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Approximate Optimal Control of Internal Combustion Engine Test Benches

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***Abstract*— This paper proposes the design of an optimal simultaneous control of speed and torque for dynamic internal combustion engine test bench by means of a *dynamic extension* to the state of the system which allows to avoid the explicit solution of the Hamilton-Jacobi-Bellman partial differential equation. To this end, a dynamic control law is firstly designed based on a simplified and approximate model of the test bench and then modified in order to cope with non-modeled dynamics and nonlinearities. The control is designed on this basis and its performance of the control law is tested and validated on an accurate simulator of the internal combustion engine test bench.**

I. Introduction

The purpose of a dynamic test bench is the reproduction of the load conditions an internal combustion engines would undergo in a passenger car, truck or other vehicle without the vehicle. A test bench guarantees reproducible conditions in terms of temperature and pressure, just to name a few, and reduces the costs and time required for development and configuration.

In a vehicle the velocity and the rotational speed ω_E of the engine are a result of the engine torque T_E and the load torque, this being the direct consequence of the road and vehicle conditions. In addition to a speed control loop, these load conditions have to be computed using models and enforced by a dynamometer connected to the engine crankshaft in a test bench. In industrial practice, this is usually accomplished by two separate control loops. In the case of passenger car simulation, in most cases the torque is controlled by the accelerator pedal position α_{CE}, while the rotational speed ω_E is controlled by the loading machine. For larger internal combustion engines, usually the accelerator pedal position α_{CE} is directly related to the engine speed while the torque is applied by the load machine.

Of course, the significance of the experiments on the test bench is a direct consequence of the precision of the control system. As a consequence, the subject has received attention in different ways. For instance, a digital torque controller for a turbocharged diesel engine as well as a direct current dynamometer was developed in [1] using a closed-loop pole assignment technique to test an internal combustion engine with the EPA transient cycle (see *e. g.* [2]). [3] presents the application of the model referenced adaptive control (MRAC) approach to the engine speed and torque control problem, where the Lyapunov stability theory was used to derive the parameters update law.

More recent approaches use a multi-variable control of the engine-dynamometer system. In [4] the closed loop reference tracking is maximized by balancing the bandwidths of the loop transfer functions to avoid excessive loop interactions in the closed loop. In [5] a robust inverse tracking method is applied to control an internal combustion engine test bench to achieve a high tracking performance.

Differently from the *inverse optimal control* problem [6], adopted in [7] – which consists in fixing the structure of the solution of the Hamilton-Jacobi-Bellman PDE and then computing the actual cost that is optimized by the resulting control law – in the standard optimal control problem, which is considered herein, the opposite approach is pursued, *i. e.* a reasonable cost functional is decided *a priori* and then a solution to the partial differential equation is sought [8], [9], [10]. Unfortunately, the explicit solution of the HJB PDE may be hard or even impossible to determine in practical cases, hence several methodologies to approximate the solution of the HJB partial differential equation in a neighborhood of the origin with a desired degree of accuracy have been proposed, see for instance [11], [12], [13].

A control design methodology for an internal combustion engine test bench, within the framework of multi-input multi-output controllers, is proposed in this work. A novel approach to approximate the solution of the well-known Hamilton-Jacobi-Bellman (HJB) partial differential equation, namely by means of a *dynamic extension*, is tested for this specific problem. A modified control law is then implemented on an accurate model of the internal combustion engine test bench, showing good performance in simulation.

The rest of the paper is organized as follows: Section II gives attention to internal combustion engine test benches, Section III deals with basic definitions and notation that will be used throughout the paper and the dynamic control law. The modified control and simulation results are presented in Section IV and Section V, respectively. The paper is concluded with comments on the proposed methodology and future outlook in Section VI.

II. Internal combustion engine test benches

A typical setup of a test bench is shown in Fig. 1. The engine under test is connected via a shaft with a second main power unit. This may be a purely passive brake whereas on the other hand electric machines, for instance, offer the possibility of an active and transient operation. The accelerator pedal position α_{CE} of the internal combustion

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engine and the set value $T_{D,\,set}$ of the dynamometer torque provide the inputs to the test bench.

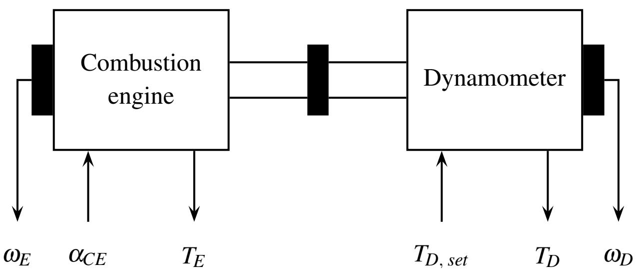

Fig. 1. Setup of an internal combustion engine test bench.

T_x denotes the measured or estimated torque, while ω_x represents the measured rotational speed, where x may be equal to E or D, denoting the internal combustion engine and the dynamometer, respectively.

The modeling of the test bench is covered in detail in [5] and [7]. Therefore only the equations necessary for the control design are summarized here. To this end the entire mechanical design – a two-mass oscillator – can be described by equations of the form

$$\begin{aligned} \Delta\dot{\varphi} &= \omega_E - \omega_D, \\ \theta_E \dot{\omega}_E &= T_E - c\Delta\varphi - d\left(\omega_E - \omega_D\right), \\ \theta_D \dot{\omega}_D &= c\Delta\varphi + d\left(\omega_E - \omega_D\right) - T_D, \end{aligned} \tag{1}$$

where $\Delta\varphi$ is the torsion of the connection shaft while θ_E and θ_D denotes the inertias of the internal combustion engine and the dynamometer, respectively. The inertia of the adapter flanges, the damping element, the shaft torque measurement device and the flywheel are already included in these values. c characterizes the stiffness of the connection shaft whereas d describes the damping.

The behavior of an internal combustion engine is in general rather complicated. However, a *simplified* torque model of a Diesel engine can be described by equations of the form (see [7] for more details)

$$\dot{T}_E = -\left(c_0 + c_1\omega_E + c_2{\omega_E}^2\right) T_E + m\left(\omega_E, T_E, \alpha\right). \tag{2}$$

$c_i > 0$, $i = 1, \ldots, 3$ are parameters to approximate the dynamic behaviour of the internal combustion engine, $m\left(\omega_E,\ T_E,\ \alpha\right)$ is a nonlinear static map including all parts of the static combustion engine torque.

The employed electric dynamometer is modeled by a second order low-pass filter, with dynamics significantly faster than those of the other components of the test bench, hence they can be neglected in the design. Within the range of maximum torque and maximum rate of change, the torque of the dynamometer can be described by

$$T_D = T_{D,\,set}. \tag{3}$$

Letting

$$v = m\left(\omega_E, T_E, \alpha\right),$$

the system (1)-(2) can be rewritten as

$$\dot{x} = Ax + \overline{f}\left(x\right) + Bu \tag{4}$$

with

$$A = \begin{pmatrix} -c_0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 \\ \frac{1}{\theta_E} & -\frac{c}{\theta_E} & -\frac{d}{\theta_E} & \frac{d}{\theta_E} \\ 0 & \frac{c}{\theta_D} & \frac{d}{\theta_D} & -\frac{d}{\theta_D} \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & \frac{1}{\theta_D} \end{pmatrix},$$

$$\overline{f}\left(x\right) = \begin{pmatrix} -\left(c_1\, x_3 + c_2\, {x_3}^2\right) T_E & 0 & 0 & 0 \end{pmatrix}^T,$$

the state $x = \begin{pmatrix} x_1 & x_2 & x_3 & x_4 \end{pmatrix}^T = \begin{pmatrix} T_E & \Delta\varphi & \omega_E & \omega_D \end{pmatrix}^T$ and the input $u = \begin{pmatrix} v & T_{D,\,set} \end{pmatrix}^T$. The actual control input, *i. e.* α_{CE}, to apply in order to generate the desired v is then obtained by an (approximative) inversion of the map $m\left(\cdot\right)$.

The main characteristics of the internal combustion engine and the electric machine are summarized in Table I. While the maximum torque is limited by the electric machine due to a thermal constraint, the maximum speed is given by the internal combustion engine.

TABLE I

CHARACTERISTICS OF MAIN POWER UNITS

Characteristic		Value	
Maximum dynamometer torque	$T_{D,\,max}$	295	Nm
Maximum rate of change	$\dot{T}_{D,\,max}$	59000	$\frac{\text{Nm}}{\text{s}}$
Maximum dynamometer speed	$\omega_{D,\,max}$	10000	rpm
Maximum engine torque	$T_{E,\,max}$	340	Nm
Maximum engine speed	$\omega_{E,\,max}$	4200	rpm

III. DYNAMIC CONTROL

Consider a nonlinear system, affine in the control, described by an equation of the form

$$\dot{x} = f\left(x\right) + g\left(x\right)u, \tag{5}$$

with $f : \mathbb{R}^n \to \mathbb{R}^n$ and $g : \mathbb{R}^n \to \mathbb{R}^{n\times m}$ smooth mappings, where $x\left(t\right) \in \mathbb{R}^n$ denotes the state of the system and $u(t) \in \mathbb{R}^m$ the input. The task of the control is to minimize the cost functional

$$J\left(x\left(t\right), u\left(t\right)\right) = \frac{1}{2}\int_0^\infty \left(q\left(x\right) + u^T u\right) \mathrm{d}t, \tag{6}$$

where $q : \mathbb{R}^n \to \mathbb{R}_+$ is positive semi-definite, subject to the dynamical constraint (5), the initial condition $x\left(t_0\right) = x_0$ and the requirement that the zero equilibrium of the closed-loop system be locally asymptotically stable. Herein we consider formally the problem of approximating the solution of the regional dynamic optimal control problem, the definition of which is given in the following statement.

Problem 1: Consider system (5) and the cost functional (6). The *regional dynamic optimal control* problem consists in determining an integer $\tilde{n} \geq 0$, a dynamic control law of the form

$$\begin{aligned} \dot{\xi} &= \alpha\left(x, \xi\right), \\ u &= \beta\left(x, \xi\right), \end{aligned} \tag{7}$$

with $\xi\left(t\right) \in \mathbb{R}^{\tilde{n}}$, $\alpha : \mathbb{R}^n \times \mathbb{R}^{\tilde{n}} \to \mathbb{R}^{\tilde{n}}$, $\beta : \mathbb{R}^n \times \mathbb{R}^{\tilde{n}} \to \mathbb{R}^m$ and a set $\bar{\Omega} \subset \mathbb{R}^n \times \mathbb{R}^{\tilde{n}}$ containing the origin of $\mathbb{R}^n \times \mathbb{R}^{\tilde{n}}$ such that the closed-loop system

$$\begin{aligned} \dot{x} &= f(x) + g(x)\beta(x,\xi), \\ \dot{\xi} &= \alpha(x,\xi), \end{aligned} \tag{8}$$

has the following properties:

(i) The zero equilibrium of the system (8) is asymptotically stable with region of attraction containing $\bar{\Omega}$.

(ii) For any $\bar{u}(t)$ and any (x_0, ξ_0) such that the trajectory of the system (8) remains in $\bar{\Omega}$

$$J((x_0,\xi_0),\beta) \le J((x_0,\xi_0),\bar{u}).$$

◇

It is well-known that if the scalar function $V : \mathbb{R}^n \to \mathbb{R}_+$ is a solution of the Hamilton-Jacobi-Bellman (HJB) partial differential equation

$$V_x f(x) - \frac{1}{2} V_x g(x) g(x)^T V_x^T + \frac{1}{2} q(x) = 0, \tag{9}$$

then the static state feedback $u_o = -g(x)^T V_x^T$ solves the regional dynamic optimal control with $\tilde{n} = 0$. Unfortunately, the explicit solution of the HJB PDE may be hard or impossible to find in specific situations. Therefore, we consider herein a different notion of the solution of (9), as detailed in the following definition.

Definition 1: Consider system (5). A $\mathscr{C}^1$ mapping $P(x) : \mathbb{R}^n \to \mathbb{R}^{1\times n}$, zero at zero, is said to be an *algebraic $\bar{P}$ solution* of (9) if there exists $\sigma(x) \triangleq x^T\ \Sigma(x)\ x > 0$, for all $x \in \mathbb{R}^n \setminus \{0\}$, with $\Sigma(x) : \mathbb{R}^n \to \mathbb{R}^{n\times n}$, $\Sigma(0) = 0$, such that

$$P(x) f(x) + \frac{1}{2} q(x) - \frac{1}{2} P(x) g(x) g(x)^T P(x)^T + \sigma(x) \le 0, \tag{10}$$

and $P(x)$ is tangent at $x = 0$ to the symmetric positive definite solution of the algebraic Riccati equation associated with the linearized problem, *i. e.*

$$\left.\frac{\partial P(x)^T}{\partial x}\right|_{x=0} = \bar{P}.$$

◇

Proposition 1: [14] Let $P(x)$ be an *algebraic $\bar{P}$ solution* with $\Sigma(0) > 0$. Then, there exist a matrix $R > 0$, a neighborhood of the origin $\Omega \subseteq \mathbb{R}^{2n}$ and $\bar{k}$ such that for all $k \ge \bar{k}$ the function

$$V(x,\xi) = P(\xi)x + \frac{1}{2}\|x-\xi\|_R^2, \tag{11}$$

is positive definite and satisfies the partial differential inequality

$$V_x f(x) + V_\xi \dot{\xi} + \frac{1}{2} q(x) - \frac{1}{2} V_x g(x) g(x)^T V_x^T \le 0, \tag{12}$$

with $\dot{\xi} = -kV_\xi^T$, for all $(x,\xi) \in \Omega$. □

As discussed in the previous section, a simplified model of the test bench can be described by equations as in (4) with $\bar{f}(x) = \begin{bmatrix} \bar{f}_1 & 0 & 0 & 0 \end{bmatrix}$. To begin with, without loss of generality, let the *algebraic $\bar{P}$ solution* be of the form $P(x) = x^T \bar{P} + Q(x)$, where $Q : \mathbb{R}^4 \to \mathbb{R}^{1\times 4}$ contains higher-order polynomials of the state variable x. In other words, the linear solution is modified, adding the term $Q(x)$, in order to dominate the nonlinear terms of the algebraic inequality (10) associated with the system (4).

In the following, exploiting the specific structure of the term $\bar{f}$ in system (4), we let $Q(x)$ be defined as $Q(x) = \begin{bmatrix} Q_1(x) & 0 & 0 & 0 \end{bmatrix}$. Let $q(x) = x^T \mathscr{Q} x$ in the cost functional (6), where the positive definite matrix $\mathscr{Q}$ weights the relative errors for the different components of the state. Then, the algebraic Hamilton-Jacobi-Bellman inequality (10) is solved with respect to the unknown $Q_1(x)$ obtaining a solution of the form

$$Q_1(x) = \frac{N(x)}{D(x)}.$$

It can be shown that the resulting solution is indeed well-defined for all the values of *interest* of the state variables, exploiting in particular physical constraints on x, namely x_1 must be positive, the minimum value for the speeds of the engine ω_E and of the dynamometer ω_D is defined by the idle speed of the internal combustion engine and must be greater than $\min\{\omega_E\} = 630\,\text{rpm} = 65.9\,\frac{\text{rad}}{\text{s}}$, *i. e.* $x_3 \ge 65.9$, $x_4 \ge 65.9$, and finally the torsion of the shaft is considerably smaller than the values of the other components, $x_2 \ll x_i$, $i = 1,3,4$.

Finally, as stated in Proposition 1, the computation of an *algebraic $\bar{P}$ solution* is enough to obtain the dynamic control law

$$\begin{aligned} \dot{\xi} &= -kV_\xi^T(e,\xi), \\ u &= -B^T V_x^T(e,\xi), \end{aligned} \tag{13}$$

where e_i denotes the tracking error for the variable x_i with respect to the corresponding reference value, namely $e_i = x_i - x_{i,set}$, $i = 1,\ldots,4$. The function V is defined as in (11) with $P(x)$ introduced above. The control law (13) approximates the solution of the regional dynamic optimal control problem for the system (4) with respect to the instantaneous cost $q(x) = x^T \mathscr{Q} x$.

IV. Modified Control

The dynamic control law (13) proposed in the previous section is designed considering a rather *simplified* model of the internal combustion engine test bench. Therefore the control action needs to be modified, as explained in the following, in order to cope with some of the nonlinearities of the internal combustion engine which are not included in the simplified model, such as saturations and hysteresis, to name just a few.

In particular, an integral action is added to the control law designed in the previous section. More specifically, letting ν_i denote the i-th component of the dynamic control law (13), $i = 1,2$, we define the actual control inputs implemented on the accurate internal combustion engine model as

$$u_i = \nu_i(e,\xi) + k_i \int \nu_i(e,\xi)\,\mathrm{d}t, \tag{14}$$

for $i = 1,2$.

Additionally we let the gain k_2 be a function of the derivative – which is implemented in the simulations as $\frac{s}{\eta s+1}$

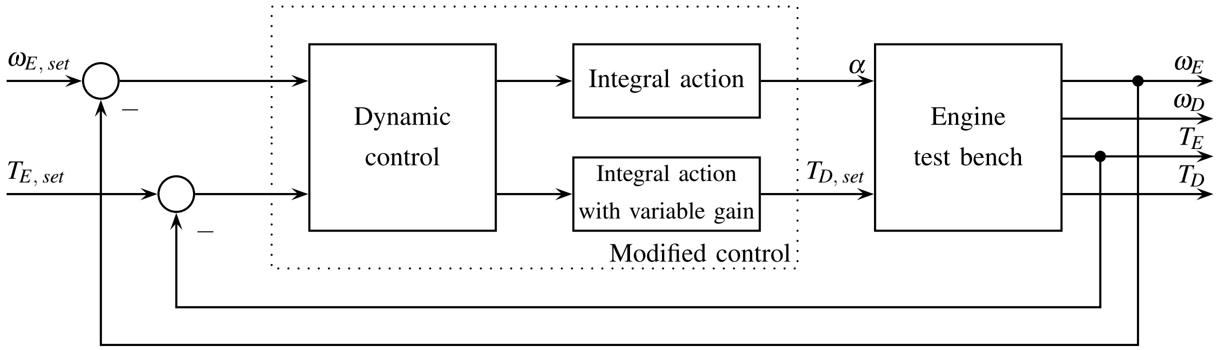

Fig. 2. Modified internal combustion engine test bench control.

with $\eta > 0$ – of the tracking error for the speed of the engine, namely $k_2(\dot{e}_3)$. In particular, the gain is defined such that, when the error on the tracking for the engine speed changes *too fast*, the integral action is negligible compared to the other components of the control signal. This choice is reasonable, as shown in simulations, and needed in order to avoid an excessively aggressive reference profile for the torque of the dynamometer.

Large errors between the desired and the measured engine speed and engine torque, due to changes in the references, are mainly, and relatively rapidly, compensated by the action provided by the dynamic control developed in the previous section. Therefore, the effect of the integrators is limited due to the short *transient responses* of the state variables when the reference value is modified. In fact, roughly speaking, this implies that the tracking errors belong to a neighborhood of the origin after a relatively short amount of time and consequently the integrals of the errors are small compared to the action of the dynamic control law.

However, the integral actions are more effective in the neighborhood of the origin mentioned above, *i. e.* close to zero tracking error, where they are actually needed in order to avoid a constant error in the *steady-state* response to the desired reference. Note additionally that, locally around the origin, the linear part of the dynamic control law is dominant with respect to higher-order terms, hence the integrals provide a *standard* integral action.

To summarize the design, the dynamic control law contains nonlinear terms that allow to obtain an extremely fast response to reference changes, whereas the linear part of the dynamic control law itself together with the integral action, with constant gains, provide accurate tracking precision close to zero tracking error.

Fig. 2 shows the closed loop composed of the internal combustion engine test bench and the modified control. The reference for engine speed $\omega_{E,\,set}$ and engine torque $T_{E,\,set}$ are given. While the measurement of engine speed ω_E, dynamometer speed ω_D and torque T_D of an electric machine is relatively easy, the measurement of the engine torque T_E is unfortunately very difficult. Shaft torque measurement devices are expensive, at test benches for development of test bench controls they are sometimes present. At most test benches a shaft torque measurement is not available and has to be estimated using an observer. A comparision between an Extended Kalman Filter (EKF), a High Gain Observer and a Sliding Mode Observer (SMO) to estimate the engine torque T_E is given in [15]. However, in simulation the engine torque T_E can be accessed directly.

V. Results

The developed controller has been tested and validated on a high quality simulator of a test bench. This simulator has already been used to design a robust inverse control (see [5]) or model based observers for torque estimation (see [15]). In addition to the dynamics of the entire mechanical description the simulator also includes a more precise, nonlinear data-based model of the internal combustion engine, the already mentioned dynamics and limitations of the dynamometer as well as disturbance effects. For example, the internal combustion engine model takes the dynamics of the accelerator pedal and combustion oscillations into account. Additive white Gaussian noise well reproducing the measurement noise known from our test bench is superimposed to the simulated rotational speeds.

The developed controller is compared with the controller developed in [7]. This controller has already a much better performance than existing standard implementations with two separate control loops.

The output of the engine – the engine torque T_E and the engine speed ω_E – is depicted in Fig. 3 and Fig. 6 for two different, rather simple test cases. In the first simulation the engine torque T_E is increased from $T_E = 100\,\mathrm{Nm}$ to $T_E = 200\,\mathrm{Nm}$, while the engine speed ω_E is kept constant. Both the engine torque T_E and the engine speed ω_E are reduced at the same time in a further simulation. The engine torque T_E is decreased from $T_E = 150\,\mathrm{Nm}$ to $T_E = 100\,\mathrm{Nm}$, the engine speed ω_E from $250\,\frac{\mathrm{rad}}{\mathrm{s}}$ to $220\,\frac{\mathrm{rad}}{\mathrm{s}}$. However, it is possible to define a number of similar engine operations reproducing real engine transients appearing in a vehicle.

The use of the controller developed herein leads to a significant reduced under- or overshooting respectively of the engine torque T_E when a change of the operation point occurs. The engine torque T_E shows a slightly reduced rise time, however the final value is reached about 4 times as

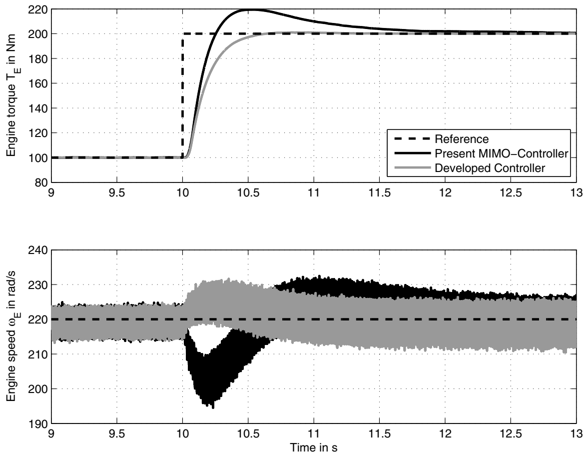

Fig. 3. Reference and simulated engine torque T_E (top) as well as engine speed ω_E (bottom) for first simulation.

fast.

Using the controller tuned in [7] the engine speed ω_E drops down by around 10 % due to the sudden increase of the engine torque T_E and the couplings. In contrast, even a small increase of engine speed ω_E results by applying the developed controller. In both cases the speed is superimposed by an additive disturbance caused by the resolution of the shaft encoders and combustion oscillations.

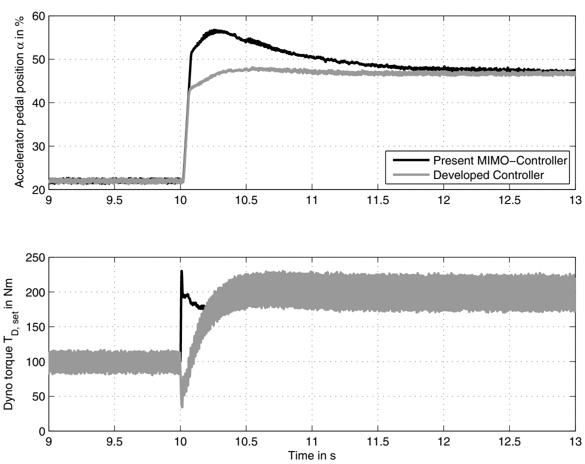

Fig. 4. Accelerator pedal position $\alpha_C E$ (top) and dynamometer torque set value $T_{D, set}$ (bottom) for first simulation.

Not only Fig. 3 shows the differences between the compared controllers. Differences are also visible in the input signals of the test bench (see Fig. 4). While both controllers increase the accelerator pedal position α_{CE} quickly at $t =$ 10 s, the developed controller avoids an overshoot of the accelerator pedal position α_{CE}. Furthermore, such overshoots can increase the polluting emissions, see *e. g.* [16].

The lower part of Fig. 4 shows the dynamometer torque $T_{D, set}$ during the first simulation. While the controller tuned in [7] as well as a decoupled single-input single-output controller increases the torque immediately and thus causes the before mentioned drop in the speed, the proposed controller prevents this drop by a slower increase of the dynamometer torque set value $T_{D, set}$.

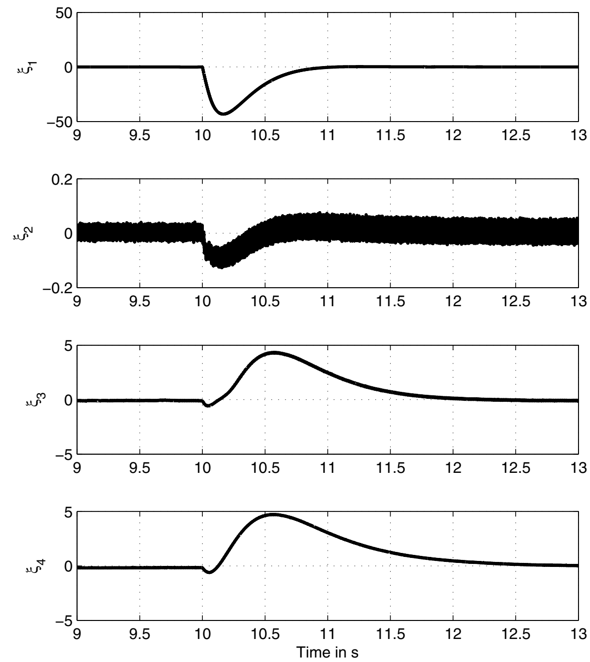

Fig. 5. Development of the states ξ during the first simulation.

Fig. 5 shows the time histories of the states ξ_i, $i = 1, \ldots, 4$ of the dynamic extension. As expected, these states converge to the origin. However, the change of the operation point at $t = 10$ s can clearly be seen. Furthermore, the size of the individual states can be recognized in these differences. Compared to the engine torque T_E, the engine and the dynamometer speed ω_E and ω_D respectively (states x_i, $i = 1, 3, 4$), the torsion of the shaft φ (state x_2) is always small. This also justifies the assumption at the end of Section III.

The states ξ_3 and ξ_4 show a very similar behaviour in the same range of values, which is not suprising, since these states are associated with the engine speed ω_E and the dynamometers speed ω_D.

The same holds for the second simulation. Again, the stationary final value of the engine torque T_E is achieved faster and without an undershoot.

The desired engine speed ω_E is achieved in a much shorter time by using the developed controller. However, in contrast to the controller developed in [7] the engine speed ω_E shows a slight undershoot limited to about 2.5 %.

Although the cost functions

$$J_{T_E} = \sqrt{\sum_{k=1}^{N} \left| \frac{T_E - T_{E,\,set}}{T_{E,\,set}} \right|^2}$$

$$J_{\omega_E} = \sqrt{\sum_{k=1}^{N} \left| \frac{\omega_E - \omega_{E,\,set}}{\omega_{E,\,set}} \right|^2}$$

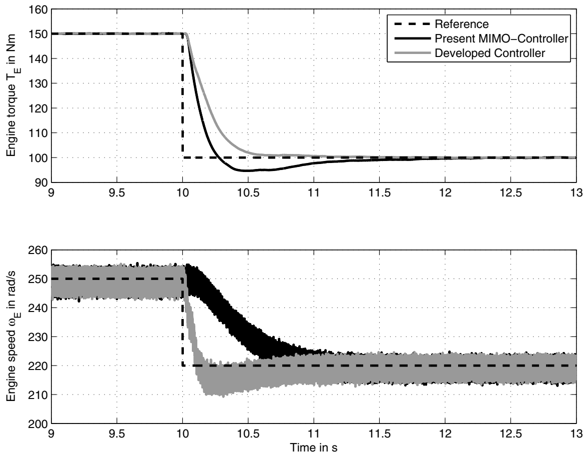

Fig. 6. Reference and simulated engine torque T_E (top) as well as engine speed ω_E (bottom) for second simulation.

have not been used to design the proposed controller, they allow an assessment of the proposed controller and controllers tuned without any or using an other cost function. N is the number of measurements. All measurements were recorded with a constant sampling time of 1 ms.

Table 3 summarizes the results for both simulations. The calculated costs are scaled to allow a better comparison.

TABLE II
COMPARISION BETWEEN EXISTING MIMO CONTROLLER AND DEVELOPED CONTROLLER

	1st simulation		2nd simulation	
	J_{T_E}	J_{ω_E}	J_{T_E}	J_{ω_E}
Existing MIMO controller	100	100	100	100
Proposed controller	101.6	66.3	117.2	46.6

While the costs according to an error in the engine torque T_E are about the same, a significant improvement in the speed characteristics can be determined. It should also not be forgotten that the comparision was not done with a standard test bench controller, instead a multi-input multi-output controller was used.

VI. CONCLUSION AND OUTLOOK

The optimal control of an internal combustion engine test bench is discussed in this paper. The state of the system is extended to avoid the calculation of the explicit solution of the corresponding Hamilton-Jacobi-Bellmann partial differential equation. The dynamic control law is afterwards modified to cope with non-modeled dynamics and nonlinearities.

The proposed controller shows a good tracking performance compared with other controls: The engine torque is achieved quickly without overshoots, the engine speed shows slight, but almost limited overshoots. The effect of couplings is limited.

In contrast to other multi-input multi-output controls, the calculation and the tuning of the proposed controller is simple, easily adopted and extended to other test benches and test benches with different setup.

Measurements applying the developed multi-input multi-output controller will be performed on the test bench for which the simulator was created. Such a controller will also be designed for a test bench with a truck engine with about 175 kW loaded by a hydrodynamic dynamometer. An ongoing project motivates the extension of the developed optimal control to internal combustion engine test benches with multiple dynamometers, namely test benches with an electric machine and a hydrodynamic dynamometer.

VII. ACKNOWLEDGMENTS

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