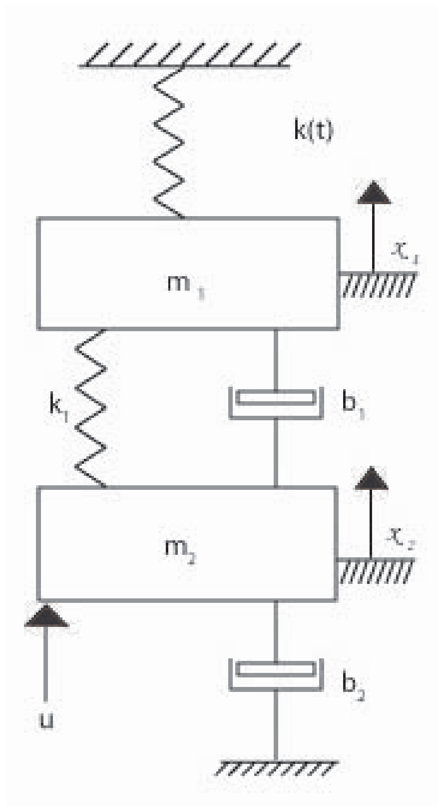

Figure 3: The pantograph model representation

$$\begin{cases} \dot{z}(t) &= A(t)z(t) + Bu(t) \\ y(t) &= C(t)z(t) \end{cases} \tag{1}$$

where

$$z = \begin{bmatrix} z_1 & z_2 & z_3 & z_4 \end{bmatrix}^T = \begin{bmatrix} x_1 & \dot{x}_1 & x_2 & \dot{x}_2 \end{bmatrix}^T,$$

$$A(t) = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_1 + k(t)}{m_1} & -\frac{b_1}{m_1} & \frac{k_1}{m_1} & \frac{b_1}{m_1} \\ 0 & 0 & 0 & 1 \\ \frac{k_1}{m_2} & \frac{b_1}{m_2} & -\frac{k_1}{m_2} & -\frac{b_1 + b_2}{m_2} \end{bmatrix},$$

$$B = \begin{bmatrix} 0 & 0 & 0 & \frac{1}{m_2} \end{bmatrix}^T, \quad C(t) = \begin{bmatrix} k(t) & 0 & 0 & 0 \end{bmatrix},$$

with (see [9])

$$k(t) = K_0 \left(1 + \nu \cos\left(\frac{2\pi V}{L} t\right)\right) \tag{2}$$

V is the train speed (m/s); L the span length (m); K_0 the average equivalent stiffness (N/m); ν is the stiffness variation coefficient in a span. Typical values of the model parameters are given in the Table 1. We also have ([9], [13])

$$\nu = \frac{K_{\max} - K_{\min}}{K_{\max} + K_{\min}}, \qquad K_0 = \frac{K_{\max} + K_{\min}}{2}$$

$K_{\max}$, $K_{\min}$ being respectively the maximum and minimum stiffness value in a span (N/m).

III. PAC ALTERNATIVE DESCRIPTION

Multiple models have been widely considered in the last decade due to their usefulness in many control engineering applications. In this section, we present a multiple model description for the PAC. It is easily seen from (2) that $k(t)$ can be bounded as

$$\underline{K} = K_0(1 - \nu) \leq k(t) \leq \overline{K} = K_0(1 + \nu)$$

inducing

$$-\frac{k_1 + \overline{K}}{m_1} \leq -\frac{k_1 + k(t)}{m_1} \leq -\frac{k_1 + \underline{K}}{m_1}.$$

Consequently, the PAC state space (1) can be rewritten in terms of a multiple model, that is

$$\begin{cases} \dot{z} &= (\mu_1 A_1 + \mu_2 A_2)z + Bu \\ y &= (\mu_1 C_1 + \mu_2 C_2)z \end{cases} \tag{3}$$

where $\mu_1 + \mu_2 = 1$, and

$$\mu_1 = \frac{\overline{K} - k(t)}{\overline{K} - \underline{K}} \geq 0, \qquad \mu_2 = \frac{k(t) - \underline{K}}{\overline{K} - \underline{K}} \geq 0 \tag{4}$$

$$\begin{cases} A_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_1 + \overline{K}}{m_1} & -\frac{b_1}{m_1} & \frac{k_1}{m_1} & \frac{b_1}{m_1} \\ 0 & 0 & 0 & 1 \\ \frac{k_1}{m_2} & \frac{b_1}{m_2} & -\frac{k_1}{m_2} & -\frac{b_1 + b_2}{m_2} \end{bmatrix} \\ C_1 = \begin{bmatrix} \underline{K} & 0 & 0 & 0 \end{bmatrix} \\ A_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_1 + \underline{K}}{m_1} & -\frac{b_1}{m_1} & \frac{k_1}{m_1} & \frac{b_1}{m_1} \\ 0 & 0 & 0 & 1 \\ \frac{k_1}{m_2} & \frac{b_1}{m_2} & -\frac{k_1}{m_2} & -\frac{b_1 + b_2}{m_2} \end{bmatrix} \\ C_2 = \begin{bmatrix} \overline{K} & 0 & 0 & 0 \end{bmatrix} \end{cases} \tag{5}$$

Description (3) is useful to apply specific methods such as LMI's techniques [15] or Takagi-Sugeno approach [16].

IV. PAC CONTROL AND OBSERVATION

A. PAC Control

In this paper, we use LMI's (Linear Matrix Inequalities) to control the PAC system modeled by the multiple model (3). LMI's have been extensively used in control theory [14], [15] due to their simple formulation and available computing solvers such as the LMI's MATLAB Toolbox.

However, the use of LMI's gives little insight on the system dynamics performance. The following proposition ensures prescribed bounds for the closed-loop eigenvalues.

Proposition 1 : Consider the general m-multiple model, $m > 0$,

$$\dot{z} = \sum_{i=1}^{m} \{\mu_i A_i\} z + Bu, \quad \sum_{i=1}^{m} \mu_i = 1,\ \mu_i > 0.$$

If there exists $X > 0$ such that

$$\begin{cases} A_i X + X A_i^T - BB^T < -2\alpha X \\ A_i X + X A_i^T - BB^T > -2\beta X \end{cases} \tag{6}$$

for some $0 \le \alpha < \beta$, then the control law $u = -Fz$ with $F = (1/2)B^T X^{-1}$, is such that $\Re(\lambda(A_c)) \in [-\beta, -\alpha]$ where λ denotes the eigenvalue, $\Re$ the real part, and A_c the closed loop state matrix given by $A_c = \sum_{i=1}^{m} \mu_i \{A_i - BF\}$.

The proof is given in the Appendix.

Remark 1 : By duality, it is easy to get a similar result to the Proposition 1 to design an observer for mutliple models with constant output matrix, of the form

$$\begin{cases} \dot{z} & = \sum_{i=1}^{m} \{\mu_i A_i\} z + \sum_{i=1}^{m} \{\mu_i B_i\} u \\ y & = Cz \end{cases} \tag{7}$$

where $\sum_{i=1}^{m} \mu_i = 1,\ \mu_i > 0$.

Now considering our PAC system (3), the control objective is to drive the output contact force $y(t)$ to a desired constant y_d, typically the desired value being $y_d = 100$ N.

To this end, we introduce the integrator

$$w(t) = \int_0^t (y_d - y(\theta))\theta$$

or equivalently

$$\dot{w}(t) = y_d - y(t) \tag{8}$$

This ensures a zero steady state error. Indeed, at the steady state where $\dot{w}(t) = 0$, one gets $y(t) = y_d$ due to equation (8).

Combining this equation with the system dynamics (3) yields

$$\begin{cases} \dot{z} & = (\mu_1 A_1 + \mu_2 A_2) z + Bu \\ y & = (\mu_1 C_1 + \mu_2 C_2) z \\ \dot{w}(t) & = y_d - y(t) = -\mu_1 C_1 - \mu_2 C_2 + y_d \end{cases}$$

which can be written more compactly under the augmented state space representaion

$$\dot{z^a} = (\mu_1 F_1 + \mu_2 F_2) z^a + Gu + My_d \tag{9}$$

where

$$z^a = \begin{bmatrix} z \\ w \end{bmatrix},\ F_i = \begin{bmatrix} A_i & 0_{4\times1} \\ -C_i & 0 \end{bmatrix},\ G = \begin{bmatrix} B \\ 0 \end{bmatrix},\ M = \begin{bmatrix} 0_{4\times1} \\ 1 \end{bmatrix}.$$

Now, we apply the proposition 1 to the augmented system (9) which gives the sufficient conditions

$$\begin{cases} F_i X + X F_i^T - GG^T < -2\alpha X, & i = 1,2 \\ F_i X + X F_i^T - GG^T > -2\beta X, & i = 1,2 \\ X > 0 \end{cases} \tag{10}$$

and the control law $u = -(1/2)G^T X^{-1} z^a$.

Remark 2: If one wants to avoid the introdution of the integrator, an alternative is to consider a feedforward term in the feedback control to be rewritten

$$u = -Fz + qy_d$$

where $q = -1/[C_0(A_0 - BF)^{-1}B]$ with

$$A_0 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\dfrac{k_1 + K_0}{m_1} & -\dfrac{b_1}{m_1} & \dfrac{k_1}{m_1} & \dfrac{b_1}{m_1} \\ 0 & 0 & 0 & 1 \\ \dfrac{k_1}{m_2} & \dfrac{b_1}{m_2} & -\dfrac{k_1}{m_2} & -\dfrac{b_1 + b_2}{m_2} \end{bmatrix},$$

$$C_0 = \begin{bmatrix} K_0 & 0 & 0 & 0 \end{bmatrix}.$$

However, this will yield to a permanent static error since $k(t)$ is approximated by its mean value K_0.

B. PAC Observer

The augmented system $\{F_i, H_i\}$ with $H_i = \begin{bmatrix} C_i & 0 \end{bmatrix}$ is not obervable whereas $\{A_i, C_i\}$, $i = 1,2$, is. Hence we consider an observer for the state vector z, the last state variale w of z^a can be estimated by a direct calculation from its dynamic equation (8) *i.e.* $\dot{w}(t) = y_d - y(t)$, $y(t)$ being the measured output at time t.

Therefore the oberver has the form

$$\begin{cases} \dot{\hat{z}} = (\mu_1 A_1 + \mu_2 A_2)\hat{z} + Gu + (\mu_1 L_1 + \mu_2 L_2)(y - \hat{y}) \\ \hat{y} = (\mu_1 C_1 + \mu_2 C_2)\hat{z} \end{cases} \tag{11}$$

where $L_i = (1/2)X^{-1}C_i^T$, i=1,2; X is a positive definite matrix satisfying the LMI's

$$\begin{cases} A_i^T X + X A_i - C_i^T C_i < 0, & i = 1,2 \\ X > 0 \end{cases} \tag{12}$$

Considering the expression (11) of $\hat{y}$ and that of C_1 and C_2 in (5), we have

$$\begin{aligned}(\mu_1 L_1+\mu_2 L_2)\hat{y} &= (\mu_1 L_1+\mu_2 L_2)(\mu_1 C_1+\mu_2 C_2)\hat{z}\\ &= (1/2)X^{-1}(\mu_1 C_1^T+\mu_2 C_2^T)(\mu_1 C_1+\mu_2 C_2)\hat{z}\\ &= (1/2)X^{-1}(\mu_1 \underline{K}+\mu_2 \overline{K})^2.\end{aligned}$$

Finally, the observer dynamics can be written

$$\left\{\begin{array}{rl}\dot{\hat{z}}= & (\mu_1 A_1+\mu_2 A_2-(1/2)X^{-1}(\mu_1 \underline{K}+\mu_2 \overline{K})^2)\hat{z}\\ & +Gu+(\mu_1 L_1+\mu_2 L_2)y\\ \hat{y}= & (\mu_1 C_1+\mu_2 C_2)\hat{z}\end{array}\right. \quad (13)$$

This dynamics should be combined with the integrator equation (8) in order to get the overall augmented state z^a in (9).

Remark 2: To speed up the observer dynamics for the local models, one can replace conditions (12) by

$$\left\{\begin{array}{l}A_i^T X+XA_i-C_i^T C_i<-2\gamma X, \quad i=1,2\\ X>0\end{array}\right. \quad (14)$$

for a chosen $\gamma \geq 0$. Hence γ gives an observer design parameter.

To summarize, we address the following control design procedure:

- *step1:* Choose suitable $0 \leq \alpha < \beta$ and solve (10) for X_c using for instance the *lmiedit* function in MATLAB.
- *step2:* Choose suitable $0 \leq \gamma$ and solve (14) giving X_o.
- *step3:* Compute the control $u=-(1/2)G^T X_c^{-1}\hat{z}^a$ where $\hat{z}^a = \begin{bmatrix}\hat{z}\\ w\end{bmatrix}$.

The dynamics of $\hat{z}$ is performed using (13) with $L_i = (1/2)X_o^{-1}C_i^T$, i=1,2, and that of w is derived using (8).

V. Simulation

Using the MATLAB LMI toolbox, conditions (10) with $\alpha=10$ and $\beta=50$ for the control and (12) for the observer have been proven to be feasible with the parameters given in [12] and reproduced in the Table 1. In Figure 4, one can see the obtained contact force corresponding to a desired value of 100 N. Figures 5 to 8 illustrate the resulting state space variables as defined for system (1). These plots clearly show satisfactory behaviors and highlight the feasibility of the proposed methods. However, it should be noted that the results corresponds to a specified constant speed V and are only valid for the chosen speed.

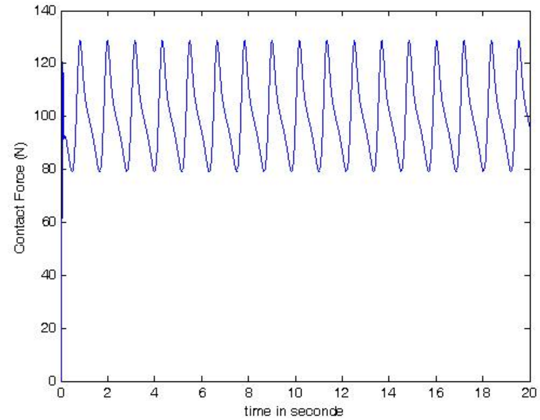

Figure 4 : The contact Force

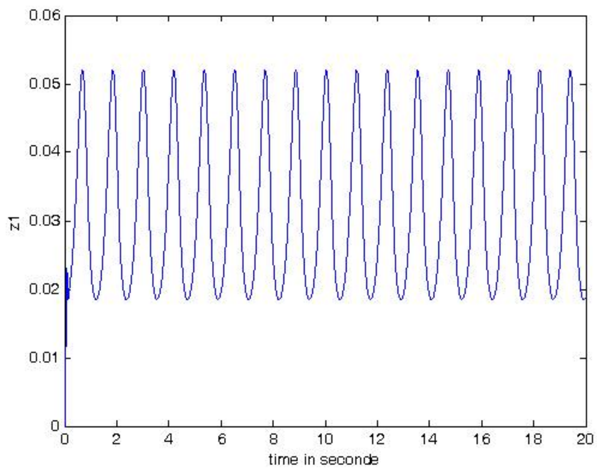

Figure 5 : The head displacement x_1

VI. Conclusion

A new formulation of the PAC model has been addressed in terms of a multiple linear model. This description allows the application of control design methods alternatively to existing ones. We have used the LMI approach to propose a control-observer scheme to drive the contact force to the desired value. Progress is under consideration to synthesize a more general control algorithm which can cope with speed variations.

VII. Appendix : Proof of Proposition 1

With $u=-Fz=-(1/2)B^T X^{-1}z$ and $\sum_{i=1}^{m}\mu_i=1$, the closed loop state matrix can be written

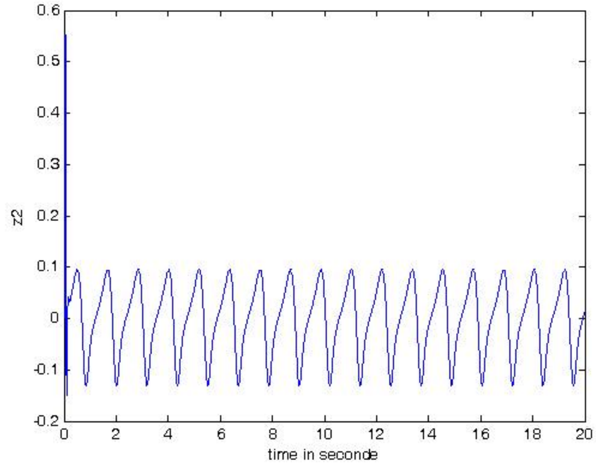

Figure 6 : The head speed $\dot{x}_1$

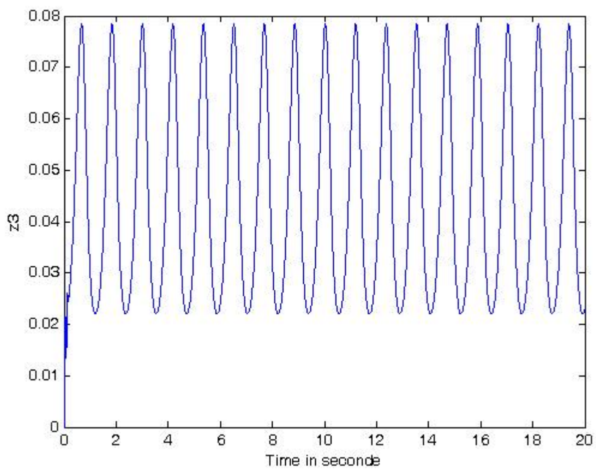

Figure 7 : The frame displacement x_2

$$\begin{aligned}
A_c = & \sum_{i=1}^{m}\{\mu_i A_i\} - BF \\
= & \sum_{i=1}^{m}\{\mu_i A_i - (1/2)BB^T X^{-1}\} \quad \text{(using } F\text{)} \\
A_c X = & \sum_{i=1}^{m}\{\mu_i A_i X - (1/2)BB^T\} \quad \text{(Postmutiplying by } X\text{)} \\
X A_c^T = & \sum_{i=1}^{m}\{\mu_i X A_i^T - (1/2)BB^T\} \quad \text{(By transposing)} \\
A_c X + X A_c^T = & \sum_{i=1}^{m}\{\mu_i A_i X + X A_i^T - BB^T\} \quad \text{By adding} \\
A_c X + X A_c^T \leq & \sum_{i=1}^{m}\{\mu_i(-2\alpha X)\} \quad \text{(Condition (6))} \\
= & -2\alpha X
\end{aligned}$$

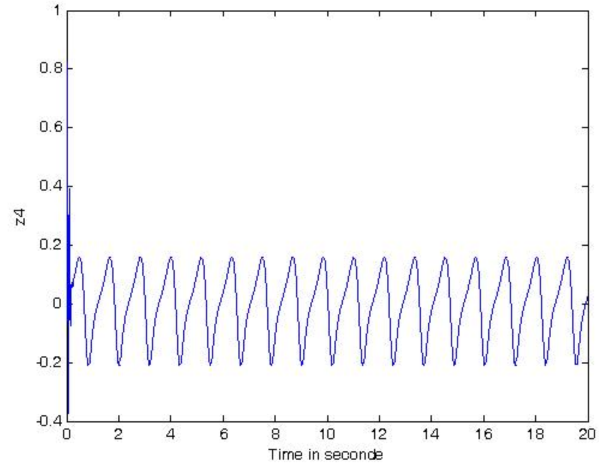

Figure 8 : The frame speed $\dot{x}_2$

TABLE I
PAC PARAMETERS

Parametres	Notations	Value
Catenary	K_0	3.6 kNm^{-1}
	α	0.5
	L	65 m
Pantograph head	m_1	8 kg
	b_1	120 N sm^{-1}
	k_1	10 kNm^{-1}
Pantograph frame	m_2	12 kg
	b_2	30 N sm^{-1}

Next let λ be an eigenvalue of A_c and v an associated eigenvector, that is $A_c v = \lambda v$. Premultiplying by v^* and postmultiplying by v the last inequality yields

$$\begin{aligned}
& v^* A_c X v + v^* X A_c^T v \leq -2\alpha v^* X v \\
\rightarrow \quad & \bar{\lambda} v^* X v + \lambda v^* X v \leq -2\alpha v^* X v \\
\text{since } X > 0, \rightarrow \quad & \Re(\lambda) < -\alpha.
\end{aligned}$$

Using similar arguments, the second condition in (6) implies $\Re(\lambda) > -\beta$. The proof is complete.

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