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Robust Feedback Linearization using Higher Order Sliding Mode Observer

S. Iqbal [1], C. Edwards [2], A. I. Bhatti [3]

Abstract— **A novel robust feedback linearization scheme is proposed in this paper based on a modified robust exact differentiator. The states and drift terms in the system are estimated simultaneously by the observer using back injection of the control effort. The estimated drift term is used in the feedback loop to compensate the disturbances and observed states are used for feedback linearization. Finite time convergence of the complete closed-loop system is proved and thus a form of separation principle is satisfied, i.e., the controller and observer can be separately designed. The design is verified through simulations and by experiments on a DC motor rig.**

I. INTRODUCTION

IT is a long standing aspiration of the control engineer to achieve a specified level of closed-loop performance and robustness from a system when only outputs are available and the system contains significant uncertainties. The popularity of PID control (Ogata, 2009; Franklin *et al,* 2010) is due to its simplicity and facility to tune its parameters without any detailed knowledge of the plant. However it is not a robust scheme. In nonlinear systems, feedback linearization (Isidori, 1995; Slotine and Li, 1991) can provide desired closed-loop performance levels, but it requires all parameters of the system to be well known; and again it is not robust. Sliding Mode Control (Utkin, 1992; Edwards and Spurgeon, 1998) can provide the desired performance and robustness, but the associated chattering effects resulting from the use of traditional discontinuous terms in the control law are an obstacle to its implementation - especially for mechanical systems.

The practical implementation of sliding mode controllers usually assumes knowledge of all system states. Generally states observers (Kalman, 1960; Luenberger, 1964; Utkin, 1999; Khalil, 2002; Davila Fridman and Levant, 2005) are used to estimate the states of the system. Results based on output information alone are less plentiful: Notable exceptions are output feedback dynamic sliding mode methods (Lu and Spurgeon, 1999), the universal SISO output-feedback controller (Levant, 2002), output feedback sliding mode controllers for linear uncertain systems (Edwards, Spurgeon and Hebden, 2003) and dynamic output feedback sliding mode control with mismatched uncertainty (Yan, Spurgeon, and Edwards, 2005).

A different approach to address the same problem is to estimate the disturbance or drift terms which constitute the combined effects of model uncertainties, unknown parameters, the influence of internal dynamics, etc; and cancel them via feedback action (Radke and Gao, 2006). For this approach the model is transformed into the Generalized Controllable Canonical Form (GCCF) (Isidori, 1995; Slotine and Li, 1991) and the state vector and drift terms are estimated via a High Gain Observer (HGO) (Khalil, 2002) or a "modified" robust exact differentiator (Levant, 1998 and Levant, 2003). On the basis of this information, a feedback linearization control (Isidori, 1995; Slotine and Li, 1991) is used to convert the system into an equivalent linear system. Examples based on HGO schemes can be seen in (Esfandiari and Khalil, 1992; Khalil, 1999; Freidovich and Khalil, 2006) and case studies with robust exact differentiator can be reviewed in (Massey and Shtessel, 2005; Hall and Shtessel, 2006; Besnard, Shtessel, and Landrum, 2007; Shtessel, Shkolnikov and Levant, 2007; Iqbal, Edwards and Bhatti, 2010; Iqbal, Edwards and Bhatti, 2011).

Robust feedback linearization has also been successfully demonstrated by various authors. In (Esfandiari and Khalil, 1992), output feedback using a High Gain Observer (HGO) was proposed which robustly estimates an appropriate number of derivatives of the output. An output feedback controller for nonlinear systems has been suggested in (Bartolini *et al*, 2002) that estimates the derivatives of the outputs with the help of the robust exact differentiator (Levant, 1998). These derivatives are then further used in the creation of a sliding surface for a second order sliding mode controller. Robust feedback linearization based on a nominal model has also been recently presented by (Freidovich and Khalil, 2006). In the design of (Freidovich and Khalil, 2006) the extended HGO is used to estimate the unmeasured derivatives of the output "plus" one. This extra derivative facilitates estimation of the uncertainties in the system. In (Benallegue, Mokhtari and Fridman, 2008), robust feedback linearization was also undertaken by using a higher order sliding mode observer (Davila, Fridman and Levant, 2005). However in the work of (Benallegue, Mokhtari and Fridman, 2008) the states and external disturbance effects were estimated based on a nominal model of the plant.

In this paper the authors propose a technique for feedback linearization on the basis of a modified robust exact differentiator (Levant 1998; Levant 2003). This observer can estimates the states as well as the drift terms based only on the available output of the system, and without detail knowledge of the mathematical model of the system. The estimated drift term is used in the feedback loop to compensate the disturbances of the system. The observed states are used to design any robust or state-space controller. The overall closed loop

1. *Sohail Iqbal*, Dept. of Electronic Engineering, Mohammad Ali Jinnah University, Islamabad. Currently, he is a Research Fellow at University of Leicester, UK; email: siayubi@gmail.com.
2. *Prof. Christopher Edwards*, Control & Instrumentation Group, Engineering Dept., University of Leicester, UK; email: chris.edwards@le.ac.uk.
3. *Prof. Aamer Iqbal Bhatti*, Control & Signal Processing Research Group, Dept. of Electronic Engineering, Mohammad Ali Jinnah University, Islamabad; email: aib@jinnah.edu.pk.

structure arising from using the proposed control law can be depicted as shown in Figure 1.1.

The underlying methodology is similar to the work of (Freidovich and Khalil, 2006) but in this paper a HOSM observer rather than a HGO is employed to estimate the required derivatives and uncertainties. The main idea here is, first transform the tacking error dynamics of the system into the Generalized Controllable Canonical Form (GCCF) (Isidori, 1995; Slotine and Li, 1991), and then to use a modified robust exact differentiator to estimate the states as well as the combined effect of the drift terms. The controller will then nullify the effects of the drift terms and impose its own closed loop dynamics based on a simple linear term. The states and the required estimates are all obtained in finite time. Consequently provided the open-loop plant does not have a finite escape time, a form of separation principle holds and the controller and observer can be designed independently.

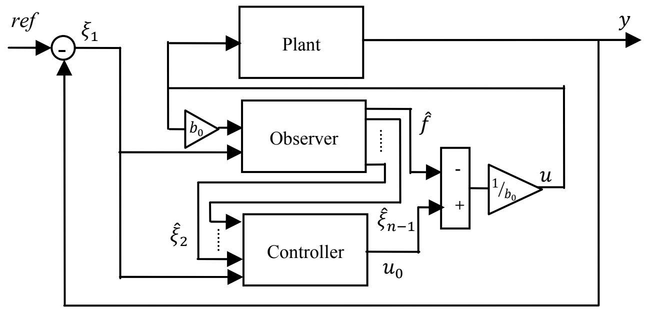

***Figure 1.1**: The Proposed Controller-Observer Structure*

The idea of estimating drift terms to cancel them via the feedback control law is from (Shtessel, Shkolnikov and Levant, 2007). In the work of (Shtessel, Shkolnikov and Levant, 2007) only relative degree one structures are considered. In (Iqbal, Edwards and Bhatti, 2010), the authors extended this idea to relative degree two systems. In the current paper the authors have further pursued this idea for more general relative degree *n* systems. One key benefit of this scheme over robust control (sliding mode or higher order sliding mode) approaches is that it does not require conservative upper bounds on the nonlinear terms and thus does not result in aggressive control action.

The remainder of the paper is structured as follows: in Section II the problem is formulated for the proposed robust feedback linearization. Section III deals with the structure of the robust exact observer and finite time stability of the estimates is proved. A case study involving a DC motor to validate the proposed technique through simulation is considered in Section IV. Experimental results arising from the implementation of these ideas on a DC motor rig are analyzed in Section V. Conclusions are drawn in Section VI.

II. THE PROBLEM FORMULATION

Consider a Single Input Single Output (SISO) dynamical system with relative degree *n* in Generalized Controllable Canonical Form (GCCF) (Isidori, 1995; Slotine and Li, 1991).

$$
\begin{aligned}
\dot{\xi}_1 &= \xi_2 \\
&\vdots \\
\dot{\xi}_n &= f(t,\xi) + b_1 u \\
y &= \xi_1
\end{aligned}
\tag{2.1}
$$

where $\xi \in \mathcal{R}^n$ is the state vector, $f(.)$ is a smooth vector field on $\mathcal{R}^n$, $u \in \mathcal{R}$ is the control input and $b_1 \in \mathcal{R}$ is the input constant of the system.

The system can also be written in the form

$$
\begin{aligned}
\dot{\xi} &= A\xi + B[f(t,\xi,\Delta_u) + b_0 u] \\
y &= C\xi
\end{aligned}
\tag{2.2}
$$

where b_0 is the nominal value of the input gain of the system, $\Delta_u = b_1 - b_0$ is uncertainty in the input channel and

$$
A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \quad C = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}^T
$$

If all the states are available and the function $f(.)$ is precisely known in (2.2), then a feedback linearization control law for the system is given by

$$u = \frac{1}{b_0}(-f(t,\xi,\Delta_u) - K\xi) \tag{2.3}$$

where $K^T \in \mathcal{R}^n$ is a fixed design gain. Using this control law in (2.2) yields linear closed loop dynamics

$$\dot{\xi} = (A - BK)\xi \tag{2.4}$$

The gain matrix K can be designed using any modern or classical state-space technique e.g. pole placement, LQR or LMI techniques etc, so that $A - BK$ is Hurwitz and the states ξ meet the desired performance objectives of the closed loop system.

However, in reality, in most engineering systems only the output of the system is available and the function $f(.)$ is unknown (or at best not known perfectly). As a result the ideal control law in (2.3) is not realizable. Instead, if the control law

$$u = \frac{1}{b_0}\left(-\hat{f} - K\hat{\xi}\right) \tag{2.5}$$

is employed where $\hat{f}$ and $\hat{\xi}$ are estimates of $f(t,\xi,\Delta_u)$ and $\xi(t)$ with the property that $\hat{f} \to f(t,\xi,\Delta_u)$ and $\hat{\xi} \to \xi(t)$ in finite time, the desired performance indicated in (2.4) can be obtained in finite time.

Assumption 1: *It is assumed that in control law (2.5)* $b_0 \neq 0$

The following sections propose an observer structure to generate the exact estimates of $\hat{\xi}$ and $\hat{f}$ in (2.5), which converge to the true values in finite time.

III. ROBUST STATE-DISTURBANCE OBSERVER

Since only the output of the system (2.2) is available, the control law (2.3) is not realistic. Also the closed loop dynamics (2.4) will be sensitive to $f(t,\xi,\Delta_u)$. The overall closed system therefore requires a good estimate of the states and the drift signals to cancel out its effects.

Assume the control $u(t)$ is Lebesgue-measurable and the drift function $f(t,\xi,\Delta_u)$ is unknown but differentiable with $|f(t,\xi,\Delta_u)| < L$, where $L > 0$ is the Lipshitz constant. Then the modified 'exact observer' structure (Levant, 1998, 2003; Shtessel, Shkolnikov and Levant, 2007) can be introduced as part of the closed loop to compensate for the undesired disturbances, and to estimate precisely the unmeasured state in finite time. The approach of (Freidovich and Khalil, 2006) is similar in terms of methodology, but is based on the use of a HGO to obtain the estimates.

The proposed observer structure can be written as follows

$$
\begin{aligned}
&\dot{\hat{\xi}}_1 = v_1\\
&v_1 = -\lambda_{n+1}L^{1/(n+1)}\left|\hat{\xi}_1 - \xi_1\right|^{n/(n+1)} sign\left(\hat{\xi}_1 - \xi_1\right) + \hat{\xi}_2\\
&\quad\vdots\\
&\dot{\hat{\xi}}_{n-1} = v_{n-1}\\
&v_{n-1} = -\lambda_3 L^{1/3}\left|\hat{\xi}_{n-1} - v_{n-2}\right|^{2/3} sign\left(\hat{\xi}_{n-1} - v_{n-2}\right) + \hat{\xi}_n\\
&\dot{\hat{\xi}}_n = \hat{f} + b_0 u,\\
&\hat{f} = -\lambda_2 L^{1/2}\left|\hat{\xi}_n - v_{n-1}\right|^{1/2} sign\left(\hat{\xi}_n - v_{n-1}\right) + \hat{\xi}_{n+1}\\
&\dot{\hat{\xi}}_{n+1} = -\lambda_1 L\, sign\left(\hat{\xi}_{n+1} - \hat{f}\right)
\end{aligned}
\quad (3.1)
$$

The parameters λ_i can be chosen recursively as suggested in (Levant, 2003). Note: the observer in (3.1) has a slightly different structure to the one in (Shtessel, Shkolnikov and Levant, 2007; Iqbal, Edwards and Bhatti, 2010) because a relative degree n system is considered in (2.2).

Theorem 1: *Suppose the parameters of the observer $\lambda_1, \lambda_2, \ldots, \lambda_{n+1}$ are properly chosen and the output of the system $\xi_1(t)$ and the input signal $u(t)$ are bounded and Lebesgue-measurable. Then in the absence of noise the following equalities are established in finite time: $\hat{\xi}_i = \xi_i, \quad \forall\, i = 1,\ldots,n$* and $\hat{\xi}_{n+1} = f(t,\xi,\Delta_u)$

Proof: Define

$$
\begin{aligned}
&\varphi_1 = \hat{\xi}_1 - \xi_1\\
&\quad\vdots\\
&\varphi_n = \hat{\xi}_n - \xi_1^{(n-1)}\\
&\varphi_{n+1} = \hat{\xi}_{n+1} - f(t,\xi,\Delta_u)
\end{aligned}
\quad (3.2)
$$

From system (2.2) and the observer described in (3.1),

$$
\begin{aligned}
&\hat{\xi}_2 - v_1 = \hat{\xi}_2 - \dot{\hat{\xi}}_1 = \hat{\xi}_2 - \dot{\xi}_1 - \dot{\varphi}_1 = \varphi_2 - \dot{\varphi}_1\\
&\quad\vdots\\
&\hat{\xi}_n - v_{n-1} = \hat{\xi}_n - \dot{\hat{\xi}}_{n-1} = \hat{\xi}_n - \xi_1^{(n-1)} - \dot{\varphi}_{n-1} = \varphi_n - \dot{\varphi}_{n-1}\\
&\hat{\xi}_{n+1} - \hat{f} = \hat{\xi}_{n+1} - \dot{\hat{\xi}}_n + b_0 u = \varphi_{n+1} - \dot{\varphi}_n
\end{aligned}
$$

By using these definitions, the observer in (3.1) can be written as

$$
\begin{aligned}
&\dot{\varphi}_1 = -\lambda_{n+1}L^{1/(n+1)}|\varphi_1|^{n/(n+1)} sign(\varphi_1) + \varphi_2\\
&\dot{\varphi}_2 = -\lambda_n L^{1/n}|\varphi_2 - \dot{\varphi}_1|^{(n-1)/n} sign(\varphi_2 - \dot{\varphi}_1) + \varphi_3\\
&\quad\vdots\\
&\dot{\varphi}_n = -\lambda_2 L^{1/2}|\varphi_n - \dot{\varphi}_{n-1}|^{1/2} sign(\varphi_n - \dot{\varphi}_{n-1}) + \varphi_{n+1}\\
&\dot{\varphi}_{n+1} \in -\lambda_1 L\, sign(\varphi_{n+1} - \dot{\varphi}_n) + [-L, +L]
\end{aligned}
\quad (3.3)
$$

The structure in (3.3) above is similar to the exact differentiator from (Levant 1998; Levant 2003). For ease of implementation, the equations can be written in such a way that the derivatives on the right hand side of each equation are excluded (Levant, 2003; Shtessel, Shkolnikov and Levant, 2007). The resulting differential inclusion can be understood in the Flippov sense (Flippov, 1988) and it is easy to see that the inclusion in (3.3) is invariant with respect to the dilation

$$t \mapsto \kappa t \text{ and } \varphi \mapsto \kappa^{n-i+1}\varphi_i \qquad \forall \kappa > 0,\ i = 0,\ldots,n$$

and therefore the system is homogenous: furthermore its homogeneity degree is equal to –1. Therefore $\varphi_i \to 0$ in finite time and the following exact equalities are obtained (in finite time):

$$
\begin{aligned}
&\varphi_1 = \hat{\xi}_1 - \xi_1 = 0 \;\Rightarrow \hat{\xi}_1 = \xi_1\\
&\quad\vdots\\
&\varphi_{n-1} - \dot{\varphi}_{n-2} = 0 \;\Rightarrow \hat{\xi}_{n-1} = \xi_{n-1}\\
&\varphi_n - \dot{\varphi}_{n-1} = 0 \Rightarrow \hat{\xi}_n - \xi_1^{(n-1)} - \dot{\hat{\xi}}_{n-1} - \xi_1^{(n-1)} \Rightarrow \hat{\xi}_n = \xi_n\\
&\varphi_{n+1} - \dot{\varphi}_n = 0 \Rightarrow \hat{\xi}_{n+1} - f(t,\xi,\Delta_u) - \dot{\hat{\xi}}_n + \xi_1^{(n)} = 0\\
&\qquad\qquad\qquad \Rightarrow \hat{\xi}_{n+1} = f(t,\xi,\Delta_u)
\end{aligned}
$$

This proves the theorem. ■

Theorem 1 assumes the inputs and outputs of the system in (2.2) are noise free. The next theorem explores the impact of noise on the estimates ξ.

Theorem 2: *Suppose the input signal u is bounded and Lebesgue-measurable, and the output noise is bounded and satisfies $\left|\hat{\xi}_1 - \xi_1\right| \le \mu_1\varepsilon$ then, the following inequalities can be established in finite time for some positive constants μ_i.*

$$
\begin{aligned}
&\left|\hat{\xi}_1 - \xi_1\right| \le \mu_1\varepsilon\\
&\left|\hat{\xi}_2 - \xi_2\right| \le \mu_2\varepsilon^{(n-1)/n}\\
&\quad\vdots\\
&\left|\hat{f} - f(t,\xi,\Delta_u)\right| \le \mu_n\varepsilon^{1/n}
\end{aligned}
\quad (3.4)
$$

where the noise $\xi \in [-\varepsilon, \varepsilon]$ and $u \in \left[-k\varepsilon^{(n-1)/n}, k\varepsilon^{(n-1)/n}\right]$

Proof: Again using the definitions in (3.2), the observer in (3.1) can be rewritten as (3.3). The structure in (3.3) is similar to the robust exact differentiator (Levant 1998; Levant 2003). If noise is present i.e. $\varepsilon \ne 0$, the output $\varphi \in [-\varepsilon, \varepsilon]$ and the input $u \in \left[-k\varepsilon^{(n-1)/n},\ k\varepsilon^{(n-1)/n}\right]$, then the bounds in (3.4) can be obtained using arguments similar to (Levant, 2003). The system (3.3) is homogenous and its homogeneity degree is equal to –1 with respect to the transformation:

$$G_\kappa: (t, \varphi, \varepsilon) \mapsto (\kappa t,\ \kappa^{n-i+1}\varphi_i,\ \kappa^{n+1}\varepsilon) \qquad \forall \kappa > 0, i = 0,..,n$$

The rest of the proof is similar to (Levant, 2003). This proves the theorem. ■

Theorem 1 and 2 demonstrate the finite time convergence of the observer given in (3.1). In the next theorem, the stability analysis for the complete closed-loop system is given.

Theorem 3: *Assume that the drift term $f(t,\xi,\Delta_u)$ in (2.2) is a smooth vector field on $\mathcal{R}^n$, and moreover, $\hat{\xi}_n$ and $\hat{f}$ are exactly estimated. Then the closed loop system (2.2) with control law in (2.5) and the observer in (3.1) is stable.*

Proof: When exact measurements of $\hat{\xi}_n$ and $\hat{f}$ are available from using the observer (3.1), the term $\hat{f}$ will exactly cancel the drift term $f(t,\xi,\Delta_u)$ and the dynamics in (2.4) will be established in finite time. Designing K by using any modern or classical state-space methods ensures that $A - BK$ is

Hurwitz and the closed loop system associated with (2.2) is stable.

As shown in Theorem 1, the observer in (3.1) can easily be transformed into structure (3.3) by using the definitions (3.2). The resulting system is homogenous and its homogeneity degree is equal to –1 by using following dilation:

$$t \mapsto \kappa t \text{ and } \varphi_i \mapsto \kappa^{n-i+1}\varphi_i \qquad \forall\kappa > 0, i = 1,..,n+1$$

Here the observer (3.1) is finite time stable and the control law (2.5) is bounded and convergent. Utilizing the separation principle, the overall closed loop system is finite time stable. This proves the theorem. ■

The remainder of the paper considers initially a simulation of the use of the proposed control law on a mathematical model of a DC motor, followed by implementation results on a laboratory rig.

IV. Simulations Studies

In this section, the proposed control scheme is demonstrated on an industrial benchmark DC motor (Utkin *et al*, 1999) with the following dynamics

$$L_0\frac{di}{dt} = u - Ri - k_e\omega$$
$$J\frac{d\omega}{dt} = k_t i - \tau_l \qquad (4.1)$$

In system (4.1), u is the input terminal voltage, whilst ω and i are the states of the system and represent shaft speed and armature current respectively. The motor load torque is defined as $\tau_l = B\omega$. All the parameters of the DC motor and their nominal values are listed in Table 1.

Name	Symbol	Values/Units
Inertia of the Motor Rotor and Load	J	0.001 Kgm^2
Armature Resistance	R	0.5 Ω
Armature Inductance	L_0	1.0 *mH*
Back-EMF Constant	K_e	0.001 *V/rad*
Torque Constant	K_t	0.008 *Nm/A*
Coefficient of Viscous Friction	B	0.01 *Nm s /rad*

Table 1*: The DC Motor Parameters*

Let ω_r be the reference shaft speed, and $e = \omega_r - \omega$ be the tracking error. Define $x_1 = e$ and $x_2 = \dot{e}$, then the error dynamical system using equation (4.1) can be represented as

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = f(t,x) + b_0 u \qquad (4.2)$$

where the function $f(t,x)$ is

$$f = -a_1x_1 - a_2x_2 + \ddot{\omega}_r + a_2\dot{\omega}_r + a_1\omega_r + \frac{R}{JL_0}\tau_l + \frac{1}{J}\dot{\tau}_l \qquad (4.3)$$

Note the drift term depends upon the reference speed and load torque. The constants in equation (4.3) above are defined as $a_1 = K_tK_e/(JL_0)$, $a_2 = R/L_0$ and $b_0 = -K_t/(JL_0)$. To ensure differentiability of $f(t,x)$, a low-pass pre-filter for the step reference signal is introduced.

A pole placement technique is chosen for the choice of the feedback gain *K*. In this paper the two poles are placed at -3 and -3. The controller parameters to achieve this are $k_1 = 9$ and $k_2 = 6$. Thus the proposed controller is given by

$$u = \frac{1}{b_0}\left(-\hat{f} - k_1x_1 - k_2\hat{x}_2\right) \qquad (4.4)$$

where $\hat{f}$ is the estimate of the drift term (4.3) and $\hat{x}_2$ is the estimate of the state x_2.

The proposed observer structure for the system is as follows:

$$\dot{\hat{x}}_1 = v_1$$
$$v_1 = -\lambda_3 L^{1/3}|\hat{x}_1 - x_1|^{2/3}\, sign(\hat{x}_1 - x_1) + \hat{x}_2$$
$$\dot{\hat{x}}_2 = \hat{f} + b_0 u$$
$$\hat{f} = -\lambda_2 L^{1/2}|\hat{x}_2 - \hat{x}_1|^{1/2}\, sign(\hat{x}_2 - \hat{x}_1) + \hat{x}_3$$
$$\dot{\hat{x}}_3 = -\lambda_1 L\, sign(\hat{x}_3 - \hat{f}) \qquad (4.5)$$

where $\lambda_1 = 2.1$, $\lambda_2 = 4.2$ and $\lambda_3 = 8.4$.

Figure 4.1 shows the simulation results of the proposed speed controller for the DC motor. The first subplot demonstrates the speed response and the second subplot displays the control effort. As illustrated, the speed tracks the reference signal very effectively. Moreover the control input does not exhibit any chattering effects.

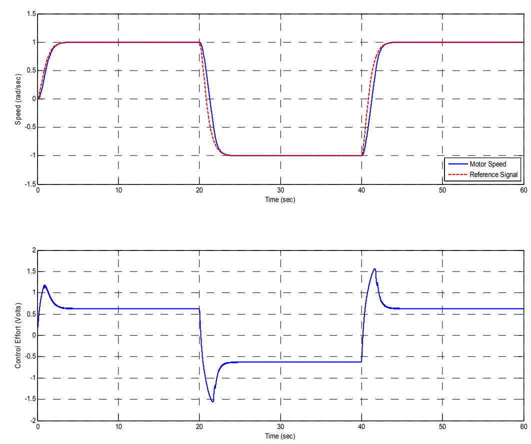

Figure 4.1*: The Speed Response of DC Motor*

Figure 4.2 shows the tracking of the drift term by the proposed observer. As shown in the figure, the observer precisely tracks the drift signal after a certain (finite) time.

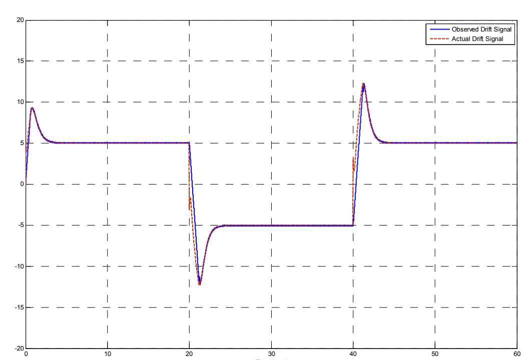

Figure 4.2*: The Actual and Observed Drift Term*

Figure 4.3 shows a comparison between the actual and observed state. It is clear from the figure that the observed state follows the actual state component x_2 accurately.

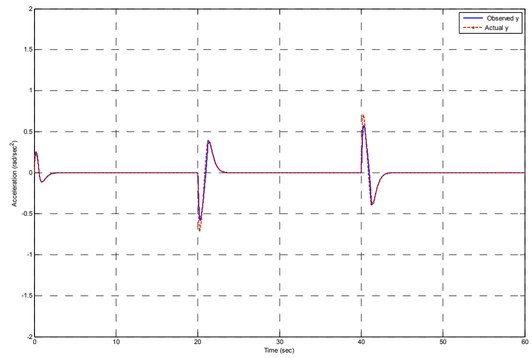

***Figure 4.3**: The Actual and Observed State x_2.*

V. Experimental Results

For stringent performance and robustness analysis of the proposed controller, an academic benchmark DC-motor (MS150) manufactured by Feedback Instrumentation has been used. Figure 5.1 shows the experimental setup.

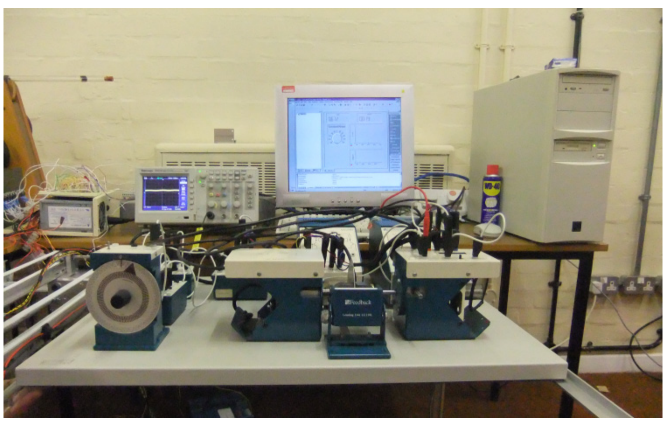

***Figure 5.1**: The DC Motor and dSPACE Setup*

The input of the motor (in Volts) is known, and the angular speed output (in radians per second) can be measured through a taco-generator. An aluminium disk is mounted on the motor shaft to increase the inertia of the motor. The disk rotates between the two poles of a magnet, to reproduce the effect of a frictional load. The key parameters of the DC-servomotor are given in Table 2 as listed by manufacturer.

Name	Symbol	Values/Units
Inertia of the Motor Rotor and Load	J	4.42x 10^{-4} *Kgm^2*
Armature Resistance	R	3.2 Ω
Armature Inductance	L_0	8.6 x 10^{-3} *H*
Back-EMF Constant	K_e	60 x 10^{-3} *V/rad*
Torque Constant	K_t	17 x 10^{-3} *Nm/A*

***Table 2**: The MS-150 DC Motor Parameters*

A dSPACE card (DS1102) was chosen as the interface for the real time implementation of the controller from the Matlab/Simulink environment. The card provides four channels of 16-bit A/D conversion and two channels of 16-bit D/A data conversion. The setup uses a TMS320C31 floating-point DSP processor with 128 K x 32-bit RAM.

For the experiment, the observer structure from (4.5) and the control law from (4.4) have been used. The values of b_0, λ_1, λ_2 and λ_3 need to be tuned. Initial guesses for these parameters were the values used in the simulations. The controller gains k_1 and k_2, are the ones described earlier to place the linear closed-loop poles at -3 and -3.

Figure 5.2 shows the experimental results of the proposed control scheme for a square reference signal. The two plots in the figure show the speed response of the DC motor and its corresponding control effort. As illustrated below, the measured speed tracks the reference precisely.

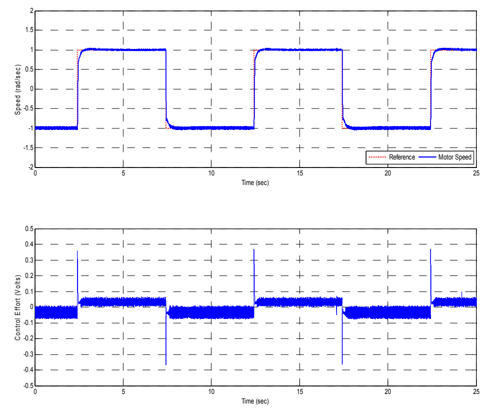

***Figure 5.2**: The Experimental Result of Square Wave.*

The performance of the proposed controller has also been examined with respect to a continuous sinusoidal waveform as a reference speed. The sinusoidal speed response of the DC motor with the sine reference signal is shown in Figure 5.3. The plots demonstrate that the trajectory is followed accurately with low control effort.

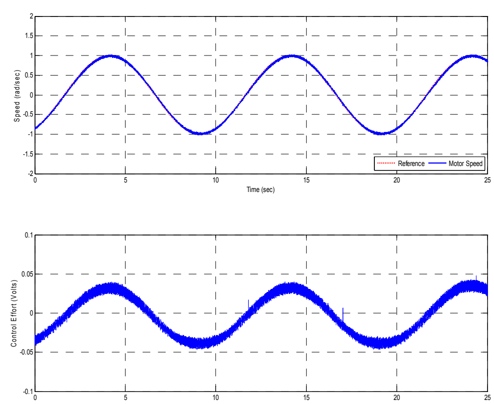

***Figure 5.3**: The Experimental Result of Sine Signal.*

To verify the robustness of the proposed controller, the friction load, with the help of a magnetic brake, has been increased by 300% during the experiment. Figure 5.4 demonstrates the speed response of the DC motor subject to this perturbation.

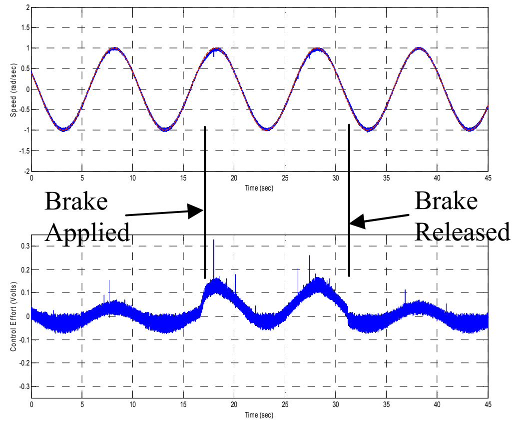

Figure 5.4: *The Response with Perturbation*

In the experiment the brake has been applied at 16.5 seconds and released after 31 seconds. The graph shows that these changes do not affect the performance of the controller significantly.

The experimental results from the proposed scheme for feedback linearization based on the modified robust exact estimator, validate the theory given in the earlier sections. The suggested strategy also offers an opportunity for achieving desired and robust performance, without the detailed knowledge of the plant system model.

VI. Conclusion

In the paper a new technique for robust feedback linearization based only on output information is proposed. A robust exact observer is used to estimate the states and drift terms of the system. Finite time stability of the observer is proved, so that the separation principle can be applied. Simulation and experimental results verify the robustness and performance levels of the proposed technique.

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