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An optimal anti-windup strategy for repetitive control systems

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Abstract— **Repetitive Control includes an Internal Model with high gain and slow time response characteristics which make it prone to the windup effect. A solution to this problem is the inclusion of an Anti-Windup compensator. Although there exist many general Anti-Windup synthesis methods in literature, some problems can arise as a result of a straightforward application of them in Repetitive Control. This paper presents the analysis and adaptation of the Model Recovery Anti-Windup strategy in the Repetitive Control frame. Thus, an optimal LQ design is proposed that looks for a deadbeat recover behaviour after saturation and a global asymptotic stability for the closed loop system. Through simulation it is shown that the propounded scheme achieves better tracking performance than other similar LQ designs.**

I. INTRODUCTION

As an Internal Model Principle (IMP) [1] based control strategy, Repetitive Control (RC) [2], [3] uses an Internal Model (IM) that characterizes the signal to be tracked or rejected. In this way, the IM of the RC provides infinite or very high gain at a given frequency an its harmonics. It is well known that, in systems with actuator saturation, a controller with these characteristics may produce a wind-up effect in which the states of the controller can grow unbounded. Even if the gain is not infinite but high, the states can overgrow significantly making harder to recover the system to the linear ideal one. Some conditions related to the boundedness of the state of the RC with actuator saturation are stated in [4].

As it is known, marginally stable or unstable controllers are prone to originate the unbounded growing of the controller state. Thus, using the pole analysis, it can be noted that the IM used in standard RC is marginally stable and those used in High Order Repetitive Control (HORC) [5], [6] have poles over the unit circle with multiplicity equal or greater than two which can yield Bounded Input Bounded Output (BIBO) unstable IM [7]. Additionally, the IM generally imposes a slow transient response for the closed loop which worsens the actuator saturation effect. Therefore, since the linear design of the repetitive control does not include the saturation in the actuator, it is convenient to include an Anti-Windup (AW) compensator. A recent review of standard AW techniques can be found in [8] and [9].

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In [9], modern AW proposals have been classified in two groups: Direct Linear Anti-Windup (DLAW) and Model Recovery Anti-Windup (MRAW). The DLAW approach seeks to find an AW compensator that assures specific performance and stability properties for the closed loop system. The MRAW approach selects the AW filter in such a way that it makes invariant the compensator-plant system. However, due to the characteristics of the RC, most of the standard AW designs should not be applied straightforward since some difficulties might appear during design or implementation. As a result, it would be necessary to adapt the generic AW strategies in order to be applied in RC.

In the DLAW scheme, some strategies are based on solving a Linear Matrix Inequality (LMI) problem [10]; however, complications usually arise since the size of this LMI depends mostly on the IM order which is usually large. Thus, for the RC case, the implementation of this scheme depends on whether the LMI is computationally solvable or not. Although the DLAW scheme allows us to obtain an AW compensator of order 0, the solution includes elements that yield a large number of on-line calculations, thus increasing the computational burden.

The MRAW scheme uses the model of the plant in its structure. Although the order of the plant could be large, it is usually significantly smaller than the IM order. Furthermore, the procedure to find the feedback gains does not depend on the controller dynamics, therefore the related LMI is always solvable. The computational load of the MRAW scheme implementation is the lowest one in comparison with the other strategies.

The proposals in [11], [4] and [12], are three examples of AW design for repetitive control. In [11], an AW law is derived for Iterative Learning Control (ILC) and also a extension to RC is briefly described. However, the AW strategy is derived for a specific plant and the described repetitive controller does not correspond to the standard architecture since the filters for stability and robustness are not included. In [4], the AW scheme cancels out the dynamics of the IM during saturation and adds a structure to shape the transients when the system saturates and gets back from saturation. However, the IM cancellation implies that, in addition to the repetitive controller order, it is necessary to implement an AW filter which has at least the order of the IM. Therefore, this scheme will be cost restrictive since it depends on a suitable implementation platform. The work in [12], can be categorized as a DLAW design. The strategy is derived in continuous time domain. It is an extension to RC of the general AW design in [10], where the case of delayed systems is described. Also in this approach, unlike the RC

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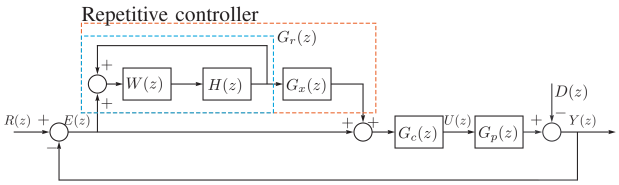

Fig. 1. Block-diagram of the repetitive controller plug-in approach.

design that will be described here, the filters for robustness and stability are designed together with the DLAW synthesis.

In view of the analysis above, the MRAW appears as a good strategy when taken into account the computational solvability and load in the design and implementation of RC. Thus, in this work a MRAW scheme is presented in which the recovery of the system is achieved using the approximation to a deadbeat transition (see [13] for the deadbeat concept). The advantage of selecting a deadbeat over other designs is shown as well as the design method.

The paper is organized as follows, Section II presents the basics of the repetitive control including some design issues and the stability conditions. Section III describes the general MRAW scheme. Section IV analyses the stability of the system and propounds an optimal design. In Section V the experimental results are shown and finally the conclusions are presented in Section VI.

II. Digital Repetitive Control

Digital repetitive control uses an IM which introduces infinite/high gain at a selected fundamental frequency and its harmonics [5]. This IM has the following transfer function:

$$G_r(z) = \frac{W(z)H(z)}{1 - W(z)H(z)}, \tag{1}$$

where $W(z) = z^{-N}$ and $H(z)$ is a null-phase FIR low-pass filter in charge of provide robustness at high frequencies. With $H(z) = 1$, IM (1) provides infinite gain at frequencies $\omega = (2k-1)2\pi/N$, with $k = 1, 2, \ldots, (N/2)+1$, where $N = \frac{T_p}{T_s}$ is the discrete period of the signal, T_p being the period of the signal to be tracked/rejected and T_s being the sampling period.

Also, in order to provide robustness against frequency uncertainty/variation a HORC technique has been developed. This version of RC uses the IM (1) with $W(z) = \sum_{k=1}^{M} w_k z^{-kN}$ and $\sum_{k=1}^{M} w_k = 1$ [6].

Besides the IM, which assures steady state performance, repetitive controllers include a stabilizing filter, $G_x(z)$, which assures closed-loop stability. Traditionally, repetitive controllers are implemented in a “plug-in” fashion, i.e. the repetitive compensator is used to augment an existing nominal controller, $G_c(z)$ (Figure 1). This nominal compensator is designed to stabilize the plant, $G_p(z)$, and provides disturbance attenuation across a broad frequency spectrum.

The closed-loop system of Figure 1, using (1) as the IM, is stable if the following conditions are fulfilled ([14]):

1) The closed loop system without the repetitive controller is stable, i.e. $G_o(z) = \frac{G_c(z)G_p(z)}{1+G_c(z)G_p(z)}$ is stable. It is advisable to design the controller $G_c(z)$ with a high enough robustness margin.
2) $\| \; W(z)H(z)(1 - T_o(z)G_x(z)) \; \|_\infty < \; 1$, where $H(z)$ and $G_x(z)$ must be selected to meet this condition. A trivial structure[1] which is often used for minimum phase systems is ([15]): $G_x(z) = k_r.(G_o(z))^{-1}$. As argued in [16], k_r must be designed looking for a trade-off between robustness and transient response.

The transfer function of the complete controller (see Figure 1) results:

$$G_{rc}(z) = \frac{U(z)}{E(z)} = (1 + G_r(z)G_x(z))G_c(z) \tag{2}$$

Additionally, the transient or convergence time is dominated by the IM dynamics, which is in general much slower than the closed loop only with the controller $G_c(z)$ [17], [18].

III. The general MRAW scheme

Figure 2 shows the MRAW structure, where $G_p(z)$ is the plant, $G_{rc}(z)$ is the controller (2), $sat(\cdot)$ is the saturation function and $C_{aw}(z)$ is the AW compensator. In the MRAW strategy, the mismatch between the saturated control action and the non-saturated one is fed back to the controller by means of the AW compensator, which is designed to be the model of the plant, $\sigma_{1,k}$ being the output that is used with this purpose. Additionally, another feedback signal, $\sigma_{2,k}$, is added with the aim of improving the behaviour of the system when it gets out from saturation. Thus, the design of this feedback involves different approaches. The Internal Model Control (IMC) AW strategy [19], turns out to be the particular case where $\sigma_{2,k} = 0$. This causes that when getting out of saturation the system recovery relies on the plant poles, which can yield a non appropriated performance. A strategy based on Predictive Control which seeks an l_2 performance criterion can be found in [20], an optimization procedure using the Linear Quadratic (LQ) approach is proposed in [21] and a fully nonlinear strategy is described in [22].

In this work, the signal $\sigma_{2,k}$ is designed to be a linear feedback of the AW compensator state. This is aimed at finding a simple linear solution to the AW problem in case of RC, also avoiding the algebraic loop that can be created using the feedback of the control action mismatch, as in [21]. Furthermore, we analyse the benefits of designing a deadbeat behaviour in the AW filter in case of repetitive control. Also, as previously mentioned, the advantage of using the MRAW scheme for the repetitive control case is that the design does not depend on the IM order. Additionally, as will be described later on, the error and control signals are the ideal ones (as if the system had no saturation in the actuator), which isolates the controller from the saturation effects.

[1]There is no problem with the improperness of $G_x(z)$ because the IM provides the repetitive controller with a high positive relative degree.

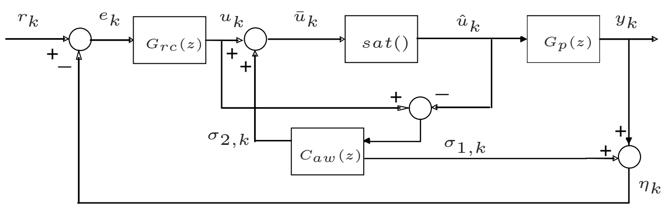

Fig. 2. The MRAW scheme in RC.

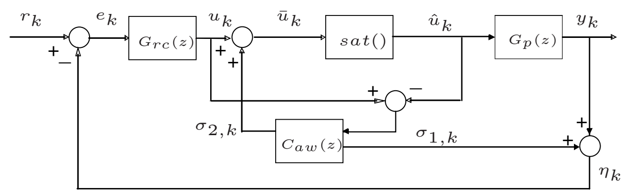

Fig. 3. The invariant part of the MRAW scheme.

A. Selected MRAW scheme

Consider the MRAW scheme depicted in Figure 2. Let the discrete-time and asymptotically stable linear plant $G_p(z)$ be

$$\begin{aligned} x_{k+1} &= Ax_k + Bsat(\bar{u}_k) \\ y_k &= Cx_k \end{aligned} \tag{3}$$

where

$$sat(\bar{u}_k) = \begin{cases} u_{min} & \bar{u}_k < u_{min} \\ \bar{u}_k & u_{min} \le \bar{u}_k \le u_{max} \\ u_{max} & \bar{u}_k > u_{max} \end{cases} \tag{4}$$

with $u_{min} < 0$ and $u_{max} > 0$.

The state-space representation of the repetitive controller $G_{rc}(z)$ is:

$$\begin{aligned} \bar{x}_{k+1} &= A_{rc}\bar{x}_k + B_{rc}e_k \\ u_k &= C_{rc}\bar{x}_k + D_{rc}e_k \end{aligned} \tag{5}$$

The AW filter $C_{aw}(z)$ is defined from the plant model (3) as:

$$\begin{aligned} \chi_{k+1} &= A\chi_k + B(u_k - sat(u_k + \sigma_{2,k})) \\ \sigma_{1,k} &= C\chi_k \end{aligned} \tag{6}$$

and

$$\sigma_{2,k} = K\chi_k \tag{7}$$

where K is the design parameter of the AW filter.

It can be noticed that while the input in system (3) is the saturated control action, the input in system (6) is the difference between the saturated and non-saturated control action. This fact, together with

$$\eta_k = y_k + \sigma_{1,k}, \tag{8}$$

helps to determine the system invariance. Thus, defining $\xi_k = x_k + \chi_k$, noticing that $\bar{u}_k = u_k + \sigma_{2,k}$ and adding equations (3) with (6) we have:

$$\begin{aligned} \xi_{k+1} &= A\xi_k + Bu_k \\ \eta_k &= C\xi_k \end{aligned} \tag{9}$$

In this way, from the input u_k to the output η_k, the system in Figure 3 can be seen as a Linear Time Invariant (LTI) one with the dynamics of the plant.

This means that η_k is the ideal plant output in the sense that it would be the plant output in a system without actuator saturation. Furthermore, in the closed loop of Figure 2, the control action action u_k is the ideal control action, i.e. u_k is the same control signal as the one in a system without actuator saturation. This fact isolates the controller from the saturation effects, allowing us to reduce the analysis to the behaviour of the invariant part shown in Figure 3, including its internal stability.

Remark 1: In this scheme the deviation from the ideal performance can be measured trough $\sigma_{1,k}$, since $\sigma_{1,k}$ is the difference between the ideal behaviour and the plant output $\sigma_{1,k} = \eta_k - y_k$.

Proposal 1: Given a RC design, the smallest possible $\sigma_{1,k}$ corresponds to the best possible performance in case of saturation (the smallest deviation from the ideal behaviour). Therefore, the problem formulation is to find the design parameter K such that $\sigma_{1,k}$ is small enough to obtain a good tracking performance.

It is important that the AW design aims at achieving good tracking performance since RC is a technique which is intended to obtain null steady-state tracking error. Also due to this RC feature, we are interested in the saturation effect produced in steady state even though it also can occur in transient state.

IV. MRAW PROPOSAL FOR RC: DESIGN AND STABILITY

The proposal is based on the idea of having a deadbeat recover once the system gets back from saturation. The goal is to obtain a $C_{aw}(z)$ AW filter such that during saturation takes the form of the plant model, and additionally, when the control action gets back from saturation, the outputs of $C_{aw}(z)$ vanish in a finite number of samples.

To obtain a deadbeat behaviour during recovery it is needed that the feedback loop created by $\sigma_{k,2}$ relocates all the poles of $C_{aw}(z)$ to $z = 0$, which can be done using the pole placement procedure, thus obtaining the gains vector K. However, the internal stability of the system must be verified.

A. Stability

Remark 2: The closed loop stability of the system in Figure 2 is established by the design of the RC and additionally by the internal stability of the system in Figure 3.

Moreover, from the facts that: 1) we are assuming an asymptotically stable plant and 2) from input u_k to output η_k the system in Figure 3 can be seen as a LTI one with the dynamics of the plant, we have that the internal stability of this system can be established analysing only the stability of the interconnection between the saturation block and $C_{aw}(z)$. As a result, in order to check the internal stability of the system in Figure 3 the following condition should be verified for the system in equations (6) and (7):

$$V(\chi_{k+1}) - V(\chi_k) + \Psi < 0, \tag{10}$$

$V(\chi_k) = \chi_k^T P \chi_k$ being the candidate Lyapunov function and $\Psi = 2(u_k - sat(u_k))W(sat(u_k))$ the sector condition[2] (see [23]), $P > 0$ and $W > 0$ being symmetric matrices to be found. Since $\Psi \geq 0$, following the S-procedure we can find that verifying condition (10) implies $V(\chi_{k+1}) - V(\chi_k) < 0$ and as a consequence the internal stability is established.

B. Optimal design

In order to obtain a design as close as possible to the deadbeat behaviour explained previously and also assuring global asymptotic stability, condition (10) can be put together with an optimal LQ design in a LMI form.

Remark 3: In this case the LQ design is used to find the deadbeat gain K which is shown to be an optimal when the weight matrix $Q_p = T^T T$, T being the linear transformation of the system (6) into the controllable canonical form (see [24]).

Other results related with the deadbeat design as an optimal LQ solution can be found in [25]. Thus, the problem formulation is to find K such that the stability of the interconnection between C_{aw}, equations (6) and (7), and the saturation block is preserved and additionally, solve the constrained LQ problem:

$$
\begin{array}{l}
\min_K \sum_{k=0}^{\infty} \chi_k^T Q_p \chi_k \\
\text{subject to} \\
\begin{array}{rcl}
\chi_{k+1} & = & A\chi_k + B\sigma_{1,k} \\
\sigma_{1,k} & = & K\chi_k.
\end{array}
\end{array}
$$

The complete problem can be formulated as an LMI minimization problem:

$$
\begin{array}{l}
\min \gamma \\
\text{s.t.} \\
\begin{bmatrix} -Q & AQ & BU \\ \star & -Q & X_1^T \\ \star & \star & -2U \end{bmatrix} < 0 \\[2ex]
\begin{bmatrix} \gamma I & I \\ I & -Q \end{bmatrix} > 0 \\[2ex]
\begin{bmatrix} -Q & \star & \star \\ AQ + BX_1 & 0 & \star \\ Q_p Q & 0 & -Q_p \end{bmatrix} < 0
\end{array}
$$

where $Q = Q^T > 0$, $U = U^T > 0$, $\gamma > 0$ and $X_1 = KQ$.

Is it worth to say that there exists some conservativeness in the sector condition Ψ which is applied to non-linearities belonging to the sector $[0, 1]$. In general, this fact yields a gain K that is an approximation to the deadbeat solution.

V. Simulations Results

This section shows the results found by simulation using the AW design presented previously and a comparison with other optimal LQ design together with the IMC AW strategy.

[2]In this case the memoryless function $sat()$ is said to belong to the sector $[0, 1]$ since $sat(t, u)\,[u - sat(t, u)] \geq 0$, which is called the sector condition.

A. Simulation setup

With the purpose of comparing the AW strategies described in this work, a linear repetitive controller design will be given. Thus, consider the following discrete-time linear stable plant:

$$
G_p(z) = \frac{2.146z + 0.7585}{z^2 - 0.9945z + 0.03498} \tag{11}
$$

The controller is constructed from model (11), for $N = 100$ and sampling period of $T_s = 5$ ms. According to Section II, the following design issues have been taken into account:

- $G_c(z) = 0.5$ provides a very robust inner loop.
- The first order linear-phase FIR filter
$$H(z) = 0.02z + 0.96 + 0.02z^{-1}$$
provides sufficient robustness in the present case.
- The fact that $G_p(z)$ is minimum-phase allows $G_x(z) = k_r G_0^{-1}(z)$, with $k_r = 0.75$.

Also, a second order HORC has been designed for comparison purposes. Thus, $M = 2$, $w_1 = 2$ and $w_2 = -1$ have been selected.

Given the state space discrete-time system: (A, B, C, D), and its reachability matrix $W_A = \begin{bmatrix} B & AB & \cdots & A^{n-1}B \end{bmatrix}$, then the matrix $T = \begin{bmatrix} v_n^T & v_n^T A & \cdots & v_n^T A^{n-1} \end{bmatrix}$ with v_n^T the last row of W_A^{-1}. Thus, for this example:

$$
A = \begin{bmatrix} 0.0244 & -0.1251 \\ 0.0903 & 0.9701 \end{bmatrix}, B = \begin{bmatrix} 0.0903 \\ 0.0216 \end{bmatrix},
$$
$$
C = \begin{bmatrix} 0 & 99.5450 \end{bmatrix}, D = 0,
$$
$$
T = \begin{bmatrix} -8.1858 & 34.2736 \\ 2.8936 & 34.2736 \end{bmatrix}.
$$

Using the optimal MRAW approach described in Section IV-B, the parameters that have been found to be a feasible solution are: $K_{db} = \begin{bmatrix} 3.1823 & 24.8267 \end{bmatrix}$, using $Q_p = T^T T$ for a deadbeat approximation and $K_{st} = \begin{bmatrix} 1.0747 & 8.1270 \end{bmatrix}$, using $Q_p = 10^5 \mathbb{I}$ as the LQ design used for comparison purposes. The idea behind the last LQ design is to find a solution that keeps small the state of the AW compensator.

B. Simulation results

This section analyses the saturation in steady state. The saturation limits have been chosen to be $u_{min} = -5$ and $u_{max} = 2$. The reference signal r_k is depicted in Figure 4 together with the system plant output y_k and control signal u_k when the settings for HORC have been applied without actuator saturation. As can be seen, in this case, the repetitive controller successfully tracks the reference signal.

Figure 5 shows the control action u_k, i.e. the control action provided by the repetitive controller. The system with actuator saturation but without AW mechanism is called SAT and the system without saturation is called Ideal. For the SAT scheme two options are depicted, SAT RC and SAT HORC, for standard and HORC respectively. As can be seen, both SAT RC and SAT HORC control signals present an undesirable wind-up effect, the SAT HORC being the worse case. This phenomena is due to the pole multiplicity in the IM for HORC which also makes it slower than the RC one. As has been pointed out, when the MRAW scheme is included, the control signal u_k corresponds with the ideal one. The rest of the examples will be carried out using only the second order HORC.

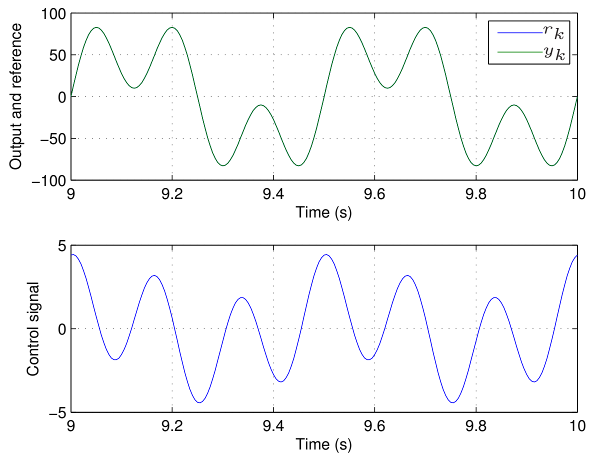

Fig. 4. Steady state reference r_k, output y_k and control signal u_k without saturation.

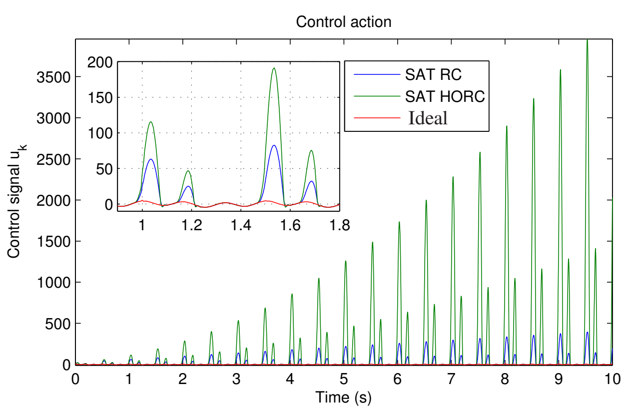

Fig. 5. Control action u_k for RC and HORC with actuator saturation and without AW filter.

Figure 6 depicts the plant output y_k and saturated control signal $\hat{u}_k$ using the proposed AW design. Thus, the optimal design proposed here denoted by K_{db} are compared with K_{st} and $K = 0$ which corresponds to the response using a standard LQ design and $K = 0$ (IMC AW) respectively, as described in the previous section. It is shown that for $K = 0$ the system recover is too much slow; in fact, in this example, the ideal output is hardly reached again before getting into saturation again. On the other hand, the saturated control action approximates very well the ideal one, except when $u_k > u_{max}$, but as can be seen, this is not the desirable behaviour. For K_{st}, the plant output is closer to the ideal one and also it is seen that its corresponding control action remains saturated longer than in the previous case. Finally, using K_{db} to approximate a deadbeat behaviour, it can be noticed that the plant output gets closer to the ideal output and its control action remains saturated longer.

Figure 7, shows the output of the AW compensator $\sigma_{1,k}$, which, as mentioned before, can be seen as the deviation from the ideal response. It is shown that during the time the three systems are in saturation the deviation is similar; however the response is quite different once the systems get out of saturation, the response for K_{db} being the smallest one. It is worth to notice that, since for this example the RC and the HORC design have the same tracking performance, $\sigma_{1,k}$ in Figure 7 is the same in both cases.

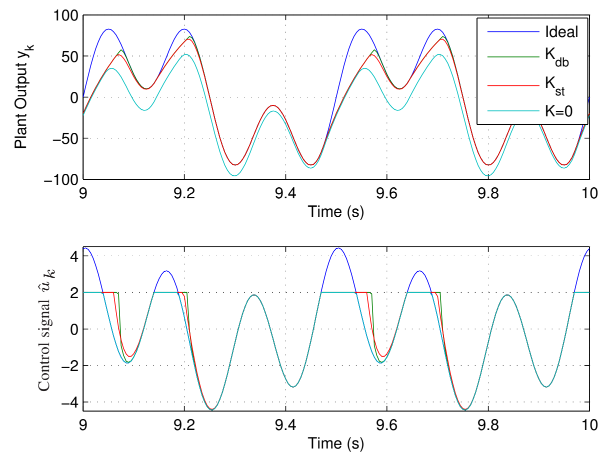

Fig. 6. Steady state saturation behaviour.

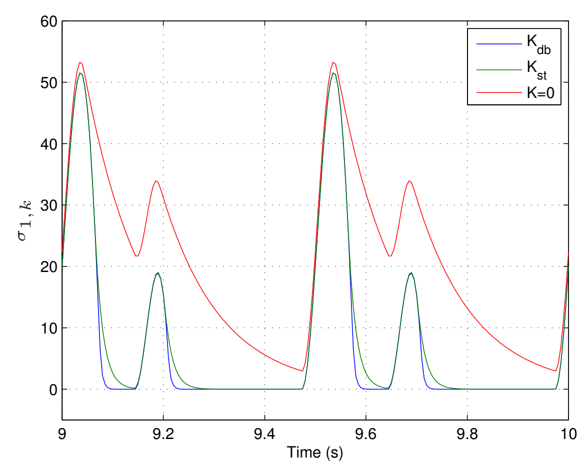

Fig. 7. Evolution of $\sigma_{1,k}$ (the deviation from the ideal behaviour).

VI. Conclusions

In this paper, the Model Recovery Anti-Windup scheme is studied and adapted to the Repetitive Control case. An optimal LQ design has been proposed aimed at finding a deadbeat recover behaviour and assuring the global asymptotic stability of the closed loop system. Through simulation results it is shown that the proposed AW scheme gets better performance in the deviation from the ideal plant output compared with other similar LQ designs. The future research includes the inclusion of less-restrictive sector conditions for the nonlinear saturation function in order to better approximate the deadbeat design.

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