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Robust Adaptive Estimation of Arbitrary Time-Varying Actuator Faults

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***Abstract*— We design a robust fault estimation technique for arbitrary time-varying faults based on an adaptive observer with enhanced speed of adaptation. The construction of the observer is carried out through a transformation of the plant model into its special coordinate basis (SCB) form. This transformation allows a decoupling of the fault estimates from the disturbances. A key feature of the overall scheme is that the adaptive mechanism is viewed as a filtering process which allows easy parameter tuning to achieve faster and exact fault estimation in the presence of disturbances and plant-model uncertainties.**

I. INTRODUCTION

As failures might have drastic consequences on plant operation, it becomes mandatory to increase significantly the reliability of automated systems with devices capable of a continuous monitoring of the plant states. For economic and hardware weight reasons, the core of such monitoring schemes is based on a mathematical model of the controlled plant which allows faults characterization in plant-components from a comparison of the signals generated by this plant-model with the available actual plant-measurements. A key problem with analytical models is that the real plants are subject to unknown disturbances and models are never perfect and may be imprecisely know. Fortunately, some unmeasurable perturbations can often be incorporated in plant models by viewing them as unknown inputs. Hence, when the unknown inputs are certain causes such as faults in actuators or other plant-components, thanks to the above view, unknown input estimation may be carried out directly for the purpose of fault detection and isolation. Amongst many methods and techniques for fault detection and estimation which have been developed over the past several years, the unknown input observer (UIO) is one of the most attractive techniques [4], [6], [15]. For *uncertain* systems, FDI based on unknown input observers which are sensitive to faults but insensitive to model uncertainties and/or unknown disturbances have been proposed in [8], [9], [12], [3], [4]. In reference [5], the authors proposed an input estimator in a more general setting not directly related to the FDI problem, however their technique can be viewed as an approach for fault estimation, albeit no distinction is made between disturbances and fault signals. The main drawback of the technique is that the existence of the proposed estimator depends on very strict conditions and moreover it cannot achieve *exact* asymptotic estimation of the unknown inputs but only asymptotic estimation of the unknown inputs to some degree of accuracy. The authors in reference [14] proposed a Luenberger-type adaptive observer to simultaneously estimate the state and the unknown fault. The fault was viewed as the adjustable (unknown) parameter of the observer and the adaptive law for updating the fault estimation was chosen judiciously in order to ensure stability and convergence of the fault estimation algorithm. However, an adaptive observer based on the nominal plant-model might have some troubles in estimating faults when the real plant is uncertain and subject to disturbances. In order to cope with this issue related to plant uncertainties/disturbances, references [18], [17], [16] presented a nice solution with a fault diagnosis observer able to produce residuals which are insensitive to the disturbances. However, this solution can only deal with constant/slowly time-varying unknown inputs which are quite restrictive as classes of faults in the context of system diagnosis. In this paper, we rely on the above mentioned results to construct a robust adaptive observer able to estimate *arbitrary* time-varying actuator faults with *enhanced* estimation speed despite the presence of disturbances and plant uncertainties.

The main contribution of this paper is twofold: first, the class of actuator faults, viewed as unknown inputs, is enlarged to that of *arbitrary* time-varying unknown inputs and the derived estimation technique can achieve *exact tracking* of the unknown fault. Second, the adaptive law synthesis for fault estimation is presented from a *new* and different angle of view, i.e. as a filtering process in which the fault estimate is the output of a filter driven by the residuals. The reward of this new viewpoint is consequential in that it allows for a straightforward proof of the closed-loop stability and the convergence of the adaptation mechanism through a passivity property and as a byproduct it yields, without any additional effort, a *general* adaptive law to simultaneously achieve stability, convergence and enhanced speed for fault estimation mechanisms. In addition to the above points, the robust estimator is designed under less restrictive existence conditions, as compared to those of the standard UIO [4].

II. PRELIMINARY MATERIALS

A. Plant description

We consider a nominal plant subject to actuator faults and represented by the following linear time-invariant model :

$$S \equiv \begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + Ef(t) \\ y(t) = Cx(t) \end{cases} \tag{1}$$

where $x(t) \in \Re^n$, $u(t) \in \Re^m$ and $y(t) \in \Re^p$ are the state, control input and output vector respectively and $f(t) \in$

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$\Re^h$ represents the actuator faults. The matrices A, B and C are known constant matrices of appropriate dimensions and E denotes the distribution matrix for actuator faults. Furthermore, we assume that E is of full column rank and the pair (A, C) is observable. Note that full rank assumption on E is necessary for estimation of $f(t)$.

B. The adaptive diagnostic observer

Using the plant model (1), the standard adaptive diagnostic observer proposed in [14] is written as

$$\begin{aligned}\dot{\hat{x}}(t) &= A\hat{x}(t) + Bu(t) + E\hat{f}(t) + K(y(t) - \hat{y}(t)) \\ \hat{y}(t) &= C\hat{x}(t)\end{aligned} \tag{2}$$

where $\hat{f}(t) \in \Re^h$ is an estimate (to be determined) of the actuator fault $f(t)$. From the observability of the pair (A, C), the observer gain K can be selected such that $(A - KC)$ is stable. Set $\tilde{x}(t) = x(t) - \hat{x}(t), \tilde{y}(t) = y(t) - \hat{y}(t), \tilde{f}(t) = f(t) - \hat{f}(t)$, then the error dynamics is given by

$$\begin{aligned}\dot{\tilde{x}}(t) &= (A - KC)\tilde{x}(t) + E\tilde{f}(t) \\ \tilde{y}(t) &= C\tilde{x}(t)\end{aligned} \tag{3}$$

We quote the following proposition from [14] for computing the fault estimate.

Proposition 1: If there exist symmetric positive definite (SPD) matrices P,$Q \in \Re^{n\times n}$ and two matrices $K\in \Re^{n\times p}$ and $F\in \Re^{h\times p}$ such that the following conditions hold

$$\begin{aligned}P(A - KC) + (A - KC)^T P = -Q, \\ E^T P = FC\end{aligned} \tag{4}$$

then the adaptive observer given by

$$\begin{aligned}\dot{\hat{x}}(t) &= A\hat{x}(t) + Bu(t) + E\hat{f}(t) + K(y(t) - \hat{y}(t)) \\ \hat{y}(t) &= C\hat{x}(t)\end{aligned} \tag{5}$$

$$\text{with adaptation law } \dot{\hat{f}}(t) = -\beta F(y(t) - \hat{y}(t)) \tag{6}$$

where $\beta \in \Re^{h\times h}$ is a SPD matrix, achieves $lim_{t\to\infty}\tilde{x}(t) = 0$ and $lim_{t\to\infty}\tilde{f}(t) = 0$.

Note that the above adaptive diagnostic observer is derived on the basis of the nominal plant model (1) subject to actuator faults. This adaptive observer will get into trouble in case the plant experiences input disturbances and model uncertainties. The actuator fault estimate may be seen as the output of an integrator, and clearly such estimation scheme will be only able to deal with constant or very slow time-varying fault.

III. Problem Statement

Consider the plant subject to actuator faults and uncertainties. These uncertainties might be possibly external disturbances acting on the plant and/or dynamics uncertainties due to modelling errors. All these uncertainties can be summarized as unknown inputs acting on the plant. Thus, the plant is described with the following model

$$\begin{cases}\dot{x}(t) = Ax(t) + Bu(t) + Ef(t) + Gd(t) \\ y(t) = Cx(t)\end{cases} \tag{7}$$

where $x(t) \in \Re^n$ is the state, $u(t) \in \Re^m$ is the control input, $y(t) \in \Re^p$ is the output. The matrices A, B, C are the nominal plant matrices. Signal $f(t) \in \Re^h$ with $h \leq p$ is the vector of actuator faults with E their distribution matrix. Vector $d(t) \in \Re^q$ denotes *all* the unknown inputs and $G\in \Re^{n\times q}$ the corresponding distribution matrix. It is worth noting that both model uncertainties and input disturbances have been grouped together in the additive unknown input $d(t)$ and act on the plant dynamics through the Gd term in the above formulation. This representation obtained by transforming at the outset model uncertainties into unknown inputs is a standard practice for tackling the robustness issue with UIO's in the FDI framework [4], [7], [12]. Throughout, it is assumed that the matrix G is perfectly known and full rank. The issue we address is:

> How to design an adaptive observer to estimate *arbitrary* time-varying actuator faults despite model-uncertainties and unknown input disturbances?

In order to solve this problem, our road map will be the following. First, we need to design a disturbance-decoupling residual in order that the observer fault estimation error be independent from the uncertainties and disturbances. Second, if this decoupling is feasible, the estimation updating mechanism should be devised as a function of the residual signal, and this function should take the form of a dynamical filter. To proceed with these requirements, we consider an adaptive observer for simultaneous residual generation and fault estimation having the following structure

$$\begin{aligned}\dot{\hat{x}}(t) &= A\hat{x}(t) + Bu(t) + E\hat{f}(t) + K(y(t) - \hat{y}(t)) \\ \hat{y}(t) &= C\hat{x}(t) \\ r(t) &= W(y - \hat{y}(t)) \\ \hat{f}(t) &= g(t) * r(t)\end{aligned} \tag{8}$$

where $\hat{x}$ is the state estimation, $\hat{f}$ is the fault estimation of the real fault f and $g * r$ is the convolution of the residual r with the impulse response g of some linear filter to be designed. In order to increase the degree of design freedom, the residual r is defined as a "post-filtering" of the output estimation error by a constant matrix W. Substracting the real dynamics (7) with that provided by the observer (8) gives the following errors dynamics

$$\begin{aligned}\dot{\tilde{x}}(t) &= (A - KC)\tilde{x}(t) + E\tilde{f}(t) + Gd(t) \\ \tilde{y}(t) &= C\tilde{x}(t)\end{aligned} \tag{9}$$

It is clearly seen that the errors dynamics is corrupted by the uncertainties and unknown disturbances $d(t)$. The residual vector can be written $r(t) = W\tilde{y}(t) = WC\tilde{x}(t)$. This yields the relationship between residual and faults signal and unknown inputs which we write in transfer operator form

$$r(s) = G_{rd}(s)d(s) + G_{rf_a}(s)\tilde{f}(s) \tag{10}$$

with $G_{rd}(s) = WC(sI-A+KC)^{-1}G$ the transfer from disturbance to residual and,$G_{rf_a}(s) = WC(sI-A+KC)^{-1}E$ the transfer from actuator fault to residual. In the light of the above adaptive observer structure, the tasks to be done are to find

1) suitable matrices K and W such that $A-KC$ is Hurwitz and, *independently* of the unknown inputs d, the residual r should be zero (or close to zero) in the absence of faults and different from zero in case of fault occurence
2) an "adaptive law" for continuously updating $\hat{f}$, where $\hat{f}$ is seen as the output of a to-be-designed filter $g(t)$ which ensures convergence and speed of the fault estimation.

The above two points are solved in the next sections.

IV. SCB-BASED ROBUST RESIDUAL GENERATION

The so-called structural decomposition basis (SCB) allows an equivalent representation of a dynamical system [13], [11, Chap. 3] and it has emerged as a key tool for many aspects of system analysis and design. Furthermore, there is now available toolkits with algorithms for computing SCB[10]. Roughly speaking, this representation provides deep insight into the internal structure of systems by revealing their distinctive parts which are directly associated with their zero dynamics as well as their left and right invertible dynamics. Here, we summarize the structural decomposition of a linear system in the following main theorem and gives some of its key properties [11, Chapter 3].

Theorem 1: For any given linear system described by the following equations

$$\Sigma \equiv \begin{cases} \dot{x}(t) = Ax(t) + Gd(t) \\ y(t) = Cx(t) + Dd(t) \qquad \text{with } D = [0_{p\times q}] \end{cases} \tag{11}$$

there exists

1) coordinate free nonnegative integers n_a^-, n_a^+, n_b, n_c, n_d, $m_d \le p$ and $q_i, i = 1, 2, ...m_d$, $n_d = \sum_{i=1}^{m_d} q_i$
2) nonsingular state, output and input transformation Γ_1, Γ_2 and Γ_3 which take the given Σ into a special coordinate basis (SCB) that displays explicitly both the finite and infinite zero structures of Σ. The SCB is described by the following set of equations :

$$\bar{x} = \Gamma_1^{-1}x, \bar{y} = \Gamma_2^{-1}y, \bar{d} = \Gamma_3^{-1}d$$

$$\bar{x} = [(x_a^-)^T, (x_a^+)^T, x_b^T, x_c^T, x_d^T]^T, \bar{y} = [(y_d)^T, (y_b)^T]^T,$$

$$\bar{d} = [(d_d)^T, (d_b)^T]^T$$

$$\bar{\Sigma} \equiv \begin{cases} \dot{\bar{x}}(t) = \bar{A}\bar{x}(t) + \bar{G}\bar{d}(t) \\ \bar{y}(t) = \bar{C}\bar{x}(t) \end{cases} \tag{12}$$

where $\bar{A}, \bar{G}, \bar{C}$ are given by

$$\bar{A} := \Gamma_1^{-1}A\Gamma_1$$

$$= \left[\begin{array}{cc|c|c|c} A_{aa}^- & 0 & L_{ab}^-C_b & 0 & L_{ad}^-C_d \\ 0 & A_{aa}^+ & L_{ab}^+C_b & 0 & L_{ad}^+C_d \\ \hline 0 & 0 & A_{bb} & 0 & L_{bd}C_d \\ \hline G_cE_{ca}^- & G_cE_{ca}^+ & L_{cb}C_b & A_{cc} & L_{cd}C_d \\ \hline G_dE_a^- & G_dE_a^+ & G_dE_b & G_dE_c & A_d \end{array}\right] \tag{13}$$

$$\bar{G} := \Gamma_1^{-1}G\Gamma_3 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & G_c \\ G_d & 0 \end{bmatrix} \tag{14}$$

$$\bar{C} := \Gamma_2^{-1}G\Gamma_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & C_d \\ 0 & 0 & C_b & 0 & 0 \end{bmatrix} \tag{15}$$

Some of the key properties revealed by the system SCB form, which are of interest to us, are listed below

Property 1:
System Σ is right invertible if and only if x_b and hence y_b do not exist, i.e., n_b is equal to zero; it is left invertible if and only if x_c and hence d_c do not exist, i.e., n_c is equal to zero.

Property 2:
$n_a = n_a^- + n_a^+$, where n_a^-, n_a^+ are the number of stable and unstable transmission zeros of system Σ respectively.

Property 3:
n_d is the number of infinite zeros with $n_d = \sum_{i=1}^{m_d} q_i$, where q_i is the number of infinite zeros of order i and m_d is the highest order of an infinite zero, also it is the row number of C_d.

Property 4:
$n = n_a + n_b + n_c + n_d$

Property 5:
(A_{bb}, C_b) forms an observable pair.

The following proposition which is the basis for realizing the disturbance decoupling (DDR) residual is from [18] and the proof is sketched for the reader to get some insight into the DDR algorithm.

Proposition 2: [18] There exists matrices K and W such that the observer given in equation (8) satisfies the disturbance decoupling property, with $A-KC$ being Hurwitz, if and only if the pair (A, C) is detectable and system (A, G, C) is not right invertible.

Proof: Assume (A, G, C) is transformed in SCB form. From the requirement of disturbance decoupling residual, the transfer from disturbance to residual

$$G_{rd}(s) = WC(sI - A + KC)^{-1}G \tag{16}$$

must be equal to zero and this can be achieved if and only if WC has the form

$$WC = W_c = [W_{ca}^- \quad 0 \quad W_{cb} \quad 0 \quad 0] \tag{17}$$

where W_{cb} has n_b columns. Thanks to the SCB form of matrix C, partition W as $W = [W_1 \quad W_2]$ where W_1 and W_2 have compatible dimensions with C then

$$WC = [0 \quad 0 \quad W_2C_b \quad 0 \quad W_1C_d] \tag{18}$$

Comparing (17) and (18) implies that W_1C_d must be zero. The term W_2C_b would be nonzero if and only if $n_b \neq 0$. In this case, we can therefore find a matrix W such that $WC \neq 0$ to satisfy the requirement $G_{rd}(s) = 0$ and a matrix K to ensure stability of $(A-KC)$.

V. ROBUST ADAPTIVE FAULT ESTIMATION DESIGN WITH ENHANCED ADAPTATION SPEED

In the observer (8), the unknown parameter representing the actuator fault is replaced by its estimate. By means of the updating law, the parameter should therefore be adjusted in real time such that its value quickly converges to the actual fault. Since we assume that the fault might be an arbitrary time-varying signal, the adaptive law should be such that the fault estimate *tracks closely* the actual fault signal. From classical control theory, it might be expected that faster adaptation rule can be achieved with filter having *proportional* and integral terms, i.e., the updating law may have the form of a filtering process

$$\hat{f}(t) = \{g(t) * F\tilde{y}(t)\} \tag{19}$$

$$\text{with} \qquad g(t) = -(\beta_1\delta(t) + \beta_2\mathbf{1}(t)) \tag{20}$$

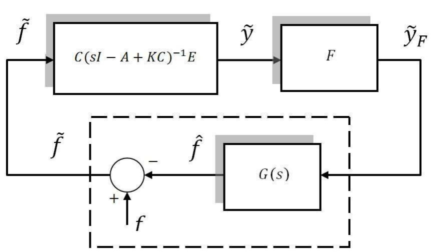
where F is a matrix to be determined, g the impulse response of the filter and β_1, β_2 are filter tuning matrix parameters (of dimension $h \times h$) which are assumed to be positive definite matrices. The symbols $\delta(t)$ and $\mathbf{1}(t)$ denotes the Dirac impulse and the Heaviside function respectively.

Let us disregard, for the moment, the unknown disturbance d and assume that F is computed as in proposition 1. Then, from the Kalman-Yakubovitch-Popov lemma [1], the error-dynamics system with output $\tilde{y}_F$ defined by

$$\dot{\tilde{x}}(t) = (A - KC)\tilde{x}(t) + E\tilde{f}(t) \tag{21}$$

$$\tilde{y}_F(t) = FC\tilde{x}(t) \tag{22}$$

is *strictly* positive real (SPR). Now, consider the adjusting law (19) in connection with the error dynamics system (21)-(22) as depicted in the figure below.



We claim the following result.

Proposition 3: Assume that matrix F is computed from conditions (4) of proposition 1 . Then, under the positive definiteness of matrices β_1 and β_2, the closed-loop system (21)-(22)-(19)-(20) is *asymptotically* stable, that is, $\lim_{t\to\infty}\tilde{y} = 0$ and $\lim_{t\to\infty}\tilde{f} = 0$.

Proof: From the positive definiteness of matrices β_1 and β_2, it follows that the filter with impulse response $g(t)$ is a positive real system, that is, a passive system. Therefore, the closed-loop (21)-(22)-(19)-(20) is a negative feedback interconnection of a strictly passive system and a passive one. The result follows as a direct application of the passivity theorem for linear systems [1, Chapter 2]. ■

Remark 1: A similar proportional and integral adaptation law has been used in [19]. It was only shown there that the adaptive observer achieves uniformly bounded estimation errors $\tilde{y}$ and $\tilde{f}$ but the authors failed to prove exact reconstruction of the fault. Proposition 3 is a stronger result which shows that proportional and integral adaptation laws really achieves *exact asymptotic* estimation of the fault.

Remark 2: Note that under the positive definiteness of the filter matrix parameters β_1 and β_2, the passive property of the filter $g(t)$ is *independent* of the "magnitude" of the elements of these matrices and, consequently the asymptotic stability of the closed-loop system (21)-(22)-(19)-(20) is always guaranteed. Therefore, with large "magnitude" elements of the proportional term β_1, it can be expected that *fast* adaptation can be achieved.

It is clearly seen from proposition 3 that a more general convergent adaptation scheme can be envisioned as a filtering of the output-error $\tilde{y}_F$, provided that the filter is *passive.* An example of such a passive filter might be a proportional-integral-and-derivative (PID) filter leading to an updating law of the form

$$\dot{\hat{f}} = \beta_1\dot{\tilde{y}}_F + \beta_2\tilde{y}_F + \beta_3\ddot{\tilde{y}}_F \tag{23}$$

where β_1,β_2 and β_3 are positive definite. This general result is stated in the next proposition.

Proposition 4: Assume that matrix F is computed from conditions (4) of proposition 1 . Then any adaptation law obtained as a filtering of the F-weighted output-estimation-error $\tilde{y}_F$ with a filter having a *positive real* transfer matrix $G(s)$ achieves *exact asymptotic* fault estimation. Furthermore, the closed-loop error-dynamics of the adaptive observer is *asymptotically* stable.

Now, we are in a position to derive the robust adaptive fault estimation algorithm with enhanced adaptation speed. We simply combine the disturbance decoupling residual generation with an updating mechanism law of PI or PID type. The algorithm proceeds through the following steps.

1) STEP 1 - Compute K and W using "SCB transformation"
Find nonsingular state, output and input transformation Γ_1, Γ_2 and Γ_3 which take the given $\Sigma = \{A, G, C\}$ into the form of (SCB) given by :

$$\bar{A} := \Gamma_1^{-1}A\Gamma_1,\ \bar{G} := \Gamma_1^{-1}G\Gamma_3 \text{ and } \bar{C} := \Gamma_2^{-1}G\Gamma_1$$

$$\bar{A} = \begin{bmatrix} A_{aa} & L_{ab}C_b & 0 & L_{ad}C_d \\ 0 & A_{bb} & 0 & L_{bd}C_d \\ G_cE_{ca} & L_{cb}C_b & A_{cc} & L_{cd}C_d \\ G_dE_a & G_dE_b & G_dE_c & A_d \end{bmatrix},$$

$$\bar{G} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & G_c \\ G_d & 0 \end{bmatrix}, \quad \bar{C} = \begin{bmatrix} 0 & 0 & 0 & C_d \\ 0 & C_b & 0 & 0 \end{bmatrix} \tag{24}$$

where the row dimension of C_d is m_d, and row dimension of C_b is the maximum dimension of residual, that is, $m_r = p - m_d$
1-a) If $n_b = 0$, stop
1-b) Otherwise continue

a) Since (A_{bb}, C_b) forms an observable pair, choose a gain K_{bb} such that eigenvalue $\lambda(A_{bb} - K_{bb}C_b)$ are placed at desired location in $\mathbb{C}^-$

b) Define matrices $\mathcal{A}$ and $\mathcal{C}$ as

$$\mathcal{A} = \begin{bmatrix} A_{aa} & 0 & L_{ad}C_d \\ G_cE_{ca} & A_{cc} & L_{cd}C_d \\ G_dE_a & G_dE_c & A_d \end{bmatrix} \quad (25)$$

$$\mathcal{C} = [0 \quad 0 \quad C_d] \quad (26)$$

As the pair (A, C) is detectable, $(\mathcal{A}, \mathcal{C})$ is also detectable. Choose a gain $\mathcal{K}$ such that eigenvalues $\lambda(\mathcal{A} - \mathcal{K}\mathcal{C})$ are assigned at desired location in $\mathbb{C}^-$. Partition $\mathcal{K}$ as $\mathcal{K} = [\mathcal{K}_a^T \mathcal{K}_c^T \mathcal{K}_d^T]$ and form the matrix $\bar{K}$ as

$$\bar{K} = \begin{bmatrix} \mathcal{K}_a & L_{ab} \\ L_{bd} & K_{bb} \\ \mathcal{K}_c & L_{cb} \\ \mathcal{K}_d & 0 \end{bmatrix} \quad (27)$$

Select matrix $\bar{W} = [W_d \quad W_b]$ such that $W_dC_d = 0$, and W_b is any nonsingular matrix with compatible dimension with C_b such that $W_bC_b \neq 0$. Note that we can choose W_b=I_{m_r}. Finally, compute the desired K and W as

$$K = \Gamma_1 \bar{K} \Gamma_2^{-1}, \quad W = \bar{W}\Gamma_2^{-1} \quad (28)$$

2) STEP 2 - Compute $\hat{f}$ using the estimation law (19) where $g(t)$ is the impulse response of a *passive* filter (e.g., PI or PID-type filter)

a) Matrix F is computed from conditions (4)

b) The updating fault estimation mechanism is derived from $\hat{f}(t) = \{g(t) * Fr(t)\}$ where $F = [\bar{F} \quad *]$ and $\bar{F}$ has compatible dimension with W and $r(t) = W\tilde{y}(t)$

Remark 3: Note that, apart from the existence of matrix F imposed by conditions (4), the algorithm is conditioned to the *non* right invertibility of the triplet (A, G, C) which is the condition $n_b \neq 0$.

Remark 4: In step 1 of the above algorithm, the complete structural decomposition yielding the $\Gamma's$-matrices as well as the transformed matrices $\bar{A}, \bar{G}, \bar{C}$ and the integers m_d, n_b is entirely performed by the linear systems toolkit [2, Chapter 12] (also available at `http://linearsystemskit.net/`).

VI. A SIMULATION EXAMPLE

The algorithm derived in the previous section is applied to realize a robust adaptive fault estimation with enhanced speed using a PI-type filter for the updating estimation mechanism. We illustrate the effectiveness of this algorithm with a double-effect pilot evaporator subject to an actuator fault in its first input channel [16]. The plant model is given by the state space matrices A, B, G, C as follows

$$A = \begin{bmatrix} 0 & 0 & -0.0034 & 0 & 0 \\ 0 & -0.041 & 0.0013 & 0 & 0 \\ 0 & 0 & -1.1471 & 0 & 0 \\ 0 & 0 & -0.0036 & 0 & 0 \\ 0 & 0.094 & 0.0057 & 0 & -0.051 \end{bmatrix}$$

$$B = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & -0 \\ -0 & 0 & 0.948 \\ 0.916 & -1 & 0 \\ -0.598 & 0 & 0 \end{bmatrix}, G = \begin{bmatrix} 0 & 1 \\ 0.062 & -0.132 \\ 0 & -7.189 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

This plant has three inputs and three outputs and we assume that a fault occurs in the first input channel with a distribution matrix given by $E = [-1 \quad 0 \quad 0 \quad 0.916 \quad -0.598]^T$. There are three independent output measurements and one fault. Then, it is possible to estimate the actuator fault because $rank(C) > rank(E)$. The standard UIO-based models in reference [4], [6] are not applicable for this plant with regard to the decoupling of the unknown input disturbances, because $rank(CG) < rank(G)$ (i.e, $1 < 2$). The test for disturbance decoupling using SCB transformation produces $n_b = 2$ and the pair (A, C) is detectable. Therefore, this system satisfies the existence conditions of our algorithm. From the SCB transformation, we get A_{bb}, C_b, A_d and C_d as

$$A_{bb} = \begin{bmatrix} -1.1715 & 0 \\ -0.0036 & 0 \end{bmatrix}, C_b = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

and $A_d = 0.0244, \quad C_d = 1$. The row dimension of C_d is $m_d = 1$. Then, the maximum dimension of the residual vector is $m_r = p - m_d = 3 - 1 = 2$. The matrices K and W are designed as

$$K = \begin{bmatrix} 0.8500 & 0 & 0 \\ -6.5395 & 0.0009 & 0 \\ 19.05092.32850 & & \\ 0 & -0.0036 & 10 \\ 0.7593 & 0.0057 & 0 \end{bmatrix}$$

$$W = \begin{bmatrix} 7.189 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

and $\bar{F} = [-205.3002 \quad 0.2605]$ and the corresponding PI parameters are given by $\beta_1 = -0.0016, \beta_2 = -0.01$. Figures 2, 4 and 5 show the fault estimation results (the actuator fault estimate is shown in solid line). It is observed that the robust adaptive fault estimator exactly reconstructs the waveform of the unknown fault. Clearly, for arbitrary time-varying faults, the robust adaptive fault estimator provides better performance as compared to the robust estimator approach given in [16] on the same double-effect pilot evaporator example.

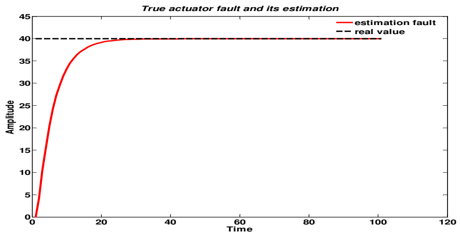


Fig. 1. Robust fault estimation with the algorithm in [16]: constant fault.

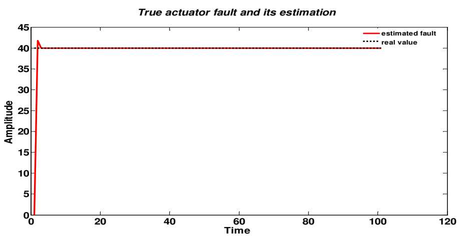


Fig. 2. Robust adaptive fault estimation (our algorithm): constant fault.

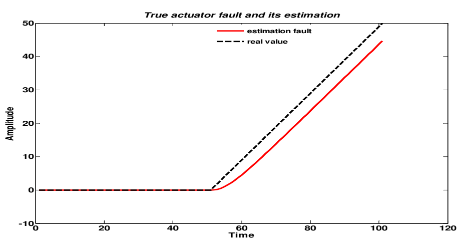


Fig. 3. Robust fault estimation with the algorithm in [16]: drift fault.

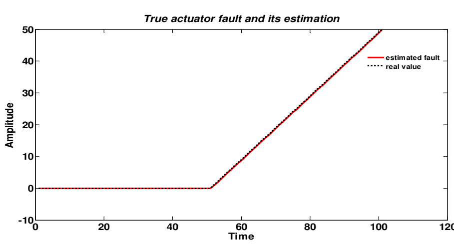


Fig. 4. Robust adaptive fault estimation (our algorithm): drift fault.

VII. Conclusion

In this paper, a robust adaptive observer for deterministic and uncertain systems has been designed using a disturbance decoupling residual generator combined with a fairly generalized adaptation mechanism for actuator fault estimation. The adaptive observer is a full Luenberger-type observer and the adaptation mechanism is *viewed* as an error filter which can be any dynamical filter provided it has positive real properties. It is shown that the algorithm assures a perfect disturbance decoupling residual generation, an asymptotic *exact* estimation and may improve drastically the estimation speed through particular structures of the filter, as e.g., filters with a proportional term. The application of this approach to a double-effect pilot evaporator model has been performed to validate the effectiveness of the designed robust adaptive observer.

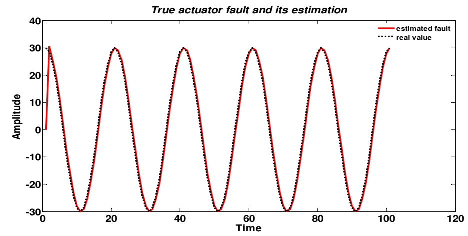


Fig. 5. Robust adaptive fault estimation (our algorithm): time-varying fault.

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