

Control of an Electric Power Assisted Steering System Using Reference Model

A. Marouf, C. Sentouh, M. Djemaï and P. Pudlo

Abstract—This paper presents a new control strategy of Electric Power Assisted Steering (EPAS) systems to ensure several control objectives. First, a reference model is employed to generate ideal motor angle that can guarantee the control objectives, assist torque generation, fast response to driver's torque command, vibration attenuation, supplying road information to the driver, improving steering wheel returnability and free control performance. Second, a sliding mode control law is used to track the desired motor angle. Finally, a sliding mode observer with unknown inputs and robust differentiators are designed to implement the reference model. Simulation results show that the proposed control structure can satisfy the desired performance.

I. INTRODUCTION

ELECTRIC Power Assisted Steering Systems (EPAS) are replacing hydraulic power steering in many new vehicles today. They have many advantages over traditional hydraulic power steering systems in engine independence/ fuel economy energy, tunability of steering feel, modularity/quick assembly, compact size and environmental compatibility, [1].

There are several research and developments on EPAS systems. Badawy et al. proposed in [1] a reduced order model which has been used to analyze various closed loop effects and to understand the basic compromises of the system and to set the requirements for the control design. Zarembo et al. in [2] proposed a procedure of designing fixed structure optimal controller to stabilize the system with high assist gain and minimize the torque vibrations. The controller is combined of classical lead and lag compensators networks. In [3], the authors proposed a method of transmitting useful information of low frequency dynamics of the tire/road contact to the driver in order to improve steering feel. In [4], a two-controller structure has been proposed to attenuate the road disturbance. The structure consists of two controllers : an H-infinity controller used to address the driver's feeling and regulate the motion response, and a proportional-integral (PI) controller used to produce the assist torque according to the command from H-infinity controller. An optimal LQR controller for dual pinion EPAS is proposed in [5]. The controller is designed for two inputs, driver torque and motor terminal voltage, to ensure the stability and reduce torsion vibrations in response to the driver torque. To improve steering maneuverability and steering wheel returnability,

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Kurishige et al, in [6] proposed a control algorithm (steering angle feedback control) based on the road reaction torque estimation. The steering feedback control gain is adjusted according to the estimated reaction torque. In [7], a robust H-infinity controller was developed to provide robust stability and minimize the effects of disturbances on the assist torque. Also, a driver torque estimator is introduced using as measurement, the pinion torque, the control signal and the transfer functions of the EPAS model. The estimator employs improper transfer functions which are approximated by a stable and a proper transfer functions. In [8], a fault tolerant control of EPAS system is designed based on the road reaction torque estimation. The estimator relies on several data such as tire slip angle and mechanical trail.

So, the control of EPAS system is a challenging task, several objectives must be satisfied, such as assisted torque generation, attenuation of vibrations, supplying road information to the driver, improving steering wheel returnability and robustness with respect to parameters uncertainties, modeling errors, nonlinearity and disturbances. Also, several information are required, such as steering wheel angle, motor speed, driver torque and road reaction torque. To achieve these several objectives without the need for different control algorithms, tuning several parameters, rules to switch between these algorithms and also reduce the number of sensors, a new control method is proposed in this paper.

The remainder of this paper is organized as follows. Section II presents the EPAS system model. In section III, the control objectives are presented. Section IV describes the controller design. First, the reference model is presented, then the choosing of the reference model parameters is discussed and finally the second order sliding mode control to track the desired motor angle is presented. To implement the controller, the sliding mode observer with unknown inputs is designed in section V. The simulation results to validate the performance of the control method are shown in section VI, while conclusion can be found in section VII.

II. SYSTEM MODELING

A. Dynamic Model of EPAS System

The dynamic model of the EPAS establishes relation between steering mechanism, electric dynamics of the motor and tire/road contact forces. Figure. 1 shows the model of steering mechanism equipped with brushed DC motor. According to Newton laws of motion, the equations of motion can be written as follows :

$$J_c \ddot{\theta}_c = -K_c \theta_c - B_c \dot{\theta}_c + K_c \frac{\theta_m}{N} - F_c \text{sign}(\dot{\theta}_c) + T_d \quad (1)$$

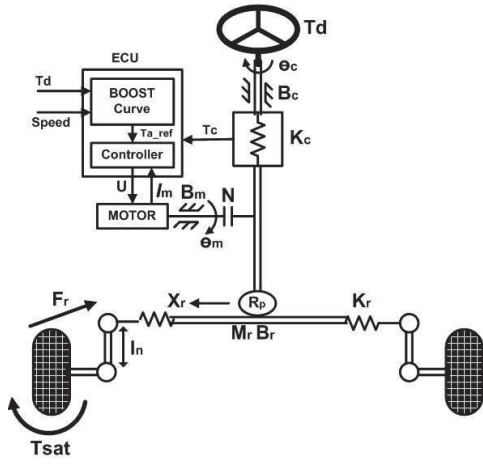


Fig. 1. EPAS Dynamic model

$$J_{eq}\ddot{\theta}_m = K_c \frac{\theta_c}{N} - \left(\frac{K_c}{N^2} + \frac{K_r R_p^2}{N^2} \right) \theta_m - B_{eq} \dot{\theta}_m + K_t I_m - F_m \text{sign}(\dot{\theta}_m) - \frac{R_p}{N} F_r \quad (2)$$

Electric dynamics of the motor is given by :

$$L\dot{I}_m = -RI_m - K_t \dot{\theta}_m + V \quad (3)$$

where: $J_{eq} = J_m + \frac{R_p^2}{N^2} M_r$, $B_{eq} = B_m + \frac{R_p^2}{N^2} B_r$ and $\theta_m = \frac{N x_r}{R_p}$. T_d is the driver torque and θ_c , θ_m , x_r are, respectively, the steering hand wheel angle, the angular position of the motor and the rack position. Table I defines and quantifies the other EPAS model parameters. The linear model of EPAS system can be expressed in the following state space form :

$$\begin{cases} \dot{x} = Ax + B_1 u_1 + B_2 u_2 \\ z = C_2 x \\ y = Cx = [y_1 \ y_2]^T \end{cases} \quad (4)$$

where $x = [\theta_c \ \dot{\theta}_c \ \theta_m \ \dot{\theta}_m \ I_m]^T$ is the state vector of the EPAS system, $u_1 = V$ is the voltage of the DC motor, $u_2 = [T_d \ T_r]^T$ represents the vector of unknown inputs which is composed with driver torque and road reaction torque, $z = T_c = K_c (\theta_c - \frac{\theta_m}{N})$ is the steering torque which acts on the steering column, used as indicator of the steering feel because it acts directly on the driver's hands via the steering wheel, [1][3], $y = [\theta_c \ \theta_m]^T$ is the measurement signal.

B. Vehicle Model

The vehicle model will be used to generate the road reaction force which acts on the rack. The vehicle lateral dynamics is modeled using single track, or bicycle model with states of sideslip angle, β , at the center of gravity (CG) and yaw rate, γ . Small angle approximations are used and lateral tire force is assumed to be proportional to the tire slip angle, so that the linear model of the vehicle lateral dynamics

TABLE I
NOMENCLATURE DEFINITIONS

Symbol	Description	Value [units]
J_c	steering column moment of inertia	$0.04Kg.m^2$
B_c	steering column viscous damping	$0.072N.m./(rad/s)$
K_c	steering column stiffness	$115N.m/rad$
F_c	steering column friction	$0.027N.m$
M_r	mass of the rack	$32kg$
B_r	viscous damping of the rack	$3820N/(m/s)$
R_p	steering column pinion radius	$0.007m$
K_r	tire spring rate	$43000N./m$
J_m	motor moment of inertia	$0.0004Kg.m^2$
B_m	motor shaft viscous damping	$0.0032N.m./(rad/s)$
F_m	motor friction	$0.056N.m$
K_t	motor torque, voltage constant	$0.05N.m/A$
L	motor inductance	$0.0015H$
R	motor resistance	0.37Ω
N	motor gear ratio	13.65

is given by :

$$\begin{cases} \dot{\beta} = -\frac{(C_f + C_r)}{mV} \beta + \left(-1 + \frac{(l_r C_r - l_f C_f)}{mV^2} \right) \gamma + \frac{C_f}{mV} \delta_f \\ \dot{\gamma} = \frac{(l_r C_r - l_f C_f)}{I_z} \beta - \frac{(l_r^2 C_r + l_f^2 C_f)}{I_z V} \gamma + \frac{l_f C_f}{I_z} \delta_f \end{cases} \quad (5)$$

The self-aligning torque, T_{sat} , the lateral force of the front

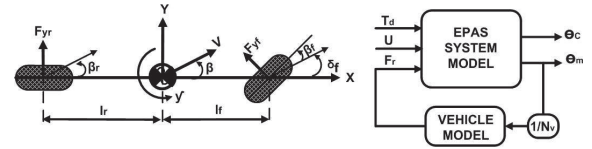


Fig. 2. Single track vehicle model (left), System block diagram (right)

tires, F_{yf} , and the reaction force acts on the rack, F_r , are defined as :

$$T_{sat} = (T_p + T_m) F_{yf} = (T_p + T_m) C_f \beta_f \quad (6)$$

$$F_r = \frac{T_{sat}}{l_n} = \frac{(T_p + T_m)}{l_n} C_f \beta_f \quad (7)$$

where C_f is the cornering stiffness coefficient, β_f is the tire slip angle, $\delta_f = \theta_m / N_v$ is the steering angle of the front tires, T_p is the pneumatic trail, T_m is the mechanical trail and l_n represents the knuckle arm.

III. CONTROL OBJECTIVES

A. Assistance Torque Generation

The main goal of the EPAS control is to generate assist torque. Figure. 3 shows the assist boost curves as a function of driver torque and vehicle speed. The system response must be fast so that the driver does not feel a lack of assistance.

B. Vibration Attenuation

The EPAS is a resonance system comprising the torsion and motor as spring and mass. The resonance can produce vibrations transmitted to steering wheel and deteriorates the steering feel. Figure. 6 shows the resonance characteristics, maximum peak at 63 rad/sec (10 Hz), in open loop. It can be

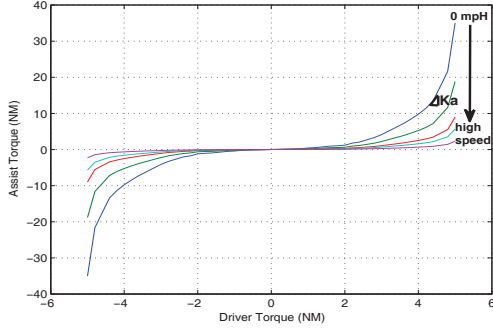


Fig. 3. Assist boost curves characteristics

seen that the resonance is a function of driver torque, reaction torque and control input. It is necessary to compensate the vibrations caused by each of these system inputs in order to ensure good performance in all operating conditions, variety of assist torque, driver's steering inputs and road conditions.

C. Supplying Road Information

The forces generated at the tire/road contact point (road information) largely determine the vehicle motion and the driver's maneuvers. To achieve good steering feel, the driver must receive appropriate amount of road information, [3].

D. Improve The Steering Wheel Returnability

At low vehicle speed, the steering wheel can't return to exact center position due to the friction torque. Whereas high vehicle speed, the self-aligning torque increases and makes the steering wheel return to center with excessive overshoot and oscillations. This characteristic leads to generate an unexpected yaw motion of the vehicle. For these reasons, the return control is needed to ensure that the steering wheel comes back to the center position quickly and without excessive overshoot and avoid free control oscillation, [1].

The driver torque, in practice, is not usually measured so that the measured torque is used as approximation of driver torque to determine the assistance torque. To show the difference between driver torque and torque sensor amplification ($T_a = K_a T_d$, $T_a = K_a T_c$), we compute the transfer function ($G = \frac{\theta_m}{T_d}$, $H = \frac{T_c}{T_d}$) using (1), (2) and neglecting the electric dynamics and the internal control of the motor which implies that the motor electromagnetic torque equal to motor voltage $V = T_a/N$.

- Case 1 : $T_a = K_a T_c$

$$G_1 = \frac{\theta_m}{T_d} = \frac{\frac{K_c}{N} (1 + K_a)}{(F_m + \frac{K_c^2}{N^2} K_a) F_c - \frac{K_c^2}{N^2} (1 + K_a)} \quad (8)$$

$$H_1 = \frac{T_c}{T_d} = \frac{K_c}{F_c} \left[1 + \frac{(K_c - F_c)}{N} G_1 \right] \quad (9)$$

- Case 2: $T_a = K_a T_d$

$$G_2 = \frac{\theta_m}{T_d} = \frac{\frac{1}{N} (F_c K_a + K_c)}{F_c F_m - \frac{K_c^2}{N^2}} \quad (10)$$

$$H_2 = \frac{T_c}{T_d} = \frac{K_c}{F_c} \left[1 + \frac{(K_c - F_c)}{N} G_2 \right] \quad (11)$$

where: $F_c = J_c s^2 + B_c s + K_c$, $F_m = J_{eq} s^2 + B_{eq} s + \frac{K_c + R_p^2 K_r}{N^2}$. It can be seen that, when we amplify the torque measured by the torque sensor (case 1), the value of the poles varies according to the assist gain, thus the speed of the system and its oscillatory behaviour is related to the assist gain. This control method increases the control requirements and the complexity of the design. The control must ensure the stability of the system and attenuate the vibrations during the variation of the assist gain, and also in the case of high levels of assist gain values and fast response time, [2]. With the driver torque amplification (case 2), the value of the poles of the system does not depend on the assist gain. Thus the control design is simpler, the maximum assist torque that can be generated is a function of motor and gearbox characteristics and the system response is faster. Other important reasons for driver torque estimation, such as the difference between driver torque and measured torque in the transition dynamics and the difficulty to determine exactly the values of steering column stiffness and steering column inertia, are reported in [7]. So, it is highly desirable to estimate the driver torque.

IV. CONTROL DESIGN

The main problem with the torque control is the need of the compensation loops, such as inertia compensation and return control, to achieve the desired performance of the whole system. This comes from that the current controller is inside the armature loop of the motor and the disturbances are out of the control loop (see figure. 4) which makes it difficult to achieve robust command of the motor angle. Thus the changes in the load torque, without compensation, will be reflected on the change of motor velocity and acceleration, and consequently on the change of steering wheel angle, speed and acceleration. Therefore, compensation algorithms are used, which lead to several control loops and large programs. Accordingly, a straightforward consideration allows to consider the motor angle as the reference signal. We will see that this is possible. This consideration leads to the control structure shown in fig. 5.

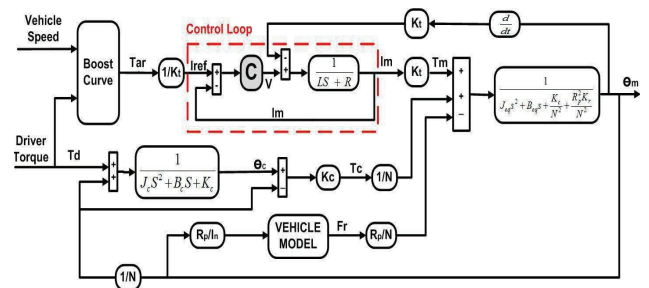


Fig. 4. Block diagram of EPAS system

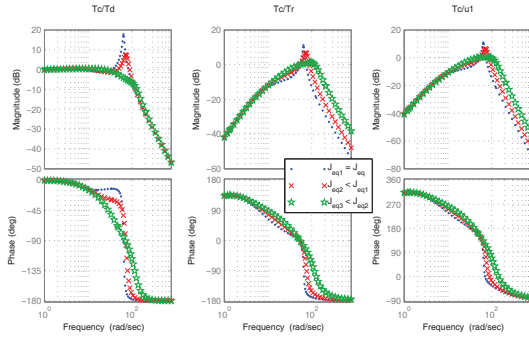


Fig. 6. Influence of equivalent inertia parameter on frequency response

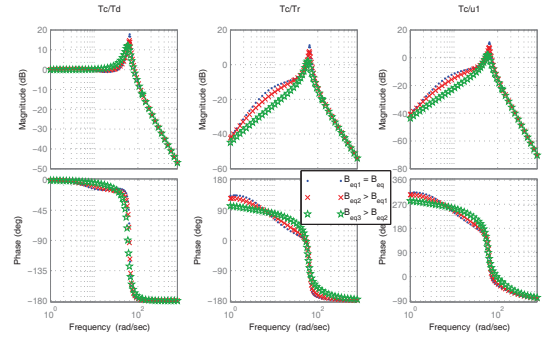


Fig. 9. Influence of equivalent damping parameter on frequency response

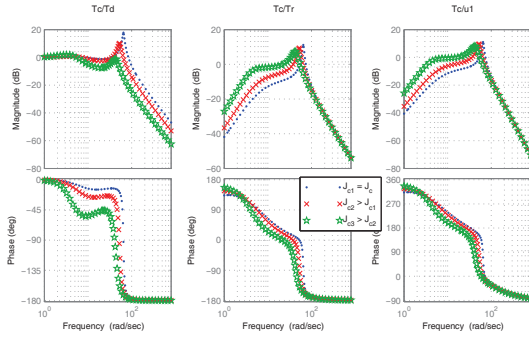


Fig. 7. Influence of steering column inertia parameter on frequency response

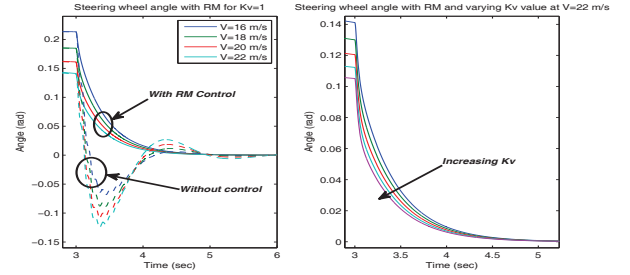


Fig. 10. Steering wheel angle returnability

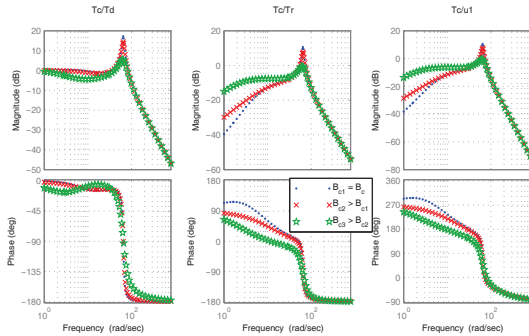


Fig. 8. Influence of steering column damping parameter on frequency response

C. Second order sliding mode controller

Sliding mode control (SMC) method is used for its robustness against parameter uncertainty, insensitivity to bounded disturbances, fast dynamic response and to the relative ease of implementation, [10][11]. The objective of the control based on the super twisting algorithm is to track the desired angle generated by the reference model. The tracking errors e and the sliding surface are defined as :

$$e = \theta_{mr} - \theta_m, \quad S = ce + \dot{e} \quad (14)$$

Differentiating (14), and considering (1), (2) and (3), the time derivative of the sliding surface can be expressed as :

$$\dot{S} = g(t) + \frac{cK_t}{RJ_{eq}}V \quad (15)$$

with: $g(t) = C_1\theta_m + C_2\dot{\theta}_m + C_3\ddot{\theta}_m + C_4\theta_c + C_5F_r$

$$\ddot{S} = \dot{g}(t) + \frac{cK_t}{RJ_{eq}}\dot{V} \quad (16)$$

Taking into account the driver torque, reaction force, current and voltage limitations, it is possible to evaluate a positive constant Φ such that : $(|\dot{g}(t)| \leq \Phi)$. Under this condition, it has been proved in [11] that the application of the controller (17) allows us to steer both S, \dot{S} to zero in finite time.

$$\begin{cases} V = -\lambda |S|^{0.5} \text{sign}(S) + V_1 \\ V_1 = -W \text{sign}(S) \end{cases} \quad (17)$$

with λ, W verifying the following inequality :

$$\lambda > \Phi, \quad W^2 \geq 4\Phi \frac{\lambda + \Phi}{\lambda - \Phi} \quad (18)$$

The control algorithm reported in (17) is referred to as the "Super-Twisting " algorithm, (see [11] for more details).

V. OBSERVER DESIGN

In order to implement the reference model and for practical consideration, we need to estimate the inputs of the reference model. For this purpose, we use the observer proposed in [12]. The proposed observer is based on the sliding

mode observer, Walcott and Zak [13], used in conjunction with second order sliding mode differentiators to estimate the additionally outputs since the system doesn't satisfy the observer matching condition. The additionally outputs needed to construct the observer are calculated using the algorithm proposed in [14]. The advantages of this method is to optimize the number of the sensors and parameters, we don't need to use vehicle model and adding new sensors.

In order to estimate the states and the unknown inputs of the system (4). The following sliding mode observer is proposed, (see [12] for more details):

$$\dot{\hat{x}} = A\hat{x} + B_1u_1 + L(y_a - \hat{y}_a) - B_2E(\hat{y}_a, y_a) \quad (19)$$

with $y_a = [y_1 \ y_{12} \ y_2 \ y_{22}]^T$, (y_1, y_2) are the measurement signals (steering wheel angle and motor angle), (y_{12}, y_{22}) are the additionally outputs (steering wheel speed and motor speed) and $E(\hat{y}_a, y_a)$ is the discontinuous output injection:

$$E(\hat{y}_a, y_a) = \begin{cases} \eta \frac{F(\hat{y}_a - y_a)}{\|F(\hat{y}_a - y_a)\|_2} & \text{for } F(\hat{y}_a - y_a) \neq 0 \\ 0 & \text{for } F(\hat{y}_a - y_a) = 0 \end{cases}$$

η is the positive constant larger than the upper bound of u_2 and the pair of matrices (P, F) satisfying (20) and (21) for some L and Q, where P and Q are matrices symmetric positive definite (see [13] for more details).

$$(A - LC_a)^T P + P(A - LC_a) = -Q \quad (20)$$

$$FC_a = B_2^T P \quad (21)$$

The differentiator is used to generate the additionally outputs (y_{12} , y_{22}) and estimate the motor acceleration and steering wheel acceleration in order to implement the reference model. Consider a signal $x(t)$ measured in real time whose second derivative has a known Lipschitz X . The 2-nd order sliding mode differentiator proposed by Levant, [15] is given by:

$$\begin{aligned} \dot{z}_0 &= v_0 \\ v_0 &= -\lambda_2 X^{1/3} |z_0 - x(t)|^{2/3} \text{sign}(z_0 - x(t)) + z_1 \\ \dot{z}_1 &= v_1 \\ v_1 &= -\lambda_1 X^{1/2} |z_1 - v_0|^{1/2} \text{sign}(z_1 - v_0) + z_2 \\ \dot{z}_2 &= -\lambda_0 \text{sign}(z_2 - v_1) \end{aligned} \quad (22)$$

where: λ_0 , λ_1 , λ_2 are positive design parameters and after finite time the following equalities are verified, [15]:

$$z_0 = x(t), \quad z_1 = \dot{x}(t), \quad z_2 = \ddot{x}(t) \quad (23)$$

The accuracy for the first and second derivative (z_1 , z_2) are proportional, respectively, to $\varepsilon^{2/3}$ and $\varepsilon^{1/3}$, where ε is the magnitude of measurement noises, [15]. The differentiator design requires computing four positive constants (four for each differentiator). One possible choice of the differentiator parameters is ($\lambda_0 = 1.1$, $\lambda_1 = 1.5$, $\lambda_2 = 2.$) given by Levant, [15]. Taking into account driver torque, reaction force, current and voltage limitations, it is possible to evaluate two constants such that: $|\ddot{\theta}_c| < X_c$, $|\ddot{\theta}_m| < X_m$.

VI. SIMULATION RESULTS

The purpose of this section is to validate and demonstrate the effectiveness and the robustness of the proposed approach. Figure. 5 illustrates the simulation block diagram implemented using Matlab/Simulink. White noise has been added to the measurement signals. The noise magnitude is 5% of the measurement signal magnitude with the sampling time of 0.001s. Also, white noise has been added to the driver torque and road reaction torque, noise magnitude is 10% of the signals magnitude with the sampling time of 0.01s. Figure. 11 shows the performance of the observer and the differentiators in response to the driver's torque command. It can be seen that the unknown inputs (driver torque and road reaction torque), additionally outputs (steering column velocity and motor speed) and the accelerations (steering wheel and motor accelerations) can be estimated with good accuracy. As depicted in figure. 11, there is a time delay between the plant speed and acceleration values, and their estimated values, caused by the necessary time for noise filtering. In fact, we have found in the simulations that a high Lipschitz X value reduces the time delay, but with the cost of increasing the sensibility to measurement noise. These curves show the robustness of the observer and the second order differentiators versus measurement noise. Figure. 12 shows the response of the EPAS plant to the driver's torque command with the proposed control structure. In figure. 12-a, we can see the good tracking of the desired motor angle even in the presence of measurement noises. The figure. 12-b shows the comparison between the reference assist torque and the generated assist torque. It can be seen that the control method provides good response to the reference assist torque. We can see also, that there is a delay between the reference assist torque and the generated assist torque. This difference is due to the difference between real and estimated driver torque and to the great difference between the damping values in original plant and reference model. However, this delay is not significant and thus will not be sensed by the driver. The figure. 12-c shows the relationship between the driver torque and the desired assist torque, produced by the proposed structure. It can be seen that the shape is close to the desired boost curve. In order to test the robustness to parameters uncertainty, we have performed another simulation considering a variation of 20% of the parameters (J_c , B_c , J_m , B_m , M_r , B_r , K_t , R , L) values. The simulation results depicted in figure 13, show the good performance of the proposed structure even with the parameters uncertainty.

VII. CONCLUSION

In this paper, a new control method for EPAS system was proposed. Where a reference model is employed to generate ideal motor angle that can guarantee the desired performance and then a second order sliding mode controller is used to track the desired motor angle. The model is built using the dynamic model of EPAS system and the model parameters are tuned to attenuate the vibrations and to improve the steering feel. To implement the reference model without adding new sensors, a sliding mode observer with unknown inputs

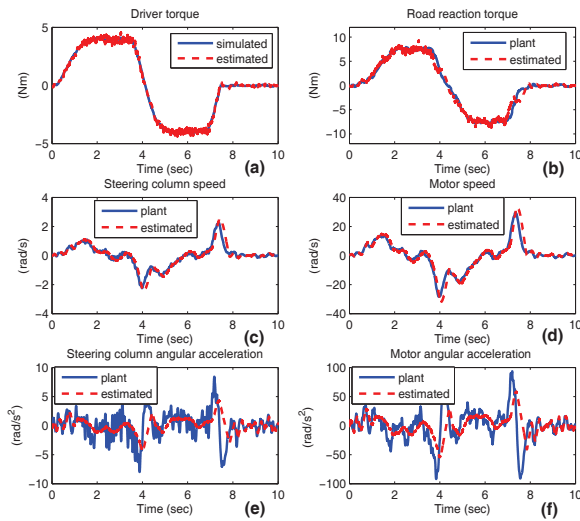


Fig. 11. Estimation performance

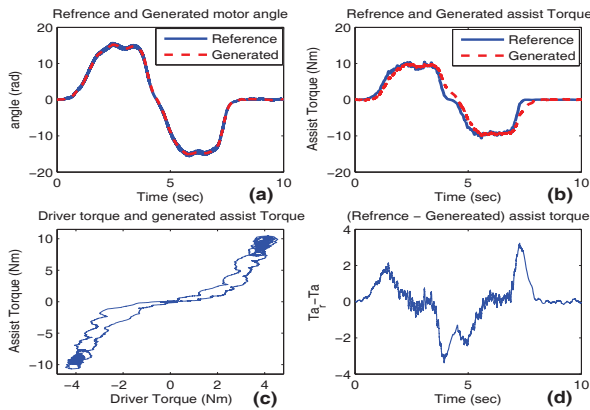


Fig. 12. Control performance

and second order sliding mode differentiators are designed. The simulation results illustrate the effectiveness and the robustness of the proposed structure. The proposed control structure allows to improve performance of the system, to reduce the system cost and also can be adapted to other kinds of driver assistance. In future work, our proposed control method for EPAS system will be tested and validated using EPAS platform actually under development.

VIII. ACKNOWLEDGMENTS

This work has been supported by ANR "Agence Nationale de la Recherche", projet VOLHAND "ANR-09-VTT-14-01/06" and by International Campus on Safety and Intermodality in Transportation, the European Community, the Delegation Regionale a la Recherche et a la Technologie, the Ministere de l'Enseignement supérieur et de la Recherche,

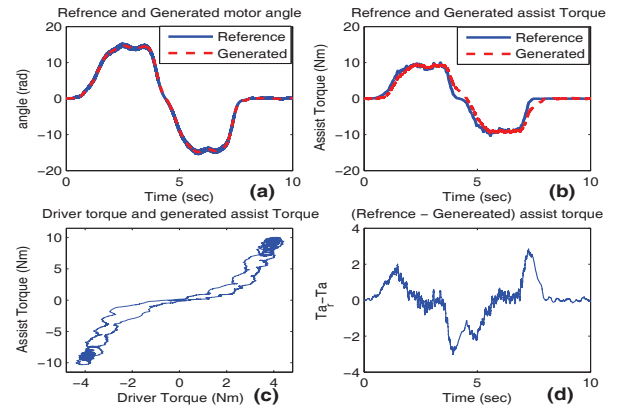


Fig. 13. Control performance with 20% variation of the parameters

the Region Nord Pas de Calais and the Centre National de la Recherche Scientifique.

REFERENCES

- [1] A. Badawy, J. Zuraski, F. Bolourchi, and A. Chandy, "Modeling and analysis of an electric power steering system," *SAE Technical Paper, 1999-01-0399*, 1999.
- [2] A. T. Zaremba, M. K. Liubakka, and R. M. Stuntz, "Control and steering feel issues in the design of an electric power steering system," *Proc. of the ACC*, pp. 36–40, 1998.
- [3] N. Sugitani, Y. Fujuwara, K. Uchida, and M. Fujita, "Electric power steering with h-infinity control designed to obtain road information," *Proc. of the ACC*, pp. 2935–2939, 1997.
- [4] X. Chen, X. Chen, and K. Li, "Robust control of electric power-assisted steering," *IEEE Vehicle Power and Propulsion Conference*, pp. 97 – 102, 2005.
- [5] M. Parmar and J. Y. Hung, "A sensorless optimal control system for an automotive electric power assist steering system," *IEEE Transactions on Industrial Electronics*, vol. 51, no. 2, pp. 290–298, 2004.
- [6] M. Kurishige, N. m. H. Tanaka, K. Tsutsumi, and T. Kifuku, "An eps control strategy to improve steering maneuverability on slippery roads," *SAE Paper, 2002-01-0618*, 2002.
- [7] R. C. Chabaan and L. Y. Wang, "Control of electrical power assist systems: h_∞ design, torque estimation and structural stability," *JSAE Review*, vol. 22, no. 4, pp. 435–444, 2001.
- [8] S. Cholakkal and X. Chen, "Fault tolerant control of electric power steering using robust filter-simulation study," *IEEE Vehicle Power and Propulsion Conference*, pp. 1244 – 1249, 2009.
- [9] H. Tanaka and M. Kurishige, "The torque controlled active steer for eps," *AVEC'04*, pp. 501 – 506, 2004.
- [10] V. I. Utkin, J. Gulder, and J. Shi, *Sliding Mode Control in Electromechanical Systems*. London, UK: Taylor and Francis, 1999.
- [11] L. Fridman and A. Levant, "Higher-order sliding modes," in *In W. Perruquetti and J. P. Barbot, Sliding mode control in engineering. Control engineering*, New York/Basel, USA, 2002, pp. 53–101.
- [12] A. Marouf, M. Djemaï, C. Sentouh, and P. Pudlo, "Driver torque and road reaction force estimation of an electric power steering using sliding mode observer with unknown inputs," *IEEE Conference on Intelligent Transportation Systems*, pp. 354 – 359, 2010.
- [13] B. L. Walcott and S. Zak, "State observation of nonlinear uncertain dynamical systems," *IEEE Trans. Automat. Contr.*, vol. AC-32, No. 2, pp. 166–170, 1987.
- [14] T. Floquet, C. Edwards, and S. Spurgeon, "On sliding mode observers for systems with unknown inputs," *International Journal of Adaptive Control and Signal Processing*, vol. 21, pp. 638–656, 2007.
- [15] A. Levant, "Higher-order sliding mode differentiation and output-feedback control," *International journal of Control*, vol. 76, pp. 924–941, 2003.