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Column Formation Control of Multi-robot Systems with Input Constraints

Xiaohan Chen, Yingmin Jia, Junping Du and Fashan Yu

***Abstract*— This paper is concerned with the column formation control of multi-robot systems that are subject to input constraints. A new leader-follower setup is proposed, under which the Lyapunov method can provide a simple controller in terms of two feedback parameters. It is shown that using an elliptic approximation to input constraints, the two feedback parameters can be obtained directly with a geometric analysis method. Moreover, a sufficient condition is presented for the leader robot to guarantee that the desired column formation can be achieved, with which a particular controller is designed by selecting two appropriate functions for the feedback parameters. Simulation results are included to verify the effectiveness of the proposed theoretical results.**

***Index Terms*— Column formation control; multi-robot systems; leader-follower; input constraints; elliptic approximation.**

I. INTRODUCTION

Formation control of mobile robots has attracted considerable attention in recent years due to its wide applications in, e.g., exploration, surveillance, search and rescue, mapping of unknown environments, and the transportation of large objects. Usually, the formation control is involved to control the relative position and orientation of the robots in a group while allowing the group to move as a whole. There are various strategies to address the formation control problem, such as behavior-based [1]-[2], virtual structure [3]-[4], and leader-follower approaches [5]-[6]. For the leader-follower formation, the robot which is designated as the leader moves along a predefined trajectory, and the follower tracks the leader with desired relative pose. Although there is the disadvantage that the formation is sensitive to perturbation because of no explicit feedback to it [7], the leader-follower based formation control is employed in this paper due to its flexibility and scalability.

In practice, the linear and angular velocities of mobile robots are always bounded in limited ranges due to the mechanical limitation. That is to say mobile robots are subject to input constraints. To our best knowledge, few works [8]-[9] have been done for formation control of mobile robots with input constraints. In [8], the main characteristic of the control strategy used to deal with the input constraints was that the position of the follower varied in proper circle arcs centered in the leader reference frame rather than being fixed with respect to the leader. In [9], the effect of input constraints on the admissible trajectories of the leader has been studied. Although the theoretic results obtained in [8] and [9] work well, their applicabilities for real mobile robots may be limited due to the complexities of the controllers.

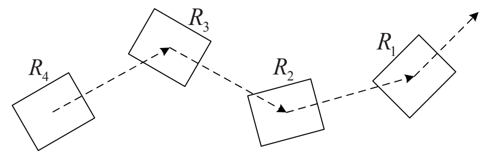

Fig. 1: The column formation.

In this paper, the approach used to address the problem of formation control of mobile robots which are subject to input constraints are different from the existing methods. The main features of our approach are in the following two aspects.

First, a leader-follower setup built on the axis of the follower with a separation from its center is proposed, which is alternative to those in the existing literature. In leader-follower architectures, the conventional $l-\psi$ setup is represented in polar coordinates which leads to singularity [10]. Meanwhile, in order to tackle the nonholonomic constraint on the mobile robot, an off-axis marker placed on the follower should be considered as the handling point that degrades the precision of the formation [11]. However, the proposed setup in this paper eliminates both the singularity and use of the marker. Furthermore, it gives a simple formation controller with two undetermined feedback parameters directly by Lyapunov method.

Second, an ellipse is used to approximate the input constraints of the mobile robot which simplifies the selection of parameters associated to the formation controller. With the motivation from [12] which employed an ellipse to bound a mobile robot, we propose an elliptic approximation of input constraints, with which a sufficient condition for the leader is provided to ensure that the desired formation can be achieved. In particular, two feedback parameters are selected by geometric analysis method. The formation controller with this specific parameters guarantees the achievement of the desired formation as well as the satisfaction of the input constraints.

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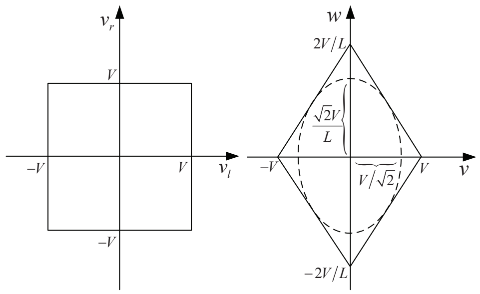

Fig. 2: Elliptic approximation of input constraints.

It is worth noting that only column formation control is considered in this paper for its important application in intelligent transport system [13]. With a little modification to the definition in [14], the column formation for a group of robots in this paper means that the robot acting as a leader is in the heading direction of its follower with desired separation (see Fig. 1).

The rest paper is organized as follows. In Section II, we state the formation control problem of mobile robots with input constraints. In Section III, a new leader-follower setup is proposed in Cartesian coordinates, and the controller design is discussed in Section IV. In Section V, simulation results are provided. Finally, Section VI gives the conclusions.

II. Problem Statement

A. Elliptic Approximation of Input Constraints

The real mobile robots can not run as fast as we wish due to their limited mechanic properties. Considering a differential drive mobile robot for instance, the velocities of the two drive wheels are restricted by the inherent properties of the drive motors, i.e., $v_l,\ v_r \in [-V, V]$, where V is a positive constant, and $v_l,\ v_r$ are the velocities for the left and right drive wheels, respectively.

Usually, the linear velocity v and angular velocity w are taken as the control inputs. For the differential drive mobile robot, the linear and angular velocities can be deduced by v_l and v_r that

$$v = \frac{v_r + v_l}{2}$$
$$w = \frac{v_r - v_l}{L}$$

where L is the distance between the two drive wheels. Then, v and w are bounded in

$$\begin{cases} -\dfrac{2(V+v)}{L} \leqslant w \leqslant \dfrac{2(V+v)}{L}, & v \in [-V, 0) \\ -\dfrac{2(V-v)}{L} \leqslant w \leqslant \dfrac{2(V-v)}{L}, & v \in [0, V]. \end{cases}$$

As depicted in Fig. 2, the boundary of the feasible domain for (v, w) is piecewise smooth. To obtain a smooth boundary, the maximum inscribed ellipse is used to approximate the bounded region of (v, w), i.e.,

$$\frac{2v^2}{V^2} + \frac{L^2 w^2}{2V^2} \leqslant 1. \tag{1}$$

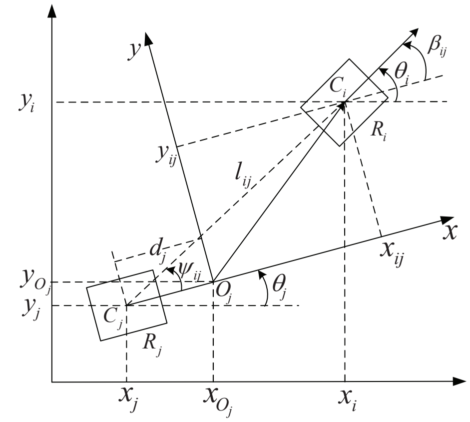

Fig. 3: Leader-follower setup.

By simple computation, the area enclosed by the ellipse is $3/4$ of the feasible domain. Although the elliptic approximation is conservative, it gives much convenience for formation controller design which will be shown in Section IV.

Although inequality (1) is derived from the differential drive mobile robot, it remains a reasonable approximation for some other mobile robots which are unicycle-type. As for safe driving the faster the linear velocity is, the lower the angular velocity should be.

B. Column Formation Control of Multiple Robots

In this paper, the column formation control of mobile robots with unicycle model is considered. Define the index set $\mathcal{I} = \{1, 2, 3, ..., n\}$ for n robots. The kinematics of robot $R_j,\ j \in \mathcal{I}$, is

$$\dot{x}_j = v_j \cos\theta_j, \dot{y}_j = v_j \sin\theta_j, \dot{\theta}_j = w_j \tag{2}$$

where (x_j, y_j, θ_j) is the generalized coordinates for R_j with (x_j, y_j) being the position and θ_j being the orientation in the world frame. v_j and w_j are linear and angular velocities, respectively.

Assume that each robot R_j suffers the following input constraints

$$\frac{{v_j}^2}{{a_j}^2} + \frac{{w_j}^2}{{b_j}^2} \leqslant 1, \tag{3}$$

where a_j and b_j are positive constants depending on the property of individual robot. The imposed input constraints present particular challenges for formation control. Now, the research problem addressed in this paper is to design controller for each follower robot to maintain the desired column formation under the input constraints (3).

III. Leader-Follower System

In this paper, the relative pose between the leader and follower is represented in a Cartesian coordinates in which the singularity and handling point no longer need to be considered. As shown in Fig. 3, robot R_i is assigned as the leader to robot R_j, where $i, j \in \mathcal{I}$ and $i \neq j$. The reference frame $O_j xy$ is set on the heading orientation of R_j with separation d_j from the its center C_j. The position of R_i in

$O_j xy$ is denoted by (x_{ij}, y_{ij}). The generalized coordinates of R_i and R_j in the world frame are (x_i, y_i, θ_i) and (x_j, y_j, θ_j), respectively. Denote (x_{O_j}, y_{O_j}) as the coordinates of O_j in world frame. Then we get

$$\begin{aligned} x_{O_j} &= x_j + d_j \cos\theta_j \\ y_{O_j} &= y_j + d_j \sin\theta_j. \end{aligned}$$

In view of (2), differentiating both sides of the above equations leads to

$$\dot{x}_{O_j} = v_j \cos\theta_j - d_j w_j \sin\theta_j \tag{4}$$
$$\dot{y}_{O_j} = v_j \sin\theta_j + d_j w_j \cos\theta_j. \tag{5}$$

The positions of R_i, (x_i, y_i) in the world frame and (x_{ij}, y_{ij}) in the reference frame $O_j xy$, are related by the following transformation

$$\begin{aligned} x_i &= x_{ij}\cos\theta - y_{ij}\sin\theta + x_{O_j} \\ y_i &= x_{ij}\sin\theta + y_{ij}\cos\theta + y_{O_j}. \end{aligned}$$

Then, we get

$$\begin{aligned} x_{ij} &= (x_i - x_{O_j})\cos\theta + (y_i - y_{O_j})\sin\theta \\ y_{ij} &= -(x_i - x_{O_j})\sin\theta + (y_i - y_{O_j})\cos\theta. \end{aligned}$$

According to (2), (4) and (5), differentiating both sides of the above equations yields

$$\dot{x}_{ij} = -v_j + v_i \cos\beta_{ij} + w_j y_{ij} \tag{6}$$
$$\dot{y}_{ij} = -d_j w_j + v_i \sin\beta_{ij} - w_j x_{ij} \tag{7}$$

where $\beta_{ij} = \theta_i - \theta_j$ is the relative orientation between R_i and R_j satisfying

$$\dot{\beta}_{ij} = w_i - w_j. \tag{8}$$

Remark 1: For the leader-follower system (6) and (7), x_{ij} and y_{ij} can not be measured by sensors directly as O_j is a virtual point. However, x_{ij} and y_{ij} can be deduced from

$$x_{ij} = l_{ij}\cos\psi_{ij} - d_j \tag{9}$$
$$y_{ij} = l_{ij}\sin\psi_{ij} \tag{10}$$

where l_{ij} and ψ_{ij} are separation and relative bearing between R_i and R_j which are measurable (see Fig. 3).

IV. Controller Design

Assume that l_{ij}^d and ψ_{ij}^d are the desired separation and relative bearing between R_i and R_j, respectively. Since only the column formation control is discussed in this paper, ψ_{ij}^d equals zero (see Fig. 1). For convenience, let d_j be l_{ij}^d. According to (9) and (10), then the desired value for (x_{ij}, y_{ij}) is $(0, 0)$. That is to say x_{ij} and y_{ij} can be considered as formation errors. As the leader shares no attention to maintain the formation, v_i and w_i in (6) and (7) can be treated as exogenous inputs. Now the control objective is to find the inputs v_j and w_j satisfying the input constraints (3) to maintain the desired column formation. Choose the control inputs as

$$v_j = k_{j_1}(t) x_{ij} + v_i \cos\beta_{ij}, \tag{11}$$
$$w_j = \frac{1}{d_j}\left[k_{j_2}(t) y_{ij} + v_i \sin\beta_{ij}\right]. \tag{12}$$

where $k_{j_1}(t) > 0$, $k_{j_2}(t) > 0$ for $\forall t > 0$ are feedback parameters.

Lemma 1: Assume that $v_i(t) > 0$ for $\forall t > T$, $|\beta_{ij}(T)| < \pi$ and $|k_{j_2}(t)| < K$, where T is some finite time and K is a positive constant. If the inputs (11) and (12) are used to control the leader-follower system (6) and (7), then x_{ij}, y_{ij} will converge to zero and β_{ij} will be bounded.

Proof: Substituting (11) and (12) into (6) and (7) yields

$$\dot{x}_{ij} = -k_{j_1}(t) x_{ij} + w_j y_{ij} \tag{13}$$
$$\dot{y}_{ij} = -k_{j_2}(t) y_{ij} - w_j x_{ij}. \tag{14}$$

Let the Lyapunov function be

$$V = \frac{1}{2}({x_{ij}}^2 + {y_{ij}}^2).$$

With (13) and (14), differentiate both sides of the Lyapunov function to get

$$\dot{V} = \dot{x}_{ij} x_{ij} + \dot{y}_{ij} y_{ij} = -k_{j_1}(t) {x_{ij}}^2 - k_{j_2}(t) {y_{ij}}^2.$$

Clearly, $V \leqslant 0$, and V equals 0 if and only if $x_{ij} = y_{ij} = 0$. Therefore, x_{ij} and y_{ij} will converge to zero under the control inputs (11) and (12). Now, we turn to the stability analysis of β_{ij}. According to (8) and (12), we obtain

$$\dot{\beta}_{ij} = -\frac{v_i}{d_j}\sin\beta_{ij} - \frac{k_{j_2}(t)}{d_j} y_{ij} + w_i.$$

Consider the nominal system

$$\dot{\beta}_{ij} = -\frac{v_i}{d_j}\sin\beta_{ij}.$$

If $v_i(t) > 0$ holds for $\forall t > T$ and $|\beta_{ij}(T)| < \pi$, then $\beta_{ij} = 0$ is an exponentially stable equilibrium point. Since $y_{ij} \to 0$ as $t \to \infty$, $|k_{j_2}(t)| < K$ and $|w_i| \leqslant b_i$, there must be some positive constant σ that

$$\left| -\frac{k_{j_2}(t)}{d_j} y_{ij} + w_i \right| \leqslant \left| -\frac{k_{j_2}(t)}{d_j} y_{ij} \right| + |w_i| < \sigma.$$

By using of the stability theory of perturbed systems [15], we come to the conclusion that β_{ij} will be bounded. ■

Remark 2: Notice that the separation d_j in (12) satisfying $d_j = l_{ij}^d$ is a positive constant, once the desired column formation is determined.

It is worthwhile to mention that any positive constants can be chose as the feedback parameters in (11) and (12), if no input constraints are imposed on the mobile robots. However, it is difficult to get the feasible feedback parameters under the input constraints. With geometric analysis, the following theorems will discuss how to obtain feedback parameters which make the control inputs (11) and (12) meet the input bounds (3).

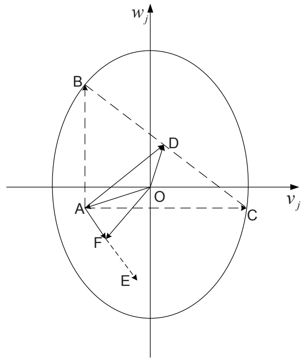

Fig. 4: Geometric analysis of controller design for R_j.

Theorem 1: Consider the leader-follower system (6) and (7), and let the input constraints be described by (3). If

$$a_i < \min\{a_j, d_j b_j\},$$

then there exist suitable feedback parameters $k_{j_1}(t)$ and $k_{j_2}(t)$ such that the control inputs (11) and (12) can satisfy the input constraints.

Proof: Set $k_{j_1}(t) = k_{j_2}(t) = \epsilon(t)$. Then (11) and (12) can be rewritten as

$$(v_j, w_j) = \left(v_i \cos\beta_{ij}, \frac{v_i \sin\beta_{ij}}{d_j}\right) + \epsilon(t)\left(x_{ij}, \frac{y_{ij}}{d_j}\right).$$

As represented in Fig. 4, set $\overrightarrow{OA} = (v_i \cos\beta_{ij}, v_i \sin\beta_{ij}/d_j)$, $\overrightarrow{AE} = (x_{ij}, y_{ij}/d_j)$, and $\overrightarrow{AF} = \epsilon(t)\overrightarrow{AE}$. Then

$$(v_j, w_j) = \overrightarrow{OA} + \epsilon(t)\overrightarrow{AE} = \overrightarrow{OA} + \overrightarrow{AF} = \overrightarrow{OF}.$$

If $a_i < \min\{a_j, d_j b_j\}$, it is clear that

$$\frac{(v_i \cos\beta_{ij})^2}{{a_j}^2} + \frac{\left(\frac{v_i \sin\beta_{ij}}{d_j}\right)^2}{{b_j}^2} \leqslant \frac{(a_i \cos\beta_{ij})^2}{{a_j}^2} + \frac{(a_i \sin\beta_{ij})^2}{{d_j}^2 {b_j}^2} < \cos^2\beta_{ij} + \sin^2\beta_{ij} = 1. \quad (15)$$

Inequality (15) means that $\overrightarrow{OA}$ lies in the elliptic boundary of input constraints. Therefore, regardless of the magnitude and angle of $\overrightarrow{AE}$, there is always an $\epsilon(t) > 0$ for $\forall t > 0$ which makes $\overrightarrow{OF}$ stay in the ellipse (see Fig. 4). That is to say there are suitable feedback parameters such that controller (11) and (12) can meet the input constraints (3). ■

Remark 3: In view of (15), there is a particular case that if $a_i = \min\{a_j, d_j b_j\}$ and $a_j \neq d_j b_j$, Theorem 1 still holds.

Remark 4: As stated in Theorem 1, $a_i < a_j$. It means that the robot with better performance should be designated as the follower for the best use of the robots.

Theorem 1 provides a sufficient condition on the leader to guarantee that the feasible feedback parameters can be obtained. The following theorem will give an instance of the selection of the two feedback parameters for (11) and (12).

Theorem 2: Consider the leader-follower system (6) and (7) with the input constraints (3). If $a_i < \min\{a_j, d_j b_j\}$, then the following control inputs

$$v_j = \frac{1}{2}\left(v_i \cos\beta_{ij} + \mathrm{sign}(x_{ij}) a_j \sqrt{1 - \left(\frac{v_i \sin\beta_{ij}}{d_j b_j}\right)^2}\right) \quad (16)$$

$$w_j = \frac{1}{2}\left(\frac{v_i \sin\beta_{ij}}{d_j} + \mathrm{sign}(y_{ij}) b_j \sqrt{1 - \left(\frac{v_i \cos\beta_{ij}}{a_j}\right)^2}\right) \quad (17)$$

can satisfy the input constraints and maintain the desired column formation.

Proof: Control inputs (11) and (12) can be reshaped as

$$(v_j, w_j) = \left(v_i \cos\beta_{ij}, \frac{v_i \sin\beta_{ij}}{d_j}\right) + k_{j_1}(t)\,(x_{ij}, 0) + k_{j_2}(t)\left(0, \frac{y_{ij}}{d_j}\right). \quad (18)$$

As depicted in Fig. 4, line $v_j = v_i \cos\beta_{ij}$ intersects with the elliptic input boundary of R_j at

$$w_j = \pm b_j \sqrt{1 - \left(\frac{v_i \cos\beta_{ij}}{a_j}\right)^2}.$$

Similarly, line $w_j = \frac{v_i \sin\beta_{ij}}{d_j}$ intersects with the elliptic input boundary at

$$v_j = \pm a_j \sqrt{1 - \left(\frac{v_i \sin\beta_{ij}}{d_j b_j}\right)^2}.$$

Denote B as the vertical intersection

$$\left(v_i \cos\beta_{ij}, \mathrm{sign}(y_{ij}) b_j \sqrt{1 - (\frac{v_i \cos\beta_{ij}}{a_j})^2}\right),$$

and C as the oriental intersection

$$\left(\mathrm{sign}(x_{ij}) a_j \sqrt{1 - (\frac{v_i \sin\beta_{ij}}{d_j b_j})^2}, \frac{v_i \sin\beta_{ij}}{d_j}\right).$$

Define D as the midpoint of the line segment BC. From the definition of $\overrightarrow{OA}$ in Theorem 1, we get

$$\overrightarrow{AB} = \left(0, \mathrm{sign}(y_{ij}) b_j \sqrt{1 - \left(\frac{v_i \cos\beta_{ij}}{a_j}\right)^2} - \frac{v_i \sin\beta_{ij}}{d_j}\right)$$

$$\overrightarrow{AC} = \left(\mathrm{sign}(x_{ij}) a_j \sqrt{1 - \left(\frac{v_i \sin\beta_{ij}}{d_j b_j}\right)^2} - v_i \cos\beta_{ij}, 0\right).$$

Assume $x_{ij} \neq 0$ and $y_{ij} \neq 0$, and choose $k_{j_1}(t)$ and $k_{j_2}(t)$ as

$$k_{j_1}(t) = \frac{\text{sign}(x_{ij}) a_j \sqrt{1 - \left(\frac{v_i \sin\beta_{ij}}{d_j b_j}\right)^2} - v_i \cos\beta_{ij}}{2x_{ij}} = \frac{a_j \sqrt{1 - \left(\frac{v_i \sin\beta_{ij}}{d_j b_j}\right)^2}}{2|x_{ij}|} - \frac{v_i \cos\beta_{ij}}{2x_{ij}} \quad (19)$$

$$k_{j_2}(t) = \frac{\text{sign}(y_{ij}) b_j \sqrt{1 - \left(\frac{v_i \cos\beta_{ij}}{a_j}\right)^2} - \frac{v_i \sin\beta_{ij}}{d_j}}{2\frac{y_{ij}}{d_j}} = \frac{d_j b_j \sqrt{1 - \left(\frac{v_i \cos\beta_{ij}}{a_j}\right)^2}}{2|y_{ij}|} - \frac{v_i \sin\beta_{ij}}{2y_{ij}}. \quad (20)$$

Then $k_{j_1}(t)(x_{ij}, 0) = \overrightarrow{AC}/2$ and $k_{j_2}(t)(0, y_{ij}/d_j) = \overrightarrow{AB}/2$. Now, (18) can be rewritten as

$$\begin{aligned}(v_j, w_j) =& \overrightarrow{OA} + \frac{1}{2}\overrightarrow{AB} + \frac{1}{2}\overrightarrow{AC} = \overrightarrow{OA} + \overrightarrow{AD} = \overrightarrow{OD} \\ =& \frac{1}{2}\left(v_i \cos\beta_{ij} + \text{sign}(x_{ij}) a_j \sqrt{1 - \left(\frac{v_i \sin\beta_{ij}}{d_j b_j}\right)^2},\right. \\ & \left.\frac{v_i \sin\beta_{ij}}{d_j} + \text{sign}(y_{ij}) b_j \sqrt{1 - \left(\frac{v_i \cos\beta_{ij}}{b_j}\right)^2}\right). \end{aligned} \quad (21)$$

As shown in Fig. 4, it is obvious that D is an inner point of the ellipse, i.e., control inputs (16) and (17) satisfy the input constraints.

Now we will show the feedback parameters defined by (19) and (20) are positive functions. According to (15) in Theorem 1, we get

$$\left|\frac{v_i \cos\beta_{ij}}{x_{ij}}\right| < \frac{a_j \sqrt{1 - \left(\frac{v_i \sin\beta_{ij}}{d_j b_j}\right)^2}}{|x_{ij}|} \quad (22)$$

$$\left|\frac{v_i \sin\beta_{ij}}{y_{ij}}\right| < \frac{d_j b_j \sqrt{1 - \left(\frac{v_i \cos\beta_{ij}}{a_j}\right)^2}}{|y_{ij}|}. \quad (23)$$

Inequalities (22) and (23) imply that $k_{j_1}(t) > 0$ and $k_{j_2}(t) > 0$. By Lemma 1, control inputs (16) and (17) can make x_{ij} and y_{ij} converge to zero. ■

However, in (16) and (17) sign is a discontinuous function which brings oscillations to control inputs when x_{ij} and y_{ij} are small. To avoid discontinuities, (16) and (17) can be modified as

$$v_j = \frac{\lambda(x_{ij})}{2}\left(\text{sign}(x_{ij}) a_j \sqrt{1 - \left(\frac{v_i \sin\beta_{ij}}{d_j b_j}\right)^2} - v_i \cos\beta_{ij}\right) + v_i \cos\beta_{ij} \quad (24)$$

$$w_j = \frac{\lambda(y_{ij})}{2}\left(\text{sign}(y_{ij}) b_j \sqrt{1 - \left(\frac{v_i \cos\beta_{ij}}{a_j}\right)^2} - \frac{v_i \sin\beta_{ij}}{d_j}\right) + \frac{v_i \sin\beta_{ij}}{d_j} \quad (25)$$

where $\lambda(\cdot)$ satisfies

$$\lambda(x) = \begin{cases} 1, & |x| > 1 \\ |x|, & |x| \leqslant 1. \end{cases}$$

The following corollary will show the effectiveness of control inputs (24) and (25).

Corollary 1: Consider the leader-follower system (6) and (7) with the input constraints (3), and let (24) and (25) be the control inputs. If $a_i < \min\{a_j, d_j b_j\}$, then the input constraints can be satisfied, and x_{ij}, y_{ij} will converge to zero.

Proof: From the proof of Theorem 2, (24) and (25) can be rewritten as

$$(v_j, w_j) = \overrightarrow{OA} + \lambda(y_{ij})\frac{\overrightarrow{AB}}{2} + \lambda(x_{ij})\frac{\overrightarrow{AC}}{2}.$$

Since $0 \leqslant \lambda(\cdot) \leqslant 1$, (v_j, w_j) is an inner point of the elliptic input constraints (see Fig. 4), i.e., control inputs (24) and (25) satisfy the input constraints. In view of (19) and (20), (24) and (25) can be rewritten as

$$\begin{aligned} v_j &= v_i \cos\beta_{ij} + \lambda(x_{ij}) k_{j_1}(t) x_{ij} \\ w_j &= \frac{v_i \sin\beta_{ij}}{d_j} + \lambda(y_{ij}) k_{j_2}(t) y_{ij}. \end{aligned}$$

According to Lemma 1, x_{ij} and y_{ij} will converge to zero by using the control inputs (24) and (25). ■

V. Simulation Results

Three robots are employed in this simulation. The velocities of the leader robot R_1 are

$$\begin{aligned} v_1 &= 0.5\sin(t) + 0.5 \\ w_1 &= \begin{cases} \frac{v_1}{2} & 0s \leqslant t < 7.5s \\ -\frac{v_1}{2} & t \geqslant 7.5s \end{cases}. \end{aligned}$$

Robot R_2 is the follower of R_1 and the leader of R_3. The desired separations are $l_{12}^d = l_{23}^d = 1m$. R_1, R_2 and R_3 suffer the same input constraints $v^2/2 + w^2/32 \leqslant 1$. The initial states for R_1, R_2 and R_3 are $(0, 0, \pi/2)$, $(-1.5, -1.5, 0)$ and $(-3, -0.5, -\pi/2)$, respectively. Control inputs (24) and (25) are used. Fig. 5 depicts the trajectories of the three robots in the world frame. From Fig. 6, we can see the formation errors E_{12} and E_{23} gradually converge to zero as time increases, where $E_{12} = \sqrt{x_{12}^2 + y_{12}^2}$ and $E_{23} = \sqrt{x_{23}^2 + y_{23}^2}$. Fig. 7 represents the velocities of R_2 and R_3, which are continuous. Fig. 8 shows that control inputs of R_2 and R_3 are always bounded in the elliptic boundary of input constraints.

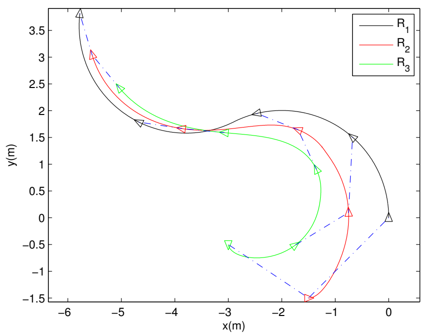

Fig. 5: Trajectories of R_1, R_2, and R_3.

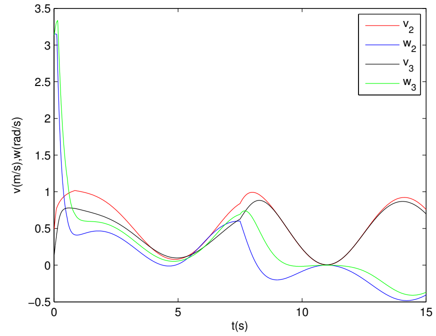

Fig. 7: Velocities for R_2 and R_3.

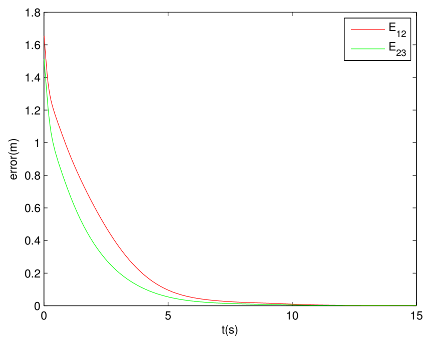

Fig. 6: Formation errors.

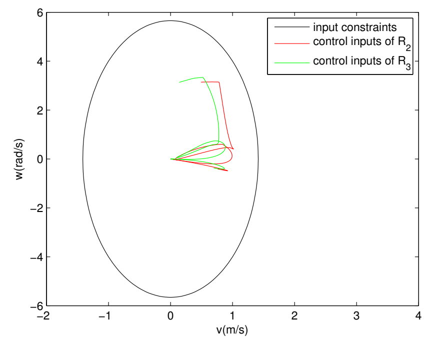

Fig. 8: Elliptic input constraints and control inputs of R_2 and R_3.

VI. Conclusions

In this paper, the column formation control of multiple robots with input constraints has been discussed. The major contribution of this work includes two aspects: First, the proposed leader-follower setup has given a simple controller in term of two feedback parameters directly by Lyapunov method. Second, the elliptic approximation of input constraints has introduced convenience to the selection of feedback parameters. A sufficient condition on the leader has been provided to ensure the desired formation can be achieved. In particular, the formation controller with specific selection of the feedback parameters has been proposed, and its effectiveness has been verified by simulation results.

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