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Smart Management of Actuator Saturation in Integrated Vehicle Control

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***Abstract*— Integrated control design to guarantee vehicle stability is one of the main topics in vehicle control. Actuators saturation is a main concern when dealing with this issue. In this work, we propose a fully integrated control by means of three actuators: Active Front Steering (AFS), Rear Torque Vectoring (RTV) and Semi–Active Suspensions (SAS). A feedback controller is used to ensure, in absence of input saturation, the exponential tracking of the reference trajectories. On top of that, a smart saturation management scheme detects when the actuators approach the occurrence of saturation, and determines a policy to avoid actuator saturations. The performance of the resulting control strategy, that is hybrid in general, is tested in simulation and compared with some classical solutions.**

***Index Terms*— Attitude vehicle control, Actuator saturation, Integrated control.**

I. INTRODUCTION

Physical systems operate under input constraints, since actuators are subject to saturation. It is well known that actuator saturation degrades the performance of the control system and may lead to instability. Actuator saturation has received increasing attention from the research community. Roughly speaking, there are two strategies for dealing with this problem. The first is to neglect the saturation in the first stage of the control design and then to add some problem–specific schemes to deal with the adverse effects caused by saturation. These schemes, known as anti–windup schemes, introduce additional feedback in such a way that the actuators stay within the limits. Recently, a number of anti–windup schemes have been proposed, providing stability and performance achievements, for linear plants with constraints on the input. Several approaches for minimizing the performance loss for linear systems have been developed. For more comprehensive overviews of modern anti–windup approaches, see the works [1], [2] and [3]. The second strategy takes into account the saturation at the outset of the control design. It analyzes the closed loop system under actuator saturation systematically, and redesigns the controller in such a way that the performance is retained while stability is improved. In [4] a notion of process directionality in input–contrained processes is defined, and an optimal directionality compensator for nonlinear processes with actuator saturation is presented.

In the automotive case, the problem of actuator saturation is emphasized due to the presence of different subsystems, individually developed to provide some desired functionality. With respect to some years ago, a much larger computational power is available, due to the improvement of the electronics and to the increasing number of available customer features and technologies. This allows designers to cope with many kinds of requests and constraints, pushing towards subsystem integration. This philosophy presents a clear advantage for saturation avoidance. In fact, in an integrated control structure more power is available for control, thus potentially limiting the saturation occurrences. In all cases, it is important to manage critical situations, whenever actuators are not physically able to apply the required input.

The design of active attitude controls is one of the main research topics in vehicle control area. Active devices modify the vehicle dynamics, imposing forces or moments to the vehicle body in different ways (see, e.g. [6], [16], [10], [17]), and can now make use of smart sensors (for example, the so–called intelligent tires [21]), allowing precise and distributed measurements from the environment, to increase the performance of the control action, the vehicle stability, and the safety and comfort of the driver. On top of that, hierarchical and hybrid structures guarantee increased performance and robustness of control strategies, taking into account the interactions among vehicle, driver and environment, considered in parallel in one core algorithm.

In some previous works, the authors have addressed vehicle attitude control by using active front steering and rear torque vectoring [11], [12], [13], [14], [15]. The application of an adaptive feedback linearization control has been proposed in [18] to improve stability in the presence of deviations of the vehicle parameters from the nominal values, and of rapid variations of road conditions. In the aforementioned works, roll motion has been neglected and no countermeasures for actuator saturations have been considered.

As far as roll control is concerned, a large number of technologies regarding electronically controlled active and semi–active suspension systems have been developed in the last twenty years, oriented both on comfort and handling improvement (see e.g. [7], [9]). Another research line has tackled the problem of control of linear and nonlinear

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systems with input constraints and saturations (e.g. see [19], [20]). In [15] a management of the actuator saturation has been proposed, in which different priorities are assigned to the fulfillment of the tracking objectives of the state variable (yaw, lateral, and roll velocities) and, when the actuators saturate, only the tracking of the variables with higher priority is ensured. Following this work, in the present paper a hybrid controller for actuator saturation management is proposed. The resulting hybrid saturation management relies on a mechanism to prevent the occurrence of input saturation, based on input limiting functions, and/or on a modification of the reference signal.

The paper is organized as follows. In Section 2, the mathematical model of the vehicle is presented, and the control problem is stated. In Section 3, the saturation management is described. In Section 4, the proposed controller is tested with simulations and comparisons. Some comments conclude the paper.

II. Mathematical Model and Problem Formulation

For simplicity, we consider the model of a rear–wheel driven vehicle. The actuators considered in this work are

1. Active Front Steer (AFS), which imposes an incremental steer angle on top of the driver's input. The control is then actuated through the front axle tire characteristic.
2. Rear Torque Vectoring (RTV), which distributes the torque in the rear axle, usually to improve vehicle traction, handling and stability. The control is then actuated through the rear axle tire characteristics.
3. Semi–active suspensions (SAS), which are able to change the damping coefficient of the shock absorber in a continuous interval, differently from passive systems.

The mathematical model is derived under the following assumptions, which are verified in a large number of situations and which mitigate the complexity of the vehicle dynamics

– The vehicle moves on a horizontal plane;
– The longitudinal velocity is constant, so that vehicle shaking/pitch motions can be neglected;
– The steering system is rigid, so that the angular position of the front wheels is uniquely determined by the steering wheel position;
– The wheels masses are much lower than the vehicle one, so the steering action does not affect the position of the centre of mass of the entire vehicle;
– The vehicle takes large radius bends and the road wheel angles are "small" (less than 10°);
– The aerodynamic resistance and the wind lateral thrust are not considered;
– The tire vertical loads are constant;
– The actuators are ideally modelled.

As a consequence of the previous assumptions, the vehicle model has four degrees of freedom: the lateral velocity v_y, the yaw rate ω_z, the roll angle α_x and the roll velocity ω_x. Following [15], one can consider the mathematical model hereinfafter

$$m(\dot{v}_y + v_x\omega_z) = \mu(F_{y,f} + F_{y,r}) + m_s h_d \dot{\omega}_x \tag{1}$$

where m is the vehicle mass, $h_d = h - d$, h is the center of gravity height, d is the roll center height, m_s is the sprung mass, μ is the road–tire friction coefficient, and $F_{y,f}$, $F_{y,r}$ the lateral (front/rear) tire forces. The lateral forces $F_{y,f} = F_{y,f}(\alpha_f)$, $F_{y,r} = F_{y,r}(\alpha_r)$ depend on the slip angles

$$\begin{aligned}
\alpha_f &= \delta_c + \alpha_{f,0} = \delta_c + \delta_d + \gamma_f\alpha_x - \frac{v_y + l_f\omega_z}{v_x}\\
\alpha_r &= \gamma_r\alpha_x - \frac{v_y - l_r\omega_z}{v_x}
\end{aligned}$$

where δ_c, δ_d are the control, driver components of the wheel angle, γ_f, γ_r are front/rear sensitivities with respect to the roll angle α_x, and l_f, l_r are the distances from the center of gravity to the front/rear axle. A simple but accurate representation of the lateral force functions is given by [5]

$$\begin{aligned}
F_{y,f}(\alpha_f) &= C_{y,f}\sin(A_{y,f}\arctan(B_{y,f}\alpha_f))\\
F_{y,r}(\alpha_r) &= C_{y,r}\sin(A_{y,r}\arctan(B_{y,r}\alpha_r))
\end{aligned} \tag{2}$$

where $A_{y,f}$, $A_{y,r}$, $B_{y,f}$, $B_{y,r}$, $C_{y,f}$, $C_{y,r}$ are experimental constants.

The vehicle yaw dynamics can be expressed considering the presence of RTV actuators

$$J_z\dot{\omega}_z = \mu(F_{y,f}l_f - F_{y,r}l_r) + M_z + J_{zx}\dot{\omega}_x \tag{3}$$

where J_z is the vehicle inertia momentum about the z axis, J_{zx} is the product of inertia about the axes z, x, and M_z is RTV yaw moment.

The vehicle roll angular acceleration can be expressed as

$$\begin{aligned}
\dot{\alpha}_x &= \omega_x\\
J_r\dot{\omega}_x &= -b_x\omega_x - (k_x - m_s g h_d)\alpha_x + J_{zx}\dot{\omega}_z\\
&\quad + m_s h_d(\dot{v}_y + v_x\omega_z)
\end{aligned} \tag{4}$$

where J_x is the vehicle inertia momentum about the x axis, $J_r = J_x + m_s h_d^2$, b_x is the suspension roll damping, k_x is the suspension roll stiffness, and g is the gravity acceleration constant.

From (1), (3), (4) we get the mathematical model of a vehicle with yaw, lateral and roll dynamics

$$\begin{aligned}
\dot{\omega}_z &= \frac{1}{J_{z,e}}\Big[\mu(F_{y,f}l_f - F_{y,r}l_r) + M_z\Big]\\
&\qquad + k_m k_z h_e \mu(F_{y,f} + F_{y,r})\\
&\qquad - k_m k_z\Big[b_x\omega_x + (k_x - m_s g h_d)\alpha_x\Big]\\
\dot{v}_y &= -\omega_z v_x + \frac{1}{m_e}\mu(F_{y,f} + F_{y,r})\\
&\qquad + k_m k_z h_e\Big[\mu(F_{y,f}l_f - F_{y,r}l_r) + M_z\Big]\\
&\qquad - k_m h_e\Big[b_x\omega_x + (k_x - m_s g h_d)\alpha_x\Big]\\
\dot{\omega}_x &= -k_m\Big[b_x\omega_x + (k_x - m_s g h_d)\alpha_x\Big]\\
&\qquad + k_m k_z\Big[\mu(F_{y,f}l_f - F_{y,r}l_r) + M_z\Big]\\
&\qquad + k_m h_e \mu(F_{y,f} + F_{y,r})\\
\dot{\alpha}_x &= \omega_x
\end{aligned} \tag{5}$$

where $k_m = 1/J_{x,e}$ and

$$J_{x,e} = J_r - \frac{J_{zx}^2}{J_z} - \frac{m_s^2 h_d^2}{m} \qquad J_{z,e} = \frac{J_z}{1 + k_m k_z J_{zx}}$$

$$m_e = \frac{m}{1 + m_s k_m h_d h_e}, \qquad h_e = \frac{m_s}{m}h_d, \quad k_z = \frac{J_{zx}}{J_z}.$$

As anticipated, we consider v_x constant. Moreover, the control inputs that we consider are M_z, and the differences

$$\begin{aligned} \varDelta F_{y,f} &= F_{y,f} - F_{y,f,0}, \qquad F_{y,f,0} := F_{y,f}(\alpha_{f,0}) \\ \varDelta b_x &= b_x - b_{x,0} \end{aligned}$$

with $b_{x,0}$ the damping when the SAS system is not active. Clearly, the real active front input is the control angle δ_c which can be determined by inverting (2), obtaining

$$\delta_c = \begin{cases} -\delta_d + \frac{v_y + l_f\omega_z}{v_x} + F_{y,f}^{-1}(\bar{F}) & \text{if } |\bar{F}| \le F_{y,f}(\alpha_{f,\mathrm{sat}}) \\ -\delta_d + \frac{v_y + l_f\omega_z}{v_x} \pm \alpha_{f,\mathrm{sat}} & \text{otherwise} \end{cases}$$

with $\bar{F}$ a fixed value to be imposed by the AFS.

The control aim is to track exponentially some bounded references, with bounded derivatives, for ω_z, v_y, α_x and ω_x. More precisely, the reference generator is

$$\begin{aligned} \dot{\omega}_{z,\mathrm{ref}} &= \frac{1}{J_{z,\mathrm{ref}}}\Big[\mu_{\mathrm{ref}}(F_{y,f,\mathrm{ref}}l_f - F_{y,r,\mathrm{ref}}l_r)\Big] \\ &\quad + k_m k_z h_e \mu_{\mathrm{ref}}(F_{y,f,\mathrm{ref}} + F_{y,r,\mathrm{ref}}) \\ &\quad - k_m k_z\Big[b_{x,\mathrm{ref}}\omega_{x,\mathrm{ref}} + (k_{x,\mathrm{ref}} - m_s g h_d)\alpha_{x,\mathrm{ref}}\Big] \\ \dot{v}_{y,\mathrm{ref}} &= -\omega_{z,\mathrm{ref}}v_x + \frac{1}{m_e}\mu_{\mathrm{ref}}(F_{y,f,\mathrm{ref}} + F_{y,r,\mathrm{ref}}) \\ &\quad + k_m k_z h_e\Big[\mu_{\mathrm{ref}}(F_{y,f,\mathrm{ref}}l_f - F_{y,r,\mathrm{ref}}l_r)\Big] \\ &\quad - k_m h_e\Big[b_{x,\mathrm{ref}}\omega_{x,\mathrm{ref}} + (k_{x,\mathrm{ref}} - m_s g h_d)\alpha_{x,\mathrm{ref}}\Big] \\ \dot{\omega}_{x,\mathrm{ref}} &= -k_m\Big(b_{x,\mathrm{ref}}\omega_{x,\mathrm{ref}} + k_{x,\mathrm{ref}}\alpha_{x,\mathrm{ref}}\Big) \\ &\quad + k_m k_z\Big[\mu_{\mathrm{ref}}(F_{y,f,\mathrm{ref}}l_f - F_{y,r,\mathrm{ref}}l_r)\Big] \\ &\quad + k_m h_e \mu_{\mathrm{ref}}(F_{y,f,\mathrm{ref}} + F_{y,r,\mathrm{ref}}) \\ \dot{\alpha}_{x,\mathrm{ref}} &= \omega_{x,\mathrm{ref}} \end{aligned} \tag{6}$$

where $J_{z,\mathrm{ref}}$, μ_{ref}, $k_{x,\mathrm{ref}}$, $b_{x,\mathrm{ref}}$ are appropriate parameters and $F_{y,f,\mathrm{ref}}$, $F_{y,r,\mathrm{ref}}$ are ideal curves, depending on

$$\begin{aligned} \alpha_{f,\mathrm{ref}} &= \delta_d + \gamma_f\alpha_{x,\mathrm{ref}} - \frac{v_{y,\mathrm{ref}} + l_f\omega_{z,\mathrm{ref}}}{v_x} \\ \alpha_{r,\mathrm{ref}} &= \gamma_r\alpha_{x,\mathrm{ref}} - \frac{v_{y,\mathrm{ref}} - l_r\omega_{z,\mathrm{ref}}}{v_x}. \end{aligned}$$

In particular, we set

$$\begin{aligned} &F_{y,j,\mathrm{ref}}(\alpha_{j,\mathrm{ref}}) = \\ &= \begin{cases} F_{y,j}(-\bar{\alpha}_{j,\mathrm{ref}}) + F'_{y,j}(\bar{\alpha}_{j,\mathrm{ref}})(\alpha_{j,\mathrm{ref}} + \bar{\alpha}_{j,\mathrm{ref}}) & \alpha_{j,\mathrm{ref}} < -\bar{\alpha}_{j,\mathrm{ref}} \\ F_{y,j}(\alpha_{j,\mathrm{ref}}) & |\alpha_{j,\mathrm{ref}}| \le \bar{\alpha}_{j,\mathrm{ref}} \\ F_{y,j}(\bar{\alpha}_{j,\mathrm{ref}}) + F'_{y,j}(\bar{\alpha}_{j,\mathrm{ref}})(\alpha_{j,\mathrm{ref}} - \bar{\alpha}_{j,\mathrm{ref}}) & \alpha_{j,\mathrm{ref}} > \bar{\alpha}_{j,\mathrm{ref}} \end{cases} \end{aligned}$$

with $j = f, r$, where $\bar{\alpha}_{j,\mathrm{ref}}$ is a limit value for the slip angle, above which the control action is needed.

III. Design of a Control Law with Saturation Management

Equations (5) are in the form

$$\begin{aligned} \dot{x}_1 &= f_1(x) + g_1(x)u \\ \dot{x}_2 &= f_2(x) + g_2(x)u \\ \dot{x}_3 &= f_3(x) + g_3(x)u \\ \dot{x}_4 &= x_3 \end{aligned}$$

where f_1, f_2, f_3, g_1, g_2, g_3 are appropriate functions, and $x = (\omega_z \quad v_y \quad \omega_x \quad \alpha_x)^T$, $u = (\varDelta F_{y,f} \quad M_z \quad \varDelta b_x)^T$ are the state and the input vectors. Similarly, the reference equations (6) are in the form

$$\begin{aligned} \dot{x}_{1,\mathrm{ref}} &= f_{1,\mathrm{ref}}(x_{\mathrm{ref}}) + g_{1,\mathrm{ref}}(x_{\mathrm{ref}})w \\ \dot{x}_{2,\mathrm{ref}} &= f_{2,\mathrm{ref}}(x_{\mathrm{ref}}) + g_{2,\mathrm{ref}}(x_{\mathrm{ref}})w \\ \dot{x}_{3,\mathrm{ref}} &= f_{3,\mathrm{ref}}(x_{\mathrm{ref}}) + g_{3,\mathrm{ref}}(x_{\mathrm{ref}})w \\ \dot{x}_{4,\mathrm{ref}} &= x_{3,\mathrm{ref}} \end{aligned}$$

where $x_{\mathrm{ref}} = (\omega_{z,\mathrm{ref}} \quad v_{y,\mathrm{ref}} \quad \omega_{x,\mathrm{ref}} \quad \alpha_{x,\mathrm{ref}})^T$ are the reference states and $w = (\varDelta F_{y,f,\mathrm{ref}} \quad 0 \quad 0)^T$ is the exogenous input, imposed by the driver through the steering wheel.

Physical limitations on actuators impose that $u \in U \subset \mathbb{R}^3$, where

$$U = \big[u_{1,\min}, u_{1,\max}\big] \times \big[u_{2,\min}, u_{2,\max}\big] \times \big[u_{3,\min}, u_{3,\max}\big]$$

is a compact set, and $u_{i,\min}$, $u_{i,\max}$, $i = 1, 2, 3$, are the lower and upper–bounds of the AFS, RTV, SAS actuators. Clearly, when $u \in U$, a feedback ensuring the exponential stability of the tracking error $e = x - x_{\mathrm{ref}}$ is [15]

$$\begin{aligned} u = \beta(x, x_{\mathrm{ref}}, w) &= g^{-1}(x)\Big(- f(x) + f_{\mathrm{ref}}(x) \\ &\quad + g_{\mathrm{ref}}(x)w + K(x - x_{\mathrm{ref}})\Big) \end{aligned} \tag{7}$$

where $f = (f_1 \quad f_2 \quad f_3)^T$, $f_{\mathrm{ref}} = (f_{1,\mathrm{ref}} \quad f_{2,\mathrm{ref}} \quad f_{3,\mathrm{ref}})^T$, $g = (g_1 \quad g_2 \quad g_3)^T$, $g_{\mathrm{ref}} = (g_{1,\mathrm{ref}} \quad g_{2,\mathrm{ref}} \quad g_{3,\mathrm{ref}})^T$, $K = \mathrm{diag}\{k_1, k_2, k_3\}$, $k_i > 0$, $i = 1, 2, 3$. The remainder of the paper will deal with the case in which $u \notin U$, i.e. when actuator saturations occur. Two solutions will be given. In the first, the input will be modified by limiting functions; in the second, a modification of the reference signal x_{ref} is proposed when the saturation occurs. In both cases, one prevents the occurrence of input saturation.

A. Saturation Prevention with Limiting Functions

In practical cases, the control (7) is not the law that is really implemented. The implemented control is rather determined by a limiting function σ. We first introduce the one–sided version of limiting function, whose domain is over positive real numbers, and then its extension to two–sided limiting functions.

Definition 1 (One–sided limiting function): Given a real number $c > 0$, a function $\sigma_c : \mathbb{R}_0^+ \to \mathbb{R}_0^+$ of class C^1 is limiting if the following conditions are satisfied.

a. $\sigma_c(0) = 0$ and $\sigma_c(u) > 0$ for any $u > 0$;
b. $\lim_{u \to +\infty} \sigma_c(u) = c$;

c. $\sigma_c'(0) = 1$ and $\sigma_c'(u) \geq 0$ for any $u > 0$;
d. $\sigma_c''(u) \leq 0$ for any $u \geq 0$. ◇

Note that the previous properties imply that $\sup_{u \geq 0} \sigma_c(u) = c$, so the input value is actually limited. The monotonicity properties on the derivatives are needed to ensure an appropriate behavior in working conditions. Examples of one–sided limiting functions are the following

$$\sigma_c(x) = \frac{cx}{x + c}$$
$$\sigma_c(x) = \frac{2c}{\pi} \arctan \frac{\pi x}{2c}$$
$$\sigma_c(x) = c \sin\left(b \arctan\left(\frac{x}{bc}\right)\right), \quad b \in (0, 1].$$

A limiting function is an extension of the previous definition, taking into account negative input values, and with a further parameter α, representing the value where the limiting action actually starts.

Definition 2 (Limiting function): For any chosen parameters $c_{\min} < 0 < c_{\max}$, $\alpha \in [0, 1)$, a function $\sigma^{\alpha}_{(c_{\min}, c_{\max})} : \mathbb{R} \to \mathbb{R}$ is a limiting function if it has the following form

$$\sigma^{\alpha}_{(c_{\min}, c_{\max})}(u) = \begin{cases} -\sigma_{(\alpha-1)c_{\min}}(-u + \alpha c_{\min}) + \alpha c_{\min} & u < \alpha c_{\min} \\ u & \alpha c_{\min} \leq u \leq \alpha c_{\max} \\ \sigma_{(1-\alpha)c_{\max}}(u - \alpha c_{\max}) + \alpha c_{\max} & u > \alpha c_{\max} \end{cases}$$

where $\sigma_{(\alpha-1)c_{\min}}$ and $\sigma_{(1-\alpha)c_{\max}}$ are one–sided limiting functions.

The previous definition implies that the function is actually limited between the given values $c_{\min}$ and $c_{\max}$, so the input value gets actually bounded. Note that a limiting function is C^1. The parameter α takes the role of "percentage of actuation", above which the behavior of this function is "smoothed" with respect to the linear saturation function

$$\sigma_{(c_{\min}, c_{\max})}(u) = \begin{cases} c_{\min} & \text{if } u < c_{\min} \\ u & \text{if } u \in [c_{\min}, c_{\max}] \\ c_{\max} & \text{if } u > c_{\max} \end{cases}$$

The function $\sigma_{(c_{\min}, c_{\max})}$ is the classical saturation characteristic of a real actuator: the actuator gives the maximal control action that is capable to supply. Although this is not always the best choice to prevent actuator saturation [4], the linear saturation function can be approximated arbitrarily well by any limiting function $\sigma^{\alpha}_{(c_{\min}, c_{\max})}$, if α is sufficiently close to 1

$$\lim_{\alpha \to 1^-} \sup_{u \in \mathbb{R}} \left| \sigma^{\alpha}_{(c_{\min}, c_{\max})}(u) - \sigma_{(c_{\min}, c_{\max})}(u) \right| = 0.$$

B. Reference Modification

When the actuators are approaching saturation, it is not possible to ensure the asymptotic tracking of the reference. This occurs, for instance in the case of the control (7), when $|g(x)|$ approaches zero. On top of limiting functions, introduced in the previous section, a modification of the reference to be tracked is now considered, given by

$$\bar{x}_{\text{ref}} = \Lambda x_{\text{ref}}$$

where $\Lambda = \operatorname{diag}\{\lambda_1, \lambda_2, \lambda_3\}$, and

$$0 \leq \lambda_{i,\min} \leq \lambda_i \leq 1, \quad i = 1, 2, 3. \tag{8}$$

Note that the choice $\lambda_{i,\min} = 0$ turns the tracking problem into a stabilization problem for the i^{th} state. The dynamics of the new tracking error $e = x - \bar{x}_{\text{ref}}$ is

$$\begin{aligned} \dot{e} &= f(x) + g(x)\bar{u}(x, x_{\text{ref}}, w) \\ &\quad - \Lambda\big(f_{\text{ref}}(x_{\text{ref}}) + g_{\text{ref}}(x_{\text{ref}})w\big) - \dot{\Lambda} x_{\text{ref}} \end{aligned}$$

when a control law $u = \bar{\beta}$ is applied, for instance the control (7), or its limited version. In the case of actuator saturation, the dynamics for $\lambda = (\lambda_1 \quad \lambda_2 \quad \lambda_3)^T$ is imposed to be

$$\begin{aligned} \dot{\lambda} &:= \gamma(\Lambda, x, x_{ref}, w) \\ &= R^{-1}\Big[Ke + f(x) + g(x)\beta(x, x_{\text{ref}}, w) \\ &\qquad - \Lambda\big(f_{\text{ref}}(x_{\text{ref}}) + g_{\text{ref}}(x_{\text{ref}})w\big)\Big] \end{aligned} \tag{9}$$

with K a fixed Hurwitz matrix, and $R = \operatorname{diag}\{x_{\text{ref},1}, x_{\text{ref},2}, x_{\text{ref},3}\}$. These dynamics are well defined provided that $|R| \neq 0$, i.e. the references are nonzero. This is the case during a maneuver, except in time instants of null measure, in which an appropriate threshold can be used to prevent the singularity.

The use of (9) during actuator saturation ensures that the tracking error tends to zero exponentially. Since the dynamics of λ might be unstable, or the components of λ could assume negative values (and this has to be avoided, in order to prevent vehicle behaviors not physically acceptable), one imposes the constraint (8). The resulting saturation management is illustrated by the hybrid system of Figure 1, showing the application of both limiting functions and reference modification.

IV. Simulation Results

In this section, we provide simulation results of the proposed control technique. We consider two simulation sets. First, we compare the proposed integrated solution with a linearizing feedback controller [11], [12] based on linear and limiting saturation functions. Second, we show the improvement ensured by the hybrid saturation management scheme with respect to a classical fixed saturation scheme.

The parameters of the vehicle are

$m = 1550$ kg	$m_s = 150$ kg	$\mu = 1$
$l_f = 1.17$ m	$l_r = 1.43$ m	$h_d = 0.5$ m
$J_z = 2300$ kg m^2	$J_x = 350$ kg m^2	$J_{zx} = 50$ kg m^2
$A_{yf} = 1.81$	$B_{yf} = 7.2$	$C_{yf} = 8854$
$A_{yr} = 1.68$	$B_{yr} = 11$	$C_{yr} = 8394$
$k_x = 150,000$ Nm/rad	$b_{x,0} = 7,000$ N m rad/s	
$\gamma_f = -0.05$	$\gamma_r = 0.05$	

while for the reference generation we have considered the following set of values

$$\begin{aligned} &J_{z,\text{ref}} = J_z & &\bar{\alpha}_{f,\text{ref}} = 0.08 \\ &\bar{\alpha}_{r,\text{ref}} = 0.04 \;\; k_{x,\text{ref}} = k_x & &b_{x,\text{ref}} = 11800. \end{aligned}$$

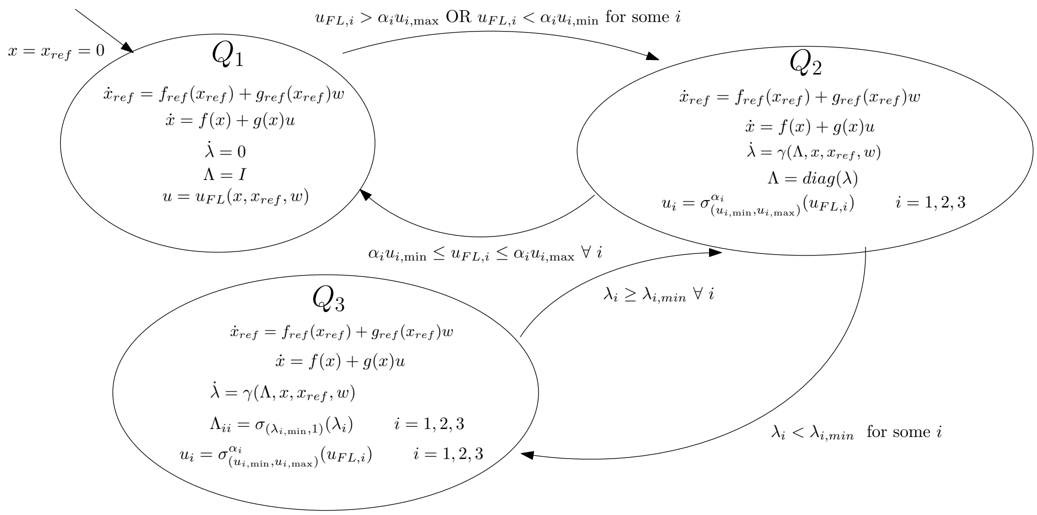

Fig. 1. Hybrid Model of vehicle controlled with linearizing feedback, limiting functions and reference modification. In the first state, the nominal reference is tracked exponentially. In the second state, at least one of the actuators is getting to saturation, so the reference is modified accordingly and the linearizing feedback is limited. In the third case, the reference modification reaches its limiting point for at least one of the state variables.

The control inputs are restricted to the following intervals (see [8])

$$\Delta F_{y,f} \in [-0.95C_{yf} - F_{y,f,0}, 0.95C_{yf} - F_{y,f,0}] \text{ N}$$
$$M_z \in [-10000, 10000] \text{ Nm}$$
$$\Delta b_x \in [-2500, 35000] \text{ N m rad/s.}$$

The parameters $\alpha_1 = \alpha_2 = \alpha_3$ are fixed to 0.8.

A. Simulation 1

The first test considered is a double step steer of $120°$ with longitudinal velocity of 33 m/s (118 km/h), that is a maneuver causing actuators saturation. The errors of the controllers on the three state variables with respect to the reference values are shown in Figure 2. They show that, outside of the linear part of the limiting functions, the errors decrease. Figure 3 shows the control inputs: when using the limiting functions, their values are lower than in the linear function case, even if the trends are similar. This first simulation shows that, when actuators work in saturation conditions, the limiting functions improve the performances.

B. Simulation 2

In this subsection we compare the proposed controller, that uses limiting functions and variable references, with the nominal controller. The maneuver considered is a step steer of $100°$ with longitudinal velocity of 30 m/s (108 km/h). To show that the proposed controller changes dynamically the reference, and thus the control that is calculated and applied, we consider a parametric variation with respect to the nominal case. The road–tire friction coefficient μ is a parameter that appears in all the dynamic equations of the vehicle, and its variation provides important information about the robustness of the controller. We suppose a variation of this coefficient in the vehicle equal to -40% with respect

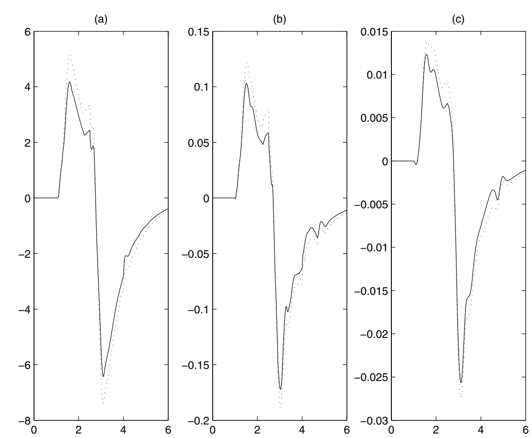

Fig. 2. (a) $e_{\omega_z} = \omega_{z,\text{ref}} - \omega_z$ with limiting function (solid), $e_{\omega_z} = \omega_{z,\text{ref}} - \omega_z$ with linear saturation function (dotted) [deg/s] vs time [s], (b) $e_{v_y} = v_{y,\text{ref}} - v_y$ with limiting function (solid), $e_{v_y} = v_{y,\text{ref}} - v_y$ with linear saturation function (dotted) [m/s] vs time [s], (c) $e_{\omega_x} = \omega_{x,\text{ref}} - \omega_x$ with limiting function (solid), $e_{\omega_x} = \omega_{x,\text{ref}} - \omega_x$ with linear saturation function (dotted) [m/s] vs time [s]

to the value used by the controller. Figure 4 shows that the lateral velocity trends are very close and the proposed controller varies the reference, and that it can pursue the ideal state trajectory. The new reference is perfectly tracked, while the nominal controller presents large tracking errors.

Conclusions

In this paper, we have proposed a hybrid controller for actuator saturation management, in the attitude control of a vehicle with roll dynamics. The resulting hybrid saturation management relies on a mechanism to prevent the occurrence of input saturation, based on input limiting functions, and/or on a modification of the reference signal. This hybrid control can manage critical conditions, when multiple actuator saturations occur. The simulations results show the improvement

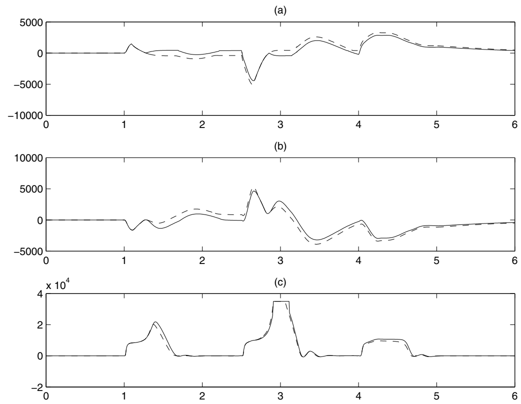

Fig. 3. (a) $F_{y,f}$ with limiting function (solid), $F_{y,f}$ with linear saturation function (dashed) [N] vs time [s]; (b) M_z with limiting function (solid), M_z with linear saturation function (dashed) [Nm] vs time [s]; (c) b_x with limiting function (solid), b_x with linear saturation function (dashed) [Nm rad/s] vs time [s]

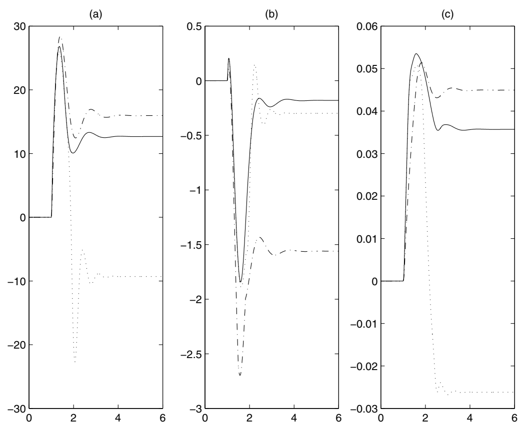

Fig. 4. (a) $\omega_{z,\mathrm{ref}}$ (dash–dot), ω_z with limiting function and variable reference (solid), ω_z with linear saturation function (dotted) [deg/s] vs time [s]; (b) $v_{y,\mathrm{ref}}$ (dash–dot), v_y with limiting function and variable reference (solid), v_y with linear saturation function (dotted) [m/s] vs time [s]; (c) $\omega_{x,\mathrm{ref}}$ (dash–dot), ω_x with limiting function and variable reference (solid), ω_x with linear saturation function (dotted) [m/s] vs time [s]

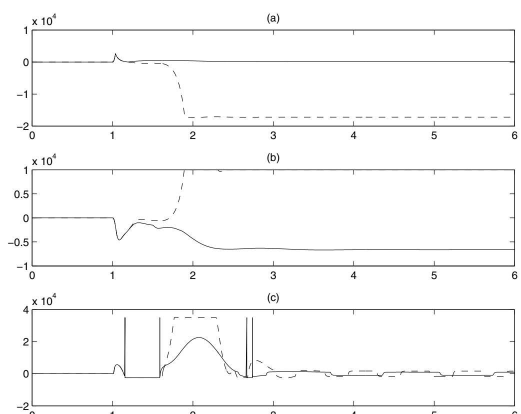

Fig. 5. (a) $F_{y,f}$ with limiting function and variable reference (solid), $F_{y,f}$ with linear saturation function (dashed) [N] vs time [s]; (b) M_z with limiting function and variable reference (solid), M_z with linear saturation function (dashed) [Nm] vs time [s]; (c) b_x with limiting function and variable reference (solid), b_x with linear saturation function (dashed) [Nm rad/s] vs time [s]

of the proposed solution with respect to existing controllers and simpler saturation schemes.

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