2011 50th IEEE Conference on Decision and Control and
European Control Conference (CDC-ECC)
Orlando, FL, USA, December 12-15, 2011

State Estimation for MIMO Hybrid Dynamical Model

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Abstract— **It is well-known that the parameter estimation is difficult in many real-world application where continuous non-linear dynamics interact with discrete-event ones. In this paper, we address the problem of state estimation in hybrid systems exhibited by a mix of continuous time dynamics, discrete-time and discrete-event dynamics. We also demonstrate that a set of parameters is identifiable if the operating modes are detectable and observable. An application will illustrate the results proposed.**

Index Terms— **MIMO Hybrid system, state estimation, observability, identifiability, detectability, persistently exciting.**

I. Introduction

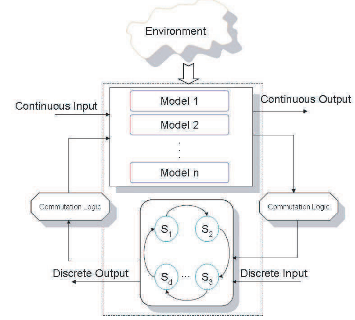
The question of identifiability is central to system identification [1], as it sets the boundaries of applicability of any system identification method. In other words, no system identification algorithm can properly estimate the parameters of a system which is not identifiable. For instance, in power systems [2], system-wide measurements of disturbances are frequently used in post-mortem analysis to gain a better understanding of system behaviour [3].

For the hybrid dynamical systems (HDS), the parameter estimation is one more complex task due to the interactions between continuous (smooth or none) and discrete dynamics [4], [5], [6]. Therefore, two alternative approaches is distinguished in the literature for HDS: either the operating conditions are fixed a priori, or they are estimated both with the sub-models. In the first case, finding a model representing the behavior of the system by observing a set of input-output data is straightforward, because data classification is easy to build, and estimation of the sub-models can be carried out by standard linear identification techniques as it shown in [8] and also argued in [9]. In the second case, the regions must be shaped to the clusters of data, and the strict relation among data classification, parameter estimation and regions for estimation must be verified.

In the literature we can find two subclasses of models to approximate continuous phenomena in nonlinear dynamics, namely Hinging hyperplanes ARX (HHARX) [12], [11], [7], and Piece Wise ARX (PWARX) models. These can be also solved via Mixed-Integer Linear Programming/ Mixed-Integer Quadratic Programming (MILP/MIQP) [13].

In this paper, a hybrid system defined by a set of linear continuous model interconnected with discrete event atomic model is proposed and analyzed for observation and identification of nonlinear hybrid and/or switched systems (figure 1). The first model is necessary to describe the continuous dynamical of the real process and the second one is used to identify the operating modes of global system. So, we focus on the identifiability and state reconstruction.

Fig. 1. Hybrid Complex System Representation



This paper is organized as follows. Section 2 introduces the class of hybrid systems which interests us and reports several formulations of the identification problem for each model which defines the real process. Section 3 presents the parameter estimation procedure adopted for the class of the system considered. Section 4 discusses on the observability and identifiability of each sub-model. Section 5 gives some experimental results for the identification of an greenhouse system. Conclusions are given in Section 6.

II. Hybrid Dynamical System Models

It is well-known that hybrid dynamical systems are systems composed of several sub-systems interconnected, introducing correlations, being subjected to commutations or to transitions (continuous or discrete). So, we can suggest some structures for the sub-models and nonlinear functions combining commutations and switching between structures. Then we have to define some methods for supervision and control of the main partial models. In this case, several simple sub-models (SS_{m_i}), defined by the following relations, needs to identify :

$$SS_{m_i} : \begin{cases} x(k+1) = f_{m_i}(x,u,w,k), & k \in \mathbb{R} \\ y(k) = g_{m_i}(x,k) \end{cases}, \tag{1}$$

for $m_i = 1,2,...s$ and that each one of these models is valid in some state subspaces $\Omega_{m_i} \subset \mathbb{R}^n$. u, y and w is respectively the set of input, output and noise term of the system. k defines the discrete time.

Let's consider that the nominal behavior of this system is driven by multiple models orchestrated by events governing

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978-1-61284-799-3/11/$26.00 ©2011 IEEE

by system operation. This corresponds to some operating conditions for the main components or sub-systems in coordination of the discrete events. Thus, equation (1) can be writting as follows :

$$SS_{m_i} : \begin{cases} x(k+1) = f_{m_i}(x,u,w,k), \quad k \in \mathbb{R} \\ y(k) = g_{m_i}(x,k) \\ m_i = \langle Mode, I, O, \delta_{int}, \delta_{ext}, \lambda, t_a \rangle \end{cases}, \tag{2}$$

where: f_{m_i} and g_{m_i} are non linear functions, $Mode$ is the set of sequential operating modes. I is the set of external events. O is the set of internal events. $\delta_{int} : Mode \to Mode$ is the internal state transition function. $\delta_{ext} : Mode_{-} \times I \to Mode_{+}$ is the external state transition function. $\lambda : Mode \to O$ is the output function and t_a is the time advance function.

In the linear and stationary cases, the class of studied system is composed by three elements. The first one is a set of behavioral linear models, the second one is a transition or commutation device and the last one is a discrete events supervisor managing the different commutation or transition.

This choice allows the relaxation of the partitioning number of the state-space (studied in [8]) thanks to the use of discrete events atomic models, and increase the possibility of deal with multi-variable inputs and outputs using a piecewise affine models in state-space form. These coupled models can be described by the following relations :

$$\begin{cases} x(k+1) = & A_{m_i}x(k) + B^e_{m_i}e(k) + B^w_{m_i}w(k) + v(k) \\ y(k) = & C_{m_i}x(k) + D^e_{m_i}e(k) + D^w_{m_i}w(k) + \mu(k) \\ m_i = & \langle Mode, I, O, \delta_{int}, \delta_{ext}, \lambda, t_a \rangle \end{cases}, \tag{3}$$

where $v(k) \in \mathbb{R}^n$ and $\mu(k) \in \mathbb{R}^p$ are noise/error terms. $w(k)$ is the input which define the external environment. The real matrices/vectors $A_{m_i}, B^e_{m_i}, B^w_{m_i}, C_{m_i}$, $D^e_{m_i}$ and $D^w_{m_i}$, with appropriate dimensions, describe each affine dynamics. $Mode$ is the operating modes of the system (i.e. the operating zone where each sub-model is available). The discrete system is in $Mode_{m_i}$ when operation is in the region Ω_{m_i}. I is the set of input event values (i.e. all the values that input event can take). The events may be considered as defined by state $x(k)$ of the system and any internal $x(k)$-dependent variable. Then, events depend on input values of the mi associated to valid model noted $I(k)$ [5]. As consequence, the validity domain Ω_{m_i} depends also on this input. O is the set of output event values. λ is the output function which warrants the activity execution. δ_{int} is the internal transition function. It ensures mode switches when no exogenous events come out before elapsed time t_i and time advance of this mode ta [14] : $Mode_j = \delta_{int}(Mode_i, t_i + ta)$. δ_{ext} is the external transit function. This function is used when exogenous events come out [14] : $Mode_j = \delta_{ext}(Mode_i, t_i, I_i)$.

Here, one see that the hybrid nature of this structure requires at each time instant to determine the active local model, estimation of the time switching between two local models, estimation of the current state of the system. Thus, new forms of analysis like observability, and identifiability [15] is necessary because discrete changes are not handled well by continuous algorithm.

III. Identifiability of HDS

This section addresses the problem of state estimation with the structure defined by the expression (3). Let's recall that for behavioral modeling, it is important to know if the structure is well defined, in other words, all the parameters in the model can be estimated accurately with data provided [21]. In the sequel, this observation is now formalized in terms of model identifiability.

Furthermore, it is well-known that the identifiability property is based on the observability one. But the verification of the observability property is another challenging problem [26], [27] due to the complex interactions between the discrete and continuous behavior exhibited by hybrid systems [10] [17] [18]. Thus, in the following paragraph, we define identifiability for HDS and we discuss its relationship with discrete state observations and continuous state output.

A. Discrete State Observability

To check the discrete state observability the following notions is introduced.

Definition 1: A discrete state is observable if the current continuous state can be determined uniquely from the current continuous state output and either the last or next event (resp. $Mode^{-}$ or $Mode^{+}$).

Thus, assume that:

Definition 2: The set of all operating modes $R^p_{I_i}$ where the system can evolve after an event is defined by

$$R^p_{I_i} = \{Mode_i \in Mode \;\; | \;\; \forall Mode_j \in Mode,$$

$$\exists I_i \in \Xi, \quad Mode_j = \delta_{ext}(Mode_i, t_i, I_i)\}$$

Definition 3: The set of all operating modes $R^n_{I_j}$ representing all operating modes reached within an event is defined by

$$R^n_{I_j} = \{Mode_j \in Mode \;\; | \;\; \forall Mode_i \in Mode,$$

$$\exists I_j \in \Xi, \quad Mode_j = \delta_{ext}(Mode_i, t_i, I_i)\}$$

so,

Proposition 1: the discrete event model is observable if:

(i) $\lambda(Mode_i) \neq \lambda(Mode_j)$ if $Mode_i \in \delta_{ext}(Mode_i, t_i, I_i)$ with $i \neq j$, and

(ii) $\left| Mode \cap R^p_{I_i} \right| = 1$, i.e the previous event is available.

(iii) $\left| Mode \cap R^n_{I_i} \right| = 1$, i.e the next event is available.

(iv) $\left| R^p_{I_i} \cap Mode \cap R^n_{I_i} \right| = 1$ i.e the next and the previous events are available.

Proof: The first condition states that the unobservable events can be detected and thus validates that the current state output and the last (or the next) event are available to determine current state. If condition (ii) (resp. (iii), (iv)) is true, there is only one state which has the current state output is reachable through the last event. Thus, the current state can be uniquely reconstructed. ∎

B. Persistent Mode and Detectability

We have seen that all operating modes can be detected (observable modes and/or unobservable modes). Now, if the sequence of discrete mode and switching times are observed and switching signal has suitable large dwell time, then the linear switched systems can be identified by identifying each linear subsystem separately. Therefore, it is well-known that the input/output signals, classified into the mode $Mode_i$, must be changed sufficiently in order to excite the system for the identifiability of each sub-system. This observation can be associated with detectability notion of a persistent mode. Thus, we characterize this notion as follows for the class of system defined by the expression (3).

Definition 4: A persistent operating mode is detectable if we can determine, after a finite number of switching signal observations, the current continuous state of the system for some trajectories of the system.

However, before deriving the Persistent Excitation (PE) constraint for the class of system considerate, some definitions on excitation of input signals are given.

Definition 5: A scalar input signal u is strongly persistently exciting for order n if for all k there exists an integer T such that

$$\rho_1 I > \sum_{i=k}^{k+T} \begin{bmatrix} u_{i+n} \\ u_{i+n-1} \\ \vdots \\ u_{i+1} \end{bmatrix} \begin{bmatrix} u_{i+n} \\ u_{i+n-1} \\ \vdots \\ u_{i+1} \end{bmatrix}' > \rho_0 I, \tag{4}$$

where $\rho_0, \rho_1 > 0$

Definition 6: A scalar input signal u is weakly persistently exciting for order n if

$$\rho_1 I \geq lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} \begin{bmatrix} u_{i+n} \\ u_{i+n-1} \\ \vdots \\ u_{i+1} \end{bmatrix} \begin{bmatrix} u_{i+n} \\ u_{i+n-1} \\ \vdots \\ u_{i+1} \end{bmatrix}' \geq \rho_0 I, \tag{5}$$

where $\rho_0, \rho_1 > 0$

According to the definition 5, a suitable constraint for MIMO-HDS is derived. The implementation is a priori straightforward as follows:

$$\rho_1 I > C_{mi} = \sum_{i=1}^{m} \sum_{j=0}^{N_{mi}} \begin{bmatrix} u^{mi}_{i,j+n} \\ u^{mi}_{i,j+n-1} \\ \vdots \\ u^{mi}_{i,j+1} \end{bmatrix} \begin{bmatrix} u^{mi}_{i,j+n} \\ u^{mi}_{i,j+n-1} \\ \vdots \\ u^{mi}_{i,j+1} \end{bmatrix}' > \rho_0 I, \tag{6}$$

where $\rho_0, \rho_1 > 0$. So if the input u is bounded, there always exists a positive scalar ρ_1, so equation (6) is equivalent to:

$$C_{mi} = \sum_{i=1}^{m} \sum_{j=0}^{N_{mi}} \begin{bmatrix} u^{mi}_{i,j+n} \\ u^{mi}_{i,j+n-1} \\ \vdots \\ u^{mi}_{i,j+1} \end{bmatrix} \begin{bmatrix} u^{mi}_{i,j+n} \\ u^{mi}_{i,j+n-1} \\ \vdots \\ u^{mi}_{i,j+1} \end{bmatrix}' > \rho_0 I, \tag{7}$$

and in a more compact form, we can write :

$$C_{mi} = \sum_{i=1}^{m} \sum_{j=0}^{N_{mi}} \Phi_{mi} \Phi'_{mi} > \rho_0 I, \tag{8}$$

In this equation, m defines the number of the input variable. Well, to tackle with extended PE constraint for HDS, we can formulate the following proposition:

Proposition 2: Each persistent operating mode m_i is detectable, according to the excitation input signal u if:

(i) the discrete event model is observable,

(ii) there exists an integer $\rho_0 > 0$ such that

$$C_{mi} = \sum_{i=1}^{m} \sum_{j=0}^{N_{mi}} \Phi_{mi} \Phi'_{mi} > \rho_0 I,$$

is satisfied.

Proof: This proposition relates the possibility to reconstruct each model associated to each operating mode. In fact, if first condition (i) is satisfied, all operating modes are observable and guaranteed the exact reconstruction of the continuous state. The second condition (ii) states for persistently of whole operating modes, in other words the experimental data contains enough information about the dynamics of the system. Otherwise, a small-scale model can be defined to preserve the detectability and observability while eliminating modes not feasible. ■

C. Continuous State Observability

In this section, we consider observability properties regarding the continuous model. To verify this properties, we recall main results given in [10] [17] [18].

A switching system is observable if:

Theorem 1: $(C(k) - C(k'))A(k)$ is full rank for all $k \neq k' \in \{1,...,N\}$ and the dwell time $\tau_k \geq \varepsilon$ for all $k \geq 0$, then the initial state the switching times are observable if and only if for all $k \neq k' \in \{1,...,N\}$ we have $rank([\mathcal{O}_\varepsilon(k) \quad \mathcal{O}_\varepsilon(k')]) = 2n$.

Unfortunately Bemporad [7] shows through counterexamples that observability properties cannot be easily deduced. Thus, an extension of this theorem is give in [25] to ensure the exact reconstruction of the state. This is based on the detectability and observability notion.

Theorem 2: A switching system is observable if the following conditions are satisfied:

(i) $S(q_i)$ is observable for any $q_i \in Q$

(ii) $\forall p \in \Omega(\gamma), \forall q_i, q_j \in \gamma^{-1}(p), \exists k \in \mathbb{N} \cup \{0\} : C_i A_i^k B_i \neq C_j A_j^k B_j$

Theorem 3: A switching system is detectable if the following conditions are satisfied:

(iii) $\forall p \in \Omega(\gamma), \forall q_i, q_j \in \gamma^{-1}(p), \exists k \in \mathbb{N} \cup \{0\}$: $C_{i1} A^k_{i11} B_{i1} \neq C_{j1} A^k_{j11} B_{j1}$

(iv) for all initial states $\begin{pmatrix} \xi_0 \\ q_0 \end{pmatrix} \in \mathcal{O}_i \times Q$, $i \in J$, and for any $\varepsilon > 0$, there exist $\mathbf{t} > t_0$ such as $\|\xi(t,j)\| \leq \xi$ for any $\mathbf{t} \geq t$.

With regard to the linear state-space model, the reconstruction of the continuous state puts down on the observability of the different behavioral models. Therefore, *Proposition 3:* the continuous state is observable if :

(vi) all linear sub-models are observable,

(vii) observability matrices $(\mathscr{O}^n)$ are full rank.

By combining three propositions (1), (2), (3), one gets the following results on the observability and identifiability of the global system.

For the class of the system considered, with the assumption that the dwell time in operating mode must be a positive number $te_k \geq \varepsilon$ for all $k \geq 0$. [16] :

Proposition 4 (Globally identifiable from measurements): A model M defined by M_i, $i = 1,..,s$ is globally identifiable at $(A^*_{mi}, B^*_{mi}, C^*_{mi}, D^*_{mi})$ from measurements (u, y) if and only if:

(1) each persistent operating mode is detectable,

(2) the switching system is observable.

Proof: The first condition states the observability and detectability of a discrete state, is equivalent to verify the observability of discrete event model and the detectability of a persistent operating mode. The second condition is associated to the observability of continuous state and validate that the whole operating modes are observable, detectable and guaranteed the exact reconstruction of the continuous state. ■

IV. EXPERIMENTAL RESULTS

In this section we give some results about the greenhouse identification. This greenhouse is equipped with many sensors which allow to measure the external and internal temperature, Te and Ti, (data expressed in ^oC), the external and internal hydrometry He and Hi (data expressed in %), the wind velocity Vv (data expressed in m/s) and the global radiation Rg (data expressed in W/m^2). Moreover, many actuators allow to act on the heating Ch, the roofing Ov, the moistening Br and the shutter Rd. We consider one day of March to identify all parameters. Therefore, from expert knowledge and with a data analyse, we retain the variables Rg and Vv like descriptive variables of the environment where our system evolves. The choice of these variables have been done in previous work [19], [20]. Moreover, these variables are necessary to construct transition table I which used for the discretization of the descriptive variables.

TABLE I

ENVIRONMENT SPECIFICATION DISCRETIZATION ($R_1 = 7.73m/s$, $R'_1 = 120W/m^2$ ET $R_2 = 250W/m^2$)

Input \ Values	Lower	Middle	Upper
Vv	$\leq R_1$	$-$	$> R_1$
Rg	$\leq R'_1$	$> R'_1$ and $\leq R'_2$	$> R'_2$

Furthermore, the wind speed will take the following values $Vv = \{VvL, VvU\}$ which are respectively *Lower* and *Upper* values of Vv while the global radiation of the sun will take $Rg = \{RgL, RgM, RgU\}$ which are respectively *Lower*, *Middle* and *Upper* values of Rg. The combination of the descriptive variables values Rg et Vv allows us to have 6 operating modes which are in table II.

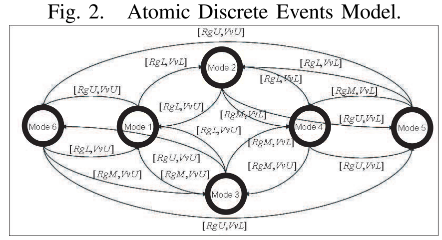
TABLE II

DISCRETE OPERATING MODES OF THE GREENHOUSE SYSTEM.

Mode	Phase	Input values
$Mode_1$	$NightWindLight$	$I_1 = \{RgL, VvU\}$
$Mode_2$	$NightWindStrong$	$I_2 = \{RgL, VvL\}$
$Mode_3$	$BreakWindLight$	$I_3 = \{RgM, VvU\}$
$Mode_4$	$BreakWindStrong$	$I_4 = \{RgM, VvL\}$
$Mode_5$	$DayWindLight$	$I_5 = \{RgU, VvL\}$
$Mode_6$	$DayWindStrong$	$I_6 = \{RgU, VvU\}$

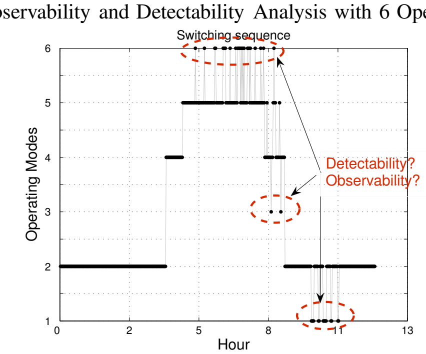
- All operating modes are defined by: $Mode = Mode_1, Mode_2, Mode_3, Mode_4, Mode_5, Mode_6$.
- The input variables defined by the set $I = \{I_1, I_2, I_3, I_4, I_5, I_6\}$, associated respectively by the input events set $ev = \{ev_1, ev_2, ev_3, ev_4, ev_5, ev_6\}$.
- The set of corresponding output events is described by $O = \{O_1, O_2, O_3, O_4, O_5, O_6\}$.

Fig. 2. Atomic Discrete Events Model.



The sub-models are in three categories : *day*, *night*, and *daybreak*. In each category they are two classes : *Cold* and *Fresh*. This leads to six sub-models associated with six operating modes. The atomic model associated with this system is illustrated in the figure (2). The simulation done with the discrete events atomic model allows us to check the detectability and observability of all operating modes of the system and the corresponding time range (figure(3)). This result shows us a priori that our system is completely observable (resp. identifiable). To according

Fig. 3. Observability and Detectability Analysis with 6 Operating Modes.



with proposition (4) given in the last section, our system looses its observability for several not persistently operating modes. For instance, modes 6, 3 and 1. However, a small-scale model can be defined to preserve the observability and the identifiability while eliminating modes not feasible.

Therefore, the discreet events atomic model is composed of operating modes defined by $Mode^* = \{Mode_1^*, Mode_2^*, Mode_3^*\}$, input variables defined by the set $I^* = \{I_1^*, I_2^*, I_3^*\}$ associated respectively by the input events set $ev^* = \{ev_1^*, ev_2^*, ev_3^*\}$ and the set of output events is described by $O^* = \{O_1^*, O_2^*, O_3^*\}$. The combination of the descriptive variables values Rg et Vv allows us to have in this case 3 persistent operating modes which are in table III.

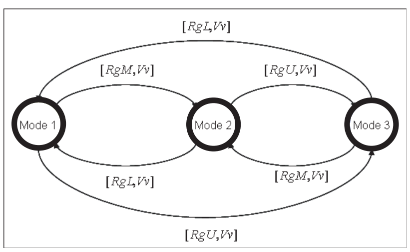
TABLE III

NOVEL DISCRETE MODE DESIGNATION OF THE SUPERVISION DEVICE

Mode	Phase	Input variable
$Mode_1^*$	*FreshNight*	$I_1^* = \{RgL, VvL\}$
$Mode_2^*$	*FreshDaybreak*	$I_2^* = \{RgM, VvL\}$
$Mode_3^*$	*ColdDay*	$I_3^* = \{RgU, VvL\}$

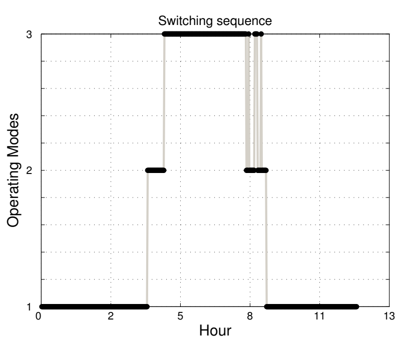
Thus, we have the following models for each mode as illustrated in figure 4.

Fig. 4. Atomic Discrete Events Model with Observability and Identifiability Constraints.



Now, we can identify the continuous dynamical models parameters according to the algorithm described above. The switching sequence used for the parameter estimation procedure is illustrated in figure 5 where the persistent operating modes are detectable.

Fig. 5. Equivalent Operating Modes Commutation for the Identification Procedure.



The identification result is given by the following:

for the mode: $Mode_1^*$

$$A_{Mode_1^*} = \begin{bmatrix} -0.2128 & 0.0819 \\ 0.1259 & -0.2162 \end{bmatrix}; \; C_{Mode_1^*} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix};$$

$$B^u_{Mode_1^*} = \begin{bmatrix} 0.1469 & 0 & 0 & 0 \\ -0.1480 & 0 & 0 & 0 \end{bmatrix};$$

$$B^v_{Mode_1^*} = \begin{bmatrix} 0.0848 & -0.0679 & -0.1323 & 0 \\ 0.0109 & 0.1365 & 0.1780 & 0 \end{bmatrix};$$

for the mode : $Mode_2^*$

$$A_{Mode_2^*} = \begin{bmatrix} -0.2694 & -0.3000 \\ 0.3795 & -0.4274 \end{bmatrix}; \; C_{Mode_2^*} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix};$$

$$B^u_{Mode_2^*} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix};$$

$$B^v_{Mode_2^*} = \begin{bmatrix} 0.3946 & 0.3160 & -0.0630 & 0.1335 \\ 0.5616 & 0.6740 & -0.1797 & -0.9603 \end{bmatrix};$$

and **for the mode :** $Mode_3^*$

$$A_{Mode_3^*} = \begin{bmatrix} -0.3235 & -0.0513 \\ -0.2581 & -1.1357 \end{bmatrix}; \; C_{Mode_3^*} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix};$$

$$B^u_{Mode_3^*} = \begin{bmatrix} 0 & -0.1304 & -0.2003 & -0.1020 \\ 0 & 0.5993 & -0.5851 & 0.0743 \end{bmatrix};$$

$$B^v_{Mode_3^*} = \begin{bmatrix} 0.3186 & 0.1036 & -0.0076 & 0.2572 \\ 0.9125 & 0.9130 & 0.2010 & -0.1596 \end{bmatrix};$$

In the experimentation, we have considered three days of March whose behaviors are not similar to validate our models. The figure 6 gives us the result of simulation between the internal temperature and hydrometry of the estimated and measured values. For the same way, we can show the effectiveness of this approach, in modeling case while considering only 3 modes where our system is observable and detectable. The performance is given by

$$VAF = 1 - \frac{var(y(k) - \hat{y}(k))}{var(y(k))} * 100, \tag{9}$$

and if we compare the performance between the hybrid dynamical model and a single global model we obtain the following results (Table IV):

TABLE IV

PERFORMANCE INDEX FOR PARAMETERS ESTIMATION

Used model	VAF-11 March	VAF-12 March	VAF-15 March
Hybrid model	80.9783	47.0203	79.6342
Single model	63.0511	0	74.3790

To talk about of the observability, one verify if $C_i * A_i^k * B_i \neq C_{i^*} * A_{i^*}^k * B_{i^*}$.

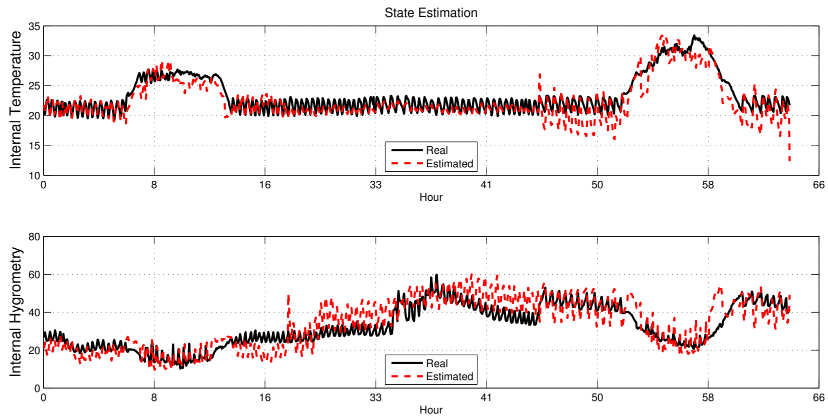
	$C_1 * A_1^k * B_1$	$C_2 * A_2^k * B_2$	$C_3 * A_3^k * B_3$
$k=1$	0.0071	0	0.1466
$k=2$	-0.003	0	-0.092
$k=3$	0.001	0	0.0824
$k=4$	-0.000337	0	-0.0876

To conclude our investigation about the observability analysis, it is necessary to check the rank of the observability matrix of each mode. One gets $rank(\mathscr{O}_{mi}^n)$ will be always equal to the sub-space dimension of the observable state $n = 2$. Thus, the system is locally observable and also globally observable between switching sequence.

V. CONCLUSIONS

In this paper an approach to modeling a MIMO hybrid dynamical system is proposed. The class of system is defined by an interconnection between discrete atomic model with a

Fig. 6. Modeling and Identification Validation.



set of state-space linear models. The atomic model is used to identify the operating modes where the system evolves, and the state-space linear models to approximate continuous phenomena in a nonlinear and not stationary dynamical system.

Also, the parameter estimation procedure is addressed for the class of system considered. Observability and identifiability properties are discussed and some results are shown to recover a best fitting and persistently model from data. Results obtained on the experimental greenhouse demonstrate the effectiveness of the methodology.

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