2011 50th IEEE Conference on Decision and Control and
European Control Conference (CDC-ECC)
Orlando, FL, USA, December 12-15, 2011

Observer design for nonlinear processes with Wiener structure

T. Á. Glaría López, D. Sbarbaro

Abstract— **Wiener models, which consider a linear dynamical model and a nonlinear output map, can represent a wide range of industrial processes. In this paper, observer design approaches for these systems are proposed. The approaches consider a Wiener structure having a Lipschitz nonlinear perturbation term and explicit and implicit nonlinear output maps. The observers gain are obtained by solving a set of LMIs which consider the Lipschitz constant associated to the nonlinear perturbation and the convex hull associated to the output map derivative. A conductivity tracking problem and pH neutralization processes illustrate the main features of the design process and the performance obtained with the proposed design approach.**

Keywords: Chemical process; Wiener structure; Observer design; Nonlinear systems; LMI solution; Nonlinear output maps; Differential Mean Value Theorem

I. Introduction

WIENER models can approximate a wide range of industrial processes. One example is motivated by state estimation in process tomography using fluid dynamics models; which are usually modeled as Markov models. A second example can be found in the pH neutralizing processes, where the nonlinearity is associated to the measuring systems. In both cases, Lipschitz nonlinearities in the process and nonlinear output map are common features in these applications. Observer designs for these processes have considered standard techniques such as Extended Kalman filter [1] and geometric methods [14].

Recent work based on Luenberger observer structures for systems with Lipschitz nonlinearities consider alternative approaches such as Linear Varying Parameters [2,6] and observer design with weighted feedback [7]. The use of the Lipschitz constant associated to the nonlinear perturbation term in observer design has been addressed in [4,13]. Less conservative results have been obtained using the one-sided Lipschitz condition [8,10,11,12] or quasi-one-side Lipschitz condition [9]. In addition, by applying the mean value theorem the error dynamic can be transformed into a LPV system [2,5,20] to design observers with improved performance. Only few of these works have addressed the problem of systems with nonlinear output maps. In [2] and [6] observers based on linear feedback are proposed, whereas in [7] a weighted feedback is considered, but this approach is limited to linear models having just one eigenvalue in the imaginary axis. In this work, these results are tailored to Wiener models and they are extended to deal with linear models having an arbitrary number of eigenvalues in the imaginary axis and also with systems having implicit output maps.

The paper is organized as follow. In Section 2, an observer design method based on [6] is proposed for systems with Lipschitz nonlinear dynamics and explicit Lipschitz nonlinear output, also a variation of this method is presented where a nonlinear term is added to the feedback. In Section 3, the method is extended to deal with systems where the nonlinear output is defined implicitly. Two examples are presented in Section 4, a conductivity tracking process and a pH process, which illustrate the proposed methods. Finally, conclusions are presented in section 5.

Manuscript received March 22, 2011. This work was supported in part by Fondecyt Project 1100272 and CONICYT.

T. Á. Glaría is with the Electrical Engineering Department, Universidad de Concepción, Concepción, Chile, (e-mail: tglaria@udec.cl).

D. Sbarbaro is with the Electrical Engineering Department, Universidad de Concepción, Concepción, Chile, (e-mail: dsbarbar@udec.cl).

II. Explicit Output

A. Luenberger observer structure

In this section, an observer design is proposed for systems with Lipschitz nonlinear dynamics and Lipschitz nonlinear output.

Consider linear systems following the form

$$\begin{cases} \dot{x} = Ax + Bu + f(x,u) \\ y = h(x) \end{cases} \tag{1}$$

where $f\colon \mathbb{R}^n \times \mathbb{R}^m \mapsto \mathbb{R}^n$ is a global Lipschitz nonlinear map with respect to $x(k)$; i.e.

$$\|f(x_1,u) - f(x_2,u)\| \leq \gamma_f \|x_1 - x_2\| \tag{2}$$

where γ_f is the Lipschitz condition constant. This can be rewritten as

$$\frac{1}{\gamma_f^2}\bar{f}^T\bar{f} \leq \bar{x}^T\bar{x} \tag{3}$$

The nonlinear function $h\colon \mathbb{R}^n \mapsto \mathbb{R}^p$ is a continuous function with bounded derivatives with respect to x.

The proposed observer follows the traditional structure:

$$\dot{\hat{x}} = A\hat{x} + Bu + K(h(\hat{x}) - y) + f(\hat{x},u) \tag{4}$$

The error is defined as $e = \hat{x} - x$ and the error dynamic is

$$\dot{e} = Ae + K\big(h(\hat{x}) - h(x)\big) + \big(f(\hat{x},u) - f(x,u)\big) \tag{5}$$

The differential mean value theorem [6] states that for a given $a, b \in \mathbb{R}^n$ and $\varphi\colon \mathbb{R}^n \to \mathbb{R}^p$, differentiable in $Co(a,b)$, then there are constant vectors $z_1, \dots, z_p \in Co(a,b)$

$$\varphi(a) - \varphi(b) = \left(\sum_{i,j=1}^{p,n} \Omega_{i,j}\frac{\partial \varphi_i}{\partial x_j}(z_i)\right)(a-b) \tag{6}$$

where $\Omega_{i,j} \in \mathbb{R}^{p\times n}$ is a real matrix whose (i,j) term is 1, and all the rest are 0.

By considering (6) for the output function $h(x)$, the error dynamic is

$$\dot{e} = Ae + K\left(\sum_{i,j=1}^{p,n} \vartheta_{i,j}(z_i)\Omega_{i,j}\right)e + \left(f(\hat{x},u) - f(x,u)\right) \tag{7}$$

where

$$\vartheta_{i,j}(z_i) = \left.\frac{\partial h_i(x)}{\partial x_j}\right|_{x=z_i} \tag{8}$$

and $z \in Co(\hat{x}, x)$.

Let's rewrite

$$\left.\frac{\partial h(x)}{\partial x}\right|_{x=z} = \sum_{i,j=1}^{p,n} \vartheta_{i,j}(z_i)\Omega_{i,j} = C(\vartheta) \tag{9}$$

Also, let $(C_i)_{1\le i\le v}$ be the convex-hull matrices such that

$$C(\vartheta) \in \{C_1, \dots, C_v\} \tag{10}$$

where 'v' is the number of convex-hulls.

Now the first theorem can be stated:

Theorem 1: If a matrix K can be chosen such that $A + KC_i, \forall\, 1 \le i \le v$ is stable, and $P = P' > 0$ is a solution of the inequality

$$\begin{bmatrix} [A + KC_i]^T P + P[A + KC_i] + I & P \\ P & -\frac{1}{\gamma_f^2} I \end{bmatrix} < 0$$

and the Lipschitz condition holds, then the proposed observer (4) yields globally asymptotically stable estimates of the observed system (1).

Proof: Consider the Lyapunov function $V(e) = e^T P e$, where $e = \hat{x} - x$ is the error, $P = P^T > 0$ and the observer defined as (3). The derivative of the Lyapunov function

$$\begin{aligned} \dot{V}(e) &= \dot{e}^T P e + e^T P \dot{e} \\ &= e^T \left[A + K\left(\sum_{i,j=1}^{p,n} \vartheta_{i,j}(z)\Omega_{i,j}\right)\right]^T P e \\ &\quad + e^T P \left[A + K\left(\sum_{i,j=1}^{p,n} \vartheta_{i,j}(z)\Omega_{i,j}\right)\right] e \\ &\quad + 2e^T P[f(\hat{x},u) - f(x,u)] \end{aligned} \tag{11}$$

Which can be rewritten as

$$\dot{V}(e) = \begin{bmatrix} e \\ f \end{bmatrix}^T \begin{bmatrix} V_{k1} & P \\ P & 0 \end{bmatrix} \begin{bmatrix} e \\ f \end{bmatrix} \tag{12}$$

With

$$V_{k1} = [A + KC(\vartheta)]^T P + P[A + KC(\vartheta)]$$

In addition, from (2) we have

$$\begin{bmatrix} e \\ f \end{bmatrix}^T \begin{bmatrix} -I & 0 \\ 0 & \frac{1}{\gamma_f^2} \end{bmatrix} \begin{bmatrix} e \\ f \end{bmatrix} \le 0 \tag{13}$$

Now, using equations (12) and (13), and the S-procedure [3], a sufficient condition to have $\dot{V}(e) < 0$ for all $x \in \mathbb{R}^n$ is attained:

$$\begin{bmatrix} [A + KC(\vartheta)]^T P + P[A + KC(\vartheta)] + I & P \\ P & -\frac{1}{\gamma_f^2} I \end{bmatrix} < 0 \tag{14}$$

$$\underline{\vartheta}_{i,j} \le \vartheta_{i,j}(x) \le \overline{\vartheta}_{i,j} \qquad \forall i, j$$

where $\underline{\vartheta}_{i,j}$ and $\overline{\vartheta}_{i,j}$, are the lower and upper bound of element $\vartheta_{i,j}(x)$ respectively.

This set of equations can be transformed into an LMI optimization problem by replacing the time-varying parameters by its convex-hulls defined by (10), which means equation (14) becomes:

$$\begin{bmatrix} [A + KC_i]^T P + P[A + KC_i] + I & P \\ P & -\frac{1}{\gamma_f^2} I \end{bmatrix} < 0 \tag{15}$$

$$\forall\, 1 \le i \le v$$

This ends the proof.■

B. Luenberger observer structure with weighted nonlinear feedback

In this section, an observer design method is proposed for systems with Lipschitz nonlinear dynamics and Lipschitz nonlinear output map, where the proposed observer follows the Luenberger structure with a weighted feedback added to it.

Let's consider the systems described by (1). The proposed observer follows the structure:

$$\begin{aligned} \dot{\hat{x}} = A\hat{x} + Bu + f(\hat{x},u) + K_1(h(\hat{x}) - y) \\ + K_2 \nabla h(\hat{x})^T (h(\hat{x}) - y) \end{aligned} \tag{16}$$

The nonlinear term $K_2 \nabla h(\hat{x})^T (h(\hat{x}) - y)$ considers the output sensitivity to adjust the feedback gain.

The error is defined as $e = \hat{x} - x$ and the error dynamics for such observer are

$$\begin{aligned} \dot{e} = Ae + \left(f(\hat{x},u) - f(x,u)\right) + K_1(h(\hat{x}) - y) \\ + K_2 \nabla h(\hat{x})^T (h(\hat{x}) - y) \end{aligned} \tag{17}$$

Using (6) and (8) to (10) a theorem addressing the stability of this kind of observer can now be stated:

Theorem 2: If a matrix K_1 can be chosen such that $A + K_1 C_i, \forall\, 1 \le i \le v$ is stable, and $P = P' > 0$ is a solution of the inequality

$$\begin{bmatrix} [A + K_1 C_i]^T P + P[A + K_1 C_i] - 2C_i^T C_i + I & P \\ P & -\frac{1}{\gamma_f^2} I \end{bmatrix} < 0$$

and the Lipschitz condition holds, then the proposed observer (16) yields locally stable estimates around $e = 0$ of the observed system (1) with $K_2 = -P^{-1}$.

Proof: Consider the Lyapunov function $V(e) = e^T P e$, where $e = \hat{x} - x$ is the error, $P = P^T > 0$, $K_2 = -P^{-1}$ and the observer defined as (16). Linearizing all terms around $e = 0$ in the error dynamics (17) but without considering the Lipschitz nonlinearity we get

$$\begin{aligned} \dot{e} = \left(A + K_1 \frac{\partial h(x)}{\partial x} + K_2 \frac{\partial h(x)}{\partial x}^T \frac{\partial h(x)}{\partial x}\right) e \\ + \left(f(\hat{x},u) - f(x,u)\right) \end{aligned} \tag{18}$$

Rewriting using (8) and (9)

$$\dot{e} = \left(A + K_1 C(\vartheta) + K_2 C(\vartheta)^T C(\vartheta)\right)e + \left(f(\hat{x},u) - f(x,u)\right) \quad (19)$$

The derivative of the Lyapunov function is

$$\begin{aligned}\dot{V}(e) &= \dot{e}^T P e + e^T P \dot{e} \\ &= e^T\left(A + K_1 C(\vartheta)\right)^T P e \\ &+ e^T P\left(A + K_1 C(\vartheta)\right)e \\ &- 2e^T C(\vartheta)^T C(\vartheta) e \\ &+ 2e^T P[f(\hat{x},u) - f(x,u)]\end{aligned} \quad (20)$$

which can be rewritten as

$$\dot{V}(e) = \begin{bmatrix} e \\ f \end{bmatrix}^T \begin{bmatrix} V_{k2} & P \\ P & 0 \end{bmatrix} \begin{bmatrix} e \\ f \end{bmatrix} \quad (21)$$

with

$$V_{k2} = [A + K_1 C(\vartheta)]^T P + P[A + K_1 C(\vartheta)] - 2C(\vartheta)^T C(\vartheta)$$

Using equations (13) and (21), and the S-procedure [3], a sufficient condition to have $\dot{V}(e) < 0$ for all $x \in \mathbb{R}^n$ is attained:

$$\begin{bmatrix} V_{k2} + I & P \\ P & -\frac{1}{\gamma_f^2} I \end{bmatrix} < 0 \quad (22)$$

$$\underline{\vartheta}_{i,j} \leq \vartheta_{i,j}(x) \leq \overline{\vartheta}_{i,j} \qquad \forall i,j$$

where $\underline{\vartheta}_{i,j}$ and $\overline{\vartheta}_{i,j}$, are the lower and upper bound of element $\vartheta_{i,j}(x)$ respectively.

This set of equations can be transformed into an LMI optimization problem by replacing the time-varying parameters by its convex-hulls defined by equation (10), which means equation (22) becomes:

$$\begin{bmatrix} [A + K_1 C_i]^T P + P[A + K_1 C_i] - 2C_i^T C_i + I & P \\ P & -\frac{1}{\gamma_f^2} I \end{bmatrix} < 0 \quad (23)$$

$$\forall\ 1 \leq i \leq v$$

This ends the proof. ■

III. Implicit Output

In this section, an observer design for systems having implicit output is proposed. Let's consider the following form

$$\begin{cases} \dot{x} = Ax + Bu + f(x,u) \\ 0 = h(x,y) \end{cases} \quad (24)$$

The proposed observer follows the traditional structure:

$$\dot{\hat{x}} = A\hat{x} + Bu + K\left(h(\hat{x},y) - h(x,y)\right) + f(\hat{x},u) \quad (25)$$

and since $h(x,y) = 0$, can be rewritten as

$$\hat{x} = A\hat{x} + Bu + Kh(\hat{x},y) + f(\hat{x},u) \quad (26)$$

The error is defined as $e = \hat{x} - x$ and the error dynamic for such observer is

$$\dot{e} = Ae + K\left(h(\hat{x},y) - h(x,y)\right) + \left(f(\hat{x},u) - f(x,u)\right) \quad (27)$$

The theorem addressing the stability of the observer is stated as follows

Theorem 3: If a matrix K can be chosen such that $A + KC_i, \forall\ 1 \leq i \leq v$ is stable, and $P = P' > 0$ is a solution of the inequality

$$\begin{bmatrix} [A + KC_i]^T P + P[A + KC_i] + I & P \\ P & -\frac{1}{\gamma_f^2} I \end{bmatrix} < 0$$

and the Lipschitz condition holds, then the observer proposed (25) yields asymptotically stable estimates of the observed system (24).

Proof: The proof for this case follows similar steps as the one presented in section 2.

IV. Some Applications

A. *Conductivity tracking*

Conductivity based sensors can be used in those applications where the conductivity of solid, liquid and gas give valuable information to monitor industrial processes [1], [15]. Electrical methods have the advantage that they do not need transparent tanks or columns. In thickeners for instance, solid concentrations, solids profiles and clear/slurry interfaces level can be determined from conductivity measurements [17]. In many column applications estimating the height of interfaces automatically has proved difficult due to the signal noise [16]. In order to cope with noisy signals and the dynamic variations of the conductivity an observer based approach has been proposed. A typical and simple conductivity probe consists of a bipolar power supply and a sensor cell connected to the power supply through a resistor.

The sensor cell considers some electrodes and a data acquisition system for collecting and processing the voltage signals. Since the relationship between conductivity and voltage is nonlinear, the values of the associated resistor of each cell must be carefully selected in order to avoid resolution problems [15]. A neural network approach was proposed in [15] to deal with this nonlinear characteristic by training the network to learn the inverse function of the nonlinearity.

In this paper, a different solution based on dynamic models and observers is proposed. The first step is to model the electrode or cell voltage variation by a simple dynamical model. One of the simplest options is to model the voltage dynamic variation by the following continuous-time state model, where the variables x are considered real values representing a stochastic process described by the following Markov model:

$$\dot{x} = Ax + B\omega_t \quad (28)$$

where A is the state transition matrix, ω is the evolution noise process having zero mean and covariance Γ_ω , B is the transition matrix of the noise process. Matrix A defines the type of evolution assumed for the state variables. For instance if $A = 0$, then a simple random walk model is obtained. In addition we also assume that the conductivity values belong to a closed set $\Omega \in \mathbb{R}^n$, where n is the dimension of x.

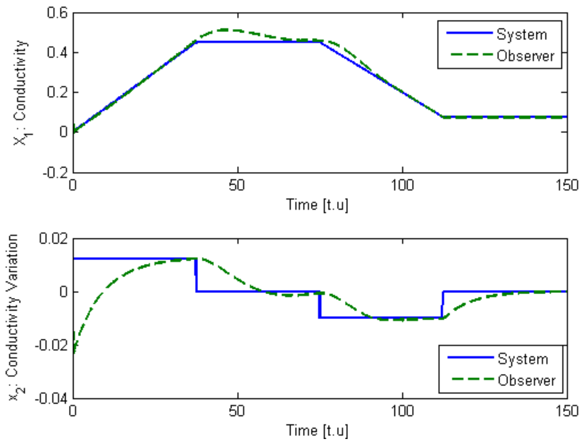

Fig. 1. Conductivity dynamics and the value estimated by the Luenberger observer

For time varying signal, a Newtonian kinematic model is more suitable, and in this case the equations representing these variations consider the following state transition matrix:

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \tag{29}$$

For a conductivity based sensor, the measurements can be described by a continuous nonlinear function $h(\cdot)$ representing the relationship between conductivity and measured voltage:

$$y_t = h(x_t) + \varepsilon_t \tag{30}$$

where y is the vector of measured voltages, and ε_t is a random gaussian noise with zero mean and covariance Γ_ε . It is assumed that the Jacobian matrix of $h(\cdot)$ satisfies the following condition:

$$a_{i,j} \leq \frac{\partial h_j}{\partial x_i} \leq b_{i,j} \tag{31}$$

Now, let's consider a conductivity application where the process and the measuring system can be described by the following model

$$\begin{cases} \dot{x} = Ax \\ y = h(x) \end{cases} \tag{32}$$

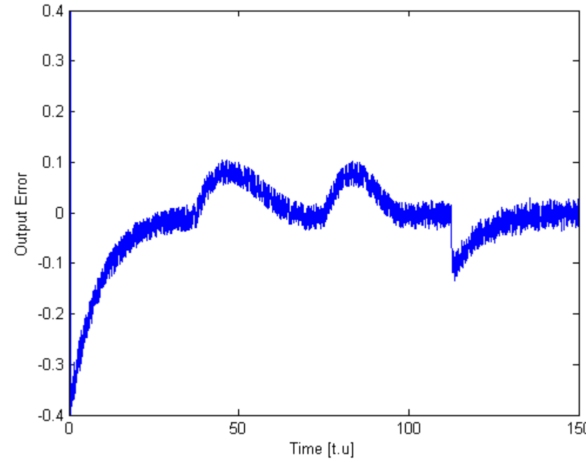

Fig. 2. Output error for conductivity tracking example using Luenberger observer.

Where A is defined as in (29), x is the conductivity, and the voltage is the output defined by the function $h(\cdot)$ described by

$$h(x) = \frac{V_i \cdot R \cdot x_1}{K + Rx_1} \tag{33}$$

with $V_i = 15\ \mathrm{V}$, $R = 50\ \Omega$ and $K = 1.5$, and whose Jacobian

$$0 < \frac{\partial h(x)}{\partial x_1} = V_i R \left[\frac{1}{K + Rx_1} - \frac{x_1}{(K + Rx_1)^2}\right] \leq \frac{V_i R}{K} \tag{34}$$

Since the conductivity $x_1 > 0$

The conductivity dynamics and their estimated values using the Luenberger observer with constant feedback gain are presented in Fig.1, whereas Fig.2 shows the output error $\tilde{y} = \hat{y} - y$.

In Fig.3, the conductivity dynamics and their estimated value given by the Luenberger observer with weighted, nonlinear feedback are presented. Fig.4 shows its output error. The use of the non-linear term increases the gain improving tracking but not filtering.

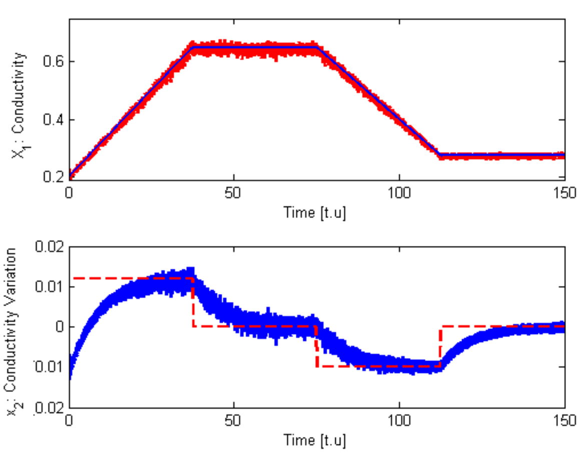

Fig. 3. Conductivity dynamics and the value estimated by the Luenberger

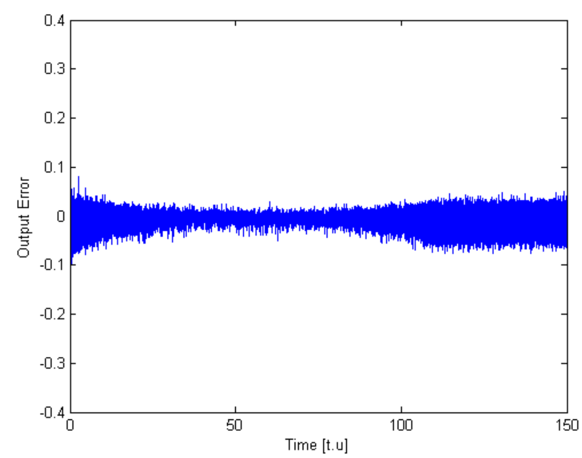

Fig. 4. Output error for conductivity tracking example using Luenberger observer with weighted feedback

B. pH process

This example considers a pH neutralization process which consists of three inputs: (u_1) a base stream, NaOH, (u_2) a buffer stream, $NaHCO_3$, and (u_3) an acid stream, HNO_3, mixed in a constant volume (V) stirring tank. The acid flow rate and the volume of the tank are assumed to be constant, and the objective usually is to control the output, the pH of the effluent solution, by manipulating the base flow $u_1$despite the unmeasured buffer flow rate u_2.

The model of this example

$$\dot{x} = \frac{1}{V}\begin{bmatrix} u_1(W_{a1} - x_1) + u2(W_{a2} - x_1) + u_3(W_{a3} - x_1) \\ u_1(W_{b1} - x_2) + u2(W_{b2} - x_2) + u_3(W_{b3} - x_2) \end{bmatrix} \quad (35)$$
$$h(x, y) = x_1 + 10^{y-14} - 10^{-y} + x_2 \frac{1 + 2 \times 10^{y - pK_2}}{1 + 10^{pK_1 - y} + 10^{y - pK_2}} = 0$$

Where the parameters pK_1 and pK_2 are the first and second disassociation constants of the weak acid H_2CO_3. The nominal operation conditions of the pH neutralization process, table 1, were used in the observer design process.

Matlab LMI toolbox was used to solve the LMI problem, obtaining

$$P = \begin{bmatrix} 6.3581 & -7.0969 \\ -7.0969 & 8.7926 \end{bmatrix}$$

$$K = \begin{bmatrix} -2.8644 \\ -2.4019 \end{bmatrix}$$

Simulation results depicted in Fig.5 show the evolution of the state variables x_1 and x_2 and its estimates $\hat{x}_1$ and $\hat{x}_2$. Note that in this case it is not necessary to solve the output map in order to build the observer, simplifying in this way the implementation of observers for this class of processes.

TABLE I
NOMINAL OPERATION CONDITIONS PARAMETERS OF THE pH NEUTRALIZATION PROCESS

V =2,900 ml
$u_1 = 15.55\ ml/s$
$u_2 = 0.55\ ml/s$
$u_3 = 16.66\ ml/s$
$W_{a1} = -3.05 \times 10^{-3}\ M$
$W_{a2} = -3 \times 10^{-2}\ M$
$W_{a3} = 3 \times 10^{-3}\ M$
$W_{b1} = 5 \times 10^{-5}\ M$
$W_{b2} = 3 \times 10^{-2}\ M$
$W_{b3} = 0\ M$
$W_a = -4.32 \times 10^{-4}\ M$
$W_b = 5.28 \times 10^{-4}\ M$
$pK_1 = 6.35$
$pK_2 = 10.25$
$y_{nom} = 7.0$

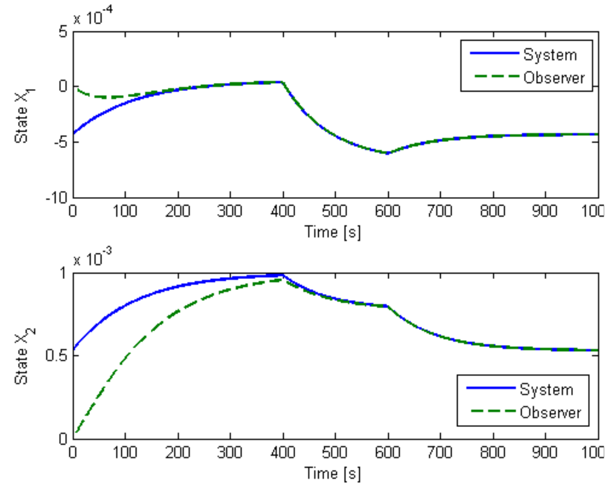

Fig.5: The state x_1 and x_2, and their estimates

V. CONCLUSION

In this paper, two observer design methods for Wiener structure have been proposed. These methods can deal with explicit and implicit nonlinear output and only require solving a LMI optimization problem for calculating the observer gain to ensure the stability of the observer.

Even though it's not addressed in this document, the presented method can easily be adapted to use one-side Lipschitz condition or quasi-one-side Lipschitz condition in the observer design, attaining less conservative results.

Future work considers more complex application such as tomography sensor and input estimation in pH neutralization process. In addition, the design of robust observer by taking into account parametric uncertainties will be also explored along the use of performance measure to shape the dynamic response.

REFERENCES

[1] Aku Seppanen, Lasse Heikkinen, Tuomo Savolainen, Arto Voutilainen, Erkki Somersalo, Jari P. Kaipio, An experimental evaluation of state estimation with fluid dynamical models in process tomography, Chemical Engineering Journal, Volume 127, Issues 1-3, 1 March 2007, Pages 23-30.

[2] Ibrir, S., "LPV approach to continuous and discrete nonlinear observer design," Decision and Control, 2009 held jointly with the 2009 28th Chinese Control Conference. CDC/CCC 2009. Proceedings of the 48th IEEE Conference on , vol., no., pp.8206-8211, 15-18 Dec. 2009

[3] D. M. Stipanović and D. D. Šiljak. "Robust stability and stabilization of discrete-time nonlinear systems: The LMI approach", International Journal of Control 74 (2001), pp. 873-879.

[4] Zemouche, A.; Boutayeb, M.; , "Observer Design for Lipschitz Nonlinear Systems: The Discrete-Time Case," Circuits and Systems II: Express Briefs, IEEE Transactions on , vol.53, no.8, pp.777-781, Aug. 2006

[5] Phanomchoeng, G.; Rajamani, R.; , "The bounded Jacobian approach to nonlinear observer design," American Control Conference (ACC), 2010 , vol., no., pp.6083-6088, June 30 2010-July 2 2010

[6] Ali Zemouche, Mohamed Boutayeb, G. Iulia Bara, "Observers for a class of Lipschitz systems with extension to H[infinity] performance analysis", Systems & Control Letters, Volume 57, Issue 1, January 2008, Pages 18-27.

[7] Andreas Johansson, Alexander Medvedev, "An observer for systems with nonlinear output map", Automatica, Volume 39, Issue 5, May 2003, Pages 909-918.

[8] Guang-Da Hu; "Observers for one-sided Lipschitz non-linear systems",IMA Journal of Mathematical Control and Information, Vol. 23, No. 4. (December 2006), pp. 395-401.

[9] Guang-Da Hu; "A note on observer for one-sided Lipschitz non-linear systems", IMA J Math Control Info first published online September 14, 2007.

[10] Zhi-Xin Tai; Xing-Cheng Wang; , "H∞ observers for one-sided Lipschitz nonlinear systems," Intelligent Control and Automation, 2008. WCICA 2008. 7th World Congress on , vol., no., pp.860-863, 25-27 June 2008

[11] Masoud Abbaszadeh, Horacio J. Marquez; "Nonlinear Observer Design for One-Sided Lipschitz Systems" American Control Conference (ACC), 2010 , vol., no., pp.5284-5289, June 30 2010-July 2 2010

[12] Yanbin Zhao, Jian Tao, Ning-Zhong Shi, "A note on observer design for one-sided Lipschitz nonlinear systems", Systems & Control Letters, Volume 59, Issue 1, January 2010, Pages 66-71.

[13] Rajamani, R.; , "Observers for Lipschitz nonlinear systems," Automatic Control, IEEE Transactions on , vol.43, no.3, pp.397-401, Mar 1998

[14] Juan C. Gomez, Enrique Baeyens, "Identification of block-oriented nonlinear systems using orthonormal bases", Journal of Process Control, Volume 14, Issue 6, September 2004, Pages 685-697.

[15] Cardenas, R., Serpa, S., Villegas, S., and Retamales, J. (1998). Conductivity sensor for mining applications. In IEEE Instrumentation and Measurement Technology, 1050–1055. Minesota, USA.

[16] Vergouw, J., Gomez, C., and Finch, J. (2004). Estimating true level in a thickener using conductivity probe. Minerals Engineering, 17, 87–88.

[17] Xu, M., Probst, A., and Finch, J. (1994). Level and solids profile detection in thickeners using conductivity. CIM Bul l, 87.

[18] J. M. Ortega and W. C. Rheinboldt, Iterative solution of nonlinear equations in several variables, SIAM ed., ser. Classics in Applied Mathematics, 2000, vol. 30.

[19] Silvina I. Biagiola and Jose´ L. Figueroa,State Estimation in Nonlinear Processes. Application to pH Process Control, Ind. Eng. Chem. Res. 2002, 41, 4777-4785

[20] Ali Zemouche, Mohamed Boutayeb, G. Iulia Bara, "Observers for nonlinear systems: An approach based on the differential mean value theorem", 44th IEEE CDC, Seville, Spain 6353-6358, 2005