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Signal Invariance and Trajectory Steering Problem for an Autonomous Wheeled Robot

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Abstract— **We give a new convenient parametrization of linear controllers that solve the problem of signal invariance (or disturbance cancellation) for MIMO plants. As an example of application of the obtained results we consider the trajectory tracking problem for non-holonomic wheeled transport robots.**

I. INTRODUCTION

The essence of the invariance problem is to design a controller that provide the closed-loop system to be stable with some of its outputs independent on the exogenous input (signal invariance) or system parameters (parametric invariance). Theory of parametric and signal invariance was pioneered by N. Minorsky [1] and G.V. Schipanov [2] and has been developed for several decades, see the works [3],[4],[5],[6],[7] just to mention a few.

The conditions providing invariance are well known nowadays, for instance, in the case of linear MIMO system an output is invariant respectively to an input if and only if the correspondent transfer function vanishes [3]. Nevertheless, even for the linear case no constructive description of all controllers providing the signal invariance seems to be known. The aim of the present paper is to give a complete parametrization of all linear controllers solving the signal invariance problems for generic MIMO plants as well as conditions for existence of such controllers. The results obtained in the paper are based on a new parametrization of the stabilizing linear controllers which is akin to the celebrated parametrizations by Youla [27] and Desoer [8],[9] but appears to be much more convenient for the description of all controllers providing desired closed-loop system transfer functions, especially for the case of minimum-phase plants.

As an example of practically important signal invariance problem, we consider the trajectory tracking problem for autonomous wheeled robot. The problems of trajectory tracking for such vehicles are investigated in a great deal of papers [29],[14],[15],[16],[17] most of which exploit methods of nonlinear control. In the vicinity of the desired path the vehicle motion may be described adequately with the linearized Ackermann model [12],[13],[23],[30] and thus may be investigated by means of linear control theory techniques. Unlike the referenced works, we consider the case of three wheeled unicycle-like robot. We reduce the trajectory steering problem in question to the signal invariance problem, the "external signal" to suppress being the curvature of the steered line and the input to be invariant is the steering error (distance to the desired path). The performance of the obtained controllers for robust trajectory steering is modeled using both MATLAB simulations and experiments with real robot assembled of the LEGO Mindstorms NXT constructor [28],[31].

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II. LINEAR PROBLEM OF SIGNAL INVARIANCE

In this section we investigate a problem of signal invariance with respect to the full system output (called also "absolute invariance") for linear time-invariant systems. Consider a MIMO plant governed by the input-output equations

$$A\left(\frac{d}{dt}\right)z(t) = B\left(\frac{d}{dt}\right)u(t) + F\left(\frac{d}{dt}\right)\varphi(t), \qquad (1)$$

where $y(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$, $\varphi(t) \in \mathbb{R}^l$ stand for the output, input and disturbance signals respectively. The matrix polynomials $A(\lambda), B(\lambda), F(\lambda)$ have dimensions $n \times n$, $n \times m$, $n \times l$ respectively and we assume that $\det A \not\equiv 0$ excluding descriptor systems from our consideration.

The problem is to find a linear controller

$$D\left(\frac{d}{dt}\right)u(t) = C\left(\frac{d}{dt}\right)y(t) + G\left(\frac{d}{dt}\right)\varphi(t), \qquad (2)$$

stabilizing the closed-loop system and providing the output $y(t)$ to be invariant of the external input $\varphi(t)$, that is

$$\lim_{t\to+\infty} y(t) = 0 \quad \forall \varphi(\cdot), \qquad (3)$$

Here $D(\lambda)$,$C(\lambda)$,$G(\lambda)$ are matrix polynomials of dimensions $m \times m$, $m \times n$, $m \times l$ respectively and $\det D \not\equiv 0$. As usual, the controller (2) is said to be stabilizing, if the matrix polynomial $\Xi = \begin{bmatrix} A & -B \\ -C & D \end{bmatrix}$ is Hurwitz, i.e. $\det \Xi(\lambda) \neq 0$ whenever $Re\,\lambda \geq 0$. Besides the stability of the closed-loop system, usually controllers are required to be *realizable* i.e. all of rational matrices $D^{-1}C$, $D^{-1}G$, $\Xi^{-1}\begin{bmatrix} F \\ G \end{bmatrix}$ are proper (bounded at ∞). Realizable stabilizing controllers do not measure the derivatives of y, φ, and provide the solutions of the closed-loop systems to depend continuously on the external signal $\varphi(\cdot)$.

Introducing the transfer matrices $W_{y/\varphi}(\lambda)$, $W_{u/\varphi}(\lambda)$ of the closed loop system (from φ to y,u respectively), given by

$$\begin{bmatrix} W_{y/\varphi} \\ W_{u/\varphi} \end{bmatrix} = \begin{bmatrix} A & -B \\ -C & D \end{bmatrix} \begin{bmatrix} F \\ G \end{bmatrix}, \qquad (4)$$

the invariance property (3) for a stabilizing controller (2) means vanishing of $W_{y/\varphi}$ [3],[4],[7]:

$$W_{y/\varphi} \equiv 0. \qquad (5)$$

Following [19], we say a stabilizing controller (2) with invariance property (3) to be *I-universal*, or universal in the sense of invariance problem. The term *universal* has the same sense as in [18] and concerns arbitrary control problem in uncertain conditions. The control law is said to be universal if it is independent of the unknown parameters (e.g. C,D,G do not depend on φ, although φ is one of controller's inputs) but for any values of those parameters achieves the desired control goal (the condition (3) for the problem in question). Thus the term "universal controller" (proposed for the invariance problem by G.V. Schipanov [2]) is close to the terms "robust controller" and "adaptive controller" but seems to be preferable since robust control problems typically deal with completely unobservable uncertain parameters and "worst-case" solutions (e.g. minimax optimal control), while the adaptive algorithms typically assume presence of parameter estimation loop in the system. More discussion on notion of the universal controller and examples of universal controllers in optimization problems with uncertain parameters can be found in [18],[21],[22].

As can be easily seen (see Lemma 2 below), any I-universal controller (2) has to measure the disturbance $\varphi(\cdot)$ or at least some of its components, i.e. $G \not\equiv 0$ (but for the trivial case $F \equiv 0$). "Partial" invariance with respect to some of output variables, for instance condition $Ky(t) \to 0$ as $t \to +\infty$ with K being some fixed matrix, sometimes may be provided without feed-forward disturbance compensation [26]. The necessary and "almost" sufficient condition for that is $rk K + rk F < n$ (where the rank of the matrix polynomial F is taken over the field of all rational functions). Below we bound ourselves with the case of invariance with respect to the full output $y(\cdot)$ and observable signal $\varphi(\cdot)$.

Our goal is to give constructive description of the class of all I-universal controllers for the fixed plant (1). In order to parametrize such controllers it is natural to take any affine parametrization of all stabilizing regulators and pick out those satisfying invariance condition (5) (a linear equation for the parameter). But despite the condition (5) looks quite simple, standard parametrizations such as Youla-Kuchera or Desoer ones [27],[8],[9],[10],[11] give typically rather "cumbersome" formulae for I-universal controllers that make it difficult, in particular, to eliminate non-realizable control laws. Also most of known parametrizations require coprime factorization of the plant transfer function.

Below we propose another affine parametrization of all stabilizing controllers (Lemma 1) which is akin to the Youla-Kuchera parametrization and allows to obtain quite simple solution of the linear invariance problem (Theorem 1). The main feature of the approach used below is quite simple formulas of I-universal controllers under assumption that the plant is minimum-phase (Corollaries 1,2). The case of minimum-phase plant is most inportant, since, as shown in Lemma 2, for non minimum-phase plants the invariance problem typically has no solution.

A. Parametrization of stabilizing controllers.

We say two controllers $D_1(p)u = C_1(p)y + G_1(p)\varphi$ and $D_2(p)u = C_2(p)y + G_2(p)\varphi$ (with $p = \frac{d}{dt}$) to be *Hurwitz-equivalent* or H-equivalent, if there exist Hurwitz $m \times m$-matrix polynomials H_1, H_2 such that

$$H_1^{-1}C_1 = H_2^{-1}C_2, H_1^{-1}D_1 = H_2^{-1}D_2, H_1^{-1}G_1 = H_2^{-1}G_2.$$

Evidently, the Hurwitz-equivalent controllers are either both stabilizing or not, realizable or not and provide the same closed-loop transfer functions $W_{y/\varphi}$, $W_{u/\varphi}$. In particular such controllers are either both I-universal or not I-universal.

The following lemma can be proved analogously to Lemma 3 of [25]. The latter result concerns delay systems, but the proof remains unchanged after replacing the word "quasipolynomial" with "polynomial". For special cases it was proved earlier in [18],[24].

Lemma 1: Suppose that C_0, D_0 are matrix polynomials (possibly, zero-valued) of dimensions respectively $m \times n$, $m \times m$ such that the matrix polynomial $\Xi_0 = \begin{bmatrix} A & -B \\ -C_0 & D_0 \end{bmatrix}$ is Hurwitz. Any controller (2) with the coefficients C,D given by

$$D = rB + \rho D_0, C = rA + \rho C_0, \qquad (6)$$

where r is $m \times n$-matrix polynomial and ρ is a scalar Hurwitz polynomial such that $\det(rB + \rho D_0) \not\equiv 0$ is stabilizing and thus any controller which is H-equivalent to it is stabilizing as well. The inverse is also true: arbitrary stabilizing controller (2) is H-equivalent to one of the controllers (6) for appropriate r, ρ.

Below we illustrate use of Lemma 1 for important special cases where the choice of C_0, D_0 is trivial and do not require coprime factorization of the plant transfer matrix

Example 1. (Stable plant) Let $A(\lambda)$ be Hurwitz matrix polynomial. Taking $C_0 = 0$, $D_0 = I_m$, the "canonical" controller (6) has the coefficients C,D as follows:

$$C = rA, D = rB + \rho I_m \qquad (7)$$

Here r, ρ are the same as in Lemma 1 (and $\det(rB + \rho i_m) \not\equiv 0$). Any stabilizing controller is H-equivalent to one of controllers (7) with appropriate r, ρ.

Example 2. (Minimum-phase plant). Let $m = n$ and $B(\lambda)$ is Hurwitz matrix polynomial. Taking $C_0 = I_n$, $D_0 = 0$, the "canonical" controller (6) is as follows:

$$C = rA + \rho I_n, D = rB \qquad (8)$$

Here r, ρ are the same as in Lemma 1 ($\det r \not\equiv 0$). Any other stabilizing controller is H-equivalent to one of the family (8) for some r, ρ.

Example 3. (Generalized minimum-phase plant). Suppose that $n < m$ (the plant is overactuated) and

$$rk B(\lambda) = n \quad \forall \lambda \in \mathbb{C} : Re \lambda \geq 0. \qquad (9)$$

The latter condition is natural generalization the minimum-phase property to the case of non-square matrix B. It is easy to see that (9) implies existence of a $(m-n)\times m$-matrix polynomial B^+ exists such that $\hat{B}=\left[\begin{smallmatrix}B\\B^+\end{smallmatrix}\right]$ is Hurwitz. Indeed, $B=[B_0,0]U_m$ where B_0 is a $n\times n$-matrix polynomial and U_m is some unimodular $m\times m$-matrix ($\det U_m(\lambda)=const\neq 0$). Due to (9), the matrix polynomial B_0 is Hurwitz therefore one can take $B^+=[0,R]S$ for any Hurwitz $(m-n)\times(m-n)$-matrix polynomial R. Let $C_0=\left[\begin{smallmatrix}0\\I_n\end{smallmatrix}\right]$, $D_0=\left[\begin{smallmatrix}B^+\\0\end{smallmatrix}\right]$, then so the controller (6) has the form

$$C=\begin{bmatrix}r_1A\\r_2A+\rho I_n\end{bmatrix},D=\begin{bmatrix}r_1B+\rho B^+\\r_2B\end{bmatrix}\qquad(10)$$

where r_1, r_2 have dimensions $(m-n)\times n$, $n\times n$ and $r=\left[\begin{smallmatrix}r_1\\r_2\end{smallmatrix}\right]$.

B. Existence and parametrization of I-universal controllers

In order to obtain criteria for I-universal controller existence we need the following lemma. We call a rational matrix-valued function S *stable* if $S(\lambda)$ is analytic whenever $Re\lambda\geq 0$.

Lemma 2: If I-universal (respectively, realizable I-universal) controller (2) there exists a stable (respectively, proper stable) rational matrix S such that

$$BS=F\qquad(11)$$

The stabilizing controller (2) is I-universal if and only if $G=-DS$, where S is stable and satisfies (11).

Proof: Indeed, let (2) be a I-universal controller, then taking $S=-W_{u/\varphi}$ (where $W_{u/\varphi}$ is the transfer function of the closed-loop system from φ to u) one obtains due to $W_{y/\varphi}=0$ that $F=AW_{y/\varphi}-BW_{u/\varphi}=BS$ and $G=DW_{u/\varphi}-CW_{y/\varphi}=-DS$ which proves the first proposition of Lemma and "only if" part of the second one. To prove "if" part consider the stabilizing controller (2) with $G=-DS$ and notice that

$$\begin{bmatrix}W_{y/\varphi}\\W_{u/\varphi}\end{bmatrix}=\begin{bmatrix}A&-B\\-C&D\end{bmatrix}^{-1}\begin{bmatrix}F\\G\end{bmatrix}=\begin{bmatrix}0\\S\end{bmatrix}$$

■

Combining Lemma 2 with the parametrization of stable controllers from Lemma 1, one easily obtains the description of all I-universal controllers.

Theorem 1: The following conditions are equivalent:

1) there exists an I-universal controller (2);
2) the equation (11) has stable solution S and there exist matrix polynomials C_0, D_0 such that the polynomial $\Xi_0=\left[\begin{smallmatrix}A&-B\\-C_0&D_0\end{smallmatrix}\right]$ is Hurwitz.

Any controller (2) with the coefficients as follows

$$C=rA+\rho C_0,D=rB+\rho D_0,G=-\rho D_0S-rF,\qquad(12)$$

where S is stable and satisfy (11), r is an $m\times n$-matrix polynomial and ρ is a scalar Hurwitz polynomial such that $\det D\not\equiv 0$, is I-universal. Any I-universal controller is H-equivalent to one of controllers (12) for appropriate r,ρ,S. If S is proper, the controller (12) is realizable if and only if $D^{-1}C$ is proper matrix.

Proof: Any controller (12) is stabilizing and satisfies $G=-DS$, therefore it is I-universal. Consider any *I*-universal controller. Since it stabilizes the plant (1), for some r,ρ is is H-equivalent to a controller (2) with C,D given by (6). Due to the Lemma 2, the latter controller should satisfy the condition $G=-DS=-rF-\rho D_0S$ where $S=W_{u/\varphi}$ is a stable solution of (11). If $S=W_{u/\varphi}=-D^{-1}G$ is proper, for realizability of the controller (12) it is necessary sufficient that $D^{-1}C$ is proper rational matrix. ■

Theorem 1 seems to be not very constructive since it supposes at least one stabilizing "controller" (possibly, degenerate) of the form $D_0u=C_0y$ to be known. In general case C_0,D_0 may be chosen in a standard way as follows. Let $A^{-1}B=B_rA_r^{-1}$ where the matrix polynomials A_r,B_r are right coprime [9]. If the greatest common left divisor L of A and B is Hurwitz (otherwise the plant is not stabilizable), then one may take C_0,D_0 in a way that $D_0A_r-C_0B_r$ is a Hurwitz polynomial. But Lemma 2 shows that the invariance problem typically has no solutions unless the plant is minimum-phase in generalized sense (9). In particular, if $rk[B(\lambda),F(\lambda)]=n$ for $Re\lambda\geq 0$ (B and F has no common left non-Hurwitz divisor) then (9) must hold for existence of I-universal controllers. In the same time, for the minimum phase plant the choice of C_0,D_0 is very simple (see Examples 2,3 from the previous paragraph). We start with the case of equal dimensions: $\dim y=n=m=\dim u$.

Corollary 1: Let $m=n$ and B is a Hurwitz matrix polynomial. Any controller (2) such that

$$C=rA+\rho I_n,D=rB,G=-rF\qquad(13)$$

with r being $n\times n$-matrix polynomial, $\det r\not\equiv 0$ and ρ scalar Hurwitz polynomial, is I-universal. Any I-universal controller is H-equivalent to one of controllers (12) for appropriate r,ρ.

Proof: Follows from Theorem 1 for $C_0=I_n$, $D_0=0$. ■

The case of overactuated plant is analogous:

Corollary 2: Suppose the condition (9) to hold and $m>n$. Any controller (2) with C,D given by (10) and

$$G=-\begin{bmatrix}\rho X\\r_2F,\end{bmatrix}$$

where X is arbitrary stable matrix, is I-universal. Any I-universal controller is H-equivalent to one of controllers described for appropriate r,ρ,X.

Proof: One can see that (11) has infinitely many solutions: $S=\left[\begin{smallmatrix}B\\B^+\end{smallmatrix}\right]^{-1}\left[\begin{smallmatrix}F\\X\end{smallmatrix}\right]$ where X is stable. Taking C_0, D_0 like in the Example 3 and applying Theorem 1, one obtains the proposition of the Corollary. ■

Notice that for concrete examples it is typically easy to find realizable controllers amongst the whole family (13) or (10). For instance, consider the minimum-phase case and controllers (13). Suppose that $A(\lambda)=\lambda^dA_d+\ldots+A_0$ has non-degenerate leading coefficient A_d. Let $N=\deg\rho$ and r is chosen in a way that $\deg C=\deg(rA+\rho I_n)<d$. Then $\deg(r^{-1})=d-N$ and thus $\deg(D^{-1}C)<\deg(B^{-1})+d-N$ thus for $N>d-\deg(B^{-1})$ the controller (13) is realizable. Here by *degree* $\deg X$ of rational matrix X we mean a number k such that $X(\lambda)\lambda^{-k}\to X_*\neq 0$ as $|\lambda|\to+\infty$.

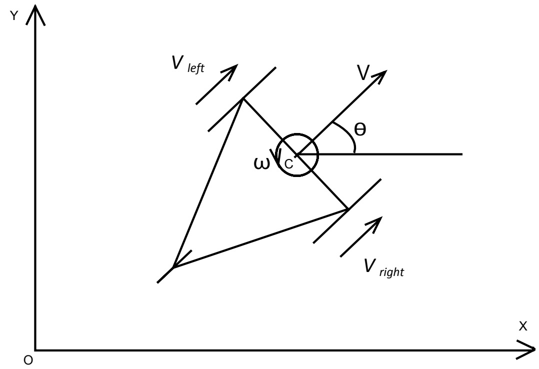

Fig. 1. Vehicle kinematics

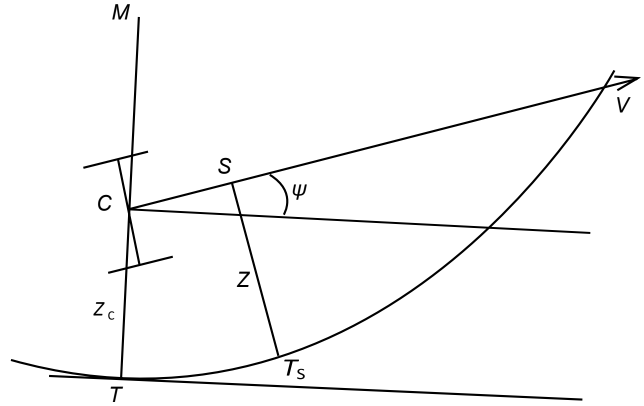

Fig. 2. Model linearization

III. I-UNIVERSAL CONTROLLERS FOR TRAJECTORY STEERING.

In the present section we apply the result obtained above to the problem of trajectory tracking for non-holonomic wheeled robot. The linearized vehicle models considered below are analogous to those proposed by Ackermann et al. [12],[13]. Unlike the Ackermann models, we consider the case of unicycle-like robots analogous to those considered in [14].

Consider a three wheel vehicle (Fig.1) with the identical parallel front wheels, that are non-deformable and controlled by separate motors (allowing to maintain fixed velocity of the vehicle while turning). The rear wheel is passive and used for steering and stabilization only. We assume the robot's center of mass C to be located in the middle of the axis connecting the front wheels, and the robot velocity vector to be orthogonal to the axis (i.e. the ground is non-slipping).

We assume the vehicle to move with constant speed $V = (V_{left} + V_{right})/2$ where V_{left}, V_{right} stand for the linear velocities of the points on the left and right wheels correspondingly. The vehicle motion is governed by the equations as follows:

$$\begin{cases} \dot{x} = \dfrac{V_{left} + V_{right}}{2} \cos\theta \\ \dot{y} = \dfrac{V_{left} + V_{right}}{2} \sin\theta \\ \omega = \dot{\theta} = \dfrac{V_{right} - V_{left}}{L}, \end{cases} \tag{14}$$

where L stands for distance between the left and right rear wheels, (x, y) are the Cartesian coordinates of the point C (in ground-fixed frame) and θ is the vehicle heading. We assume $\omega(t) = \dot{\theta}(t)$ to be the only control input of the system (14).

In general the problem of path following is solved for some point S which we assume to be on some distance $l > 0$ from the axis (see Figure 2). We assume that there is a sensor mounted at S which measures distance $z(t)$ from S to the path (the distance between S and closest point on the trajectory T_S). The motion of the robot in a sufficiently small vicinity of the desired trajectory may be analyzed analogously to [12]. The track section near the robot may be approximate with an arc of the osculating circle centered at the point M. The radial ray connecting M with the center of mass C intersects the track in the unique point T (which is close to the projection of C onto the path). Let $\psi(t)$ be the angle between the velocity of the vehicle and tangent to the curve at point T. If $\psi(t)$ is sufficiently small, then [12]

$$\dot{z}_c(t) \approx V \sin\psi(t) \approx V\psi(t) \tag{15}$$

and thus

$$\dot{z}(t) \approx V\psi(t) + l\omega \tag{16}$$

Introducing instant angular velocity of the tangent line rotation $\omega_T = VR_T$ where R_T is the curvature of the path at the point T, it is easily seen that $\dot{\psi} = \omega - \omega_T$ thus one obtains the dynamics as follows:

$$\begin{cases} \dot{z} = V\psi + l\omega \\ \dot{\psi} = \omega - VR_T. \end{cases} \tag{17}$$

So the linearized robot model may be considered as a linear plant in the input-output form as follows

$$\begin{pmatrix} \dot{\psi} \\ \dot{z} \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ V & 0 \end{pmatrix} \begin{pmatrix} \psi \\ z \end{pmatrix} + \begin{pmatrix} 1 & -V \\ l & 0 \end{pmatrix} \begin{pmatrix} \omega \\ R_T \end{pmatrix} \tag{18}$$

Our goal is to provide the invariance of the output $z(t)$ of the unknown beforehand "disturbance" $R_T(t)$ which is assumed to be observable:

$$\lim_{t\to+\infty} z(t) \to 0 \quad \forall R_T(\cdot). \tag{19}$$

Eliminating the variable ψ, the equation (18) may be reduced to the scalar input-output model of the type (1):

$$A\left(\frac{d}{dt}\right) z(t) = B\left(\frac{d}{dt}\right)\omega(t) + F\left(\frac{d}{dt}\right) R_T(t), \tag{20}$$

where

$$A(\lambda) = \lambda^2, B(\lambda) = l\lambda + V, F(\lambda) = -V^2.$$

Accordingly to the Corollary 1, any controller

$$D(\lambda)\omega = C(\lambda)z + G(\lambda)R_T, \tag{21}$$

with D, C, G given by (13) is I-universal with respect to the output $z(t)$, provides (19) and thus solves the trajectory tracking problem in question.

Fig. 3. LEGO robot with 3 wheels

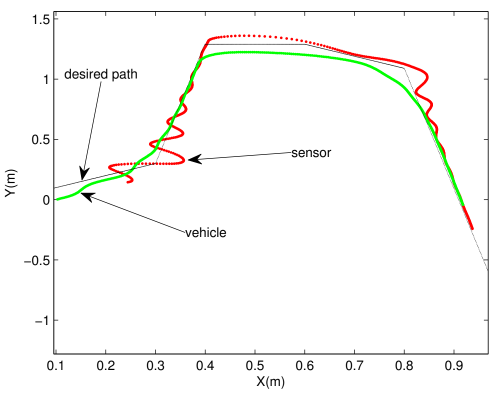

Fig. 4. Simulation

IV. PERFORMANCE EVALUATION OF THE TRAJECTORY STEERING CONTROLLER

IN this section we discuss simulation of the controller (21) for special r,ρ in the problem of trajectory steering for a real three-wheeled robot designed of LEGO Mindstorms constructor [31],[28] (see Fig.3).

Let $V = 0.187$m/s be the constant velocity the vehicle maintains, $l = 0.2$m be the length of the rod where the "sensor" (point S to be steered) is mounted, $L = 0.14$m be the distance between the wheels. We take

$$r(\lambda) = \lambda + 1, \quad \rho(\lambda) = -(\lambda^3 + \lambda^2 + 5\lambda + 1)$$

and consider the controller (21) with C,D,G given by (13) for the specified r,ρ. It's easy to show that such a controller is strictly realizable.

The Figure 4 illustrates the MATLAB simulation of motion of the described vehicle along the desired path. The dark line stands for the desired path, the red one is the trajectory of reference point (sensor), and the green line corresponds to the motion of the point C (middle of the axis). Notice that formally the desired path is not a smooth line here but a polygon, nevertheless the linear model from the previous section allows to steer this path.

The experiments with real vehicle built from LEGO Mindstorms have been fulfilled. The desired track is formed by the black adhesive tape on the smooth floor the robot (more precisely, virtual point S in front of the wheel axis) has to follow. The distance and curvature measurements may be done by either web-cameras scanning some section of the track (analyzing the geometric form of the curve one may estimate it's curvature and distance to it) or using the light-sensors moving along the line and measuring the intensity of different colors. The Figure 5 illustrates the motion of the robot along smooth self-crossing line.

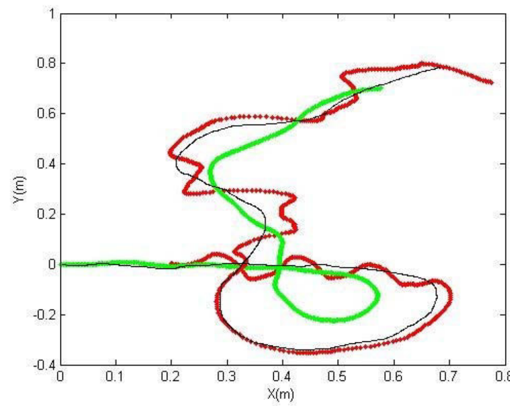

Fig.5

A number of unaccounted factors such as delays in the actuators, unknown disturbances, etc. make it impossible to achieve such an accuracy as in MATLAB modelling but the control algorithm can be seen to work.

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