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Dealing with plant variations in multi-model unfalsified switching control via adaptive memory selection

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***Abstract*— In this paper, a multi-model unfalsified adaptive switching control scheme is proposed for controlling uncertain plants subject to time variations. In the adopted approach, the switching between the candidate controllers is orchestrated according to a hysteresis logic variant wherein the memory length is adaptively selected, on the basis of the exhibited plant I/O behavior, so that past recorded data can be safely discarded. To this end, novel model-based resetting conditions are introduced. The global stability of the resulting switched closed-loop system is guaranteed provided that, at every time instant, a stabilizing candidate controller exists and that the (possibly abrupt) changes in the plant model are infrequent.**

I. INTRODUCTION

Adaptive switching control (ASC) has recently gained special attention as a promising approach for controlling plants in the presence of large model uncertainties. In switching control, a "high-level" data-driven unit, called the supervisor, switches-on at any time in feedback with the plant one element from a family of candidate controllers on the basis on past plant I/O data. These data are processed to enable the supervisor to decide whether the currently controller is adequate, and, in the negative, to replace it by a different candidate controller. ASC can therefore be seen as an adaptive version of classic gain-scheduling control. For an early overview of ASC, the reader is referred to [1].

Multiple schemes for ASC have been proposed in the literature, which can be classified on the basis of which switching logic is adopted (with possible alternatives being pre-routing [2], [3], dwell-time [4], [5], and hysteresis switching [6], [7]) and on the basis of how the selection of the controller to be switched on is carried out. As for the latter issue, the main current approaches to ASC can be subdivided into two different groups (multi-model ASC and model-free ASC) according to whether, or not, a family of dynamic nominal models of the plant be available together with the family of candidate controllers. Among the most significant approaches that use multi-model ASC architectures are those investigated in [1], [5], [8], [9], [10], [11]. On the other hand, the most relevant model-free ASC schemes are those developed by Safonov [12], [13], [14], [15] and his coworkers in the so-called *unfalsified control* framework. In unfalsified ASC, thanks to the use of the virtual reference tool, the supervisor infers the performance of the potential loop made up by each candidate controller and selects the one with the best inferred performance.

More recently [16], [17], a novel approach, multi-model unfalsified ASC (MMUASC), has been proposed which combines multi-model and unfalsified ASC by: embedding, in the unfalsified ASC framework, a family of nominal models pairwise associated with the given candidate controllers; and devising appropriate test functionals based on a percentage measure of discrepancy. The resulting MMUASC scheme retains the main positive features of both approaches. In fact, like in unfalsified ASC, stability in-the-large is guaranteed under the minimal conceivable assumption that a candidate stabilizing controller exists and, like in multi-model ASC, the magnitude and time duration of learning transient can be dramatically reduced provided that the nominal model distribution be dense enough.

Unfortunately, most of the positive features of MMUASC, as proposed in [16], [17], are lost for plants subject to persistent time variations due to the infinite memory of the test functionals and of the adopted hysteresis switching variant. The objective of this paper is to show how MMUASC schemes can be modified so as to deal with persistent plant variations by *adaptively* selecting the memory length. Such an adaptive selection is accomplished, on the basis of the exhibited plant I/O behavior, by devising an appropriate *resetting logic* whereby past recorded data can be safely discarded. This makes it possible to ensure (global) stability of the resulting closed-loop switched system despite possible time variations of the uncertain plant. Due to space constraints all the proofs are omitted.

Notations. Throughout the paper, the prime denotes transpose, $|\cdot|$ Euclidean norm, and $\mathcal{S}$ the space of all real-valued vector sequences on the set $\mathbb{Z}_+$ of nonnegative integers. For any $s \in \mathcal{S}$, and $t_0, t \in \mathbb{Z}_+$, $t_0 \leq t$, we define $s|_{t_0}^{t} := \{s(t_0), \ldots, s(t)\}$. For simplicity, if $t_0 = 0$, s^t indicates the sequence $s|_{t_0}^{t}$. Given λ, $0 < \lambda \leq 1$, we denote the λ-exponentially weighted ℓ_2-norm of $s|_{t_0}^{t}$ by $\| s|_{t_0}^{t} \|_\lambda := \sqrt{\sum_{\tau=t_0}^{t} \lambda^{2(t-\tau)} |s(\tau)|^2}$ whenever $t \geq t_0$, or the zero number otherwise. If $\lambda = 1$, we let $\| s|_{t_0}^{t} \|$ denote the ℓ_2-norm of $s|_{t_0}^{t}$. The ℓ_∞-norm of $s|_{t_0}^{t}$ is defined as $\| s|_{t_0}^{t} \|_\infty := \max_{\tau \in \{t_0, \cdots, t\}} \max_i |s_i(\tau)|$ where s_i denotes the $i-$th component of s. The sequence $s \in \mathcal{S}$ is said to be *bounded* if its ℓ_∞-norm is finite.

II. PROBLEM SETTING

We consider the adaptive control system depicted in Figure 1. Specifically, the plant P to be controlled consists

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of a discrete-time strictly causal SISO linear time-varying dynamic system, described by

$$A(t,d)\, y(t) = B(t,d)\ [u(t) + n_u(t)] \\ + A(t,d)\, n_y(t), \quad t \in \mathbb{Z}_+ \quad (1)$$

with input u, output y, input disturbance n_u and output disturbance n_y. $A(t,d)$ and $B(t,d)$ denote time-varying polynomials in the unit backward shift operator d.

The supervisor handles the plant I/O past data in order to generate the sequence σ specifying the switching controller C_σ. The latter has the one-degree-of-freedom form $u(t) = C_{\sigma(t)}[r(t) - y(t)]$, where r is the output reference, while the subscript $\sigma(t)$ identifies the specific candidate controller connected in feedback to the plant at time t. More specifically, all candidate controllers belong to a finite family $\mathscr{C} = \{C_i,\ i \in \overleftarrow{N}\}$, $\overleftarrow{N} := \{1, \cdots, N\}$, of linear time-invariant (LTI) controllers with transfer functions $C_i(d) := S_i(d)/R_i(d)$ with no unstable hidden modes, *i.e.*, the polynomials $S_i(d)$ and $R_i(d)$ have no common roots outside the open unit circle of the complex plane. Accordingly, given $\sigma(t)$ at time t, the plant input $u(t)$ is given as follows

$$R_{\sigma(t)}(d)\, u(t) = S_{\sigma(t)}(d)\ [r(t) - y(t)] \quad (2)$$

Assume that, for every $t \in \mathbb{Z}_+$, the LTI plant with transfer function $B(t,d)/A(t,d)$ have no unstable hidden modes and belong to an uncertainty set $\mathscr{P}$. Let P_* denote a generic element of $\mathscr{P}$. Given a finite family $\mathscr{C}$ of candidate controllers, $\mathscr{C}_S(P_*)$ will denote the subset of $\mathscr{C}$ composed by all controllers which (internally) stabilize P_*.

Definition 1: The switched system (1)-(2) is said to be (globally) *stable* if, for all initial conditions, any bounded exogenous input (r, n_u, n_y) produces a bounded output (u, y). The problem is said to be feasible if $\mathscr{C}_S(P_*) \neq \emptyset$, $\forall\, P_* \in \mathscr{P}$.

A. Basic Assumptions

Definition 2: A polynomial $p(d)$ is said to be a λ-*Hurwitz* polynomial (in the indeterminate d) if it has no root in the closed disk of radius λ^{-1} of the complex plane.

Definition 3: The feedback loop (P_*/C_i) composed by the time-invariant plant P_* and the controller C_i, whose transfer functions are given by $B_*(d)/A_*(d)$ and $S_i(d)/R_i(d)$, respectively, is said to be λ-*stable* if its characteristic polynomial

$$\chi_{*/i}(d) := A_*(d)\, R_i(d) + B_*(d)\, S_i(d)$$

is a λ-*Hurwitz* polynomial.

We make the following assumptions.

- A1 The plant uncertainty set $\mathscr{P}$ is compact, *i.e.*, for every $P_* \in \mathscr{P}$, the polynomials A_* and B_* have bounded orders and their coefficients belong to a compact set.
- A2 For every $P_* \in \mathscr{P}$, there always exists a candidate controller $C_i \in \mathscr{C}$ such that (P_*/C_i) is λ-*stable*.
- A3 The exogenous inputs r, n_u, and n_y are bounded.

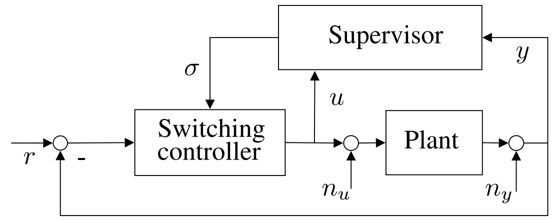

Fig. 1. Adaptive switching control scheme

III. Reference-Loop Identification and Model-Based Test Functionals

In order to decide when and how to change the controller, the supervisor embodies a family $\pi := \left\{\pi_i,\ i \in \overleftarrow{N}\right\}$ of test functionals where, in broad terms, $\pi_i(t)$ quantifies the suitability of the candidate C_i to control P, given the I/O data up to time t. In some cases, $\pi_i(t)$ might assume the meaning of a performance measure of (P/C_i). The notation (P/C_i) denotes the feedback loop composed by the plant P interconnected in feedback with the controller C_i.

In the remaining of this section, the considered multi-model test functionals will be described and some of their basic features analyzed. To this end, it will be assumed that a finite family $\mathscr{M} := \{M_i, i \in \overleftarrow{N}\}$ of N strictly causal LTI dynamic models M_i is available,

$$M_i(d) := B_i(d)/A_i(d), \quad i \in \overleftarrow{N}$$

where $B_i(d)$ and $A_i(d)$ are coprime polynomials and $A_i(0) = 1$. In a MMUASC scheme, the candidate controllers C_i, $i \in \overleftarrow{N}$, are chosen so as to satisfy at least the feasibility condition, while the M_i's, along with the associated C_i's, form a finite family $\mathscr{R} := \left\{(M_i/C_i),\ i \in \overleftarrow{N}\right\}$ of *internally* stable feedback-loops, each designed to fulfill desirable prescriptions. Hereafter, (M_i/C_i) will be referred to as the i-th "*tuned-loop*" or "*reference-loop*". Given an unknown plant $P \in \mathscr{P}$, one of the main steps in MMUASC is to carry out a *reference-loop identification* task, *viz.*, select a candidate controller C_σ in such a way that (P/C_σ) behave as closest as possible to one of the candidate reference-loops in $\mathscr{R}$.

Hence, roughly speaking, the ideal goal of the switching supervisor, can be envisaged as follows. Given an uncertain plant $P \in \mathscr{P}$, find an index $\sigma \in \overleftarrow{N}$ such that: i) (P/C_σ) is stable; and the behavioral data produced by (P/C_σ) in response to r are as closest as possible to the ones produced by (M_σ/C_σ) in accordance to the reference-loop identification criterion

$$\sigma := \arg\min_{i \in \overleftarrow{N}} \sup_{r \neq 0} \frac{\|(P/C_i)[r] - (M_i/C_i)[r]\|_\lambda}{\|(M_i/C_i)[r]\|_\lambda} \quad (3)$$

where, by the sake of simplicity, no time-argument is shown and the dependence of the behavioral data on the disturbances n_u and n_y are omitted.

On-line implementation of (3) is impossible without using pre-routing, a solution that, in general, has to be ruled out because it typically causes large and long-lasting learning transients. A way for side-stepping such a difficulty hinges upon the use of the virtual reference (VR) concept [12], [13]. At each time, and for each index $i \in \overleftarrow{N}$, one solves in real-time with respect to $v_i(t)$ the difference equation

$$S_i(d)(v_i(t) - y(t)) = R_i(d)u(t), \quad t \in \mathbb{Z}_+ . \tag{4}$$

In words, v_i^t equals the virtual reference sequence which reproduces the recorded I/O sequence (u^t, y^t) should the plant P be fed-back by the candidate controller C_i, irrespective of the way the plant input u^t is generated.

Since (4) is computed for all the indices in $\overleftarrow{N}$, it is possible to compare the performance achievable by each candidate loop (P/C_i), driven by its related v_i, to that of its corresponding reference loop (M_i/C_i). Therefore, (3) is modified as follows

$$\sigma := \arg\min_{i\in\overleftarrow{N}} \sup_{v_i \neq 0} \frac{\|(P/C_i)[v_i] - (M_i/C_i)[v_i]\|_\lambda}{\|(M_i/C_i)[v_i]\|_\lambda} \tag{5}$$

In order to specify an on-line implementable for (5), let

$$\begin{aligned} w &:= [r \quad n_u \quad n_y]' \\ w_i &:= [v_i \quad n_u \quad n_y]' \\ z &:= [u \quad y]' . \end{aligned}$$

Then, by the definition of virtual reference, one has

$$z(t) = (P/C_{\sigma(\cdot)})[w](t) = (P/C_i)[w_i](t) . \tag{6}$$

Further, the behavioral data $z_i := [u_i \quad y_i]'$ of each candidate reference-loop (M_i/C_i), driven by the corresponding v_i, are given by

$$\left.\begin{aligned} A_i(d)\, y_i(t) &= B_i(d)\, u_i(t) \\ S_i(d)\, (v_i(t) - y_i(t)) &= R_i(d) u_i(t) \end{aligned}\right\} . \tag{7}$$

Then, a convenient test functional related to the identification criterion (5) is as follows

$$\pi_i(t) := \frac{\|(z - z_i)^t\|_\lambda}{\mu + \|z_i^t\|_\lambda}, \quad t \in \mathbb{Z}_+ \tag{8}$$

where $\mu > 0$ accounts for possible non-zero initial states as well as for the exogenous disturbances n_u and n_y. Hereafter, the test functional (8) will be referred to as *virtual percentage discrepancy* relatively to the i-th reference loop.

In the arrangement of equation (5) and the subsequent developments, it is understood that all v_i's are computable, which would require that all the candidate controllers be stably casually invertible. However, it turns out that such restrictive conditions can be circumvented since it is possible to obtain (6) without direct computation of the v_i. In fact, by combining (4) with (7), the reference-loop behavioral data z_i can be computed from z as follows

$$\begin{bmatrix} -B_i(d) & A_i(d) \\ R_i(d) & S_i(d) \end{bmatrix} z_i(t) = \begin{bmatrix} 0 & 0 \\ R_i(d) & S_i(d) \end{bmatrix} z(t) , \quad t \in \mathbb{Z}_+ , \tag{9}$$

which only requires the i-th tuned loop to be stable. Accordingly, the following assumption is made.

A4 Each reference loop $(C_i/M_i) \in \mathscr{R}$ is λ-stable.

For reasons that will be clarified later on, A4 actually requires that each reference loop has a large enough stability margin. Because $\mathscr{R}$ is a finite set, this requirement can always be fulfilled by choosing λ close enough to one.

To avoid needless complications, we assume that the switching controller (2) as well as the reference loops (9) are initialized at time zero from zero initial conditions by letting r, z and $z_i, \in \overleftarrow{N}$ be zero for $t < 0$. Regarding the initialization of (1), let $(u_P,\, y_P)$ denote the sequence of actual I/O pairs of the plant P. [1] Then, we shall denote by $\xi_P := [\, u_P(-1)\; u_P(-n)\; \ldots\; y_P(-1)\; y_P(-n)\,]'$ the vector composed by the plant initial conditions, where $n := \max_{\mathscr{P}} \{\deg B_*, \deg A_*\}$, $P_* \in \mathscr{P}$.

IV. Switching Logic

The properties of test functionals as in (8) have been thoroughly analyzed and discussed in [16], [17] for time-invariant plants. In this context, test functionals as in (8) enjoys the nice property that (P/C_i) is stable if and only if π_i takes on finite values, $\forall\, r \in \mathcal{S}$ and σ. Starting from these considerations, such test functionals have been shown capable of ensuring stability despite plant uncertainties and disturbances when used together with a Hysteresis switching logic with infinite memory (HSL-∞), wherein one considers the infinite-memory test functionals

$$\Pi_i(t) := \left\|\pi_i^t\right\|_\infty , \quad i \in \overleftarrow{N} . \tag{10}$$

Unfortunately, a strategy with infinite memory is hampered in applications involving time-varying plants. In fact, to handle possible plant variations, one needs to adopt test functionals with fading memory and analyze stability under persistent switching (i.e., without relying on a finite switching as in [16], [17]).

In order to overcome such limitations, we propose to modify (10) by *adaptively* selecting the memory length of $\Pi_i(t)$. In this respect, one simple scheme is to adopt a *resetting logic*, where resetting here denotes the mechanism according to which the supervisor resets all the Π_i's to zero whenever suitable events (resetting conditions), to be specified next, occur. Specifically, the mentioned mechanism consists of: defining the adaptive-memory test functionals

$$\Pi_i(t) := \left\| \pi_i|_{t_k}^t \right\|_\infty , \quad t \in \mathbb{T}_k, \qquad \mathbb{T}_k := \{t_k, \ldots, t_{k+1} - 1\}$$

where $\{t_k\}_{k\in\mathbb{Z}_+}$, $t_0 := 0$, is the sequence of resetting instants to be specified; and then, at each step, computing the least index $i_*(t)$ in $\overleftarrow{N}$ such that $\Pi_{i_*(t)}(t) \leq \Pi_i(t)$, $\forall i \in \overleftarrow{N}$. In particular, the switching index sequence σ is generated as follows

$$\left.\begin{aligned} &\sigma(t+1) = l(\sigma(t), \Pi(t)), \qquad \sigma(0) = i_0 \in \overleftarrow{N} \\ &l(i, \Pi(t)) = \begin{cases} i, & \text{if } \;\Pi_i(t) < \Pi_{i_*(t)}(t) + h \\ i_*(t), & \text{otherwise} \end{cases} \end{aligned}\right\}$$

[1]In view of the feedback configuration in Fig. 1, the sequence $(u_P,\, y_P)$ satisfies $u_P = u + n_u$ and $y_P = y - n_y$.

where $h > 0$ is the hysteresis constant.

For clarity, from now on this logic will be referred to by HSL-R (Hysteresis Switching Logic with Resetting).

A. Admissible Resetting

The adoption of a resetting rule, which is necessary to handle possible variations, carries the consequent potential risk that the switched system become unstable due to persistent switching. To circumvent this problem and obtain stability it is important that a reset occurs only when the switched system (P/C_σ) exhibits a satisfactory behavior. In view of the arrangements of Section III, a possibility for ensuring this consists in enabling a reset only when (P/C_σ) behaves close enough to the candidate reference loop (M_σ/C_σ), both the loop being driven by the actual reference signal r.

To this end, it is convenient to consider, for each reference loop (M_i/C_i), $i \in \overleftarrow{N}$, the *actual percentage discrepancy*

$$\bar{\pi}_i(t) := \frac{\|(z-\bar{z}_i)^t\|_\lambda}{\mu + \|\bar{z}_i^t\|_\lambda}, \quad t \in \mathbb{Z}_+ \tag{11}$$

where $\bar{z}_i$ are the behavioral data of the i-th reference loop driven by the actual reference signal r. Each $\bar{z}_i$ is computed, from zero initial conditions, as follows

$$\begin{bmatrix} -B_i(d) & A_i(d) \\ R_i(d) & S_i(d) \end{bmatrix} \bar{z}_i(t) = \begin{bmatrix} 0 \\ S_i(d) \end{bmatrix} r(t), \quad t \in \mathbb{Z}_+ . \tag{12}$$

Then the following definition can be introduced.

Admissible Resetting Times: A sequence of reset times $\{t_k\}_{k\in\mathbb{Z}_+}$ is called *admissible* if, for every $k \geq 0$, we have that

$$\bar{\pi}_{\sigma(t_k)}(t_k - 1) \leq \pi_{\sigma(t_k)}(t_k - 1) + \epsilon, \quad \epsilon > 0 \tag{13}$$

To understand the rationale for (13), we note that inequality (13) guarantees that $\bar{\pi}_\sigma$ does not get much larger than π_σ, whereas the selection of σ through the HSL-R already makes sure that π_σ remains bounded (as will be shown in Section V).

In accordance with the above considerations, hereafter the following *resetting rule* will be considered (with $t_0 = 0$)

$$t_{k+1} := 1 + \min \{ t : \ t \geq t_k; \\ \bar{\pi}_{\sigma(t+1)}(t) \leq \pi_{\sigma(t+1)}(t) + \epsilon \}, \quad k \in \mathbb{Z}_+ \tag{14}$$

which, by construction, always generates an admissible resetting sequence satisfying (13).

As shown next, when π_σ remains bounded, the resetting rule embodied by eq. (14) enjoys the following two nice properties: a) the plant input/output data z are always bounded; b) the admissibility condition (13) is always attained in finite time.

B. Boundedness of the data

Consider an admissible resetting sequence $\{t_k\}_{k\in\mathbb{Z}_+}$ and let

$$\Pi^k := \min_{i\in\overleftarrow{N}} \left\{ \max_{t\in\mathbb{T}_k} \pi_i(t) \right\} + h, \qquad k \in \mathbb{Z}_+ . \tag{15}$$

In order to prove property a), we first derive an upper bound on the plant input/ouptut data over each interval $\mathbb{T}_k$ as established in the next lemma.

Lemma 1: Let the HSL-R switched system be based on the test functionals (8). Then, under assumptions A1-A4, there exists a bounded function $\mathfrak{g}$ such that, $\forall\, t \in \mathbb{T}_k$,

$$\left\|z^t\right\|_\lambda \leq \mathfrak{g}\left(\Pi^k\right) \left[\mu + |\,\xi_P\,|\, \lambda^{t+1} + \left\|w^t\right\|_\lambda \\ + \left\|z^{t_k-1}\right\|_\lambda \lambda^{t-t_k+1} \right] \tag{16}$$

holds for any resetting sequence $\{t_k\}_{k\in\mathbb{Z}_+}$. □

Notice that the bound of equation (16) depends both on the initial condition of the plant, through the term $|\xi_P|\lambda^{t+1}$, and on the plant state at the beginning of the interval $\mathbb{T}_k$ through the term $\left\|z^{t_k-1}\right\|_\lambda \lambda^{t-t_k+1}$. While the former contribution goes to zero exponentially, the latter need not vanish since $t - t_k$ is reset to zero every time a reset occurs. This means that boundedness of $\left\{\Pi^k\right\}_{k\in\mathbb{Z}_+}$ cannot *per se* ensure boundedness of the data, unless the sequence $\left\{\left\|z^{t_k-1}\right\|_\lambda\right\}_{k\in\mathbb{Z}_+}$ is bounded as well. This is precisely the point where the admissibility condition (13) comes into play. In fact, thanks to the boundedness of the actual percentage discrepancy $\bar{\pi}_\sigma$ at each reset instant, the following result can be stated.

Proposition 1: Consider the HSL-R based on (14) and let assumption A4 hold. Then for any admissible resetting sequence $\{t_k\}_{k\in\mathbb{Z}_+}$,

$$\|z^{t_k-1}\|_\lambda \leq (\Pi^{k-1} + \epsilon)\,\mu + (\Pi^{k-1} + \epsilon + 1)\, g\, \|r^{t_k-1}\|_\lambda$$

for some positive real $g \geq 1$. □

Combining Propostion 1 with Lemma 1, it is immediate to state the following theorem which represents the main result of this section.

Theorem 1: Consider the HSL-R based on (14) and let assumptions A1-A4 hold. Further, assume that $\Pi^k \leq \Pi^*$, $\forall\, k \in \mathbb{Z}_+$, for some finite constant Π^*. Then, for any admissible resetting sequence $\{t_k\}_{k\in\mathbb{Z}_+}$

$$\left\|z^t\right\|_\lambda \leq \mathfrak{g}\left(\Pi^*\right) |\,\xi_P\,|\, \lambda^{t+1} + \mathfrak{h}\left(\Pi^*\right) \left[\mu + \left\|w^t\right\|_\lambda\right], \\ \forall\, t \in \mathbb{Z}_+ \tag{17}$$

where $\mathfrak{h}\left(\Pi^*\right) := \mathfrak{g}\left(\Pi^*\right) \left[(\Pi^* + \epsilon + 1)\, g + 1\right]$. □

It follows from Theorem 1 that, for the proposed switching adaptive control scheme based on the test-functionals (8) and on the reset rule (14), boundedness of at least one test functional in each reset interval is a sufficient condition for stability. It will be shown in Section V that such a property holds provided that the plant variations are sufficiently slow.

C. Finite-Time Resetting

As should be evident from the discussion of Section IV, in order to deal with plant variations it is important that the reset condition be always attained in finite time. In fact, this is crucial to ensure that, by discarding past recorded data, the supervisor be able to promptly select a controller well-suited to the currently active plant configuration. In order to show that the reset rule (14) enjoys such a property, a key observation is that for the switched on controller the virtual percentage discrepancy π_σ converges to the actual percentage discrepancy $\bar{\pi}_\sigma$. More specifically, the following result holds.

Proposition 2: Let assumption A4 hold. Further suppose that the same controller, say C_i, is switched on from a certain time τ up to the current time t. Then

$$\|(z_i - \bar{z}_i)^t\|_\lambda \le \lambda^{t-\tau+1}\left(f_1\|z^{\tau-1}\|_\lambda + f_2\|r^{\tau-1}\|_\lambda\right) \quad (18)$$

for some positive reals f_1, f_2. □

Notice now that, under the same assumptions as in Theorem 1, the plant input/output data can be upper bounded as

$$\begin{aligned}\max_{t\in\mathbb{Z}_+}\left\|z^t\right\|_\lambda \;&\le\; (1-\lambda^2)^{-1/2}\mathfrak{h}\left(\Pi^*\right)\left[\mu + \|w\|_\infty\right] \\ &+ \mathfrak{g}\left(\Pi^*\right)|\,\xi_P\,| \;=:\; Z\left(\Pi^*\right) \qquad (19)\end{aligned}$$

where $\|w\|_\infty$ is finite in view of assumption A4. Then, combining such an upper bound with inequality (18), it is an easy matter to see that the admissibility condition (13) is eventually attained provided that the same controller is switched on for a sufficiently long time. Since over each reset interval the number of switching instants is bounded from the above by $\mathfrak{N}_* := N\left\lceil \Pi^*/h\right\rceil$, the above arguments can be exploited in order to show that the reset condition is always attained in finite time. More specifically, if one defines

$$\Delta\left(\Pi^*\right) := \left(\mathfrak{N}_*+1\right)\left\lceil \log_\lambda\left(\epsilon\,\mu\right) - \log_\lambda(\Pi^*+1) - \log_\lambda\left[f_1 Z\left(\Pi^*\right) + f_2(1-\lambda^2)^{-1/2}\|w\|_\infty\right]\right\rceil,$$

the next result can be stated.

Theorem 2: Consider the HSL-R based on (14). Then, under the same assumptions as in Theorem 1, one has

$$t_{k+1} - t_k \;\le\; \Delta\left(\Pi^*\right), \qquad (20)$$

for any $k \in \mathbb{Z}_+$. □

V. Bound on the test functionals and stability

In this section, we discuss how the HSL-R based on test functionals (8) and reset rule (14) yields stability of the switched system (1)-(2) in cases of abrupt, but infrequent, plant variations. To this end, let $\{\ell_c\}_{c\in\mathbb{Z}_+}$ denote the sequence of time instants at which a plant variation occurs, with $\ell_0 := 0$ by convention. Accordingly, we let $\mathbb{L}_c := \{\ell_c, \ldots, \ell_{c+1}-1\}$, $c \in \mathbb{Z}_+$, define the c-th time interval over which the plant is constant.

A. Analysis for time-invariant plants

In view of the results of Section IV-B, in order to prove stability it is sufficient to show that, in each reset interval there always exists at least one index i for which the test functional (8) remains bounded. For the sake of clarity, we first provide a bound on the test functional in each interval $\mathbb{L}_c$. To this end, let P^c denote the time-invariant model taken on by the plant in the interval $\mathbb{L}_c$ and let $B^c(d)/A^c(d)$ be its transfer function.

Notice now that, by means of simple manipulations, the discrepancy $z - z_i$ can be written as a function of z_i and of the exogenous disturbances n_u and n_y as follows

$$\begin{bmatrix} -B^c & A^c \\ R_i & S_i \end{bmatrix}(z-z_i)(t) = \begin{bmatrix} B^c & -A^c \\ 0 & 0 \end{bmatrix}\zeta_i(t)\,, \qquad t\in\mathbb{L}_c\,, \qquad (21)$$

where,

$$\zeta_i := z_i + \begin{bmatrix} n_u(t) \\ -n_y(t) \end{bmatrix}$$

and, here and in the following, for the sake of compactness the unit backward shift operator d is omitted. Then, the mapping from ζ_i to $z - z_i$ coincides with the generalized sensitivity matrix of the feedback loop (P^c/C_i)

$$\Omega_i^c = \frac{1}{R_iA^c + S_iB^c}\begin{bmatrix} -S_iB^c & S_iA^c \\ R_iB^c & -R_iA^c \end{bmatrix}.$$

Since, for indices i belonging to $\mathscr{C}_S(P^c)$, such sensitivity matrix is λ stable, the following upper bound can be derived.

Lemma 2: Let assumptions A1-A4 hold. Then, there exist finite positive constants κ_0, κ_1, κ_2 and κ_3 such that, for every $P^c \in \mathscr{P}$, there exists some $i \in \overleftarrow{N}$ for which

$$\begin{aligned}\pi_i(t) \quad &\le \quad \kappa_0 + \kappa_1\,|\,\xi_P\,|\,\lambda^{t+1} + \kappa_2\,\|w\|_\infty \\ &+\kappa_3\left\|z^{\ell_c-1}\right\|_\lambda\lambda^{t-\ell_c+1}, \quad \forall\, t\in\mathbb{L}_c\,. \quad (22)\end{aligned}$$

□

By virtue of Lemma 2, it is possible to readily establish stability in case the plant be time-invariant, i.e., $\mathbb{L}_0 = \mathbb{Z}_+$. Indeed, in this case, since $\ell_0 = 0$ one can conclude that there always exists at least one index i for which

$$\pi_i(t) + h \;\le\; \kappa_0 + \kappa_1\,|\,\xi_P\,| + \kappa_2\,\|\,w\,\|_\infty + h =:\; \Pi^*_{TI} \quad (23)$$

Then the following result follows from Theorem 1.

Theorem 3: Consider the HSL-R based on (14) and let assumptions A1-A4 hold. Further, let the plant be time-invariant. Then, the switched system (1)-(2) is stable. □

This result, together with Theorem 2, indicates that, when the plant is time-invariant, stability of the switched system can be achieved without relying on a finite switching stopping time (as, instead, it was enforced in [16] through the HSL-∞).

B. Stability under infrequent plant changes

In the presence of plant variations, the upper bound (23) in general does not hold. Nevertheless, in view of Lemma 2, one can see that for every $c \in \mathbb{Z}_+$ there always exists a candidate index $i \in \overleftarrow{N}$ such that

$$\pi_i(t) + h \;\le\; \Pi^*_{TI} + \kappa_3\left\|z^{\ell_c-1}\right\|_\lambda\lambda^{t-\ell_c+1}, \quad \forall\, t\in\mathbb{L}_c \quad (24)$$

where κ_3 and Π^*_{TI} are as in (22) and (23), respectively. Thus for any given accuracy ν and provided that the next plant variation instant ℓ_{c+1} be far enough, the right-hand side of (24) eventually enters a neighborhood of amplitude ν of Π^*_{TI}, the upper bound corresponding to the time-invariant case. With this respect, let

$$\ell_c^\nu := \min\{t:\ t \geq \ell_c,\ \kappa_3 \|z^{\ell_c - 1}\|_\lambda \lambda^{t-\ell_c+1} \leq \nu\}$$

and let $\mathbb{L}_c^\nu := \left\{ t \in \mathbb{L}_c :\ t \geq \ell_c^\nu \right\}$. Then, if at least two resets occur between ℓ_c^ν and ℓ_{c+1}, i.e., there exists at least one index k such that $\mathbb{T}_k \subseteq \mathbb{L}_c^\nu$, one can exploit Lemma 1 and Proposition 1 in order to conclude that at time ℓ_{c+1}, when the next plant variation occurs, the plant input/ouptut data can be upper bounded as

$$\|z^{\ell_{c+1}-1}\|_\lambda \leq Z(\Pi^*_{TI} + \nu) \tag{25}$$

with $Z(\cdot)$ as in (19). Notice that one single reset would not be sufficient since, while the bound of Lemma 1 depends only on the values of the test functionals in the current reset interval, the bound of Proposition 1 is a function of the values of the test functionals in the previous reset interval.

Combining the latter inequality with (24), an upper bound on the smallest test functional over each reset interval can be derived also in the presence of plant variations. More specifically, by means of simple induction arguments, the following result can be proved.

Lemma 3: Consider the HSL-R based on (14) and let assumptions A1-A4 hold. Further, suppose that

$$\forall\, c \in \mathbb{Z}_+ \quad \exists\, k \in \mathbb{Z}_+ \quad \text{such that} \quad \mathbb{T}_k \subseteq \mathbb{L}_c^\nu\,. \tag{26}$$

Then, for all $k \in \mathbb{Z}_+$, one has $\Pi^k \leq \Pi^*_{TV}$ with

$$\begin{aligned} \Pi^*_{TV} \quad := \quad \max \Big\{ & \Pi^*_{TI} + \kappa_3 Z(\Pi^*_{TI} + \nu); \\ & 1 + \mu^{-1} Z(\Pi^*_{TI} + \nu) + h \Big\} \end{aligned}$$

and consequently the switched system (1)-(2) is stable. □

In the light of Lemma (3), it is immediate to see that a sufficient condition for stability is that the plant *dwell time*, i.e., the minimum interval between two consecutive plant variations, be large enough to allow the fulfillment of condition (26). This amounts to requiring that, for any $c \in \mathbb{Z}_+$, ℓ_{c+1} be always greater or equal to ℓ_c^ν plus the time needed for two resets to occur. With this respect, notice that the upper bound (25) implies

$$\ell_c^\nu - \ell_c \leq \left\lceil \log_\lambda \frac{\nu}{\kappa_3\, Z(\Pi^*_{TI} + \nu)} \right\rceil .$$

Moreover, by induction arguments, if condition (26) is satisfied up to a certain ℓ_c then Π^*_{TV} is an upper bound on the smallest test functional over $\mathbb{L}_c$. This, in turn, implies that after at most $2\,\Delta\,(\Pi^*_{TV})$ steps subsequent to ℓ_c^ν the two required resets occur (see Theorem 2). Then, the following stability result can be claimed.

Theorem 4: Consider the HSL-R based on (14) and let assumptions A1-A4 hold. Then, the switched system (1)-(2) is stable provided that

$$\ell_{c+1} - \ell_c \ \geq \ \left\lceil \log_\lambda \frac{\nu}{\kappa_3\, Z(\Pi^*_{TI} + \nu)} \right\rceil + 2\Delta\,(\Pi^*_{TV})\,,$$

for any $c \in \mathbb{Z}_+$. □

VI. Conclusions

The problem of controlling uncertain plants subject to time variations has been addressed within a MMUASC framework. A novel scheme has been proposed wherein the supervisor orchestrates the switching by adaptively selecting the information, characterized in terms of past performance records, that is necessary to preserve stability. Such an approach represents a significant advance since it makes it possible to derive stability properties without relying on a finite switching stopping time (as instead it was required in existing MUASSC schemes); thus allowing for persistent plant variations.

References

[1] A. S. Morse, "Control using logic-based switching," in *Trends in Control: An European perspective*, A. Isidori, Ed. London: Springer, 1995, pp. 69–113.

[2] P. V. Zhivoglyadov, R. H. Middleton, and M. Fu, "Localization based switching adaptive control for time-varying discrete-time systems," *IEEE Trans. Automat. Control*, vol. 45, pp. 752–755, 2000.

[3] D. Angeli and E. Mosca, "Lyapunov-based switching supervisory control of nonlinear uncertain systems," *IEEE Trans. Automat. Control*, vol. 47, pp. 500–505, 2002.

[4] A. S. Morse, "Supervisory control of families of linear set-point controllers, part 1: robustness," *IEEE Trans. on Automatic Control*, vol. 42(11), pp. 1500–1515, 1997.

[5] K. Narendra and J. Balakrishnan, "Adaptive control using multiple models," *IEEE Trans. on Automatic Control*, vol. 42, pp. 171–187, 1997.

[6] A. S. Morse, D. Q. Mayne, and G. C. Goodwin, "Applications of hysteresis switching in parameter adaptive control," *IEEE Trans. on Automatic Control*, vol. 37, pp. 1343–1354, 1992.

[7] J. P. Hespanha, D. Liberzon, and A. S. Morse, "Hysteresis-based switching algorithms for supervisory control of uncertain systems," *Automatica*, vol. 39, pp. 263–272, 2003.

[8] B. D. O. Anderson, T. S. Brinsmead, F. D. Bruye, J. P. Hespanha, D. Liberzon, and A. S. Morse, "Multiple model adaptive control, part 2: switching," *Int. J. Robust Nonlinear Control*, vol. 11(5), pp. 479–496, 2001.

[9] S. Fekri, M. Athans, and A. Pascoal, "Issues, progress and new results in robust adaptive control," *International Journal of Adaptive Control and Signal Processing*, vol. 20, pp. 519–579, 2006.

[10] M. Kuipers and P. Ioannou, "Multiple model adaptive control with mixing," *IEEE Transactions on Automatic Control*, vol. 55, no. 8, pp. 1822–1836, 2010.

[11] L. Vu and D. Liberzon, "Supervisory control of uncertain linear time-varying systems," *IEEE Trans. on Automatic Control*, vol. 56, no. 1, pp. 27–42, 2011.

[12] M. G. Safonov and T. C. Tsao, "The unfalsified control concept and learning," *IEEE Trans. on Automatic Control*, vol. 42, pp. 843–847, 1997.

[13] R. Wang, A. Paul, M. Stefanovic, and M. G. Safonov, "Cost detectability and stability of adaptive control systems," *Int. J. Robust Nonlinear Control*, vol. 17, pp. 549–561, 2007.

[14] M. Stefanovic and M. G. Safonov, "Safe adaptive switching control: Stability and convergence," *IEEE Trans. on Automatic Control*, vol. 53, pp. 2012–2021, 2008.

[15] G. Battistelli, E. Mosca, M. G. Safonov, and P. Tesi, "Stability of unfalsified adaptive switching control in noisy environments," *IEEE Trans. on Automatic Control*, vol. 46, pp. 249–259, 2010.

[16] S. Baldi, G. Battistelli, E. Mosca, and P. Tesi, "Multi-model unfalsified adaptive switching supervisory control," *Automatica*, vol. 46, pp. 249–259, 2010.

[17] ——, "Multi-model unfalsified adaptive switching control: Test functionals for stability and performance," *International Journal of Adaptive Control and Signal Processing*, vol. 25, pp. 593–612, 2011.