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Cooperative Tracking For A Swarm Of Unmanned Aerial Vehicles: A Distributed Tagaki-Sugeno Fuzzy Framework Design

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Abstract— **This paper focuses on a systematic analysis for the tracking problem in swarm-based missions for Unmanned Aerial Vehicles (UAVs) with linear and angular velocity constraints. In this paper the nonlinear model of the dynamics are represented by Takagi-Sugeno (TS) fuzzy models. A distributed control law is introduced which is composed of both node and network level information. Firstly feedback gains are synthesised for the isolated UAVs ignoring interconnections. The resulting common Lyapunov matrix is utilised at network level, to incorporate into the control law the relative differences in the states of the agents, to induce cooperative behaviour. Eventually stability is guaranteed for the entire swarm. The control synthesis is all performed subject to design criteria, posed as Linear Matrix Inequalities (LMIs). An illustrative example based on a UAV tracking scenario is included to outline the potential of the analysis.**

I. Introduction

The use of a single Unmanned Aerial Vehicle (UAV) can jeopardise a mission since any degradation in performance can have a serious effect on the objectives. For that reason, there is an increasing interest in co-operative control which can benefit swarm-based missions in many ways. For the interested reader, reference [1] includes an exhaustive list of work concerning different architectures used in cooperative control.

Large-scale multi-agent systems can be represented accurately by nonlinear models in a large domain of operation. However this coupled with the dimensionality of the network means the task of designing a control law may be a far from trivial task. Most of the existing work has focused on the interconnection of systems with linear dynamics. For example, consensus was examined for multi-agent systems with general linear dynamics in [2], [3]. In [4], [5] consensus for agents with single/double or higher integrator dynamics were studied. In reference [6] the authors focused on the stabilisation of a network of identical agents with linear dynamics. Unlike the previous methodologies which consider first or higher order linear models for the vehicles' motion, in this work, a nonlinear representation of the dynamics of a group of UAV systems with constraints on angular and linear velocity is investigated.

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In particular, motivated by work in [7] where the global stabilisation of a complex network of agents is considered by applying local decentralised output feedback control law, reference [8] developed a distributed control law for nonlinear systems based on a two step procedure. This allowed a decoupled design procedure at both node and network level and offered a systematic analysis for stabilisation/tracking problems in a reasonably large class of networks of nonlinear systems represented in the Takagi-Sugeno (TS) framework [9]. In this paper, the work in [8] is extended for a more general case of nonlinear systems with a focus on tracking for a swarm of UAVs. Through a special choice of the gain matrix in the relative state information term, the utilisation of the procedure suggested by the authors in [8] is adopted. Due to the structure of the TS model, which is a fuzzy *blending* of linear local models, this allows a systematic analysis for proving stability, in a Lyapunov sense, of a general class of nonlinear systems. Interesting work that addresses the design aspects for Takagi-Sugeno controllers exists in the literature: see for example, [10].

In this work, the model under investigation is the error dynamics of the UAV as developed in references [11] and [12]. At the first step of the procedure, the error dynamics of the UAV system are isolated, and a node level control law is designed ignoring interconnections. The node level control law utilises a Parallel Distributed Compensation (PDC) structure as suggested in [13] and the feedback gains are synthesised, subject to certain design criteria posed as Linear Matrix Inequalities (LMIs). Subsequently in the second step, now including dependencies among the UAVs, a distributed control law is introduced and it is shown that stability is guaranteed for the entire swarm.

The benefit of the proposed approach is that the analysis and design is performed at node level, thus the problem of stabilisation/tracking is decoupled from the network's scale, topology, and complexity. Also the methodology can be applied to a reasonably large class of nonlinear systems.

The remainder of the paper is structured as follows: in Section II the graph theory tools which are used, and their relevance to a network of systems is presented. In Section II-B the Takagi-Sugeno model is described for a general network of nonlinear multi-agent systems. Thereafter in Section IV the architecture of the controller and the LMI conditions to stabilise the system at node and network level are described. A swarm based UAV tracking example is included in Section V-A demonstrating the proposed analysis. In Section VI concluding remarks are stated.

II. PRELIMINARIES

A. Graph theory

In this section the graph theory preliminaries and their relevance with respect to multi-agent systems are stated. Adopting the notation in [14], a graph G is an ordered pair (V, E), where V is the set of vertices or nodes ($V = \{1, \dots N\}$) and E is the set of edges, ($E = \{c_1, \dots, c_l\}$), which represent every possible connection created between a pair of nodes. In this paper a node coincides with a UAV within the group, and the set E denotes the communication links between UAVs i and j. A graph G can be represented in the form of the adjacency matrix $\mathbf{A}(G) = [\alpha_{ij}] \in \mathbb{R}^{N\times N}$ and is defined by:

$$\alpha_{ij} = \begin{cases} 1, & \forall (i,j) \in E \; and \; i \neq j \\ 0, & otherwise \end{cases} \tag{1}$$

The degree $\mathbf{D}(G) = [d_{ij}] \in \mathbb{R}^{N\times N}$ of a graph is a diagonal matrix for which $d_{ii} = \sum_{i=1}^{N} \alpha_{ij}$ and $d_{ij} = 0,\ \forall i \neq j$. The Laplacian of a graph $\mathbf{L}(G) = [\ell_{ij}] \in \mathbb{R}^{N\times N}$ is equal to:

$$\mathbf{L}(G) = \mathbf{D}(G) - \mathbf{A}(G) = [\ell_{ij}] = \begin{cases} \sum_{j=1}^{N} \alpha_{ij}, & i = j \\ -\alpha_{ij}, & i \neq j \end{cases} \tag{2}$$

According to [14], for undirected graphs (i.e. $\alpha_{ij} = \alpha_{ji}$) the Laplacian matrix is symmetric, positive semi-definite and satisfies $\sum_{j=1}^{N} \ell_{ij} = 0,\ \forall i \in V$.

B. Model description and the Takagi-Sugeno model

Consider a group of systems $i = 1, \dots, N$ described by:

$$\dot{e}^i(t) = f_i(e^i(t)) + g_i(e^i(t))u^i(t) \tag{3}$$

where $e^i(t) \in \mathbb{R}^n$, and $u^i(t) \in \mathbb{R}^m$ is the state, and input vector, respectively. Assume $f_i(e^i(t))$ and $g_i(e^i(t))$ are functions that are dependent on the state. The nonlinear model in (3) can be represented in a compact region of the state-space $\mathcal{X} \subseteq \mathbb{R}^n$ by a TS fuzzy model.

Adopting the notation in reference [10], for agent i, the TS fuzzy model is formed by κ local linear subsystems. The TS is represented by implications of $\mathbf{IF-THEN}$ form or Input-Output form. The general layout for the κ^{th} model rule is:

<u>**Model Rule**</u> κ [10]:
IF $z_1^i(t)$ is $M_{\kappa 1}$ **AND**...**AND** $z_q^i(t)$ is $M_{\kappa q}$ **THEN**

$$\dot{e}^i(t) = \mathbf{A}_\kappa e^i(t) + \mathbf{B}_\kappa u^i(t) \tag{4}$$

where $e^i(t) = col([e_1^i(t), \dots, e_n^i(t)]) \in \mathbb{R}^n$, and $\mathbf{A}_\kappa \in \mathbb{R}^{n\times n}$, $\mathbf{B}_\kappa \in \mathbb{R}^{n\times m}$ are constant matrices. The vector $z^i(t) = col([z_1^i(t), \dots, z_q^i(t)])$ is a known premise variable which may depend on the state vector. Every premise variable is a-priori bounded on a compact space (i.e. $z^i(t) \in [z^i_{min}, z^i_{max}]$) since the state is assumed to belong to $\mathcal{X}$. The symbol $M_{\kappa\mu}(z_\mu^i(t)) \in [0, 1]$ denotes the fuzzy sets and $r = 2^{|z|}$ the number of rules. The notation $|z|$ coincides with the length of the vector. The fuzzy sets $M_{\kappa\mu}(z_\mu^i(t))$ are generated utilising the sector nonlinearity approach [15].

In Input-output form, the defuzzification process of system (4) can be represented by the following polytopic form:

$$\dot{e}^i(t) = \sum_{\kappa=1}^{r} \lambda_\kappa(z^i(t))[\mathbf{A}_\kappa e^i(t) + \mathbf{B}_\kappa u^i(t)] \tag{5}$$

where the $\lambda_\kappa(z^i(t))$ are normalised weighting functions defined by:

$$\begin{aligned} \lambda_\kappa(z^i(t)) &= w_\kappa(z^i(t)) / \sum_{\kappa=1}^{r} w_\kappa(z^i(t)) \\ w_\kappa(z^i(t)) &= \prod_{\mu=1}^{q} M_{\kappa\mu}(z_\mu^i(t)) \end{aligned} \tag{6}$$

The weighting terms $\lambda_\kappa(z^i(t))$ satisfy the convex sum property for all t. Provided that bounds on the state space are a-priori known, the TS model (5) is an exact representation of the nonlinear model (3) inside $\mathcal{X}$. Motivated by work in reference [8] it will be shown in the sequel that such a structure can be utilised in the UAV context.

III. UNMANNED AERIAL VEHICLE MODELLING - ERROR POSTURE MODEL

According to [16], under certain assumptions (i.e. an electrically powered UAV flying at constant altitude and ground speed, the thrust and velocity vector are collinear, and no slip in lateral direction), the motion of the i^{th} point-mass UAV can be described by:

$$\begin{aligned} \dot{x}_c^i(t) &= v_{er}^i(t) \cos\theta_c^i(t) \\ \dot{y}_c^i(t) &= v_{er}^i(t) \sin\theta_c^i(t) \\ \dot{\theta}_c^i(t) &= w_{er}^i(t) \end{aligned} \tag{7}$$

where x_c^i, y_c^i, are the position coordinates, θ_c^i is the heading angle, and v_{er}^i, w_{er}^i the linear and angular velocity.

For the purpose of tracking, the error posture is utilised in this paper for every agent in the network, as in references [11] and [12]. In particular, the error posture model of a vehicle is generated with the aid of the reference $P_{ref}(x_{ref}, y_{ref}, \theta_{ref})$ and the current posture $P_c^i(x_c^i, y_c^i, \theta_c^i)$ utilising the kinematics in (7). The tracking error is governed by:

$$e^i(t) = \begin{bmatrix} cos(\theta_c^i(t)) & sin(\theta_c^i(t)) & 0 \\ -sin(\theta_c^i(t)) & cos(\theta_c^i(t)) & 0 \\ 0 & 0 & 1 \end{bmatrix} (P_{ref}(t) - P_c^i(t)) \tag{8}$$

where $e^i(t) = [x_e^i(t), y_e^i(t), \theta_e^i(t)]$ is the tracking error in the state for the i^{th} UAV in the $x - y$ plane and direction, respectively. Following the description in Section 3.1 of reference [12], taking the time derivative of (8), the error dynamics are generated. Hence assuming that there is no side-slip (i.e. $\dot{x}_{ref} sin(\theta_{ref}) = \dot{y}_{ref} cos(\theta_{ref})$), and applying a control action vector $u_{er}^i(t) = u_F^i(t) + u^i(t)$ (proposed in [11]), where $u_F^i(t) = [v_{ref}(t)cos(\theta_e^i(t)), w_{ref}(t)]^T$ and $u^i(t) = [v^i(t), w^i(t)]^T$, then the error dynamics satisfy:

$$\begin{bmatrix} \dot{x}_e^i(t) \\ \dot{y}_e^i(t) \\ \dot{\theta}_e^i(t) \end{bmatrix} = \begin{bmatrix} 0 & w_{ref}(t) & 0 \\ -w_{ref}(t) & 0 & v_{ref}(t)sinc(\theta_e^i(t)) \\ 0 & 0 & 0 \end{bmatrix} \times \begin{bmatrix} x_e^i(t) \\ y_e^i(t) \\ \theta_e^i(t) \end{bmatrix} + \begin{bmatrix} -1 & y_e^i(t) \\ 0 & -x_e^i(t) \\ 0 & -1 \end{bmatrix} u^i(t) \tag{9}$$

where $v_{ref}(t)$, $v^i(t)$ are the reference and current linear velocities, $w_{ref}(t)$, $w^i(t)$ the reference and current angular velocities. The actual control law $u_{er}^i(t)$ applied in (7) consists of feed-forward ($u_F^i(t)$) and feedback ($u^i(t)$) elements.

The structure of the error posture dynamics in (9) allow its representation as a Takagi-Sugeno fuzzy model [12]. Motivated by the work in [8] the two step procedure is adopted for a network of nonlinear error posture models in (9) which are structured into the TS form in (5). For the system, $u^i(t)$ is the control action vector to be calculated which is described in the next section and is designed based on TS concepts. The control law has the form referred to in the literature as PDC [13].

IV. Swarm tracking and control law description

In this section the design of the control law for the stabilisation of the error dynamics in (5) is described. The task is for the error state $e^i(t)$ for $i = 1, \dots, N$ to converge to zero asymptotically at a local level. In this work the assumption is that individual systems have common $\mathbf{A}_\kappa, \mathbf{B}_\kappa, \forall\ \kappa = 1, \dots, r$, and the communication topology is bidirectional static. As in [8], the control design for the stabilisation problem is treated in two steps.

A. Step 1 - Node level tracking

The controller $u_\tau^i(e^i(t))$, used to stabilise the error dynamics for the i^{th} UAV system at node level, is designed from the rules of the TS fuzzy model and maintains the same structure as the model rules. The κ^{th} control rule at node level has the following structure:

Control Rule κ:
IF $z_1^i(t)$ is $M_{\kappa 1}$ **AND** … **AND** $z_q^i(t)$ is $M_{\kappa q}$ **THEN** $u_\tau^i(e^i(t)) = -\mathbf{F}_\kappa e^i(t), \forall i, j = 1, \dots, N$
for $\kappa = 1, \dots, r$ and where $q = |z|$. In polytopic form the node level state feedback control law is equal to:

$$u_\tau^i(e^i(t)) = -\sum_{\kappa=1}^{r} \lambda_\kappa(z^i(t))\mathbf{F}_\kappa e^i(t) \tag{10}$$

where $\mathbf{F}_\kappa \in \mathbb{R}^{m\times n}$ are the feedback gains. By substitution of the control law (10) into (5), the node level closed-loop error dynamics are equal to:

$$\dot{e}^i(t) = \sum_{\kappa=1}^{r}\sum_{\mu=1}^{r} \lambda_\kappa(z^i(t))\lambda_\mu(z^i(t))\mathbb{A}_{\kappa\mu} e^i(t) \tag{11}$$

where $\mathbb{A}_{\kappa\mu} = \mathbf{A}_\kappa - \mathbf{B}_\kappa \mathbf{F}_\mu$. According to reference [10] it is not restrictive to expand equation (11) into:

$$\dot{e}^i(t) = \sum_{\kappa=1}^{r} \lambda_\kappa(z^i(t))^2 \mathbb{A}_{\kappa\kappa} e^i(t) \dots + 2\sum_{\kappa=1}^{r}\sum_{\kappa<\mu}^{r} \lambda_\kappa(z^i(t))\lambda_\mu(z^i(t))(\frac{\mathbb{A}_{\kappa\mu} + \mathbb{A}_{\mu\kappa}}{2})e^i(t) \tag{12}$$

The rationale behind the expansion in (12) is the use of more relaxed conditions, as argued in [10].

For the stabilisation of the node level error dynamics Lyapunov theory is utilised. The task is to determine the feedback gains $\mathbf{F}_\mu$, and a symmetric positive definite matrix $\mathbf{P} \in \mathbb{R}^{n\times n}$, such that a local performance criteria for stability (in a locally optimal sense) is satisfied. The node level fuzzy controller is designed by minimising the upper bound of an a-priori known quadratic performance function. The performance index for agent i is equal to:

$$J^i = \int_0^\infty \big(e^i(t)^T \mathbf{Q} e^i(t) + u^i(t)^T \mathbf{R} u^i(t)\big)dt \tag{13}$$

where $\mathbf{Q} = \mathbf{Q}^T > 0$ and $\mathbf{R} = \mathbf{R}^T > 0$ are given weighting matrices of appropriate dimensions. If the closed-loop system satisfies

$$\dot{\upsilon}_i(t) < -e^i(t)^T \mathbf{Q} e^i(t) - u^i(t)^T \mathbf{R} u^i(t) \tag{14}$$

where $\dot{\upsilon}_i(t)$ is the derivative of the positive definite Lyapunov function $\upsilon^i(t) = e^i(t)^T \mathbf{P} e^i(t),\ \forall i = 1, \dots, N$, then integrating (14) over the interval $[0, \infty]$ results in the inequality $J^i < \upsilon(e^i(0)) < \eta$ [10]. This denotes an upper bound on J^i with respect to the initial conditions of the state. Thus, in this paper, $\upsilon(e^i(0))$ is minimised instead of J^i. According to [10], the stabilisation of the error dynamics for the system (5) is guaranteed via the PDC control law in (10) if there exists a common Lyapunov positive definite matrix $\mathbf{P} > 0$ satisfying:

$$\begin{cases} \mathbf{P} > 0 \\ \begin{bmatrix} \mathbf{\Pi}_\kappa^1 & \mathbf{C}_\kappa^T & -\mathbf{F}_\kappa^T \\ \mathbf{C}_\kappa & -\mathbf{Q}^{-1} & 0 \\ -\mathbf{F}_\kappa & 0 & -\mathbf{R}^{-1} \end{bmatrix} < 0 \\ \begin{bmatrix} \mathbf{\Pi}_{\kappa\mu}^2 & \mathbf{C}_\kappa^T & -\mathbf{F}_\mu^T & \mathbf{C}_\mu^T & -\mathbf{F}_\kappa^T \\ \mathbf{C}_\kappa & -\mathbf{Q}^{-1} & 0 & 0 & 0 \\ -\mathbf{F}_\mu & 0 & -\mathbf{R}^{-1} & 0 & 0 \\ \mathbf{C}_\mu & 0 & 0 & -\mathbf{Q}^{-1} & 0 \\ -\mathbf{F}_\kappa & 0 & 0 & 0 & -\mathbf{R}^{-1} \end{bmatrix} < 0 \\ \kappa < \mu\ s.t\ \lambda_\kappa(z^i(t)) \cap \lambda_\mu(z^i(t)) \neq \varnothing \end{cases} \tag{15}$$

where $\kappa, \mu = 1, \dots, r$, and

$$\mathbf{\Pi}_\kappa^1 = \mathbb{A}_{\kappa\kappa}^T \mathbf{P} + \mathbf{P}\mathbb{A}_{\kappa\kappa}$$

and

$$\mathbf{\Pi}_{\kappa\mu}^2 = (\mathbb{A}_{\kappa\mu} + \mathbb{A}_{\mu\kappa})^T \mathbf{P} + \mathbf{P}(\mathbb{A}_{\kappa\mu} + \mathbb{A}_{\mu\kappa})$$

The notation $\lambda_\kappa(z^i(t)) \cap \lambda_\mu(z^i(t)) \neq \varnothing$ implies that the conditions hold for $\kappa < \mu$ except if $\lambda_\kappa(z^i(t)) \times \lambda_\mu(z^i(t)) = 0$

for all $z(t)$. The conditions are valid provided that two rules are active simultaneously. Conditions in (15) are transformed into LMIs by pre/post multiplying with the block diagonal matrix $[\mathbf{X}\ I\ I]$ where $\mathbf{X} = \mathbf{P}^{-1}$, and substituting with $\mathbf{\Xi}_\mu = \mathbf{F}_\mu \mathbf{X}$. Hence combining the minimisation of the upper bound of $\upsilon(e^i(0))$ and the resulting LMIs from the transformation in (15), the stabilisation of the error dynamics for the system in (5) is guaranteed via the PDC control law in (10), and an upper bound on the performance index can be determined if there exists a symmetric positive definite matrix, $\mathbf{X} > 0$ ($\mathbf{X} \in \mathbb{R}^{n\times n}$) and matrices $\mathbf{\Xi}_\mu \in \mathbb{R}^{m\times n}$ for $\kappa, \mu = 1, \ldots, r$ such that the minimisation problem

$$\min_{\mathbf{X},\mathbf{\Xi}_1,\ldots,\mathbf{\Xi}_r} \eta \tag{16}$$

subject to the transformed conditions from (15) and constraints

$$\begin{cases} \mathbf{X} > 0 \\ \begin{bmatrix} \eta & e^i(0)^T \\ e^i(0) & \mathbf{X} \end{bmatrix} > 0 \end{cases} \tag{17}$$

is solved for $\kappa, \mu = 1, \ldots, r$. Provided that the LMIs are feasible, then a solution can be recovered from:

$$\mathbf{F}_\mu = \mathbf{\Xi}_\mu \mathbf{X}^{-1} \tag{18}$$

Provided the feedback gains $\mathbf{F}_\mu$ are chosen for a common Lyapunov matrix $\mathbf{P}$ from the solution of the minimisation problem (16) subject to the transformed conditions from (15) and (17), then (14) is satisfied. Thus stability can be guaranteed for any set of initial conditions $e^i(0) \in \mathcal{X}$. Once the node level stabilisation is completed, a second step is undertaken at a network level, as discussed in the sequel.

B. Step 2 - Tracking at network level

At a network level an additional term which represents the relative state information among neighboring UAVs and the reference trajectory is introduced in the control law so that:

$$u(e^i(t)) = -\sum_{\kappa=1}^{r} \lambda_\kappa(z^i(t))\mathbf{F}_\mu e^i(t) + \gamma \overline{\mathbf{F}} \sum_{j,\ i\neq j}^{N} \ell_{ij} e^j(t) \tag{19}$$

where $\overline{\mathbf{F}} \in \mathbb{R}^{m\times n}$ and γ a positive scalar. Using the control law in (19), at a network level the error-dynamics are equal to:

$$\dot{e}^i(t) = \sum_{\kappa=1}^{r}\sum_{\mu=1}^{r} \lambda_\kappa(z^i(t))\lambda_\mu(z^i(t)) \left(\mathbb{A}_{\kappa\mu} e^i(t) + \gamma \mathbf{B}_\kappa \overline{\mathbf{F}} \sum_{j=1}^{N} \ell_{ij} e^j(t) \right) \tag{20}$$

In a compact form the expression above can be conveniently written using the Kronecker product notation [17] as:

$$\dot{\mathrm{e}}(t) = [\mathcal{A}(z(t)) + \gamma \mathcal{B}(z(t))(\mathbf{L} \otimes I_n)]\mathrm{e}(t) \tag{21}$$

where

$$\mathcal{A}(z(t)) = \mathrm{diag}\left\{ \sum_{\kappa=1}^{r}\sum_{\mu=1}^{r} \lambda_\kappa^1 \lambda_\mu^1 \mathbb{A}_{\kappa\mu}, \ldots, \sum_{\kappa=1}^{r}\sum_{\mu=1}^{r} \lambda_\kappa^N \lambda_\mu^N \mathbb{A}_{\kappa\mu} \right\} \tag{22}$$

and

$$\mathcal{B}(z(t)) = \mathrm{diag}\left\{ \sum_{\kappa=1}^{r} \lambda_\kappa^1 \mathbf{B}_\kappa \overline{\mathbf{F}}, \ldots, \sum_{\kappa=1}^{r} \lambda_\kappa^N \mathbf{B}_\kappa \overline{\mathbf{F}} \right\} \tag{23}$$

and $\mathrm{e}(t)$ is the concatenation of the state vector $e^i(t)$ so that $\mathrm{e}(t) = col([e^1(t), \ldots, e^N(t)])$.

Define a candidate Lyapunov function for the swarm as

$$V(t) = \sum_{i=1}^{N} e^i(t)^T \mathbf{P} e^i(t) \tag{24}$$

where the symmetric positive definite matrix $\mathbf{P}$ is from the earlier node level synthesis in Subsection IV-A. Taking the time derivative of (24), and substituting for (20) yields

$$\dot{V}(t) = V_1 + V_2 \tag{25}$$

where

$$V_1 = \sum_{i=1}^{N}\sum_{\kappa=1}^{r}\sum_{\mu=1}^{r} \lambda_\kappa(z^i(t))\lambda_\mu(z^i(t)) e^i(t)^T \left[\mathbb{A}_{\kappa\mu}^T \mathbf{P} + \mathbf{P}\mathbb{A}_{\kappa\mu}\right] e^i(t) \tag{26}$$

and

$$V_2 = 2\gamma \mathrm{e}^T(t)(I_N \otimes \mathbf{P})\mathcal{B}(z(t))(\mathbf{L} \otimes I_n)\mathrm{e}(t) \tag{27}$$

For the swarm of UAVs to track the virtual leader system, which is moving according to a prescribed reference trajectory, it is sufficient to show that $\dot{V}(t) < 0$. Utilising the stabilisation procedure from the first step of the design process in subsection IV-A, for the choice of a common Lyapunov matrix $\mathbf{P}$ and feedback gains $\mathbf{F}_\mu$, $V_1 < 0$. Hence all that needs to be shown is that V_2 is negative semi definite for all $\mathrm{e}(t) \neq 0$. It is evident from the TS model that the input matrix $\mathbf{B}_\kappa$ is time varying because of (9); however the first column is constant: i.e. $\mathbf{B}_\kappa = [\mathbf{B}_1, \mathbf{B}_{2\kappa}]$. Here by choice

$$\overline{\mathbf{F}} = -[\mathbf{B}_1, 0]^T \mathbf{P} \tag{28}$$

which means that $\mathcal{B}(z(t)) = I \otimes \mathbf{B}_1 \mathbf{B}_1^T \mathbf{P}$. As a result of this choice in (27)

$$V_2 = -2\gamma \mathrm{e}^T(t)(\mathbf{L} \otimes \mathbf{P}\mathbf{B}_1\mathbf{B}_1^T\mathbf{P})\mathrm{e}(t) \tag{29}$$

The physical interpretation of this is the fact that less communication load is required among the UAVs. Since the representation of the Laplacian $\mathbf{L}$ is positive semi definite, and by construction $\mathbf{P}\mathbf{B}_1\mathbf{B}_1^T\mathbf{P} \geq 0$, it follows $-(\mathbf{L} \otimes \mathbf{P}\mathbf{B}_1\mathbf{B}_1^T\mathbf{P}) \leq 0$ by Corollary 4.2.13 [17]. Thus (25) is negative definite for all $\mathrm{e}(t) \neq 0$ and the error dynamics of the swarm is stable.

V. Simulation Example

In this section a tracking scenario is considered where a swarm of UAVs is deployed to collectively follow the prescribed trajectory of a virtual leader from any initial conditions satisfying bounds on the state space. The path is assumed to be a-priori known. The reference track considered for the virtual leader in this example is referred to in the literature as the Dubins path [18].

A. Description of the Tagaki Sugeno UAV model

For the purpose of illustration consider a swarm of identical UAV models. The TS fuzzy model has been derived as described in section II-B. For the model illustrated in (9) $z_1^i(t) = w_{ref}(t)$, $z_2^i(t) = v_{ref}(t)sinc(\theta_e^i(t))$, $z_3^i(t) = y_e^i(t)$ and $z_4^i(t) = x_e^i(t)$ are chosen as the premise variables with $z_1^i(t) \in [-0.513, 0.513]$, $z_2^i(t) \in [18.0048, 20]$, $z_3^i(t) \in [-10, 10]$ and $z_4^i(t) \in [-10, 10]$. In addition $\theta_e^i(t) \in [-\pi/4, \pi/4]$. Hence the number of rules of the fuzzy system is equal to $r = 16$ and the length of the premise vector is equal to $q = 4$. It should be noted that the latter bounds are not chosen in an arbitrary manner, and are selected in order not to lose controllability of the system. Utilising the sector nonlinearity approach in [15] the membership functions $M_{\kappa\mu}(z^i(t))$ are determined and the weighting terms are calculated according to (6). All sixteen rules are developed as prescribed in Subsection II-B equation (4), where $\mathbf{A}_\kappa$ and $\mathbf{B}_\kappa$ are shown in the sequel. Finally, the defuzzification is carried out with respect to equation (5). Hence the equivalent TS fuzzy model (5) for the full nonlinear is derived. For model (5):

$$\mathbf{A}_\kappa = \begin{bmatrix} 0 & -\varepsilon_\kappa^1 w_{r,max} & 0 \\ \varepsilon_\kappa^1 w_{r,max} & 0 & \overline{\mu}_\kappa^i \\ 0 & 0 & 0 \end{bmatrix} \mathbf{B}_\kappa = \begin{bmatrix} -1 & \varepsilon_\kappa^3 e_{max}^i \\ 0 & \varepsilon_\kappa^4 e_{max}^i \\ 0 & -1 \end{bmatrix}$$

where $w_{r,max} = 0.513$ $[rad/sec]$, $e_{max}^i = 10$ $[m]$ and

$$\varepsilon_\kappa^1 = \begin{cases} -1, & for\ 1 \leq \kappa \leq 8 \\ +1, & otherwise \end{cases} \quad \varepsilon_\kappa^4 = (-1)^{\kappa+1}$$

$$\varepsilon_\kappa^3 = \begin{cases} +1, & otherwise \\ -1, & for\ \kappa \in \{1, 2, 5, 6, 9, 10, 13, 14\} \end{cases}$$

$$\overline{\mu}_\kappa^i = \begin{cases} 18.0048, & for\ 1 \leq \kappa \leq 4\ and\ 9 \leq \kappa \leq 12 \\ 20, & otherwise \end{cases}$$

B. Tracking a Virtual Leader

In this example 20 UAVs are interconnected through control law (19). The contribution factor for the global information is chosen as $\gamma = 0.7$. The task is given a reference trajectory, from the preflight planning, for the entire swarm to track the virtual leader. The Laplacian matrix has the form in (2). The graph considered here is $G(20, 184)$ and the task is for the error state $e^i(t) \to 0$ as $t \to \infty$.

Following the procedure introduced in Section IV, firstly the LMIs are synthesised at node level for the closed loop error posture model (11). This leads to the choice of the feedback gains $\mathbf{F}_\mu$ and a common positive definite matrix $\mathbf{P}$ by minimising (16) subject to the LMI conditions in (17), and the transformed conditions from (15). The gains $\mathbf{F}_\mu$ are shown in (30) and the positive definite matrix returned is:

$$\mathbf{P} = \begin{bmatrix} 0.1084 & -0.0099 & -0.2314 \\ -0.0099 & 0.0968 & 1.6264 \\ -0.2314 & 1.6264 & 56.1980 \end{bmatrix}$$

From the minimisation problem of (16) subject to LMIs (17), and the transformed conditions from (15) $\eta = 74.4979$. In (15), $\mathbf{Q} = 10^{-2} \times \text{diag}\{1, 1, 4\}$ and $\mathbf{R} = 10^{-2} \times I_{2\times2}$. Altering the elements in the $\mathbf{Q}$ and $\mathbf{R}$ matrices results in different responses of the system. This gives the designer the possibility of obtaining another control performance according to design specifications.

From Figure 3 the bounds on the state space $e^i(t)$ are not violated and thus the TS model represents exactly the nonlinear model of the error dynamics of the UAV. Utilising the stabilisation procedure at node level, $\overline{\mathbf{F}}$ is chosen as (28) at the second step according to Subsection IV-B. Hence the overall control law (19) is synthesised and is added to the feed-forward control action vector $u_F^i(t) = [v_{ref}(t)cos(\theta_e^i(t)), w_{ref}(t)]^T$ to generate $u_{er}^i(t)$. The control input $u_{er}^i(t)$ consists of the angular $w_{er}^i(t)$ and the linear $v_{er}^i(t)$ velocities which are fed to the i^{th} UAV model (7). Thereafter the measured state of the vehicle is used to calculate the tracking error as in (8). The initial conditions for each UAV were chosen in a random manner (whilst satisfying the a-priori assumed bounds on the state space).

The swarm trajectories which are converging to the virtual leader reference track are depicted in Figure 1. Figure 2 shows the heading angle of each UAV versus the virtual leader's. The states of the tracking error ($e^i(t) = [x_e^i(t), y_e^i(t), \theta_e^i(t)]^T$) are given in Figure 3. The firing of the weighting functions $\lambda_\kappa(z^i(t))$ are depicted in Figure 4. The control action vector $u_{er}^i(t)$ is given in Figure 5.

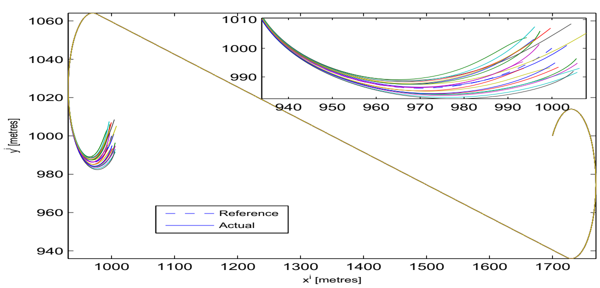

Fig. 1. Trajectories of the swarm in $x - y$ plane (solid lines) versus the virtual leader's trajectory (dashed line).

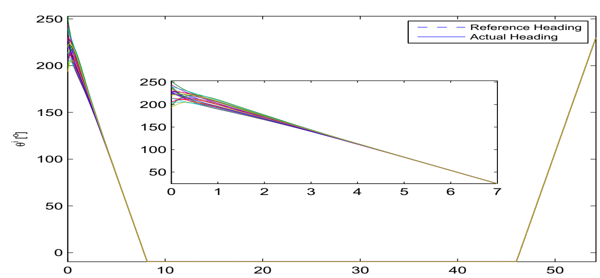

Fig. 2. Heading angle profile for each UAV (solid lines) and heading angle of the virtual leader (dashed line).

The benefit of the proposed analysis is that the design of the controller is decoupled from the size of the network and its topology. This is due to the fact that there are only r LMIs that are utilised to stabilise each node locally. Additionally, due to the decoupled structure of the network it allows a convenient choice for gain $\overline{\mathbf{F}}$. The advantage is that through the special choice of feedback gains (28) for the relative state information, the methodology can be applied to a large class of nonlinear large-scale network of systems.

$$\mathbf{F}_1 = \begin{bmatrix} -0.914 & 0.202 & 5.181 \\ 0.006 & -0.079 & -2.986 \end{bmatrix} \mathbf{F}_2 = \begin{bmatrix} -0.925 & 0.193 & 4.92 \\ 0.019 & -0.143 & -5.071 \end{bmatrix} \mathbf{F}_3 = \begin{bmatrix} -0.885 & -0.054 & -3.924 \\ 0.025 & -0.097 & -3.545 \end{bmatrix} \mathbf{F}_4 = \begin{bmatrix} -0.879 & -0.046 & -3.826 \\ 0.026 & -0.113 & -3.844 \end{bmatrix}$$
$$\mathbf{F}_5 = \begin{bmatrix} -0.917 & 0.204 & 5.257 \\ 0.006 & -0.085 & -3.208 \end{bmatrix} \mathbf{F}_6 = \begin{bmatrix} -0.926 & 0.192 & 4.938 \\ 0.019 & -0.147 & -5.232 \end{bmatrix} \mathbf{F}_7 = \begin{bmatrix} -0.883 & -0.057 & -4.035 \\ 0.025 & -0.096 & -3.49 \end{bmatrix} \mathbf{F}_8 = \begin{bmatrix} -0.881 & -0.048 & -3.864 \\ 0.027 & -0.12 & -4.119 \end{bmatrix}$$
$$\mathbf{F}_9 = \begin{bmatrix} -0.945 & 0.197 & 7.534 \\ 0.006 & -0.099 & -3.517 \end{bmatrix} \mathbf{F}_{10} = \begin{bmatrix} -0.924 & 0.193 & 7.48 \\ 0.003 & -0.082 & -2.76 \end{bmatrix} \mathbf{F}_{11} = \begin{bmatrix} -0.893 & -0.05 & -1.232 \\ 0.014 & -0.068 & -2.51 \end{bmatrix} \mathbf{F}_{12} = \begin{bmatrix} -0.91 & -0.04 & -0.967 \\ 0.016 & -0.12 & -4.27 \end{bmatrix} \quad (30)$$
$$\mathbf{F}_{13} = \begin{bmatrix} -0.946 & 0.20 & 7.61 \\ 0.005 & -0.1 & -3.557 \end{bmatrix} \mathbf{F}_{14} = \begin{bmatrix} -0.926 & 0.193 & 7.51 \\ 0.004 & -0.09 & -3.069 \end{bmatrix} \mathbf{F}_{15} = \begin{bmatrix} -0.893 & -0.052 & -1.327 \\ 0.015 & -0.072 & -2.63 \end{bmatrix} \mathbf{F}_{16} = \begin{bmatrix} -0.91 & -0.04 & -1.025 \\ 0.017 & -0.124 & -4.42 \end{bmatrix}$$

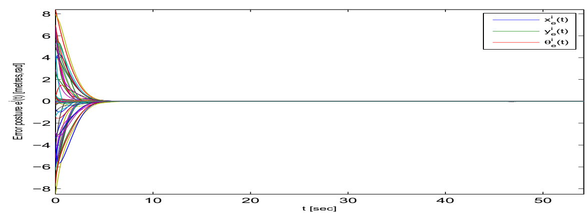

Fig. 3. Tracking error $e^i(t) = [x_e^i(t), y_e^i(t), \theta_e^i(t)]^T$ of every UAV within the swarm.

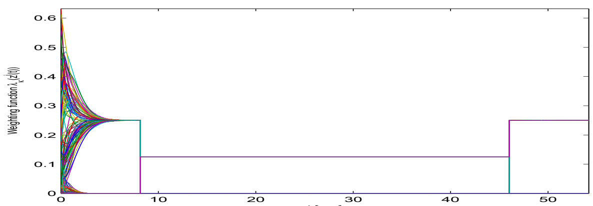

Fig. 4. Weighting functions $\lambda_\kappa(z_i(t))$.

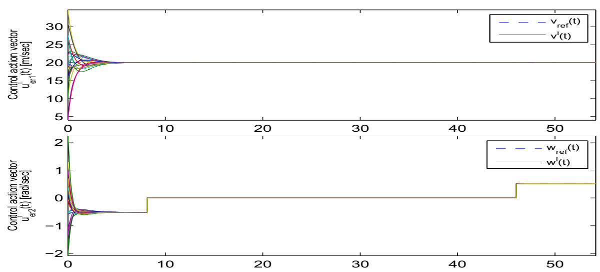

Fig. 5. Control action vector $u_{er}^i(t) = [v_{er}^i(t), w_{er}^i(t)]$.

VI. Conclusions

This work proposes a systematic analysis for tracking problems in swarm-based UAV missions with linear and angular velocity constraints. The communication topology among the UAVs is represented using graph theory tools. The intermediate step of representing a network of nonlinear systems with TS models circumvents the difficulty in designing a control law when dependencies among the UAVs are considered. This is possible due to the structure of the TS representation as it allows a decoupling of the network into node level dynamics, which simplifies the stability analysis and allows established tools from linear control theory to be applied. A special choice of feedback gains for the relative state information allows the methodology to be applied to a reasonably large class of nonlinear systems. The distributed control law which is proposed, is composed of both node and network level information. The two step design procedure is performed subject to criteria, posed as Linear Matrix Inequalities (LMIs). The methodology proposed shows that the design of the controller is decoupled from the size and topology of the network, and it allows a convenient choice of feedback gains for the network level dynamics. An illustrative example, where a swarm of UAVs is deployed to follow the track of a virtual leader, was included to demonstrate the potential of the analysis.

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