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A Fault Detection Scheme for Discrete-Time Markov Jump Linear Systems

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Abstract— **In this paper, the fault detection (FD) problems for discrete-time Markov jump linear systems (MJLS) are addressed. A scheme for solving FD problem for discrete-time MJLS, which is subject to the Gaussian disturbances, is proposed. The Kalman filter (KF) is used as a residual generator. Once a residual signal is generated, it will be evaluated whether faults occur or not. Residual evaluation function is selected such that the maximum fault detection rate (FDR) is achieved, for a given false alarm rate (FAR). Finally, threshold is computed by using an estimation of the variance of evaluation function in the fault-free case. To demonstrate the performance of this proposed method, a numerical example is given.**

I. INTRODUCTION

Nowadays, more and more fault diagnosis units are integrated into the complex dynamic systems in order to increase safety and reliability. The tasks of fault diagnosis are to detect, isolate and identify faults that occur inside systems. The first step of the fault diagnosis is fault detection (FD). The task of FD is to detect whether faults occur in the systems or not. Among various FD techniques, analytical redundancy approaches have been intensively studied and investigated due to the interesting properties. The main idea of this FD technique is to generate the so-called residual signal, which contains the information of faults. Applications of this FD technique to various dynamic systems have been reported in literatures, e.g. [8], [15] and references therein.

MJLS has been studied for decades in both continuous- and discrete-time domains. This kind of system modeling has a great success to model the behavior of the physical systems that change their structure abruptly in a random manner, e.g. [13], [21]. Recently, there are also numerous works dealing with FD in MJLS, [23] and [24] proposed FD for MJLS under the assumption that the transition probabilities are known, whereas, [22] proposed FD using partially known of these probabilities. [20] designed robust fault detection filter for MJLS and recently [4] proposed FD for MJLS with parameter varying.

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Early work on FD for MJLS studies concentrated on residual generation such as observer design or KF, e.g. [22], [23] and references therein. When the system is pure stochastic, the abrupt change detection is evaluated by the hypothesis testing. The Generalized Likelihood Ratio (GLR) [10] is of the most popular. However, this test requires the density of residual signal to be known before, and after the faults occur [7]. As this information is difficult to be obtained from MJLS, other schemes for residual evaluation are of interest and still be an open issue.

Based on the dynamic behavior of residual signal, which is obtained from KF, the variance of its evaluation function can be estimated. From this information, it is possible to compute the threshold for solving FD problem.

The main objectives of this paper are to

- utilize KF as the residual generator for discrete-time MJLS.
- compute a threshold based on an estimation of variance of the proposed residual evaluation function.
- illustrate the detection performance of the proposed scheme by a numerical example.

The paper is organized as follows: In Section II, preliminaries and formulation of the problem are addressed. Section III presents the residual evaluation function and threshold computation, which are the main contributions of this paper. The proposed scheme is illustrated by a numerical example in Section IV. Finally, the conclusion of the paper and future work are presented in Section V.

II. PRELIMINARIES AND PROBLEM FORMULATION

Suppose $(\Omega, \mathscr{F}, \mathscr{P})$ is a probability space, where Ω is the sample space, $\mathscr{F}$ is the algebra of events and $\mathscr{P}$ is the probability measure, which is defined on $\mathscr{F}$. A discrete-time MJLS, which is subject to faults and disturbances, is described as follows:

$$\begin{aligned} x(k+1) &= A(m_k)x(k) + B(m_k)u(k) + G(m_k)w(k) \\ &\quad + E_f(m_k)f(k) \qquad (1) \\ y(k) &= C(m_k)x(k) + D(m_k)u(k) + v(k) \\ &\quad + F_f(m_k)f(k) \qquad (2) \end{aligned}$$

where $k \in Z^+$ is the time index, $x(k) \in \mathscr{R}^n$ is the state vector, $u(k) \in \mathscr{R}^{n_u}$ is the control input vector, $f(k) \in \mathscr{R}^{n_f}$ is the fault vector to be detected, $y(k) \in \mathscr{R}^q$ is the output vector.

Disturbances are modeled as process and measurement noise, $w(k) \in \mathscr{N}(0,W)$ and $v(k) \in \mathscr{N}(0,V)$, respectively.

These noises are assumed to be stationary, uncorrelated and independent of initial state vector $x(0)$.

Parameter m_k represents a system mode at any time instant k, which is modeled by a discrete homogeneous Markov chain taking value in a finite set $\mathscr{M} = \{1,\ldots,M\}$. The transitions between modes are governed by the transition probability matrix:

$$\Pi = [\pi_{ij}] \tag{3}$$

where

$$\pi_{ij} = Pr[m_{k+1} = j | m_k = i], \forall i, j \in \mathscr{M} \tag{4}$$

and follows the restrictions, $\pi_{ij} > 0$ and $\sum_{i=1}^{M} \pi_{ij} = 1$ for any $i, j \in \mathscr{M}$

Define the probabilities of mode at any time instant k as a vector:

$$\Lambda(k) = \begin{bmatrix} \lambda_1(k) & \lambda_2(k) & \ldots & \lambda_M(k) \end{bmatrix} \tag{5}$$

where

$$\lambda_i(k) = Pr[m_k = i], \forall i \in \mathscr{M} \tag{6}$$

and follows the restrictions, $0 \leq \lambda_i(k) \leq 1$ and $\sum_{i=1}^{M} \lambda_i(k) = 1$ for any $i \in \mathscr{M}$.

Mode probabilities at any time instant k is computed from

$$\Lambda(k) = \Lambda(k-1)\Pi \tag{7}$$

If $\lim_{k\to\infty} \Pi^k =$ constant , then $\lim_{k\to\infty} \Lambda(k) = \Lambda(\infty)$. This implies that mode probabilities are time invariant and independent of mode transition probabilities, i.e. $\Lambda(0) = \Lambda(\infty)$. Hence, Markov chain can be considered into two groups:

- Stationary, if $\Lambda(0) = \Lambda(\infty)$
- Non-stationary, if $\Lambda(0) \neq \Lambda(\infty)$

In this paper, the MJLS under consideration is governed by the stationary Markov chain with known real constant system matrices associated with the $i-th$ mode.

An KF for system (1) and (2), $\forall m_k \in \mathscr{M}$ is formulated as follows:

$$\hat{x}(k+1) = A(m_k)\hat{x}(k) + B(m_k)u(k) + K(m_k)(y(k) - \hat{y}(k)) \tag{8}$$

$$\hat{y}(k) = C(m_k)\hat{x}(k) + D(m_k)u(k) \tag{9}$$

where $K(m_k)$ is the Kalman gain, $\hat{x}(k) \in \mathscr{R}^n$ and $\hat{y}(k) \in \mathscr{R}^q$ are state and output vector estimation, respectively.

Denote state estimation error by $e(k) = x(k) - \hat{x}(k)$, then the dynamic behavior of residual signal $r(k)$ is governed by:

$$\begin{aligned} e(k+1) &= \bar{A}(m_k)e(k) + \bar{E}_f(m_k)f(k) + G(m_k)w(k) \\ &\quad -K(m_k)v(k) \end{aligned} \tag{10}$$

$$r(k) = C(m_k)e(k) + F_f(m_k)f(k) + v(k) \tag{11}$$

where $r(k) = y(k) - \hat{y}(k)$, $\bar{A}(m_k) = A(m_k) - K(m_k)C(m_k)$ and $\bar{E}_f(m_k) = E_f(m_k) - K(m_k)F_f(m_k)$

For the sake of simplicity, the system matrices and Kalman gain associated with the $i-th$ mode are denoted as

$$\begin{aligned} A(m_k = i) &= A_i, \quad B(m_k = i) = B_i \\ C(m_k = i) &= C_i, \quad D(m_k = i) = D_i \\ E_f(m_k = i) &= E_{fi}, \quad F_f(m_k = i) = F_{fi} \\ G(m_k = i) &= G_i, \quad K(m_k = i) = K_i \end{aligned}$$

with appropriate dimension.

Throughout this paper, the following assumptions are made for the discrete-time MJLS.

Assumption 1: The discrete-time MJLS reaches a steady state before any faults occurred.

Assumption 2: The transition probability matrix (Π) and mode probability (Λ) are assumed to be known.

The following definition will be useful in the paper.

Definition 1: [1] System (1) and (2) with $u(k) = 0$ and $f(k) = 0$, $\forall k \geq 0$ is mean square stable (MSS) if for any initial condition $x_0 \in \mathscr{R}^n$ and initial mode $m_0 \in \mathscr{M}$, the state trajectory of (1) and (2) satisfies

$$\lim_{k\to\infty} E[||x(k, x_0, m_0)||^2] = 0$$

where $E[\cdot]$ is the expected value operator.

Solving FD problem for discrete-time MJLS begins with residual generation. To this end, the following theorem is introduced.

Theorem 1:[14] Suppose that system (1) and (2) with $f(k) = 0$ is MSS. Given a set of symmetric positive definite matrices $\{Y_i = Y_i^T\}$, $\forall i \in \mathscr{M}$, there exist two sets of symmetric positive definite matrices $\{P_i = P_i^T\}$ and $\{Q_i = Q_i^T\}$ such that $\{P_i\}$ and $\{Q_i\}$ are stabilizing solutions of the following two sets of coupled Riccati equations

$$\begin{aligned} A_i^T \bar{P}_i A_i - P_i + A_i^T \bar{P}_i W^{1/2}(I - W^{1/2}\bar{P}_i W^{1/2})^{-1} W^{1/2} \bar{P}_i A_i + Y_i &= 0 \\ \hat{A}_i \bar{Q}_i \hat{A}_i^T - Q_i - \hat{A}_i \bar{Q}_i C_i^T (C_i \bar{Q}_i C_i^T + V)^{-1} C_i \bar{Q}_i \hat{A}_i^T + M_i &= 0 \end{aligned}$$

where

$$\bar{P}_i = \sum_{j=1}^{M} \pi_{ij} P_j, \quad \bar{Q}_i = \sum_{j=1}^{M} \pi_{ij} Q_j$$

$$\begin{aligned} \hat{A}_i &= A_i + W(\bar{P}_i^{-1} - W)^{-1} A_i \\ M_i &= W + W(\bar{P}_i^{-1} - W)^{-1} W \end{aligned}$$

Then the KF in (8) and (9) with gain

$$K_i = \hat{A}_i \bar{Q}_i C_i^T (C_i \bar{Q}_i C_i^T + V)^{-1}$$

is MSS with guaranteed cost

$$E[e^T(k)e(k)] \leq \sigma_i := \max_{i\in\mathscr{M}} \text{trace}(Q_i)$$

Proof: It follows the proof of [14], when model uncertainties are neglected. □

The solutions of coupled Riccati equations arise in this theorem can be founded in [2] ,[9] or [16].

To achieve a successful FD, the generated residual signal will be evaluated and compared with the predefined threshold.

It can be seen that, the covariance of residual signal in (11) can be computed from

$$S(k) = E[r(k)r^T(k)] \tag{12}$$

which is affected from fault $f(k)$. Thus, it is possible to use the covariance as the residual evaluation function for solving FD problem in MJLS.

The following definition and lemma are fruitful to the proof of the main result.

Definition 2: [1] Given the probability space $(\Omega, \mathscr{F}, \mathscr{P})$, the indicator function 1_F is defined for any $F \in \mathscr{F}$ and $\omega \in \Omega$ as

$$1_F(\omega) = \begin{cases} 1 & \text{if} \quad \omega \in F \\ 0 & \text{if} \quad \omega \notin F \end{cases} \tag{13}$$

Lemma 1:[3] For a given random number x and constant $\varepsilon > 0$ satisfying $\varepsilon^2 \geq E[x - \bar{x}\ ]^2$, Chebyshev's inequality states that

$$Pr[|x - \bar{x}\ | \geq \varepsilon] \leq \frac{E[x - \bar{x}\ ]^2}{\varepsilon^2} \tag{14}$$

where $\bar{x} = E[x]$

In the next section, residual evaluation and threshold computation, which is the main contribution of this paper, will be addressed.

III. RESIDUAL EVALUATION AND THRESHOLD COMPUTATION

As mentioned in the last section that the application of GLR for FD problem in discrete-time MJLS is difficult due to the lack of distribution knowledge of residual signal after faults occurred. Finding alternative detection scheme is of interest and will be proposed in this section.

The attempt to detect a fault in discrete-time MJLS starts with the residual evaluation. Finally, this value is compared with the threshold, and the FD can be obtained from the following decision logic:

$$\begin{aligned} J > J_{th} \quad &: \quad \text{Fault} \\ J \leq J_{th} \quad &: \quad \text{Fault-free} \end{aligned}$$

Performance of this FD scheme is evaluated by fault alarm rate (FAR) and fault detection rate (FDR), which are defined as follows:

Definition 3:[3] Given an evaluation function J and threshold J_{th}, the FAR is defined as

$$Pr[J > J_{th} | f(k) = 0] \tag{15}$$

Definition 4:[3] Given an evaluation function J and threshold J_{th}, the FDR is defined as

$$Pr[J > J_{th} | f(k) \neq 0] \tag{16}$$

To this end, the problem of FD is generally comprised of:

- Selection of a residual evaluation function.
- For a selected residual evaluation function and an allowed FAR, compute a threshold value.

It can be seen that residual signal, which is generated from KF in (8) and (9), is a stochastic process. Thus, based on the Chebyshev's inequality given in Lemma 1, the FAR is estimated according to [3] as

$$Pr[J - E[J] > J_{th} | f(k) = 0] \leq \alpha$$

Then it turns out that

$$\frac{E[(J - E[J])^2]}{J_{th}^2} \leq \frac{E[J^2]}{J_{th}^2} \leq \alpha \tag{17}$$

where $\alpha := FAR$ is used as the detection performance index.

Hence, the allowed FAR is fulfilled by the determination of $E[J^2]$ and J_{th}.

In the first step, evaluation function of residual signal is defined according to [3], in order to reduce FAR and maximize FDR. This requirement can be achieved by using the evaluation function

$$J = \sqrt{\left(\frac{1}{N}\sum_{a=1}^{N} \tilde{r}(k-a)\right)^T \left(\frac{1}{N}\sum_{a=1}^{N} \tilde{r}(k-a)\right)} \tag{18}$$

where $\tilde{r}(k-a) = (\sqrt{S(k-a)})^{-1} r(k-a)$ is the normalized residual and $\tilde{S}(k-a)$ is its covariance matrix, $\forall (k-a) \in Z^+$

Corresponding to evaluation function (18), the threshold is generally set according to [3]:

$$J_{th} = \sqrt{\beta(N) \cdot \text{trace}(\tilde{S}_{max})} \tag{19}$$

where $\beta(N) > 0$ and $\tilde{S}_{max} = \max\limits_{k} \tilde{S}(k)$ is the covariance matrix of the normalized residual.

From (17) and (19), it obtains

$$\beta(N) \geq \frac{E[J^2]}{\alpha \cdot \text{trace}(\tilde{S}_{max})} \tag{20}$$

To this end, threshold computation problem is reduced to find $E[J^2]$, which can be obtained by the following theorem.

Theorem 2: Given system (1) and (2), KF in (8) and (9) when $f(k) = 0$, allowed FAR and the residual evaluation in (18), the variance of evaluation function follows an inequality

$$E[J^2] \leq \frac{\gamma}{N} + \frac{\phi}{N^2} \tag{21}$$

where $\gamma > 0$ and $\phi > 0$ are some constants and the threshold can be set as (19).

Proof: Note that for $b > a > 0$,

$$E[J^2] = \frac{1}{N^2}\text{trace}\Big\{\sum_{a=1}^{N} E[\tilde{r}(k-a)\tilde{r}^T(k-a)] + \sum_{b=2}^{N}\sum_{a=1}^{b-1}\big(E[\tilde{r}(k-a)\tilde{r}^T(k-b)] + E[\tilde{r}(k-b)\tilde{r}^T(k-a)]\big)\Big\}$$

It can be seen that

$$\sum_{a=1}^{N} E[\tilde{r}(k-a)\tilde{r}^T(k-a)] = \sum_{a=1}^{N}(\sqrt{S(k-a)})^{-1}E[r(k-a)r^T(k-a)](\sqrt{S(k-a)})^{-1}$$

From the aid of Definition 2 and results from Theorem 1,

$$\begin{aligned} &E[r(k-a)r^T(k-a)] \\ &= E[C(m_{k-a})e(k-a)e^T(k-a)C^T(m_{k-a}) + v(k-a)v^T(k-a)] \\ &= \sum_{i=1}^{M}\lambda_i(C_iQ_iC_i^T + V) \\ &\leq C_jQ_jC_j^T + V \end{aligned}$$

where $Q_i = E[e(k-a)e^T(k-a)]$, $\forall (k-a) \in Z^+$ and $Q_j = \max_{i\in\mathscr{M}} Q_i$

It turns out

$$\sum_{a=1}^{N} E[\tilde{r}(k-a)\tilde{r}^T(k-a)] \leq N(\sqrt{S_j})^{-1}S_j(\sqrt{S_j})^{-1}$$

where $S_j = C_jQ_jC_j^T + V$

Consider that

$$\sum_{b=2}^{N}\sum_{a=1}^{b-1} E[\tilde{r}(k-a)\tilde{r}^T(k-b)] = \sum_{b=2}^{N}\sum_{a=1}^{b-1}(\sqrt{S(k-a)})^{-1}E[r(k-a)r^T(k-b)](\sqrt{S(k-b)})^{-1}$$

From Definition 2 and results from Theorem 1,

$$\begin{aligned} &E[r(k-a)r^T(k-b)] \\ &= \sum_{i_N=1}^{M}\sum_{i_{N-1}=1}^{M}\cdots\sum_{i_1=1}^{M}\lambda_{i_N}\lambda_{i_{N-1}}\cdots \\ &\quad \cdots\lambda_{i_1}C_{i_N}(A_{i_{N-1}} - K_{i_{N-1}}C_{i_{N-1}})\cdots \\ &\quad \cdots(A_{i_2} - K_{i_2}C_{i_2})(A_{i_1}Q_{i_1}C_{i_1}^T - K_{i_1}(C_{i_1}Q_{i_1}C_{i_1}^T) + V) \\ &\leq C_j(A_j - K_jC_j)(A_jQ_jC_j^T - K_j(C_jQ_jC_j^T) + V) \end{aligned}$$

It turns out

$$\sum_{b=2}^{N}\sum_{a=1}^{b-1} E[\tilde{r}(k-a)\tilde{r}^T(k-b)] \leq (\sqrt{S_j})^{-1}(\sum_{b=2}^{N}(N-b+1)C_j\bar{A}_j^{b-2}U_j)(\sqrt{S_j})^{-1}$$

where $\bar{A}_j = A_j - K_jC_j$ and $U_j = A_jQ_jC_j^T - K_jS_j$

The previous procedures also leads to

$$\sum_{b=2}^{N}\sum_{a=1}^{b-1} E[\tilde{r}(k-b)\tilde{r}^T(k-a)] \leq (\sqrt{S_j})^{-1}(\sum_{b=2}^{N}(N-b+1)(C_j\bar{A}_j^{b-2}U_j)^T)(\sqrt{S_j})^{-1}$$

Finally,

$$E[J^2] \leq \frac{\gamma}{N} + \frac{\phi}{N^2}$$

where

$$\gamma = \text{trace}((\sqrt{S_j})^{-1}S_j(\sqrt{S_j})^{-1}) = \text{trace}(I_{q\times q})$$

and

$$\phi = \text{trace}((\sqrt{S_j})^{-1}(\sum_{b=2}^{N}(N-b+1)[C_j\bar{A}_j^{b-2}U_j + (C_j\bar{A}_j^{b-2}U_j)^T](\sqrt{S_j})^{-1})$$

This theorem is thus proved. □

With the aid of Theorems 1 and 2, the FD problem for the discrete-time MJLS can be solved by the following algorithm:

Algorithm 1: FD for the discrete-time MJLS

- Step 1: Compute the variance of residual evaluation function $E[J^2]$ according to (21).
- Step 2: Compute $\beta(N)$ according to (20) and set threshold J_{th} according to (19).
- Step 3: Construct the residual evaluation function (18) and compare with the threshold J_{th}.

Remark 1: It can be seen that a lower allowed FAR requires a larger $\beta(N)$ and leads to high value of J_{th}. However, increasing the number of sampling data N significantly reduces the value of J_{th} while remains a preferable FAR.

Remark 2: The estimation value of $E[J^2]$ depends on the number of sampling data N. For a given FAR, large number of N leads to small value of J_{th} and increases FDR.

Remark 3: By using normalized residual $\tilde{r}(k)$ in the evaluation function ensures the maximum FDR for a given FAR and threshold J_{th} when the number of sampling data N is fixed.

In the next section, the numerical example will be presented in order to illustrate the performance of the proposed residual evaluation scheme for the discrete-time MJLS.

IV. A NUMERICAL EXAMPLE

This section presents some numerical examples of the proposed residual evaluation for the discrete-time MJLS.

Consider the two-mode discrete-time MJLS with $m(k) = i \in \{1,2\}, \forall k \geq 0$ described by (1) and (2) with the following matrices:

$$A_1 = \begin{bmatrix} 0.5 & 0.4 \\ 0.3 & 0.1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0.2 & 0.7 \\ 0.1 & 0.3 \end{bmatrix}$$
$$B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$
$$C_1 = \begin{bmatrix} 0.7 & 0.5 \\ 0.1 & 0.2 \end{bmatrix}, \quad C_2 = \begin{bmatrix} 0.5 & 0.2 \\ 0.7 & 0.5 \end{bmatrix}$$
$$D_1 = \begin{bmatrix} 0.5 \\ 1 \end{bmatrix}, \quad D_2 = \begin{bmatrix} 1 \\ 0.8 \end{bmatrix}$$
$$E_{f1} = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \quad E_{f2} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$
$$F_{f1} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad F_{f2} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
$$G_1 = \begin{bmatrix} 0.1 & 0.5 \\ 0.5 & 0.1 \end{bmatrix}, \quad G_2 = \begin{bmatrix} 0.5 & 0.1 \\ 0.1 & 0.5 \end{bmatrix}$$
$$W = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.7 \end{bmatrix}, \quad V = \begin{bmatrix} 0.5 & 0 \\ 1 & 0.9 \end{bmatrix}$$

The mode transition probabilities are known and given by

$$\Pi = [\pi_{ij}], \quad \forall i,j \in \{1,2\}$$

where $\pi_{11} = 0.2, \quad \pi_{12} = 0.8, \quad \pi_{21} = 0.2, \quad \pi_{22} = 0.8$

Also assume that the mode probabilities are known as

$$\Lambda = \begin{bmatrix} \lambda_1 & \lambda_2 \end{bmatrix}$$

where $\lambda_1 = 0.2$ and $\lambda_2 = 0.8$

Finally, assume that $x(0) = \begin{bmatrix} 0 & 0 \end{bmatrix}^T$ and $m(0) = 2$ where $m(k)$ changes stochastically between two modes as shown in figure (1).

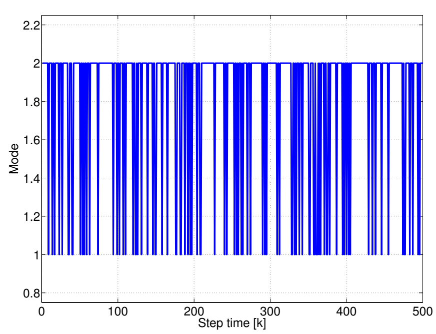

Fig. 1. Change of mode, $m(k)$

Design KF according to Theorem 1, then the following results are obtained:

$$P_1 = \begin{bmatrix} 0.5515 & 0.0400 \\ 0.0400 & 0.4699 \end{bmatrix}$$
$$P_2 = \begin{bmatrix} 0.4228 & -0.0061 \\ -0.0061 & 0.3557 \end{bmatrix}$$
$$Q_1 = \begin{bmatrix} 0.2046 & -0.0063 \\ -0.0063 & 0.4753 \end{bmatrix}$$
$$Q_2 = \begin{bmatrix} 0.1837 & -0.0704 \\ -0.0704 & 0.4417 \end{bmatrix}$$

with Kalman gains

$$K_1 = \begin{bmatrix} 0.2150 & 0.0358 \\ 0.1007 & 0.0107 \end{bmatrix}, K_2 = \begin{bmatrix} 0.0989 & 0.1540 \\ 0.0528 & 0.0798 \end{bmatrix}$$

a) Threshold computation when number of sampling data, N is fixed

The first simulation shows an effect of allowed *FAR* to the threshold (J_{th}) when the number of sampling data (N) is kept constant. Given number of sampling data $N = 20$ and allowed $FAR = 5\%$, threshold can be obtained from (19) as $J_{th} = 1.4227$ and if allowed $FAR = 10\%$, yields $J_{th} = 1.0060$. Finally, the results when the system is subject to fault $f(k) = \begin{bmatrix} 0 & 3 & 0 \end{bmatrix}^T$ at $k = 250$ with $FAR = 5\%$ and $FAR = 10\%$ are shown in figure (2) and (3), respectively.

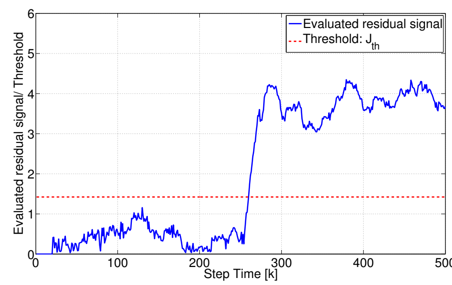

Fig. 2. Evaluated residual signal when FAR = 5%

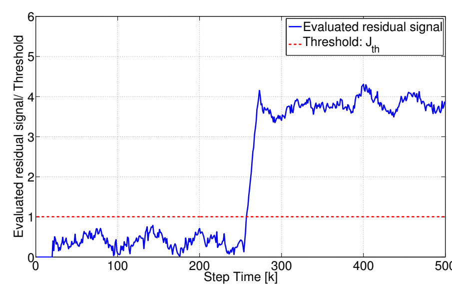

Fig. 3. Evaluated residual signal when FAR = 10%

It can be seen from this simulation that J_{th} is set higher when allowed *FAR* is decreased. However, when required *FAR* is set to low, J_{th} level is very high and may reduce the FDR.

b) Threshold computation when FAR is fixed

The second simulation illustrates a situation when allowed *FAR* is given with the difference number of sampling data (N). Given $FAR = 5\%$, threshold with $N = 20$ is computed from (19) as $J_{th} = 1.4227$ and if the number of sampling data is reduced to $N = 10$, the threshold is computed as $J_{th} = 2.0081$. Finally, the results when the system is subject to fault $f(k) = \begin{bmatrix} 3 & 0 & 0 \end{bmatrix}^T$ at $k = 250$ with $N = 20$ and $N = 10$ are shown in figure (4) and (5), respectively.

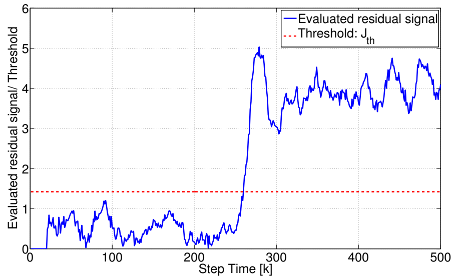

Fig. 4. Evaluated residual signal when N = 20

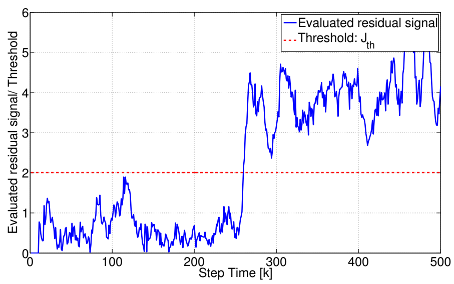

Fig. 5. Evaluated residual signal when N = 10

From the results, number of sampling data (N) affects the system such that when it is small, FDR will be reduced as shown in figure (5). It can be seen also that reducing N yields faster fault detection time.

V. CONCLUSIONS AND FUTURE WORKS

The problem of FD for the discrete-time MJLS has been addressed in this paper. Due to the lack of residual density after faults occur, the alternative FD scheme is proposed. KF is used as the residual generator and FD problem is formulated as the inequality relation between the variance of residual evaluation function and threshold via the Chebyshev's inequality. By estimating the variance of residual evaluation function, threshold can be set according to a given FAR. Finally, numerical example is given in order to illustrate an efficiency of this proposed FD scheme. The future work will investigate the FD problem when model uncertainties are taking into account.

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