2011 50th IEEE Conference on Decision and Control and
European Control Conference (CDC-ECC)
Orlando, FL, USA, December 12-15, 2011

A Gait Generation Method for the Compass-type Biped Robot on Slopes via Discrete Mechanics

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***Abstract*— In this paper, we develop a discrete mechanics approach to gait generation on slopes for the compass-type biped robot. We formulate a optimal gait generation problem for the discrete compass-type robot and show a solving method of it by the sequential quadratic programming to calculate a discrete control input. Then, we propose a transformation method from a discrete control input into a continuous zero-order hold input based on discrete Lagrange-d'Alembert principle. As a result of numerical simulations, it is confirmed that stable gaits on both downward and upward slopes can be generated for the continuous compass-type robot by the proposed method.**

I. INTRODUCTION

In the fields of robotics and control theory, humanoid robots have become attractive research objects and a lot of work on them have been done so far. In particular, the compass-type biped robot has been mainly studied as one of the simplest models of humanoid robots. For example, theoretical analysis of passive walking [8], researches associated with nonlinear mechanics such as Poincáre section and limit cycles [9], gait pattern generation based on ZMP (zero-moment point) [11], self-motivating acquirement of gaits by learning theory and evolutionary computing. However, in general, it is quite difficult to realize stable gaits for humanoid robots in terms of nonlinear problems, and hence there is still a lot of problems left to solve.

In almost every work on humanoid robots, a model based on continuous-time mechanics is used. On the other hand, *discrete mechanics*, which is a new discretizing tool for nonlinear mechanical systems and is derived by discretization of basic principles and equations of classical mechanics, has been focused on [1], [2], [3], [4], [5], [6]. a discrete model (the discrete Euler-Lagrange equations) in discrete mechanics has some interesting characteristics; (i) less numerical error in comparison with other numerical solutions such as Euler method and Runge-Kutta method, (ii) it can describe energies for both conservative and dissipative systems with less errors, (iii) some laws of physics such as Noether's theorem are satisfied. (iv) simulations can be performed for large sampling times. Hence, discrete mechanics has a possibility of analysis and controller synthesis with high compatibility with computers.

We have focused on discrete mechanics and considered its applications to control theory. In [12], [13], [14], we applied discrete mechanics to control problems for the cart-pendulum system, and confirmed the application potentiality to control theory. Moreover, in [15], [16], we have considered a gait generation problem for the compass-type biped robot and confirmed that the proposed method can generate stable gaits on flats. However, the method cannot be applied to gait generation problems on more complex grounds such as slopes, stairs and irregular grounds.

In this paper, we deal with gait generation problems for the compass-type biped robot on slopes from the standpoint of discrete mechanics. The contents of this paper is as follows. In Section II, some fundamental concepts on discrete mechanics are summed up. Next, we derive the continuous and discrete compass-type biped robots based on both continuous and discrete mechanics, respectively in Section III. Then, in Section IV, we formulate a gait generation problem for the discrete compass-type biped robot and propose a solving method of it by the sequential quadratic programming to calculate a discrete control input. Furthermore, a transformation method from a discrete control input into a continuous zero-order hold input based on discrete Lagrange-d'Alembert principle is developed. In Section V, we show some numerical simulations on gait generation on downward and upward slopes for the continuous compass-type biped robot in order to confirm the effectiveness of our method.

II. DISCRETE MECHANICS

This section summarizes fundamental concepts of discrete mechanics. See [1], [2], [3], [4] for more details. Let Q be an n-dimensional configuration manifold and $q \in \mathbf{R}^n$ be a generalized coordinate of Q. We also refer to T_qQ as the tangent space of Q at a point $q \in Q$ and $\dot{q} \in T_qQ$ denotes a generalized velocity. Moreover, we consider a time-invariant Lagrangian as $L^c(q, \dot{q}) : TQ \to \mathbf{R}$. We first explain about the discretization method. The time variable $t \in \mathbf{R}$ is discretized as $t = kh \ (k = 0, 1, 2, \cdots)$ by using a sampling interval $h > 0$. We denote q_k as a point of Q at the time step k, that is, a curve on Q in the continuous setting is represented as a sequence of points $q^d := \{q_k\}_{k=1}^N$ in the discrete setting. The transformation method of discrete mechanics is carried out by the replacement:

$$q \approx (1-\alpha)q_k + \alpha q_{k+1}, \ \ \dot{q} \approx \frac{q_{k+1} - q_k}{h}, \tag{1}$$

where q is expressed as a internally dividing point of q_k and q_{k+1} with an internal division ratio $\alpha \ (0 < \alpha < 1)$ We then define *a discrete Lagrangian*:

$$L_\alpha^d(q_k, q_{k+1}) := hL\left((1-\alpha)q_k + \alpha q_{k+1}, \frac{q_{k+1} - q_k}{h}\right), \tag{2}$$

T. Kai is with Faculty of Information Science and Electrical Engineering, Kyushu University, JAPAN kai@ees.kyushu-u.ac.jp. T. Shintani is with Kyocera Corporation, JAPAN and *a discrete action sum*:

$$S^d_\alpha(q_0, q_1, \cdots, q_N) = \sum_{k=0}^{N-1} L^d_\alpha(q_k, q_{k+1}). \tag{3}$$

We next summarize the discrete equations of motion. Consider a variation of points on Q as $\delta q_k \in T_{q_k}Q$ $(k = 0, 1, \cdots, N)$ with the fixed condition $\delta q_0 = \delta q_N = 0$. In analogy with the continuous setting, we define a variation of the discrete action sum (3) as

$$\delta S^d_\alpha(q_0, q_1, \cdots, q_N) = \sum_{k=0}^{N-1} \delta L^d_\alpha(q_k, q_{k+1}). \tag{4}$$

The discrete Hamilton's principle states that *only a motion which makes the discrete action sum (3) stationary is realized*. Calculating (4), we have

$$\delta S^d_\alpha = \sum_{k=1}^{N-1} \{D_1 L^d_\alpha(q_k, q_{k+1})\delta q_k + D_2 L^d_\alpha(q_{k-1}, q_k)\}\delta q_k, \tag{5}$$

where D_1 and D_2 denotes the partial differential operators with respect to the first and second arguments, respectively. Consequently, from the discrete Hamilton's principle and (5), we obtain *the discrete Euler-Lagrange equations*:

$$D_1 L^d_\alpha(q_k, q_{k+1}) + D_2 L^d_\alpha(q_{k-1}, q_k) = 0, \quad k = 1, \cdots, N-1 \tag{6}$$

with the initial and terminal equations:

$$\begin{aligned} &D_2 L^c(q_0, \dot{q}_0) + D_1 L^d_\alpha(q_0, q_1) = 0 \\ &- D_2 L^c(q_N, \dot{q}_N) + D_2 L^d_\alpha(q_{N-1}, q_N) = 0. \end{aligned} \tag{7}$$

It turns out that (6) is represented as difference equations which contains three points q_{k-1}, q_k, q_{k+1}, and we need q_0, q_1 as initial conditions when we simulate (6).

Then, we consider a method to add external forces to the discrete Euler-Lagrange equations. By an analogy of continuous mechanics, we denote discrete external forces by $F^d : Q \times Q \to T^*(Q \times Q)$, and discretize continuous Lagrange-d'Alembert's principle as

$$\delta \sum_{k=0}^{N-1} L^d_\alpha(q_k, q_{k+1}) + \sum_{k=0}^{N-1} F^d(q_k, q_{k+1}) \cdot (\delta q_k, \delta q_{k+1}) = 0, \tag{8}$$

where we define *right/left discrete external forces*: F^{d+}, $F^{d-} : Q \times Q \to T^*Q$ as

$$\begin{aligned} F^{d+}(q_k, q_{k+1})\delta q_k &= F^d(q_k, q_{k+1}) \cdot (\delta q_k, 0), \\ F^{d-}(q_k, q_{k+1})\delta q_{k+1} &= F^d(q_k, q_{k+1}) \cdot (0, \delta q_{k+1}), \end{aligned} \tag{9}$$

respectively. By right/left discrete external forces, a continuous external force $F^c : TQ \to T^*Q$ can be discretized as

$$\begin{aligned} F^{d+}(q_k, q_{k+1}) &= (1-\alpha)hF^c\left((1-\alpha)q_k + \alpha q_{k+1}, \frac{q_{k+1} - q_k}{h}\right), \\ F^{d-}(q_k, q_{k+1}) &= \alpha h F^c\left((1-\alpha)q_k + \alpha q_{k+1}, \frac{q_{k+1} - q_k}{h}\right). \end{aligned} \tag{10}$$

Therefore, by calculating variations for (8), we obtain *the discrete Euler-Lagrange equations with discrete external forces*:

$$\begin{aligned} &D_1 L^d(q_k, q_{k+1}) + D_2 L^d(q_{k-1}, q_k) \\ &\quad + F^{d+}(q_k, q_{k+1}) + F^{d-}(q_{k-1}, q_k) = 0, \\ &\quad k = 1, \cdots, N-1, \end{aligned} \tag{11}$$

with the initial and terminal equations:

$$\begin{aligned} &D_2 L^c(q_0, \dot{q}_0) + D_1 L^d_\alpha(q_0, q_1) + F^{d+}_\alpha(q_0, q_1) = 0 \\ &- D_2 L^c(q_N, \dot{q}_N) + D_2 L^d_\alpha(q_{N-1}, q_N) \\ &\quad + F^{d-}_\alpha(q_{N-1}, q_N) = 0. \end{aligned} \tag{12}$$

III. COMPASS-TYPE BIPED ROBOT

A. Setting of Compass-type Biped Robot

In this subsection, we first give a problem setting of the compass-type biped robot. In this paper, we consider a simple compass-type biped robot which consists of two rigid bars (Leg 1 and 2) and a joint without rotational friction (Waist) as shown in Fig. 1. In Fig. 1, Leg 1 is called *the supporting leg* which connects to ground and Leg 2 is called *the swing leg* which is ungrounded. Moreover, for the sake of simplicity, we give the following assumptions; (i) the supporting leg does not slip at the contact point with the ground, (ii) the swing leg hits the ground with completely inelastic collision, (iii) the compass-type biped robot is supported by two legs for just a moment, (iv) the length of the swing leg gets smaller by infinitely small when the swing leg and the supporting leg pass each other. Let θ and ϕ be the angles of Leg 1 and 2, respectively. We also use the notations: m: the mass of the legs, M: the mass of the waist, I: the inertia moment of the legs, a: the length between the waist and the center of gravity, b: the length between the center of gravity and the toe of the leg, $l\ (= a + b)$: the length between the waist and the toe of the leg.

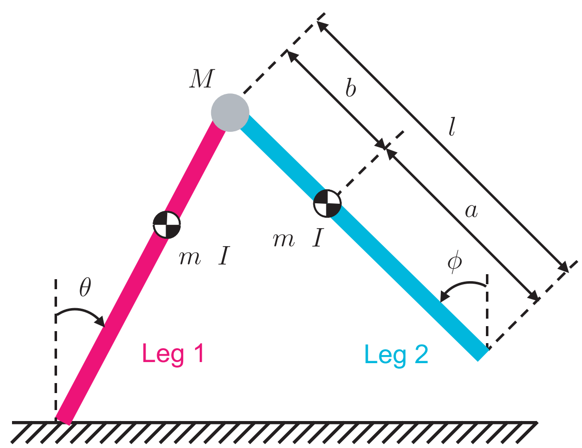

Fig. 1 : Compass-Type Biped Robot

In the walking process of the compass-type biped robot, there exist two modes: *the swing phase* and *the impact phase*. In the swing phase the swing leg is ungrounded, and in the impact phase the toe of the swing leg hit the ground. As shown in Fig. 2, it is noted that the swing phase and the impact phase occur alternately and the swing leg and the supporting leg switch positions with each other with respect to each collision. We denote the order of the swing phase and the impact phase by $i = 1, 2, \cdots, L$ and $i = 1, 2, \cdots, L-1$, respectively. In addition, we assume that Leg 1 is the swing leg and Leg 2 is the supporting leg in odd-numbered swing phases, and Leg 1 is the supporting leg and Leg 2 is the swing leg in even-numbered swing phases.

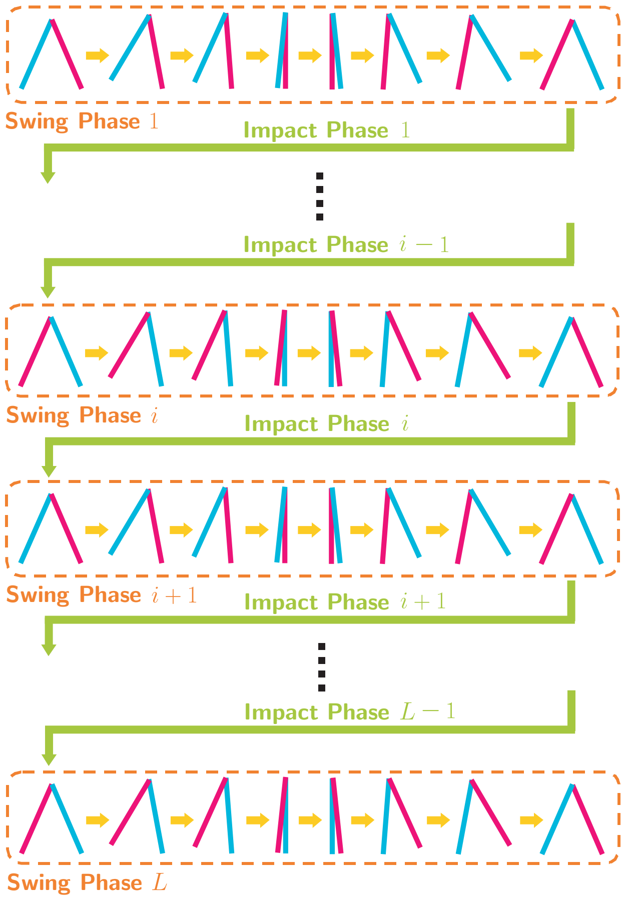

Fig. 2 : Gait of Compass-type Biped Robot

B. Continuous Compass-type Biped Robot (CCBR)

In this subsection, we derive a model of *continuous compass-type biped robot (CCBR)* via usual continuous mechanics. We denote the angles of Leg 1 and 2 in the i-th swing phase by $\theta^{(i)}$, $\phi^{(i)}$, respectively. In addition, $\dot{\theta}^{(i)}$, $\dot{\phi}^{(i)}$ denote their angular velocities.

First, we consider a model of the CCBR in the i-th swing phase where Leg 1 is the supporting leg and Leg 2 is the swing leg. We assume that the torque at the waist can be controlled, and denote it by $v^{(i)} \in \mathbf{R}$. The Lagrangian of this system L^c is given by (13). Substituting the Lagrangian (13) into the Euler-Lagrange equations and adding the control input to the right-hand sides of them, we have the model of the CCBR in the i-th swing phase as (14), (15).

We then derive a model of the CCBR in the i-th impact phase. It is assumed that the swing leg hits the ground with completely inelastic collision, and $\theta^{(i)} = \theta^{(i+1)}$, $\phi^{(i)} = \phi^{(i+1)}$ holds because of an instantaneous impact. Hence, calculating the principle of conservation of angular momentum for the CCBR, we obtain the model of the CCBR in the i-th impact phase as (16), where $a^-, a^+ \in \mathbf{R}^{2\times 2}$ are coefficient matrices.

C. Discrete Compass-type Biped Robot (DCBR)

Next, we derive a model of *discrete compass-type biped robot (CCBR)* by discrete mechanics in this subsection. We here use the notations; h: the sampling time, $k = 1, 2, \cdots, N$: the time step, $i = 1, \cdots, L$: the order of the swing phases, $\alpha = 1/2$: the internal division ratio in discrete mechanics, $\theta_k^{(i)}$, $\phi_k^{(i)}$: the angles of Leg 1 and 2 at the k-th step in the i-th swing phase.

In this paper, we use only the model of the DCBR in the swing phases, and hence we will derive it. By using the transformation law from a continuous Lagrangian into a discrete Lagrangian (2), we obtain the discrete Lagrangian as (17) from (13). Since the left and right discrete external forces (9) satisfy $F^{d+}(q_k, q_{k+1}) = F^{d-}(q_k, q_{k+1})$ for $\alpha = 1/2$, we set a discrete control input that consists of only the left discrete external force F^{d-} as

$$u_k^{(i)} := F^{d-}(q_k, q_{k+1}),\ \ k = 1, \cdots, N-1. \tag{18}$$

Then, substituting (17) into the discrete Euler-Lagrange equations (11), (12) and using the discrete control input (18), we have the model of the DCBR in the i-th swing phase as (19)–(24).

For the impact phases, we use the model of the CCBR (16), and we rewrite it with the terminal variables of the i-the swing phase $\theta_N^{(i)}$ $\phi_N^{(i)}$, $\dot{\theta}_N^{(i)}$ $\dot{\phi}_N^{(i)}$ and the initial variables of the $(i+1)$-the swing phase $\theta_1^{(i+1)}$ $\phi_1^{(i+1)}$, $\dot{\theta}_1^{(i+1)}$ $\dot{\phi}_1^{(i+1)}$ as (25). This representation (25) will be utilized in the next section.

IV. Gait Generation Method on Slopes

A. Setting of Slopes

In this subsection, we first give the problem setting of general walking surfaces including downward and upward slopes. As shown in Fig. 3, we set the x and z axes to the horizontal and vertical directions, respectively, and P_0 denotes the origin of the xz-plane. We then set L points: $P_1, P_2, \cdots, P_L$ in the xz-plane. We represent P_i as $P_i = (r_i, \rho_i)$ by the polar coordinate with reference to P_{i-1} as illustrated in Fig. 4. It is noted that $r_i > 0, -\pi/2 < \rho_i < \pi/2$ are assumed. This problem setting can treat various walking surfaces, for examples, $\rho_i = 0$ $(i = 1, \cdots, L)$: flats, $\rho_i = \rho^- < 0$ $(i = 1, \cdots, L)$: downward slopes, and $\rho_i = \rho^- < 0$ $(i = 1, \cdots, L)$: upward slopes. The sequence of points $P_1, P_2, \cdots, P_L$ are reference grounding points for the compass-type biped robot. Based on the setting above, we consider the following problem on the gait generation for the compass-type biped robot.

$$L^c(\theta^{(i)}, \dot\theta^{(i)}, \phi^{(i)}, \dot\phi^{(i)}) = \frac{1}{2}(I + ma^2 + Ml^2 + ml^2)(\dot\theta^{(i)})^2 + \frac{1}{2}(I + mb^2)(\dot\phi^{(i)})^2 - mbl\cos(\theta^{(i)} - \phi^{(i)})\dot\theta^{(i)}\dot\phi^{(i)} - (ma + mg + Ml)g\cos\phi^{(i)} + mgb\cos\phi^{(i)} \tag{13}$$

$$\frac{d}{dt}\left(\frac{\partial L^c(\theta^{(i)}, \dot\theta^{(i)}, \phi^{(i)}, \dot\phi^{(i)})}{\partial\dot\theta^{(i)}}\right) - \frac{\partial L^c(\theta^{(i)}, \dot\theta^{(i)}, \phi^{(i)}, \dot\phi^{(i)})}{\partial\theta^{(i)}} = v^{(i)} \tag{14}$$

$$\frac{d}{dt}\left(\frac{\partial L^c(\theta^{(i)}, \dot\theta^{(i)}, \phi^{(i)}, \dot\phi^{(i)})}{\partial\dot\phi^{(i)}}\right) - \frac{\partial L^c(\theta^{(i)}, \dot\theta^{(i)}, \phi^{(i)}, \dot\phi^{(i)})}{\partial\phi^{(i)}} = -v^{(i)} \tag{15}$$

$$a^-(\theta^{(i)}, \phi^{(i)})\begin{bmatrix}\dot\theta^{(i)}\\ \dot\phi^{(i)}\end{bmatrix} = a^+(\theta^{(i)}, \phi^{(i)})\begin{bmatrix}\dot\theta^{(i+1)}\\ \dot\phi^{(i+1)}\end{bmatrix} \tag{16}$$

$$L^d(\theta_k^{(i)}, \theta_{k+1}^{(i)}, \phi_k^{(i)}, \phi_{k+1}^{(i)}) = \frac{1}{2}(I + ma^2 + Ml^2 + ml^2)\left(\frac{\theta_{k+1}^{(i)} - \theta_k^{(i)}}{h}\right)^2 + \frac{1}{2}(I + mb^2)\left(\frac{\phi_{k+1}^{(i)} - \phi_k^{(i)}}{h}\right)^2 - mbl\cos\left(\frac{\theta_k^{(i)} + \theta_{k+1}^{(i)}}{2} - \frac{\phi_k^{(i)} + \phi_{k+1}^{(i)}}{2}\right)\frac{\theta_{k+1}^{(i)} - \theta_k^{(i)}}{h}\frac{\phi_{k+1}^{(i)} - \phi_k^{(i)}}{h} - (ma + mg + Ml)g\cos\left(\frac{\phi_k^{(i)} + \phi_{k+1}^{(i)}}{2}\right) + mgb\cos\left(\frac{\phi_k^{(i)} + \phi_{k+1}^{(i)}}{2}\right) \tag{17}$$

$$D_2L^d(\theta_{k-1}^{(i)}, \theta_k^{(i)}, \phi_{k-1}^{(i)}, \phi_k^{(i)}) - D_1L^d(\theta_k^{(i)}, \theta_{k+1}^{(i)}, \phi_k^{(i)}, \phi_{k+1}^{(i)}) + u_{k-1}^{(i)} + u_k^{(i)} = 0 \tag{19}$$

$$D_4L^d(\theta_{k-1}^{(i)}, \theta_k^{(i)}, \phi_{k-1}^{(i)}, \phi_k^{(i)}) - D_3L^d(\theta_k^{(i)}, \theta_{k+1}^{(i)}, \phi_k^{(i)}, \phi_{k+1}^{(i)}) - u_{k-1}^{(i)} - u_k^{(i)} = 0 \tag{20}$$

$$D_2L^c(\theta_1^{(i)}, \dot\theta_1^{(i)}, \phi_1^{(i)}, \dot\phi_1^{(i)}) + D_1L^d(\theta_1^{(i)}, \theta_2^{(i)}, \phi_1^{(i)}, \phi_2^{(i)}) + u_1^{(i)} = 0 \tag{21}$$

$$D_4L^c(\theta_1^{(i)}, \dot\theta_1^{(i)}, \phi_1^{(i)}, \dot\phi_1^{(i)}) + D_3L^d(\theta_1^{(i)}, \theta_2^{(i)}, \phi_1^{(i)}, \phi_2^{(i)}) - u_1^{(i)} = 0 \tag{22}$$

$$-D_2L^c(\theta_N^{(i)}, \dot\theta_N^{(i)}, \phi_N^{(i)}, \dot\phi_N^{(i)}) + D_1L^d(\theta_{N-1}^{(i)}, \theta_N^{(i)}, \phi_{N-1}^{(i)}, \phi_N^{(i)}) + u_{N-1}^{(i)} = 0 \tag{23}$$

$$-D_4L^c(\theta_N^{(i)}, \dot\theta_N^{(i)}, \phi_N^{(i)}, \dot\phi_N^{(i)}) + D_3L^d(\theta_{N-1}^{(i)}, \theta_N^{(i)}, \phi_{N-1}^{(i)}, \phi_N^{(i)}) - u_{N-1}^{(i)} = 0 \tag{24}$$

$$a^-(\theta_N^{(i)}, \phi_N^{(i)})\begin{bmatrix}\dot\theta_N^{(i)}\\ \dot\phi_N^{(i)}\end{bmatrix} = a^+(\theta_N^{(i)}, \phi_N^{(i)})\begin{bmatrix}\dot\theta_1^{(i+1)}\\ \dot\phi_1^{(i+1)}\end{bmatrix} \tag{25}$$

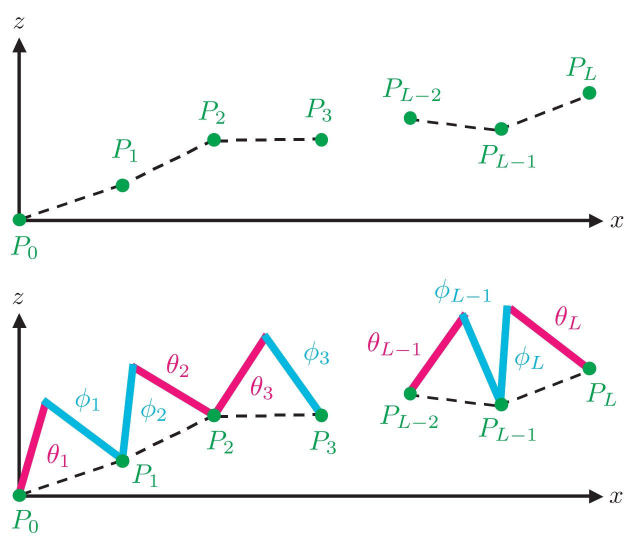

Fig. 3 : Reference Grounding Points in xz-Plane

Problem 1: For the continuous compass-type biped robot (CCBR) (14)–(16), find a control input $v^{(i)}$ $(i = 1, \cdots, L)$ such that the swing leg of the CCBR lands at the reference grounding points P_i $(i = 1, \cdots, L)$ with a stable and natural gait. ∎

In order to solve Problem 2 above, we will propose a new synthesis method based on discrete mechanics. The method consists of two steps: (i) calculation of a discrete control input by solving a finite dimensional constrained nonlinear optimization problem (Subsection IV-B), (ii) transformation a discrete control input into a zero-order hold input by discrete Lagrange-d'Alembert principle (Subsection IV-C).

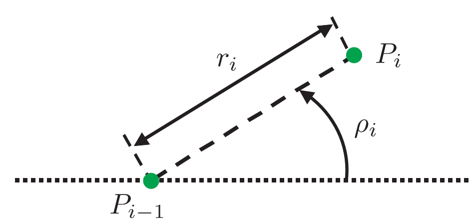

Fig. 4 : r_i and ρ_i

B. Discrete Gait Generation Problem

We next consider a problem on generation of a discrete gait for the DCBR. The discrete gait generation problem for the DCBR in the i-th swing phase is stated as follows.

Problem 2: For the discrete compass-type biped robot (DCBR) (19)–(24) in the i-th swing phase, find a control input $u_k^{(i)}$ $(i = 1, \cdots, N-1)$ such that the swing leg of the DCBR lands at the reference grounding points P_i with a stable and natural discrete gait. ∎

It is expected that in order to generate a stable gait on a slope for the CCBR, a periodic behavior in each swing phase is needed. So, we introduce a cost function of a square of difference between initial angular velocities in the i-th and $(i+1)$-th swing phases, and consider an optimization control problem for the cost function with some constraints. This problem is formulated as follows.

$$\text{min} \quad J = (\dot{\theta}_1^{(i+1)} - \dot{\phi}_1^{(i)})^2 + (\dot{\phi}_1^{(i+1)} - \dot{\theta}_1^{(i)})^2 \tag{26}$$

$$\text{s.t.} \quad (19), (20), (21), (22), (23), (24) \tag{27}$$

$$\theta_1^{(i)},\ \phi_1^{(i)}\ \dot{\theta}_1^{(i)},\ \dot{\phi}_1^{(i)} \tag{28}$$

$$\theta_N^{(i)} = \sin^{-1}\left(\frac{r_i}{2l}\right) - \rho_i,\ \phi_N^{(i)} = \sin^{-1}\left(-\frac{r_i}{2l}\right) - \rho_i \tag{29}$$

$$\phi_1^{(i)} > \phi_2^{(i)} > \cdots > \phi_N^{(i)} \tag{30}$$

In the optimization control problem (26)–(30), (29) means constraints on desired angles of Leg 1 and 2, which can be obtained from data of the reference grounding points $P_i (i = 1, \cdots, N)$, and (30) indicates constraints that prevent a reverse behavior of the swing leg and realize a natural gait. However, since the cost function (26) contains the angular velocities in the $(i+1)$-th swing phase $\dot{\theta}_1^{(i+1)}, \dot{\phi}_1^{(i+1)}$, we cannot solve the minimization problem (26)–(30). So, substituting the model of the impact phase (25) into the cost function (26), we have

$$\begin{aligned} J = &(a_{11}\theta_N^{(i)} + a_{12}\phi_N^{(i)} - \dot{\phi}_1^{(i)})^2 \\ &+ (a_{21}\theta_N^{(i)} + a_{22}\phi_N^{(i)} - \dot{\theta}_1^{(i)})^2, \end{aligned} \tag{31}$$

where

$$(a^+)^{-1}a^- =: \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}.$$

Note that the new cost function (31) does not contain $\dot{\theta}_1^{(i+1)}$, $\dot{\phi}_1^{(i+1)}$ and is represented by only variables in the i-th swing phase. We can see that the optimization control problem (31), (19)–(30) is represented as a finite dimensional constrained nonlinear optimization problem with respect to the $(3N-1)$ variables: $\theta_1^{(i)}, \cdots, \theta_N^{(i)}$, $\phi_1^{(i)}, \cdots, \phi_N^{(i)}$, $u_1^i, \cdots, u_{N-1}^i$. Therefore, we can solve it by *the sequential quadratic programming* [4], [21], and obtain a sequence of discrete control input $u_1^i, \cdots, u_{N-1}^i$.

C. Transformation to Continuous Zero-order Hold Input

In the previous subsection, we show a synthesis method of a discrete control input for the DCBR by solving a finite dimensional constrained nonlinear optimization problem. However, the obtained discrete control input cannot be utilized for the CCBR. So, we here consider transformation of a discrete control input into a continuous one.

There exist infinite methods to generate a continuous control input from a given discrete one, and a continuous control input generated from a given discrete input has to be consistent with laws of physics. Hence, in this paper, we deal with a zero-order hold input in the form:

$$v^{(i)}(t) = v_k^{(i)},\ (i-1)kh \le t < (i-1)(k+1)h, \tag{32}$$

which is one of the simplest continuous inputs. We need to derive a relationship between a discrete input $u_k^{(i)}$ $(k = 1, 2, \cdots, N-1)$ and a zero-order hold input (32). By using discrete Lagrange-d'Alembert's principle which is explained in Section II, we can obtain the following theorem.

Theorem 1: A zero-order hold input (32) that satisfies discrete Lagrange-d'Alembert's principle is given by

$$v_k^{(i)} = \frac{2}{h} u_k^{(i)}. \tag{33}$$

(Proof) For the time interval $kh \le t < (k+1)h$, Substituting (18) and (32) into the definition of the left discrete external force in (9):

$$F^{d-}(q_k, q_{k+1}) = \frac{h}{2} F^c \left((1-\alpha) q_k + \alpha q_{k+1}, \frac{q_{k+1} - q_k}{h} \right),$$

we obtain

$$u_k^{(i)} = \frac{h}{2} v_k^{(i)}.$$

Hence, we have (33). ∎

By using (33) in Theorem 1, we can easily calculate a zero-order hold input from $u_k^{(i)}$, $i = 1, \cdots, N-1$ which are obtained by solving a finite dimensional constrained nonlinear optimization problem (31), (19)–(30). In addition, it must be noted that since we use discrete Lagrange-d'Alembert's principle to prove Theorem 1, a zero-order hold input with a gain (33) is consistent with laws of physics.

V. SIMULATIONS

A. Problem Setting

In this section, we carry out some numerical simulations on continuous gait generation on slopes for the CCBR via the method proposed in the previous section, and confirm the effectiveness of our method. First, this subsection gives the problem setting. we set parameters as follows; parameters of the CCBR: $m = 2.0$ [kg], $M = 10.0$ [kg], $I = 0.167\,[\text{kgm}^2]$, $a = 0.5\,[\text{m}]$, $b = 0.5\,[\text{m}]$, $l = 1.0\,[\text{m}]$, and other parameters: $\alpha = 1/2$, $h = 0.005$ [s]. In Subsection V-B, we will perform a simulation of gait generation for the CCBR on a downward slope, and then we show a simulation on a upward slope in Subsection V-C.

B. Simulation I: Downward Slope

In this subsection, we perform a simulation on gait generation for the CCBR on a downward slope via our new method. We determine data of reference grounding points on a downward slope as $r_i = 1.0\,[\text{m}]$, $\rho_i = -\pi/6\,[\text{rad}]\,(-30\,[\text{degree}])$. Parameters of gait generation are set as $N = 50$, $L = 7$, and intial conditions are $\theta_1^{(1)} = 0$ [rad], $\phi_1^{(1)} = 1.5$ [rad], $\dot{\theta}_1^{(1)} = 0.1$ [rad], $\dot{\phi}_1^{(1)} = -0.1$ [rad].

Figs. 5–7 show the simulation results for gait generation on the downward slope. Fig. 5 illustrates the time series of Leg 1 and 2 (θ and ϕ). Fig. 6 shows the plot of solution trajectory in the phase space of $\theta - \phi$. In Fig. 7, a snapshot of the continuous gait is depicted. From these results, we can confirm that a stable gait on the downward slope for the CCBR can be generated by the proposed approach.

Moreover, we also confirm stable gaits for large numbers of L (the total number of steps in walking).

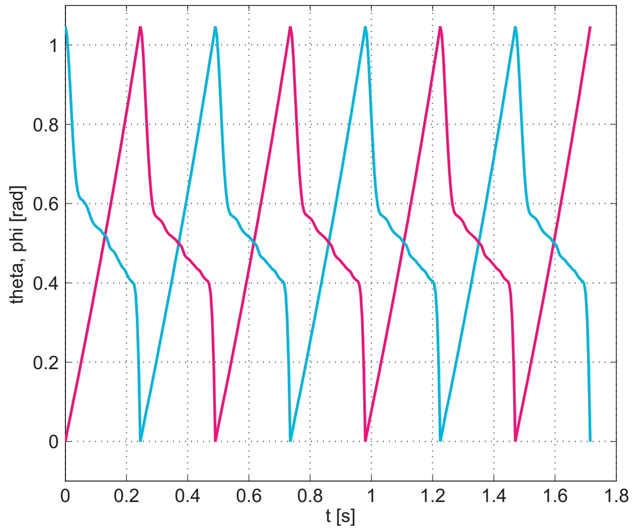

Fig. 5 : Time Series of θ and ϕ (Downward Slope)
(red line: θ, blue line: ϕ)

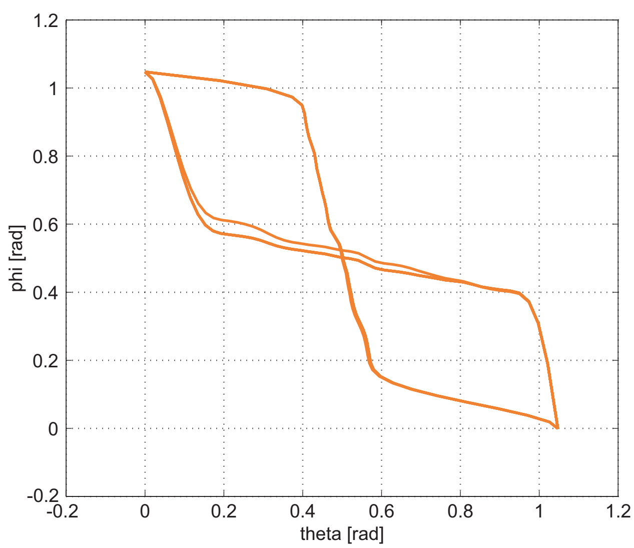

Fig. 6 : Solution Trajectory on $\theta\phi$-Space (Downward Slope)

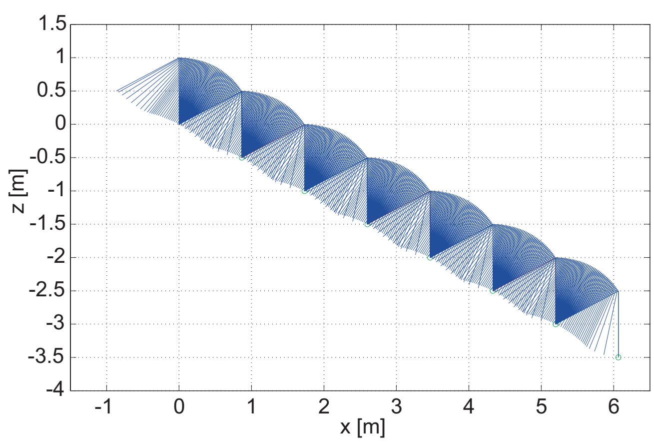

Fig. 7 : Snapshot of Gait (Downward Slope)

C. Simulation II: Upward Slope

Next, a simulation on gait generation for the CCBR on a upward slope via our new method is carried out in this subsection. We determine data of reference grounding points on a upward slope as $r_i = 1.0$[m], $\rho_i = -\pi/4$[rad](45[degree]). Parameters of gait generation are set as $N = 50$, $L = 7$, and intial conditions are $\theta_1^{(1)} = -1.3$ [rad], $\phi_1^{(1)} = -0.25$ [rad], $\dot{\theta}_1^{(1)} = 0.1$ [rad], $\dot{\phi}_1^{(1)} = -0.1$ [rad].

Figs. 8–10 show the simulation results for gait generation on the downward slope. In Fig. 8, the time series of Leg 1 and 2 (θ and ϕ) are depicted. Fig. 9 shows the plot of solution trajectory in the phase space of $\theta - \phi$. a snapshot of the continuous gait is illustrated in Fig. 10. From these results, it can be confirmed that the proposed approach can generate a stable gait on the upward slope for the CCBR. In addition, similar to simulation results on the downward slope in Subsection V-B, we also confirm stable gaits for large numbers of L.

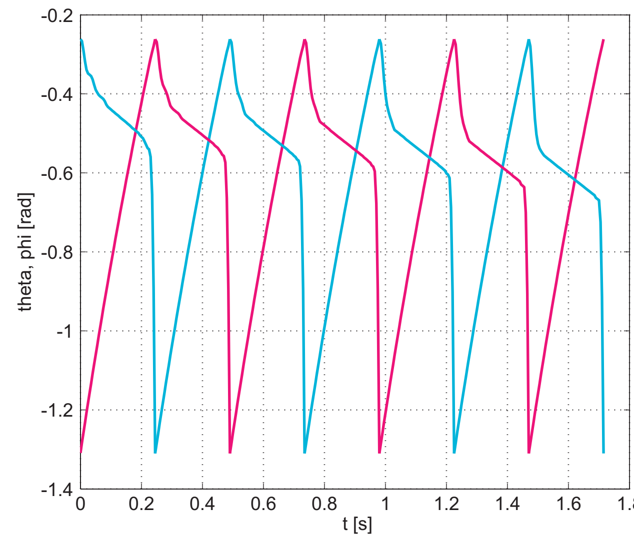

Fig. 8 : Time Series of θ and ϕ (Upward Slope)
(red line: θ, blue line: ϕ)

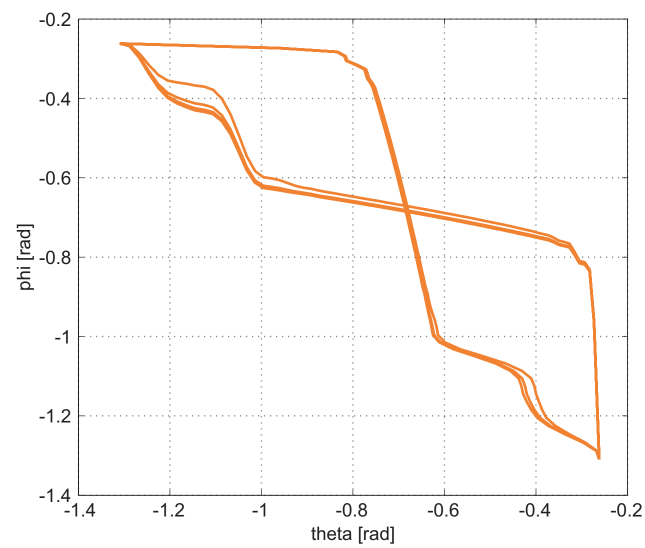

Fig. 9 : Solution Trajectory on $\theta\phi$-Space (Upward Slope)

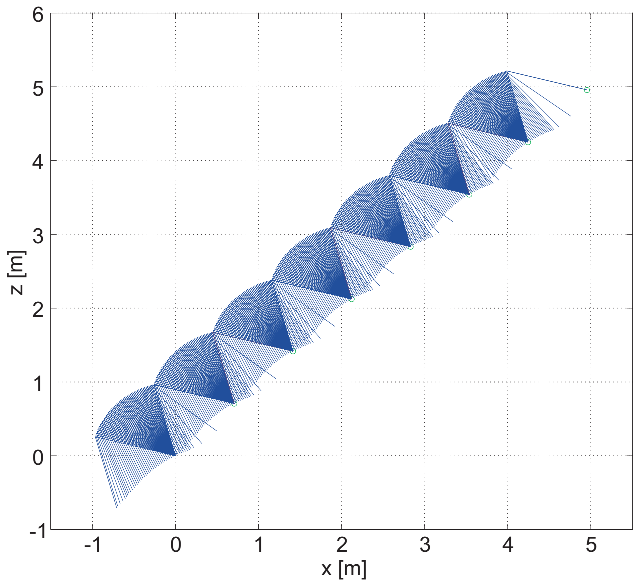

Fig. 10 : Snapshot of Gait (Upward Slope)

VI. CONCLUSION

In this paper, a discrete mechanics approach to gait generation on slopes for the compass-type biped robot based has been studied. We have formulated a discrete gait generation problem for the DCBR and developed a synthesis method of a discrete control input by solving a finite dimensional constrained nonlinear optimization problem. We also have introduced a transformation method from a discrete control input into a zero-order hold input from the viewpoint of discrete Lagrange-d'Alembert principle. Simulation results have confirmed stable gaits on downward and upward slopes and indicated the effectiveness of our new approach.

Our future work on control of humanoid robots via discrete mechanics are as follows: (i) stable gait generation of the CCBR irregular grounds, (ii) experimental evaluation of the proposed control method, (iii) applications of discrete mechanics to more human-like robots and systems represented by partial differential equations.

REFERENCES

[1] J. E. Marsden, G. W. Patrick and S. Shkoller, Multisymplectic Geometry, Variational Integrators and Nonlinear PDEs, *Comm. in Math. Phys.*, Vol. 199, pp. 351–395, 1998

[2] C. Kane, J. E. Marsden, M. Ortiz and M. West, Variational Integrators and the Newmark Algorithm for Conservative and Dissipative Mechanical Systems, *Int. J. for Numer. Meth. in Engineering*, Vol. 49, pp. 1295–1325, 2000

[3] J. E. Marsden and M. West, Discrete Mechanics and Variational Integrators, *Acta Numerica*, Vol. 10, pp.3571–5145, 2001

[4] O. Junge, J. E. Marsden and S. Ober-Blöbaum, Discrete Mechanics and Optimal Control, *in Proc. of 16th IFAC World Congress*, Praha, Czech Republic, Paper No. We-M14-TO/3, 2005

[5] A. M. Bloch, M. Leok, J. E. Marsden and D. V. Zenkov, Controlled Lagrangians and Stabilization of the Discrete Cart-Pendulum System, *in Proc. of 44th IEEE CDC-ECC*, Seville, Spain, pp. 6579–6584, 2005

[6] A. M. Bloch, M. Leok, J. E. Marsden and D. V. Zenkov, Controlled Lagrangians and Potential Shaping for Stabilization of the Discrete Mechanical Systems, *in Proc. of 45th IEEE CDC*, San Diego, U.S.A., pp. 3333–3338, 2006

[7] A. Goswami, B. Espiau and A. Keramane, Limit Cycles in a Passice Compass Gait Biped and Passivity Mimicking Control Laws, *Autonomous Robots*, Vol. 4, pp. 273–286, 1997

[8] T. McGeer, Passive Dynamic Walking, *The Int. J. of Robotics Research*, Vol. 9, No. 2, pp. 62–82, 1990.

[9] M. Garcia, A. Chatterjee, A. Ruina and M. J. Coleman, The simplest walking model: stability, complexity, and scaling, *ASME Journal of of Biomech. Eng.*, Vol. 120, No. 2, pp. 281–288, 1998

[10] J. W. Grizzle, G. Abba and F. Plestan, Asymptotically Stable Walking for Biped Robots: Analysis via Systems with Impulse Effects, *IEEE Trans. on Automatic Control*, Vol. 46, No. 1, pp. 51-64, 2001

[11] Q. Huang, K. Yokoi, S. Kajita, K. Kaneko, H. Arai, N. Koyachi and K. Tanie, Planning Walking Patterns for a Biped Robot, *IEEE Trans. on Robotics and Automation*, Vol. 17, No. 3, pp. 280–289, 2001

[12] T. Kai and Y. Yamamoto, On Analysis and Control of the Cart-Pendulum System Modeled by Discrete Mechanics, *in Proc. of NOLTA 2007*, Vancouver, Canada, pp. 485–488, 2007

[13] T. Kai and K. Bito, Solvability Analysis and Stabilization of the Cart-Pendulum Modeled by Discrete Mechanics with Friction, *in Proc. of NOLTA 2008*, Budapest, Hungary, pp. 305–308, 2008

[14] T. Kai and T. Shintani, Stabilization Control and Experiments of the Cart-Pendulum based on Discrete Mechanics, *in Proc. of NOLTA 2009*, Sapporo, Japan, pp. 82–85, 2009

[15] T. Kai and T. Shintani, Discrete Gait Generation for the Compass-Type Biped Robot Modeled by Discrete Mechanics, *in Proc. of NOLTA 2010*, Krakow, Poland, pp. 434–437, 2009

[16] T. Kai and T. Shintani, Discrete Mechanics Approach to Gait Generation for the Compass-type Biped Robot, *to appear in Proc. of IFAC World Congress 2011*, Milano, Italy, 2011

[17] H. Nijmeijer and A. J. van der Schaft, *Nonlinear Dynamical Control Systems*, Springer-Verlag, 1990

[18] J. E. Marsden and T. S. Ratiu, *introduction to Mechanics and Symmetry*, Springer-Verlag, 1994

[19] A. Isidori, *Nonlinear Control Systems 3rd Edition*, Springer-Verlag, 1995

[20] V. Jurdjevic, *Geometric Control Theory*, Cambridge University Press, 1996

[21] C. B. Gurwitz, *Sequential Quadratic Programming Methods Based on Approximating a Projected Hessian Matrix*, General Books, 2001

[22] F. Bullo and A. D. Rewis, *Geometric Control of Mechanical Systems*, Springer-Verlag, 2004

[23] W. M. Haddad and V. Chellaboina, *Nonlinear Dynamical Systems and Control*, Princeton University Press, 2008