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Robust Integral of Neural Network and Sign of Tracking Error Control of Uncertain Nonlinear Affine Systems Using State and Output Feedback

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Abstract—**This paper presents a novel state and output feedback control law for the tracking control of a class of multi-input-multi-output (MIMO) continuous time nonlinear systems with unknown dynamics and disturbance input. First the state feedback based control law is designed which consists of the robust integral of a neural network (NN) output plus the sign of the tracking error signal multiplied with an adaptive gain. The two-layer NN learns the system dynamics in an online manner while the NN residual reconstruction errors and the bounded system disturbances are overcome by the error sign signal. Both of the NN output and error sign signal are included into the integral to ensure the control input is a smooth function. Since certain states are not available in practice, subsequently, a high-gain observer is utilized to estimate the unmeasurable system states and output feedback based controller is designed. A semi-global asymptotic tracking performance is guaranteed in the case of state feedback while boundedness in the case of output feedback and the NN weights and all other signals are shown to be bounded by using the Lyapunov method. Finally, theoretical results are verified in the simulation environment.**

I. INTRODUCTION

The tracking control of nonlinear systems with unknown dynamics has attracted a great deal of interest within the control community. Various approaches have been designed for the control of several important classes of uncertain nonlinear systems [1-3]. In the recent years, due to their universal approximation properties [13-14], neural network (NN) techniques have been utilized [9-12] extensively in order to parameterize the unknown plant nonlinearities.

However, NN based control methodologies typically deliver uniformly ultimately bounded (UUB) stability results due to NN functional reconstruction errors and unknown disturbances [15]. In the recent literature, a significant effort is in place to achieve asymptotic stability. Among them, a robust term of sign function is typically used [16] to constrain the tracking error into a bounded set which could be made arbitrarily small by increasing the controller's adaptive rate. An innovative neuro-adaptive control framework is proposed in [17] to guarantee asymptotic stability with an assumption that the approximation errors of uncertain system nonlinearities lie in a bounded conic sector. On the other hand, a novel robust design to guarantee asymptotic convergence of discrete-time systems is discussed in [6].

Recently, the robust integral of the sign of the error (RISE) term originating in [18] is blended with a multilayer NN in [19] to yield semi-global asymptotic tracking performance which also generates a continuous-time control signal. Therefore, it waives the requirement of infinite bandwidth and chattering [19]. However, certain bounds on the disturbance and NN reconstruction errors have to be known for control parameter selection while the use of projection algorithm [21] demands the selection of a predefined convex set so as to force the target NN weights [20] to lie within the set which is a challenge. Moreover, only the desired system state trajectory is taken as the NN input, which may result in large control gains to compensate for the unmeasurable auxiliary terms.

In contrast, in this work, the weaknesses of the traditional RISE controller are relaxed. A two-layer NN structure is utilized where the NN input vector includes the tracking error and the control input signals so that the auxiliary terms are always bounded in an arbitrarily large compact set as long as the number of neurons is chosen sufficiently large. As a consequence, the region of attraction can be made arbitrarily large to include any initial conditions without reconfiguring the controller. Next, the derivative of the control signal is designed instead of the control input to guarantee its continuity. Meanwhile, the gain of the robust term is varied adaptively through an updating rule which in turn helps to relax the bounds of the unknown system disturbances and the NN reconstruction errors along with their derivatives. Further, a novel NN weight tuning law is developed instead of the projection algorithm to eliminate the need for the convex set.

Finally, with only outputs measurable in many practical environments, a high-gain observer is employed to estimate the unmeasurable system states, so that an output feedback control law is developed. It is shown that a semi-global asymptotic tracking performance is achieved when the states are measurable and semi-globally uniformly ultimately boundedness in the presence of a high-gain observer. The boundedness of the NN weights and other signals in the closed-loop system are also shown by using Lyapunov analysis. The separation principle is relaxed by considering the observer errors in the same Lyapunov candidate.

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II. PRELIMINARIES

A. Problem statement

Consider a class of multi-input-multi-output (MIMO) continuous-time affine nonlinear systems described by

$$\begin{aligned} x^{(n)} &= f(X) + g(X)u + d(t) \\ y &= x \end{aligned} \tag{1}$$

where $f(\cdot):\mathbb{R}^{n^2} \to \mathbb{R}^n$ and $g(\cdot):\mathbb{R}^{n^2} \to \mathbb{R}^{n\times n}$ are uncertain nonlinear smooth functions, $(\cdot)^{(i)}(t)$ denotes the ith derivative with respect to time. $x(t) \in \mathbb{R}^n$ is the system state vector, $u(t) \in \mathbb{R}^n$ is the control input vector, and $y(t) \in \mathbb{R}^n$ is the output. $X(t) \equiv \begin{bmatrix} x(t)^T & \dot{x}(t)^T & \dots & x^{(n-1)}(t)^T \end{bmatrix}^T \in \mathbb{R}^{n^2}$ is the vector of all states and their derivatives, and $d(t) \in \mathbb{R}^n$ represents the unknown bounded disturbance vector. The control objective is to drive the system output $y(t)$ track a $\mathbb{C}^{n+2}$ reference trajectory $y_d(t) \in \mathbb{R}^n$, such that $y_d^{(i)}(t) \in L_\infty$ for $i = 0,1,...,n+2$.

Assumption 1: Since $g(\cdot)$ is a $\mathbb{C}^1$ smooth nonlinear function, without loss of generality, let $g(X)$ be a positive definite matrix for all $X \in \mathbb{R}^{n^2}$ with $g_{\min} \in R^+$ and $g_{\max} \in R^+$ representing the minimum and maximum singular value of the matrix $g(X)$ respectively with $0 < g_{\min} \le g_{\max}$.

Assumption 2: The disturbance $d(t)$ and its derivatives are bounded above such that $\|d(t)\| \le d_{m0}$, $\|\dot{d}(t)\| \le d_{m1}$, and $\|\ddot{d}(t)\| \le d_{m2}$, where $d_{m0}, d_{m1}, d_{m2} \in \mathbb{R}^+$ are unknown positive constants with $\|\bullet\|$ denoting the standard Euclidean norm.

Assumptions 1 and 2 are commonly found in the control literature [18]. It also has to be noted that the bounds of the disturbance and its derivative are not required to be known.

B. Two-layer neural networks

In our controller architecture, a NN having two layers is considered. The NN output is $\hat{M}(A) = W^T\phi(V^T A) \in \mathbb{R}^{N_3}$, where $A \in \mathbb{R}^{N_1}$ is the NN input, $V \in \mathbb{R}^{N_1\times N_2}$ and $W \in \mathbb{R}^{N_2\times N_3}$ denote the hidden and output layer weights respectively. $\phi(\cdot):\mathbb{R}^{N_2} \to \mathbb{R}^{N_2}$ is the activation function in the hidden layer which is selected as the hyperbolic tangent function in this work, and the number of hidden layer nodes is denoted as N_2. Therefore, the activation function is bounded by known positive value such that $\|\phi(\cdot)\| \le \phi_m$, where $\phi_m \in \mathbb{R}^+$. The NN *universal approximation property* states that any smooth function $M(A)$ can be written as [10]

$$M(A) = W^T\phi(V^T A) + \varepsilon(A) \tag{2}$$

for some target weights W, V, with $\varepsilon(A)$ being a NN functional reconstruction error vector. It is demonstrated [23] that if the hidden layer weights, V, are chosen initially at random and held fixed, while N_2 is sufficiently large, the NN reconstruction error ε can be made arbitrarily small for all input $A \in S$ in an arbitrary compact set $S \subset \mathbb{R}^{N_1}$.

III. CONTROLLER METHODOLOGY

A. Dynamics of filtered tracking error

The tracking error $e(t) \in \mathbb{R}^n$ between the actual and desired system state is firstly defined as

$$e = x - x_d \tag{3}$$

Thereafter, define the filtered tracking error as

$$r = \lambda_{n-1}e^{(n-1)} + \lambda_{n-2}e^{(n-2)} + \dots + \lambda_0 e = \sum_{i=0}^{n-1}\lambda_i e^{(i)} \tag{4}$$

where $\lambda_0, \dots, \lambda_{n-1}$ are appropriately chosen constants such that $\lambda_{n-1}s^{n-1} + \lambda_{n-2}s^{n-2} + \dots + \lambda_0$ is Hurwitz. As a consequence, $e \to 0$ exponentially when $r \to 0$. Without loss of generality, take $\lambda_{n-1} = 1$.

Taking the derivative of (4) and using (1) yields

$$\begin{aligned} \dot{r} &= \sum_{i=0}^{n-1}\lambda_i e^{(i+1)} = \sum_{i=0}^{n-1}\lambda_i (x - x_d)^{(i+1)} \\ &= \sum_{i=0}^{n-2}\lambda_i (x - x_d)^{(i+1)} + f(X) + g(X)u(t) + d(t) - x_d^{(n)} \\ &= F(\bar{X}) + g(X)u(t) + d(t) \end{aligned} \tag{5}$$

where $\bar{X} \equiv \begin{bmatrix} x^{(0)^T} & \dots & x^{(n-1)^T} & x_d^{(0)^T} & \dots & x_d^{(n)^T} \end{bmatrix}^T \in \mathbb{R}^{2n^2+n}$ and

$$F(\bar{X}) = \sum_{i=0}^{n-2}\lambda_i (x - x_d)^{(i+1)} + f(X) - x_d^{(n)} \tag{6}$$

Define $\bar{X}_{n-1} = \begin{bmatrix} x^{(0)^T} & \dots & x^{(n-2)^T} & x_d^{(0)^T} & \dots & x_d^{(n-1)^T} \end{bmatrix}^T$. Then, by using standard matrix calculus [7], differentiating (6) gives

$$\begin{aligned} \dot{F}(\bar{X}) &= \frac{\partial F(\bar{X})}{\partial \bar{X}_{n-1}}\dot{\bar{X}}_{n-1} + \frac{\partial F(\bar{X})}{\partial x^{(n-1)}}x^{(n)} \\ &= \frac{\partial F(\bar{X})}{\partial \bar{X}_{n-1}}\dot{\bar{X}}_{n-1} + \frac{\partial F(\bar{X})}{\partial x^{(n-1)}}\left[f(X) + g(X)u(t) + d(t)\right] \end{aligned} \tag{7}$$

Denote $G(X) \equiv g^{-1}(X)$, which is also a $\mathbb{C}^1$ smooth function with $1/g_{\max}$ and $1/g_{\min}$ being the minimum and maximum singular value respectively by recalling Assumption 1. Similarly, define $X_{n-1} = \begin{bmatrix} x^{(0)^T} & \dots & x^{(n-2)^T} \end{bmatrix}^T$ and rewrite

$$\begin{aligned} \dot{G}(X) &= \frac{\partial G(X)}{\partial X_{n-1}}\dot{X}_{n-1} + \frac{\partial G(X)}{\partial x^{(n-1)}}x^{(n)} \\ &= \frac{\partial G(X)}{\partial X_{n-1}}\dot{X}_{n-1} + \frac{\partial G(X)}{\partial x^{(n-1)}}\left[f(X) + g(X)u(t) + d(t)\right] \end{aligned} \tag{8}$$

where $\frac{\partial G(X)}{\partial X_{n-1}} \in \mathbb{R}^{n\times n\times(n^2-n)\times 1}$, $\frac{\partial G(X)}{\partial x^{(n-1)}} \in \mathbb{R}^{n\times n\times n\times 1}$ are quartixes (fourth-order tensor) [7].

Remark 1: Since $x^{(n)}$ is unavailable, taking the time derivative of $F(X)$ and $g(X)$ renders their derivatives to be expressed as functions of measurable signals X, u, and the bounded disturbance d.

Therefore, differentiating (5) gives

$$\ddot{r} = \dot{F}(\bar{X}) + \dot{g}(X)u(t) + g(X)\dot{u}(t) + \dot{d}(t) \tag{9}$$

Further, define $\bar{r} = \dot{r} + \alpha r$ with $\alpha \in \mathbb{R}^+$ being a positive constant, and by utilizing (9) one has

$$\begin{aligned} G(X)\dot{\bar{r}} &= G(X)\left(\ddot{r} + \alpha\dot{r}\right) \\ &= G(X)\left[\dot{F}(\bar{X}) + \dot{g}(X)u(t) + g(X)\dot{u}(t) + \dot{d}(t)\right] \\ &\quad + \alpha G(X)\left[F(\bar{X}) + g(X)u(t) + d(t)\right] \\ &= -\frac{1}{2}\dot{G}(X)\bar{r} - r + \dot{u}(t) + G(X)\dot{d}(t) + \alpha G(X)d(t) \\ &\quad + \left[\frac{1}{2}\dot{G}(X)\dot{r} + \frac{\alpha}{2}\dot{G}(X)r + r + G(X)\dot{F}(\bar{X})\right. \\ &\quad \left. + G(X)\dot{g}(X)u(t) + \alpha G(X)F(\bar{X}) + \alpha u(t)\right] \end{aligned} \tag{10}$$

After substituting (5), (7), and (8) into (10) and combining similar terms, the following expression can be obtained

$$G(X)\dot{\bar{r}} = -\frac{1}{2}\dot{G}(X)\bar{r} - r + \dot{u}(t) + N(Y) + D(t) \tag{11}$$

where the auxiliary function is defined as

$$\begin{aligned} N(Y) &= G(X)\frac{\partial F(\bar{X})}{\partial \bar{X}_{n-1}}\dot{\bar{X}}_{n-1} + G(X)\frac{\partial F(\bar{X})}{\partial x^{(n-1)}}\left(f(X) + g(X)u\right) \\ &\quad + \frac{1}{2}\left(\frac{\partial G(X)}{\partial X_{n-1}}\dot{X}_{n-1} + \frac{\partial G(X)}{\partial x^{(n-1)}}\left(f(X) + g(X)u\right)\right) \\ &\quad \times\left(F(\bar{X}) + g(X)u + \alpha r\right) + r + \alpha G(X)F(\bar{X}) + \alpha u \end{aligned} \tag{12}$$

and

$$\begin{aligned} D(t) &= G(X)\dot{d} + \alpha G(X)d + G(X)\frac{\partial F(\bar{X})}{\partial x^{(n-1)}}d \\ &\quad + \frac{1}{2}\left(\frac{\partial G(X)}{\partial x^{(n-1)}}d\right)\left(F(\bar{X}) - g(X)u + \alpha r\right) \\ &\quad + \frac{1}{2}\left(\frac{\partial G(X)}{\partial X_{n-1}}\dot{X}_{n-1} + \frac{\partial G(X)}{\partial x^{(n-1)}}\left(f(X) + g(X)u + d\right)\right)d \end{aligned} \tag{13}$$

with

$$Y \equiv \left[\bar{X}^T \quad u^T\right]^T \in \mathbb{R}^{2n(n+1)} \tag{14}$$

B. NN approximation

Notice that the expression (12) is an unknown smooth nonlinear function. Therefore, given an arbitrary compact set $\Omega \subset \mathbb{R}^{2n(n+1)}$, for all $Y \in \Omega$, the auxiliary function can be represented by a two-layer NN as

$$N(Y) = W^T\phi(V^TY) + \varepsilon(Y) = W^T\phi(Y) + \varepsilon(Y) \tag{15}$$

The hidden layer weights V are omitted for convenience. The *universal approximation property* of NNs [10] shows that the constant target weight matrix $W \in \mathbb{R}^{N_n \times n}$ satisfies $\|W\| \le W_m$ with N_n being the number of hidden neurons, ε being NN reconstruction error satisfying $\|\varepsilon\| \le \varepsilon_{m0}$ and $\|\dot{\varepsilon}\| \le \varepsilon_{m1}$.

***Remark 2*:** The NN reconstruction error bounds ε_{m0}, ε_{m1} are considered to be unknown as opposed to [19-20]. Furthermore, due to Assumption 1 and the smoothness of the NN activation function $\phi(\cdot)$, it can be readily shown that $\dot{\varepsilon}$ is bounded above on Ω.

***Remark 3*:** The control signal $u(t)$ is also used as an input to the NN to approximate the auxiliary function $N(Y)$.

***Remark 4*:** Due to Assumption 1, $D(t)$ is continuously differentiable. Therefore, with the help of Assumption 2, the Mean Value Theorem [22] can be used to show that, given an arbitrary compact set Ω, the function $D(t)$ and $\dot{D}(t)$ are bounded above such that $\|D(t)\| \le D_{m0}$, $\|\dot{D}(t)\| \le D_{m1}$, where D_{m0} and D_{m1} are determined by d_{m0}, d_{m1}, d_{m2} and Ω.

By utilizing (15), the system dynamics in (11) can be rewritten as

$$G(X)\dot{\bar{r}} = -\frac{1}{2}\dot{G}(X)\bar{r} - r + \dot{u}(t) + W^T\phi(Y) + D(t) + \varepsilon(Y) \tag{16}$$

Since the target weights of the NN are unknown, the NN approximation of $N(Y)$ is now defined as

$$\hat{N}(Y) = \hat{W}^T\phi(Y) \tag{17}$$

where $\hat{W} \in \mathbb{R}^{N_n \times n}$ is the NN estimate of the target weight matrix W with the input vector Y defined in (14).

C. State feedback controller development

In this subsection, the system state vector $X(t)$ is considered to be measurable to design a state feedback control law. Based on the system dynamics in (16), the control law is now designed as

$$\dot{u}(t) = -(k_s + 1)\dot{r}(t) - \hat{W}^T\phi(Y) + (k_s + 1)\alpha r(t) + \beta(t)\,\mathrm{sgn}(r(t)) \tag{18}$$

which in turn renders the control input with zero initial value

$$\begin{aligned} u(t) &= -(k_s + 1)r(t) + (k_s + 1)r(0) \\ &\quad - \int_0^t\left[\hat{W}^T\phi(Y(\tau)) + (k_s + 1)\alpha r(\tau) + \beta(\tau)\,\mathrm{sgn}(r(\tau))\right]d\tau \end{aligned} \tag{19}$$

where $k_s \in \mathbb{R}^+$ is positive constant control gain, $\beta(t) \in \mathbb{R}$ is an adaptive term which will be given later, and $\mathrm{sgn}(\bullet)$ is the signum function. It has to be noted that actually a first order dynamic system in terms of the control input is built in (19). Therefore, the control signal can be fed to the NN as an input.

Substituting (18) into (16) generates the closed-loop tracking error system dynamics as

$$G(X)\dot{\bar{r}} = -\frac{1}{2}\dot{G}(X)\bar{r} - r - \tilde{W}^T\phi(Y) - (k_s + 1)\alpha\bar{r} - \beta\,\mathrm{sgn}(r) + \bar{D}(t) \tag{20}$$

where $\bar{D} \equiv D + \varepsilon$, and $\tilde{W} \equiv \hat{W} - W$ is the NN weight estimation error. It is not difficult to see that $\|\bar{D}(t)\| \le \bar{D}_{m0} \equiv D_{m0} + \varepsilon_{m0}$ and $\|\dot{\bar{D}}(t)\| \le \bar{D}_{m1} \equiv D_{m1} + \varepsilon_{m1}$.

Finally, the NN weight tuning law is designed as

$$\hat{W} = k_n\phi(Y)r^T - k_n\phi(Y(0))r^T(0) + \alpha k_n\int_0^t\phi(Y(\tau))r^T(\tau)d\tau \tag{21}$$

where $k_n \in \mathbb{R}^+$ is a positive user design parameter. Further, the adaptive term β can be written as

$$\beta = |r(t)| - |r(0)| + \alpha \int_0^t |r(\tau)| d\tau \tag{22}$$

where $|\bullet|$ stands for $\|\bullet\|_1$. The adaptive term β is updated to approach a constant value $\beta_d = \bar{D}_{m0} + \bar{D}_{m1}/\alpha$, and the approximation error is defined as $\tilde{\beta} = \beta - \beta_d$.

***Remark 5*:** Since the projection algorithm is not used, the challenge of selecting the predefined convex set is avoided.

***Remark 6*:** The control gain β is designed as a function of time and not as a constant as in [19] or a function depending on the NN reconstruction error and disturbance bounds [20].

D. Stability analysis

Lemma 1: Given an auxiliary function defined as

$$L(t) \equiv \bar{r}^T (\bar{D} - \beta_d \operatorname{sgn}(r)) \tag{23}$$

where $\beta_d \equiv |\bar{D}_{m0}| + \frac{1}{\alpha}|\bar{D}_{m1}| \in \mathbb{R}^+$ is an unknown positive constant, the following inequality is satisfied

$$\int_0^t L(\tau) d\tau \le \beta_d |r(0)| - r(0)^T \bar{D}(0) \tag{24}$$

Proof: The proof can be readily obtained by expanding and integrating (23) [8].

We now state the stability result for the proposed state feedback controller.

Theorem 1: Consider the uncertain nonlinear system given by (1) with all states available and the Assumptions 1 and 2 hold. The control law (18), NN weight update law (21) and adaptive parameter tuning law (22) ensure that all signals are bounded and the tracking error converges to zero asymptotically, i.e, $\|e(t)\| \to 0$ as $t \to \infty$.

Proof: The proof is similar to [8] and thus omitted due to space limit.

E. Output Feedback Control Design

In practical applications, when only the output is available, the output feedback control needs to be considered. First of all, a high gain observer is designed as

$$\begin{aligned} \varepsilon\dot{\xi}_1 &= \xi_2 \\ &\vdots \\ \varepsilon\dot{\xi}_n &= \xi_{n+1} \\ \varepsilon\dot{\xi}_{n+1} &= -b_1\xi_{n+1} - b_2\xi_n - \cdots - b_n\xi_2 - \xi_1 + y(t) \end{aligned} \tag{25}$$

where ε is any small positive constant, and the positive parameters $b_1, \cdots, b_n$ are chosen such that the polynomial $s^{n+1} + b_1 s^n + \cdots + b_n$ is *Hurwitz*.

Lemma 2: Consider the system (1) and the observer (25). There exist positive constants $h_k, k = 2, 3, \ldots, n+1$, and t^* such that for all $t > t^*$, we have

$$\begin{aligned} &\dot{\xi}_{k+1}/\varepsilon^k - y^{(k)} = -\varepsilon\varphi^{(k+1)} && k = 1, \ldots, n \\ &\left\|\dot{\xi}_{k+1}/\varepsilon^k - y^{(k)}\right\| \le \varepsilon D_{k+1} && k = 1, \ldots, n \end{aligned} \tag{26}$$

where $\varphi = \xi_{n+1} + b_1\xi_n + \cdots + b_n\xi_1$ and $\|\varphi^{(k)}\| \le D_k$.

Proof: The proof is similar to [4] and thus omitted.

It has to be noted that a n+1*th* order observer is constructed in order to achieve an estimate of $y^{(n)}$, which is assumed to be not measurable even in the state feedback version.

Having observer (25), following variables are defined

$$\begin{aligned} &\hat{X} = \begin{bmatrix} \hat{x}_1^T & \hat{x}_2^T & \cdots & \hat{x}_n^T \end{bmatrix}^T = \begin{bmatrix} x^T & \xi_2^T/\varepsilon & \cdots & \xi_n^T/\varepsilon^{n-1} \end{bmatrix}^T \\ &\hat{r} = \sum_{i=0}^{n-1} \lambda_i \left(\hat{x}_i - x_d^{(i)} \right) \\ &\hat{\bar{r}} = \dot{\hat{r}} + \alpha\hat{r} = \sum_{i=1}^{n} \lambda_{i-1} \left(\hat{x}_i - x_d^{(i)} \right) + \alpha \sum_{i=0}^{n-1} \lambda_i \left(\hat{x}_i - x_d^{(i)} \right) \\ &\hat{\bar{X}} = \begin{bmatrix} \hat{x}_1^T & \ldots & \hat{x}_n^T & x_d^{(0)^T} & \ldots & x_d^{(n)^T} \end{bmatrix}^T \\ &\hat{Y} = \begin{bmatrix} \hat{\bar{X}}^T & u^T \end{bmatrix}^T \end{aligned} \tag{27}$$

Further, the neural network with target weights in (15) is redefined as

$$W^T \phi(\bar{Y}) + \varepsilon(\bar{Y}) = N(Y) + \frac{1}{2}\dot{G}(X)\hat{\bar{r}} + \hat{r} - \frac{1}{2}\dot{G}(X)\bar{r} - r \tag{28}$$

where $\bar{Y} = \begin{bmatrix} Y^T & \hat{\bar{X}}^T \end{bmatrix}^T$ and (16) can be now rephrased as

$$G(X)\dot{\bar{r}} = -\frac{1}{2}\dot{G}(X)\hat{\bar{r}} - \hat{r} + \dot{u}(t) + W^T\phi(\bar{Y}) + D(t) + \varepsilon(\bar{Y}) \tag{29}$$

Since Y is not completely measurable, the target NN is defined virtually for proof purpose and its input is expanded to include $\hat{\bar{X}}$. Also let the unknown target output layer weights be upper bounded by $\|W\| \le W_m \in \mathbb{R}^+$.

Therefore, the output feedback version of the proposed controller is obtained by replacing the state $X(t)$ with $\hat{X}(t)$, which is provided by (25). In other words, the control input is now selected as

$$\begin{aligned} u(t) = &-(k_s+1)\hat{r}(t) + (k_s+1)\hat{r}(0) \\ &- \int_0^t \left[\hat{W}^T \phi(\hat{Y}(\tau)) + (k_s+1)\alpha\hat{r}(\tau) + \beta(\tau)\operatorname{sgn}(\hat{r}(\tau)) \right] d\tau \end{aligned} \tag{30}$$

The updating laws for NN and β are also changed to

$$\hat{W} = k_n \phi(\hat{Y})\hat{r}^T - k_n \phi(\hat{Y}(0))\hat{r}^T(0) + \alpha k_n \int_0^t \phi(\hat{Y}(\tau))\hat{r}^T(\tau) d\tau \tag{31}$$

$$\beta = |\hat{r}(t)| - |\hat{r}(0)| + \alpha \int_0^t |\hat{r}(\tau)| d\tau \tag{32}$$

The actual NN has the same number of neurons within the hidden layer as the ideal NN. Hence, due to boundedness of the activation function, we have

$$\left\|\tilde{\phi}(\bar{Y})\right\| \equiv \left\|\phi(\hat{Y}) - \phi(\bar{Y})\right\| = 2\phi_m \tag{33}$$

Hence, following theorem is presented.

Theorem 2: Consider the uncertain nonlinear system (1) with only output available and the high-gain observer (25).

The output feedback controller is given in (30) along with the updating laws (31) and (32). Let Assumptions 1 and 2 hold. Then, the tracking error, e, is semi globally uniformly ultimately bounded (SGUUB). Meanwhile, the other signals of the closed-loop system are also bounded.

Proof: Due to Lemma 2 and (26), one has

$$\hat{r}-r=\sum_{i=0}^{n-1}\lambda_i\left(\hat{x}_i-y^{(i)}\right)=-\sum_{i=0}^{n-1}\lambda_i\varepsilon\varphi^{(i+1)} \tag{34}$$

$$\begin{aligned}\hat{\bar{r}}-\bar{r}&=\sum_{i=0}^{n-1}\lambda_i\left(\hat{x}_{i+1}-y^{(i+1)}\right)+\alpha\sum_{i=0}^{n-1}\lambda_i\left(\hat{x}_i-y^{(i)}\right)\\&=-\sum_{i=0}^{n-1}\lambda_i\varepsilon\varphi^{(i+2)}-\alpha\sum_{i=0}^{n-1}\lambda_i\varepsilon\varphi^{(i+1)}\end{aligned} \tag{35}$$

Meanwhile, Lemma 1 can be utilized to give

$$\int_0^t\hat{L}(\tau)d\tau\le\beta_d\left|\hat{r}(0)\right|-\hat{r}(0)^T\bar{D}(0) \tag{36}$$

where

$$\hat{L}(t)\equiv\hat{\bar{r}}^T(\bar{D}-\beta_d\operatorname{sgn}(\hat{r})) \tag{37}$$

Thereafter, a Lyapunov candidate is built as

$$V\equiv\frac{1}{2}\hat{r}^T\hat{r}+\frac{1}{2}\hat{\bar{r}}^TG(X)\hat{\bar{r}}+\hat{P}+Q+\frac{1}{2}\tilde{\beta}^2 \tag{38}$$

with the auxiliary function $\hat{P}(t)\in\mathbb{R}$ is defined as

$$\hat{P}(t)\equiv\beta_d\left|r(0)\right|-r(0)^T\bar{D}(0)-\int_0^t\hat{L}(\tau)d\tau \tag{39}$$

Hence, the derivative of (35) is derived as

$$\begin{aligned}\dot{V}&=\hat{r}^T\dot{\hat{r}}+\hat{\bar{r}}^TG(X)\dot{\hat{\bar{r}}}+\frac{1}{2}\hat{\bar{r}}^T\dot{G}(X)\hat{\bar{r}}+\frac{1}{k_n}tr\left(\tilde{W}^T\dot{\hat{W}}\right)\\&\quad-\hat{\bar{r}}^T(\bar{D}-\hat{\beta}_d\operatorname{sgn}(\hat{r}))+\tilde{\beta}\dot{\hat{\beta}}\\&=\hat{r}^T\dot{\hat{r}}+\hat{\bar{r}}^TG(X)\left(\dot{\varphi}_{\bar{r}}+\dot{\bar{r}}\right)+\frac{1}{2}\hat{\bar{r}}^T\dot{G}(X)\hat{\bar{r}}-\hat{\bar{r}}^T\tilde{W}^T\phi(\hat{Y})\\&\quad-\hat{\bar{r}}^T(\bar{D}-\beta_d\operatorname{sgn}(\hat{r}))+\tilde{\beta}\left(\left|\dot{\hat{r}}(t)\right|+\alpha\left|\hat{r}(t)\right|\right)\\&=\hat{r}^T\dot{\hat{r}}-\hat{\bar{r}}^TG(X)\left(\sum_{i=0}^{n-1}\lambda_i\varepsilon\varphi^{(i+2)}+\alpha\sum_{i=0}^{n-1}\lambda_i\varepsilon\varphi^{(i+1)}\right)+\hat{\bar{r}}^T\\&\quad\times\left(-\dot{G}(X)\hat{\bar{r}}/2-\hat{r}+\tilde{W}^T\phi(\hat{Y})+W^T\tilde{\phi}(\bar{Y})-(k_s+1)\alpha\hat{\bar{r}}\right.\\&\quad\left.-\beta\operatorname{sgn}(\hat{r})+\bar{D}\right)+\hat{\bar{r}}^T\dot{G}(X)\hat{\bar{r}}/2-\hat{\bar{r}}^T\tilde{W}^T\phi(\hat{Y})\\&\quad-\hat{\bar{r}}^T(\bar{D}-\beta_d\operatorname{sgn}(\hat{r}))+\tilde{\beta}\left(\left|\dot{\hat{r}}(t)\right|+\alpha\left|\hat{r}(t)\right|\right)\\&\le\hat{r}^T\dot{\hat{r}}+D_{\hat{\bar{r}}}\left\|\hat{\bar{r}}\right\|-\hat{\bar{r}}^T\hat{r}-(k_s+1)\alpha\hat{\bar{r}}^T\hat{\bar{r}}-\beta\hat{\bar{r}}^T\operatorname{sgn}(\hat{r})\\&\quad+\hat{\bar{r}}^T\hat{\bar{D}}-\hat{\bar{r}}^T(\bar{D}-\beta_d\operatorname{sgn}(\hat{r}))+\tilde{\beta}\left(\left|\dot{\hat{r}}(t)\right|+\alpha\left|\hat{r}(t)\right|\right)\\&=-\alpha\hat{r}^T\hat{r}-\alpha\hat{\bar{r}}^T\hat{\bar{r}}-k_s\alpha\left(\left\|\hat{\bar{r}}\right\|-D_{\hat{\bar{r}}}/2k_s\alpha\right)+D_{\hat{\bar{r}}}^2/4k_s\alpha\end{aligned} \tag{40}$$

where

$$D_{\hat{\bar{r}}}=\frac{\sum_{i=0}^{n-1}\lambda_i\varepsilon D_{i+2}+\alpha\sum_{i=0}^{n-1}\lambda_i\varepsilon D_{i+1}}{g_{\min}}+2W_m\phi_m\in\mathbb{R}^+ \tag{41}$$

Define a compact set

$$\Theta_{\hat{r}}\equiv\left\{\hat{r}(t):\left\|\hat{r}\right\|\le D_{\hat{\bar{r}}}\big/2\alpha\sqrt{k_s}\right\} \tag{42}$$

$\dot{V}$ is negative as long as $\hat{r}(t)$ is outside it. Hence, r and $\hat{r}$ are bounded by definition and Lemma 2. Since $\lambda_{n-1}s^{n-1}+\lambda_{n-2}s^{n-2}+\cdots+\lambda_0$ is Hurwitz, the actual tracking error $e(t)$ is also asymptotically bounded. According to the standard Lyapunov extension theorem [15], the aforementioned analysis implies that the closed-loop system is SGUUB. In addition, because $1/k_s$ and ε can be made arbitrarily small to achieve arbitrarily small tracking error.

IV. Simulation

In this section, the proposed output feedback control design is used to control a planar two-link arm [5] with dynamics as

$$\begin{gathered}\begin{bmatrix}\eta_1+\eta_2+2\eta_3\cos q_2 & \eta_2+\eta_3\cos q_2\\ \eta_2+\eta_3\cos q_2 & \eta_2\end{bmatrix}\begin{bmatrix}\ddot{q}_1\\ \ddot{q}_2\end{bmatrix}+\begin{bmatrix}-\eta_3(2\dot{q}_1\dot{q}_2+\dot{q}_2^2)\sin q_2\\ \eta_3\dot{q}_1^2\sin q_2\end{bmatrix}\\+\begin{bmatrix}\eta_1e_1\cos q_1+\eta_3e_1\cos(q_1+q_2)\\ \eta_3e_1\cos(q_1+q_2)\end{bmatrix}+\begin{bmatrix}d_1\\ d_2\end{bmatrix}=\begin{bmatrix}\tau_1\\ \tau_2\end{bmatrix}\end{gathered} \tag{43}$$

where $\eta_1=(m_1+m_2)a_1^2$, $\eta_2=m_2a_2^2$, $\eta_3=m_2a_1a_2$, $e_1=g/a_1$. $g=9.8m/s^2$ is the gravity acceleration. m_1,m_2 represent point mass of the links at distal end while a_1,a_2 are the length of the links. The rotational angle of the joints q_1, q_2 is the system state, and the torque applied on the joints τ_1, τ_2 is the control inputs. In the simulation, only q_1 and q_2 are measurable. Therefore, an observer is constructed as

$$\begin{aligned}\begin{bmatrix}\dot{\hat{q}}_1\\ \dot{\hat{q}}_2\end{bmatrix}&=\begin{bmatrix}\hat{q}_3\\ \hat{q}_4\end{bmatrix}+\left(\alpha_1/\varepsilon\right)\begin{bmatrix}q_1-\hat{q}_1\\ q_2-\hat{q}_2\end{bmatrix}\\ \begin{bmatrix}\dot{\hat{q}}_3\\ \dot{\hat{q}}_4\end{bmatrix}&=\phi_0(\hat{X},u)+\left(\alpha_2/\varepsilon^2\right)\begin{bmatrix}q_1-\hat{q}_1\\ q_2-\hat{q}_2\end{bmatrix}\end{aligned} \tag{44}$$

TABLE 1
SUMMARY OF PARAMETERS USED IN SIMULATION OF 2-LINK ROBOT ARM

Parameter	m_1	m_2	a_1	a_2	α_1	λ_0
Value	0.8	2.3	1	1	2	diag(4,2)
Parameter	k_s	k_n	α	N_n	α_2	ε
Value	10	1	4	30	1	0.0001

To be more realistic, a bounded disturbance $d=\begin{bmatrix}d_1 & d_2\end{bmatrix}^T=\begin{bmatrix}-0.1\sin 10t & 0.1\cos 10t\end{bmatrix}^T$ is added. The initial states of the system are selected as $q_1(0)=q_2(0)=10^\circ$. Our goal is to manipulate the robot arm back to track a desired trajectory $q_{1d}=\sin t$, $q_{2d}=\cos t$. Other parameters used in this simulation are given in Table 1.

A typical system response using the proposed output feedback controller is shown in Fig. 1 including the system trajectories and control signals. Fig. 2 shows the performance of the high-gain observer. With the presence of bounded disturbance and NN reconstruction error, the actual joint angles can still track the desired values with asymptotical performance. The norm of NN weights and the adaptive term β are also demonstrated in Fig. 3. The simulation results demonstrate that the design is capable of attaining satisfactory tracking performance while all other signals bounded.

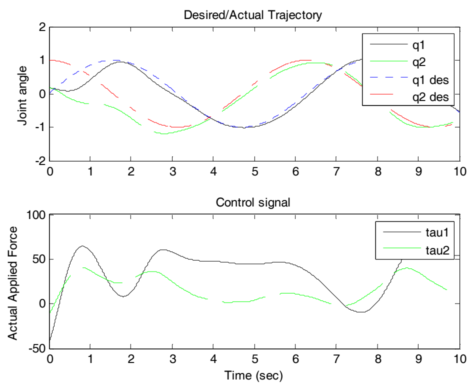

Fig. 1. Response of the output feedback controller. Top: Actual and desired joint angles. Bottom: Control inputs.

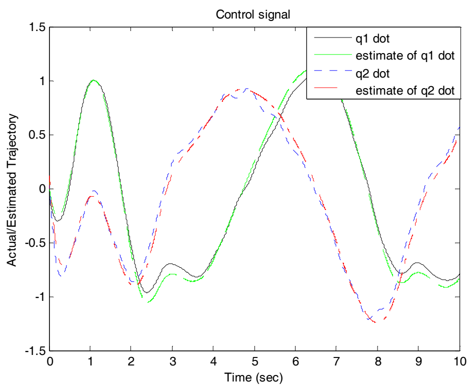

Fig. 2. The unavailable $\dot{q}_1, \dot{q}_2$ and their estimates.

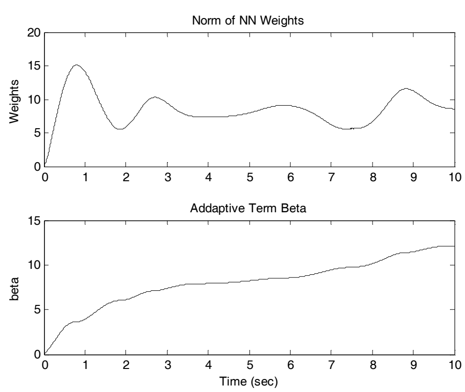

Fig. 3. Top: Norm of NN weights. Bottom: adaptive term β.

V. Conclusions

In this paper, a novel output feedback-based controller design is proposed by incorporating the integral of the neural network and robust term for a class of MIMO high-order uncertain nonlinear systems with bounded disturbances. Semi-global asymptotic tracking performance is obtained when all system states are available. When certain states are not measurable, a high-gain observer is utilized to estimate them. The output feedback controller can recover the performance of the proposed state feedback counterpart by increasing the observer gain.

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