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Optimal Decomposed Particle Filtering of Two Closely Spaced Gaussian Targets

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Abstract— **For Bayesian filtering of two closely spaced linear Gaussian targets from Gaussian observations, the paper exploits a unique decomposition of the joint conditional density into a mixture of a permutation invariant density and a permutation strictly variant density. This leads to the development of a novel particle filter which performs optimal in the sense of either minimizing track swapping or minimizing track switching, and which includes estimation of the conditional track swap probability. Through Monte Carlo simulations, it is shown that minimizing track switching has a significant advantage over minimizing track swapping, and that the novel particle filter performs remarkably better than a standard particle filter.**

I. Introduction

THE development of the Sampling Importance Resampling (SIR) particle filter [20],[14],[11] has created a complete novel approach towards approximating an exact Bayesian filter arbitrarily close, and has led to the development of a large variety of particle filters (e.g. [1], [19],[18]) that typically outperform established approaches in maintaining single target tracks. This success strongly motivated particle filter developments for multiple target tracking (e.g. [16],[23],[17]). Nevertheless, particle filtering for closely spaced targets still poses unresolved problems, e.g. [8],[13],[4],[12],[5].

The unresolved problems already surface in tracking two closely spaced linear Gaussian targets. Assume two such targets, starting out well separated and with known identity labels, move close to each other and separate again some time later. Initially, a joint particle filter evolves two clearly separated clouds of sub-particles, one for each target, and with clear identity labels per sub-cloud. However, once the targets start to move close to each other, then the two clouds of sub-particles are mixed in one large particle cloud [13]. The mixing itself is not caused by some sub-optimal behaviour of the particle filter, though represents the behaviour of an exact Bayes filter [5]. However, the SIR particle filter approximation of the exact Bayes recursion tends to dissolve this exact mixing [8].

The challenge is to handle the joint particles well when the two targets start to move away from each other. This asks for solutions to two sub-problems: 1) To handle the mixing of joint particles well; and 2) To provide a useful tracking output from the joint particles, including the estimation of track swap probability.

Regarding the first sub-problem, [6] derives a unique decomposition of the joint conditional density into a mixture of a permutation invariant and a strictly permutation variant density. Subsequently [7] uses this unique decomposition for the development of a novel decomposed particle filter. Regarding the second sub-problem, it is well known that the typical MMSE track output is sensitive to track-coalescence. Alternative track output approaches have been developed by [17],[12],[5],[15]: [17] applies particle clustering prior to Minimum Mean Square Error (MMSE) estimation per cluster; [12] and [5] apply Maximum A Posteriori (MAP) estimation; and [15] develops an algorithm for Minimum Mean OSPA Error (MMOE) estimation [22], where OSPA refers to the Optimal Sub-Pattern Assignment metric of [21]. Although these approaches avoid track coalescence, they suffer from another problem, i.e. the track output typically switches in an uncontrolled way between permutation options [5]. Moreover, none of these methods estimate track swap probability.

The aim of this paper is to exploit the unique decomposition of [6] and the decomposed particle filter cycle of [7] for the development of a particle filter for two Gaussian targets which estimates track swapping and minimizes track swapping and track switching respectively.

The paper is organized as follows. Section II formulates the track maintenance problem for two linear Gaussian targets. Section III reviews the unique decomposition results of [6]. Section IV reviews the decomposed particle filter cycle of [7]. Section V studies track outputs which minimize track swapping and track switching. Section VI provides simulation results for an example of two closely spaced targets. Section VII draws conclusions.

II. Two Gaussian Targets Tracking Problem

We consider two linear Gaussian targets, i.e.

$$x_{i,t} = a_i x_{i,t-1} + w_{i,t}, \quad i = 1,2 \tag{1}$$

where $x_{i,t}$ is the n-vectorial state of the i-th target, a_i is an $(n \times n)$-matrix and $\{w_{i,t}\}$ is a sequence of independent identically distributed (i.i.d.) zero mean Gaussian variables of dimension n', with $E\{w_{i,t}w_{i,t}^T\} = W_i$, $Det\{W_i\} > 0$, and $\{w_{1,t}\}$ and $\{w_{2,t}\}$ mutually independent, and also

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independent of $x_{1,0}$ and $x_{2,0}$.

We assume that a potential measurement originating from target i is modelled as a linear Gaussian system:

$$z_{i,t} = h_i x_{i,t} + g_i v_{i,t}, \quad i = 1,2 \tag{2}$$

where $z_{i,t}$ is an m-vector, h_i is an ($m \times n$)-matrix and g_i is an ($m \times m'$)-matrix, and $\{v_{i,t}\}$ is a sequence of i.i.d. standard Gaussian variables of dimension m' with $\{v_{1,t}\}$ and $\{v_{2,t}\}$ mutually independent. Moreover $\{v_{i,t}\}$ is independent of $x_{j,0}$ and $\{w_{j,t}\}$ for all i, j.

Stacking target states and potential measurements yields:

$$x_t = Ax_{t-1} + w_t \tag{3}$$

$$z_t = Hx_t + Gv_t \tag{4}$$

with: $x_t = \mathrm{Col}\{x_{1,t}, x_{2,t}\}$, $z_t \triangleq \mathrm{Col}\{z_{1,t}, z_{2,t}\}$,

$w_t = \mathrm{Col}\{w_{1,t}, w_{2,t}\}$, $v_t = \mathrm{Col}\{v_{1,t}, v_{2,t}\}$,

$A = \mathrm{Diag}\{a_1, a_2\}$, $W = \mathrm{Diag}\{W_1, W_2\}$

$H = \mathrm{Diag}\{h_1, h_2\}$, $G = \mathrm{Diag}\{g_1, g_2\}$,

where $\mathrm{Col}\{y_1, y_2\} \triangleq \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$ and $\mathrm{Diag}\{y_1, y_2\} \triangleq \begin{bmatrix} y_1 & 0 \\ 0 & y_2 \end{bmatrix}$.

At moment $t = 1,2,..,T$, a vector observation y_t is made of the two targets. The relation between y_t and x_t satisfies:

$$\underline{\chi}_t y_t = z_t = Hx_t + Gv_t \tag{5}$$

where $\underline{\chi}_t \triangleq \chi_t \otimes I$, with I a unit-matrix (of size m), $\otimes$ Kronecker product, and $\{\chi_t\}$ a sequence of i.i.d. 2×2 permutation matrices, which is independent of $\{x_t, v_t, w_t\}$. For two targets, χ_t either is the 2×2 unity matrix I, or its permutation $\Pi \triangleq \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.

The multi-target track maintenance problem considered is to estimate x_t in a recursive way from observations $Y_t \triangleq \{y_s; 0 \le s \le t\}$, where $y_0 = \{\}$. In spite of all linear Gaussian assumptions, the permutation matrix process $\{\chi_t\}$ in (5) makes this is a non-linear filtering problem. A recursive characterization of the exact conditional density is given in [3] for a more general problem setting.

III. Unique decomposition

This section presents the unique decomposition of [6].

Definition 1:

We say that the conditional density of the joint two-target state is permutation invariant at moment t if for all $x \in \mathbb{R}^{2n}$

$$p_{x_t|Y_t}(x) = p_{x_t|Y_t}(\underline{\Pi}x) \tag{6}$$

The standard way of working in target tracking is to use the (global) MMSE state estimation for the target tracking output, which satisfies:

$$\hat{x}_t^{MMSE} = \int_{\mathbb{R}^{Mn}} x p_{x_t|Y_t}(x)dx \tag{7}$$

If two targets have a permutation invariant joint conditional density then (6) applies, and together with (7) this yields:

$$\begin{aligned}\hat{x}_t^{MMSE} &= \int_{\mathbb{R}^{Mn}} x\left[p_{x_t|Y_t}(x) + p_{x_t|Y_t}(\underline{\Pi}x)\right]dx/2 \\ &= \left[\hat{x}_t^{MMSE} + \underline{\Pi}\hat{x}_t^{MMSE}\right]/2\end{aligned} \tag{8}$$

which implies $\hat{x}_t^{MMSE} = \underline{\Pi}\hat{x}_t^{MMSE}$. This proofs that when two targets have a permutation invariant joint conditional density, they also have equal MMSE estimated states.

When there is an $x \in \mathbb{R}^{2n}$ for which

$$p_{x_t|Y_t}(x) \neq p_{x_t|Y_t}(\underline{\Pi}x) \tag{9}$$

then we know from Definition 1 that for this x, $p_{x_t|Y_t}(x)$ is not permutation invariant. In order to capture a total lack of permutation invariance for all $x \in \mathbb{R}^{2n}$, [6] introduces the following definition.

Definition 2:

We say that the conditional density $p_{x_t|Y_t}$ of the joint two-target state is strictly permutation variant if for all $x \in \mathbb{R}^{2n}$:

$$p_{x_t|Y_t}(x).p_{x_t|Y_t}(\underline{\Pi}x) = 0 \tag{10}$$

Theorem 1 ([6], Theorem 2)

$p_{x_t|Y_t}(x)$ admits a unique decomposition in a weighted sum of a permutation invariant density $p^{\chi}_{x_t|Y_t}(x)$ and a strictly permutation variant density $p^{\not\chi}_{x_t|Y_t}(x)$, i.e. for all $x \in \mathbb{R}^{2n}$:

$$p_{x_t|Y_t}(x) = \alpha_t p^{\chi}_{x_t|Y_t}(x) + (1-\alpha_t) p^{\not\chi}_{x_t|Y_t}(x) \tag{11}$$

$$p^{\chi}_{x_t|Y_t}(x) = p^{\chi}_{x_t|Y_t}(\underline{\Pi}x) \tag{12}$$

$$p^{\not\chi}_{x_t|Y_t}(x).p^{\not\chi}_{x_t|Y_t}(\underline{\Pi}x) = 0 \tag{13}$$

[6] also characterizes the terms appearing in the unique decomposition of the joint conditional density as follows.

Theorem 2 ([6], Theorem 3)

In the unique decomposition of Theorem 1, the weight α_t and the permutation invariant density $p^{\chi}_{x_t|Y_t}(x)$ satisfy:

$$\alpha_t = \int_{\mathbb{R}^{2n}} \min\{p_{x_t|Y_t}(x), p_{x_t|Y_t}(\underline{\Pi}x)\}dx \tag{14}$$

$$p^{\chi}_{x_t|Y_t}(x) = \min\{p_{x_t|Y_t}(x), p_{x_t|Y_t}(\underline{\Pi}x)\}/\alpha_t, \text{ if } \alpha_t > 0 \tag{15}$$

$$p^{\not\chi}_{x_t|Y_t}(x) = \left[p_{x_t|Y_t}(x) - \alpha_t p^{\chi}_{x_t|Y_t}(x)\right]/(1-\alpha_t), \text{ if } \alpha_t < 1 \tag{16}$$

In [7], this unique decomposition is exploited to develop a decomposed particle filter for two Gaussian targets.

IV. DECOMPOSED PARTICLE FILTER

Following [7], we assume that the joint conditional density $p_{x_{t-1}|Y_{t-1}}(x)$ of the two targets in Section II, is an empirical density which is spanned by N_p decomposed particles: $\left\{x_{t-1}^j \in \mathbb{R}^{2n}, \mu_{t-1}^{\chi,j}, \mu_{t-1}^{\not\chi,j} \in [0,1]; j=1,...,N_p\right\}$, such that for all $j \in \{1,...,N_p\}$, $\mu_{t-1}^{\chi,j} + \mu_{t-1}^{\not\chi,j} = 1/N_p$, and $x_{t-1}^j \neq x_{t-1}^i \neq \underline{\Pi} x_{t-1}^j$ almost surely in probability for all $i \neq j$. With this the empirical density satisfies:

$$
\begin{aligned}
\hat{p}_{x_{t-1}|Y_{t-1}}(x) &= \sum_{j=1}^{N_p} \mu_{t-1}^{\not\chi,j} \delta(x - x_{t-1}^j) \\
&+ \sum_{j=1}^{N_p} \tfrac{1}{2} \mu_{t-1}^{\chi,j} [\delta(x - x_{t-1}^j) + \delta(x - \underline{\Pi} x_{t-1}^j)] \qquad (17)
\end{aligned}
$$

Starting from (17), in ([7], section 4) the following characterization of $p_{x_t|Y_t}(x)$ is derived:

$$
\begin{aligned}
\hat{p}_{x_t|Y_t}(x) &= \sum_{\chi} \sum_{j=1}^{N_p} \beta_t^{\not\chi,j}(\chi) N\{x; \hat{x}_t^j(\chi), P\} \\
&+ \sum_{\chi} \sum_{\chi'} \sum_{j=1}^{N_p} \tfrac{1}{2} \beta_t^{\chi,j}(\chi) N\{x; \underline{\chi}' \hat{x}_t^j(\chi), P\} \qquad (18)
\end{aligned}
$$

$$\beta_t^{\not\chi,j}(\chi) = \mu_{t-1}^{\not\chi,j} N\{\underline{\chi} y_t; H\bar{x}_t^j, Q\} / c_t^{\cdot} \qquad (19a)$$

$$\beta_t^{\chi,j}(\chi) = \mu_{t-1}^{\chi,j} N\{\underline{\chi} y_t; H\bar{x}_t^j, Q\} / c_t^{\cdot} \qquad (19b)$$

$$\hat{x}_t^j(\chi) = \bar{x}_t^j + K(\underline{\chi} y_t - H\bar{x}_t^j) \qquad (20)$$

with:

$$K = WH^T Q^{-1}$$
$$Q = HWH^T + GG^T$$
$$P = W - KHW$$

and $c_t^{\cdot}$ such that: $\sum_{\chi} \sum_{j=1}^{N_p} [\beta_t^{\not\chi,j}(\chi) + \beta_t^{\chi,j}(\chi)] = 1$

Equations (17)-(20) show that by starting with an empirical density $\hat{p}_{x_{t-1}|Y_{t-1}}(x)$ which is spanned by N_p decomposed particles (17), we get $\hat{p}_{x_t|Y_t}(x)$ in (18) which is a mixture of $4N_p$ Gaussians of same covariance P.

The steps of the novel decomposed particle filter are specified in Table 1. The evolution and correction steps are based on equations (18)-(20). In order to arrive at the other steps in Table 1, the key issue is how to resample N_p new decomposed particles from $\hat{p}_{x_t|Y_t}(x)$ in (18), and at the same time perform the Bayesian update step of the particle weights. Straightforward application of the unique decomposition characterization of Theorem 2 to the Gaussian mixture in (18) would lead to a very complicated exercise. Therefore [7] performs this characterization such that the unique decomposition applies at the locations of the newly sampled particles, and subsequently identifies the novel decomposed particles and evaluates their novel weights. This leads to the following Resampling and Unique decomposition approaches.

Resampling: First, we independently draw N_p times a pointer $\left(\kappa_t^i, \chi_t^i\right)$ to one of the $4N_p$ Gaussians. Subsequently we draw x_t^i from $N\{x; \hat{x}_t^{\kappa_t^i}(\chi_t^i), P\}$ for $i = 1,..,N_P$. Because $Det\{W\} > 0$ we have $Det\{P\} > 0$, which implies that almost surely $x_t^j \neq x_t^i \neq \underline{\Pi} x_t^j$ for all $i \neq j$, $i,j = 1,..,N_p$.

Unique decomposition: The next step is to perform the characterization of Theorem 2, which means that the weight $1/N_p$ is decomposed in the permutation invariant and strictly variant weights $\mu_t^{\chi,i} = \rho_t^i N_p^{-1}$ and $\mu_t^{\not\chi,i} = (1-\rho_t^i) N_p^{-1}$ respectively. Following ([7], Appendix), this yields the permutation invariant fraction ρ_t^i in Table 1.

Table 1. Cycle of Decomposed Particle Filter

$\hat{p}_{x_{t-1}\vert Y_{t-1}} \to \hat{p}_{x_t\vert Y_t}$
Particles $\left\{x_{t-1}^j \in \mathbb{R}^{2n}, \mu_{t-1}^{\chi,j}, \mu_{t-1}^{\not\chi,j} \in [0,1]; j=1,...,N_p\right\}$ At $t-1=0$ the weight values and samples are: $\mu_{t-1}^{\chi,j} = 0$ and $\mu_{t-1}^{\not\chi,j} = 1/N_p$ $x_{t-1}^j \sim p_{x_0\vert Y_0}(x)$
For $j=1,...,N_p$: Evolution: $\bar{x}_t^j = Ax_{t-1}^j$ Correction: $\beta_t^{\not\chi,j}(\chi) = \mu_{t-1}^{\not\chi,j} N\left\{\underline{\chi} y_t; H\bar{x}_t^j, Q\right\} / c_t^{\cdot}$ $\beta_t^{\chi,j}(\chi) = \mu_{t-1}^{\chi,j} N\left\{\underline{\chi} y_t; H\bar{x}_t^j, Q\right\} / c_t^{\cdot}$ with $Q = HWH^T + GG^T$ and $c_t^{\cdot}$ such that $\sum_{\chi} \sum_{j=1}^{N_p} \beta_t^{\not\chi,j}(\chi) + \beta_t^{\chi,j}(\chi) = 1$
Resampling: For $i=1,...,N_p$, draw samples, with replacement: $\left(\kappa_t^i, \chi_t^i\right) \sim \hat{p}_{\kappa_t,\chi_t\vert Y_t}(j,\chi) \triangleq \beta_t^{\not\chi,j}(\chi) + \beta_t^{\chi,j}(\chi)$ $x_t^i \sim N\{x; \hat{x}_t^{\kappa_t^i}(\chi_t^i), P\}$ with: $\hat{x}_t^j(\chi) = \bar{x}_t^j + K(\underline{\chi} y_t - H\bar{x}_t^j)$ $K = WH^T Q^{-1}$ $P = W - KHW$
The new decomposed weights become: $\mu_t^{\chi,i} = \rho_t^i / N_p$

$$\mu_t^{\not\chi,i} = (1-\rho_t^i)/N_p$$

with: $$\rho_t^i = \frac{\sigma_t^{\chi}(x_t^i) + \min\{\sigma_t^{\not\chi}(x_t^i), \sigma_t^{\not\chi}(\underline{\Pi} x_t^i)\}}{\sigma_t^{\chi}(x_t^i) + \sigma_t^{\not\chi}(x_t^i)}$$

$$\sigma_t^{\chi}(x) = \sum_{\chi}\sum_{\chi'}\sum_{j=1}^{N_p} \beta_t^{\chi,j}(\chi') N\{x; \underline{\chi}\hat{x}_t^j(\chi'), P\}$$

$$\sigma_t^{\not\chi}(x) = \sum_{\chi'}\sum_{j=1}^{N_p} \beta_t^{\not\chi,j}(\chi') N\{x; \hat{x}_t^j(\chi'), P\}$$

The new decomposed particles are:

$$\left\{x_t^i \in \mathbb{R}^{2n}, \mu_t^{\chi,i}, \mu_t^{\not\chi,i} \in [0,1],;\ i=1,...,N_p\right\}$$

These particles span the empirical density:

$$\hat{p}_{x_t|Y_t}(x) = \sum_{j=1}^{N_p} \mu_t^{\not\chi,j} \delta(x - x_t^j)$$
$$+ \sum_{j=1}^{N_p} \tfrac{1}{2}\mu_t^{\chi,j} [\delta(x - x_t^j) + \delta(x - \underline{\Pi} x_t^j)]$$

The permutation invariant fraction of the unique decomposition equals

$$\hat{\alpha}_t = \sum_{j=1}^{N_p} \mu_t^{\chi,j}$$

- Produce track output, and
- Perform next Decomposed PF cycle.

V. Minimizing Track Swap and Track Switching

For the Decomposed Particle Filter, the MMSE track output satisfies:

$$\hat{x}_t = \sum_{i=1}^{N_p} [\mu_t^{\not\chi,i} x_t^i + \tfrac{1}{2}\mu_t^{\chi,i}(\underline{I} + \underline{\Pi}) x_t^i] \quad (21)$$

This characterization clearly shows that particles having a total weight of $\mu_t^{\chi,i}$ are used to cause track coalescence behavior. In order to avoid this, in the sequel we consider track output which avoids track coalescence, i.e. for each of the particles only one permutation version is used in the track output. The latter is accomplished by considering estimated track output which is of the following form:

$$\hat{x}_t^O = \sum_{i=1}^{N_p} (\mu_t^{\not\chi,i} + \mu_t^{\chi,i}) \hat{\underline{\chi}}_t^{O,i} x_t^i = \sum_{i=1}^{N_p} \frac{1}{N_p} \hat{\underline{\chi}}_t^{O,i} x_t^i \quad (22)$$

where $\{x_t^i, \mu_t^{\not\chi,i}, \mu_t^{\chi,i};\ i=1,..,N_p\}$ is the set of decomposed particles at moment t and $\hat{\chi}_t^{O,i}$ is some optimal permutation for the i-th particle. In this section we consider the minimization of track swapping and track switching respectively, both within the setting of track coalescence avoiding eq. (22).

Under track output (22), the track swap probability becomes:

$$\hat{P}_{Swap,t}^{O} = \sum_{i=1}^{N_p} [1(\hat{\chi}_t^{O,i} = I)\hat{p}_{x_t|Y_t}(\{\underline{\Pi} x_t^i\})$$
$$+ 1(\hat{\chi}_t^{O,i} = \Pi)\hat{p}_{x_t|Y_t}(\{x_t^i\})] \quad (23)$$

where $\hat{p}_{x_t|Y_t}(\{a\})$ denotes the total mass of the empirical density $\hat{p}_{x_t|Y_t}(.)$ at $\{a\}$. Evaluation of equation (23) yields:

$$\hat{P}_{Swap,t}^{O} = \sum_{i=1}^{N_p} [1(\hat{\chi}_t^{O,i} = I)\hat{p}_{\hat{\chi}_t^i|Y_t}(\Pi) + 1(\hat{\chi}_t^{O,i} = \Pi)\hat{p}_{\hat{\chi}_t^i|Y_t}(I)]$$

$$= \sum_{i=1}^{N_p} [1(\hat{\chi}_t^{O,i} = I)\tfrac{1}{2}\mu_t^{\chi,i} + 1(\hat{\chi}_t^{O,i} = \Pi)(\mu_t^{\not\chi,i} + \tfrac{1}{2}\mu_t^{\chi,i})]$$

$$= \sum_{i=1}^{N_p} [\tfrac{1}{2}\mu_t^{\chi,i} + 1(\hat{\chi}_t^{O,i} = \Pi)\mu_t^{\not\chi,i}] \quad (24)$$

The latter implies that minimization of $\hat{P}_{Swap,t}^{O}$ is accomplished when $\hat{\chi}_t^{O,i} = I$ for all i. This leads to the track swap minimization track output equations in Table 2. The Minimum Track Swap (MTS) probability is determined by the $\frac{1}{2}\hat{\alpha}_t$ value, i.e. half the weight of the permutation invariant part of the unique decomposition.

Table 2. Decomposed Particle Filter MTS Output

Track swap probability is minimized by the track output:

$$\hat{x}_t^{MTS} = \sum_{i=1}^{N_p} \frac{1}{N_p} x_t^i$$

$$\hat{R}_t^{MTS} = \sum_{i=1}^{N_p} \frac{1}{N_p} (x_t^i - \hat{x}_t^{MTS})(x_t^i - \hat{x}_t^{MTS})^T$$

The corresponding probability of track swap equals:

$$\hat{P}_{Swap,t}^{S} = \tfrac{1}{2}\sum_{i=1}^{N_p} \mu_t^{\chi,i} = \tfrac{1}{2}\hat{\alpha}_t$$

In order to minimize track switching we define the Track Continuity Assignment (TCA) $\hat{\chi}_t^{TCA,i}$ for the i-th particle through the following minimization of jumps in track output[1]:

$$\hat{\chi}_t^{TCA,i} \triangleq \underset{\chi}{Arg\min} \| H(\underline{\chi}\hat{x}_t^i - A\hat{x}_{t-1}^{TCA}) \| \quad (25)$$

The corresponding TCA based track state output $\hat{x}_t^{TCA}$ is:

$$\hat{x}_t^{TCA} = \sum_{i=1}^{N_p} \frac{1}{N_p} \hat{\underline{\chi}}_t^{TCA,i} x_t^i \quad (26)$$

In order to avoid singularities, we assume $\hat{\chi}_t^{TCA,i} = I$ if $\hat{x}_t^i = \underline{\Pi}\hat{x}_t^i$ or $\hat{x}_{t-1}^{TCA} = \underline{\Pi}\hat{x}_{t-1}^{TCA}$.

Using (24), the probability of track swap under TCA

[1] Within the context of Multiple Hypothesis Tracking (MHT), [9] develops a related approach in minimizing Mahalanobis distances.

output becomes:

$$\hat{P}^{TCA}_{Swap,t} = \sum_{i=1}^{N_p} [\tfrac{1}{2}\mu_t^{\chi,i} + 1(\hat{\chi}_t^{TCA,i} = \Pi)\mu_t^{\cancel{\chi},i}]$$

$$= \tfrac{1}{2}\hat{\alpha}_t + \sum_{i=1}^{N_p} 1(\hat{\chi}_t^{TCA,i} = \Pi)\mu_t^{\cancel{\chi},i} \qquad (27)$$

which implies $\hat{P}^{TCA}_{Swap,t} \geq \hat{P}^{MTS}_{Swap,t}$. Application of the above approach yields the TCA track output equations in Table 3.

Table 3. Decomposed Particle Filter TCA Output

Track Continuity Assignment (TCA) track state output:

$$\hat{x}_t^{TCA} = \sum_{i=1}^{N_p} \frac{1}{N_P} \underline{\hat{\chi}}_t^{TCA,i} x_t^i$$

$$\hat{R}_t^{TCA} = \sum_{i=1}^{N_p} \frac{1}{N_p} (\underline{\hat{\chi}}_t^{TCA,i} x_t^i - \hat{x}_t^{TCA})(\underline{\hat{\chi}}_t^{TCA,i} x_t^i - \hat{x}_t^{TCA})^T$$

with $\hat{\chi}_t^{TCA,i}$ the TCA based permutation for particle *i*:

$$\hat{\chi}_t^{TCA,i} = Arg \min_{\chi} \| H(\underline{\chi} x_t^i - A\hat{x}_{t-1}^{TCA}) \|$$

The probability of track swap under TCA equals:

$$\hat{P}^{TCA}_{Swap,t} = \sum_{i=1}^{N_p} [\tfrac{1}{2}\mu_t^{\chi,i} + 1(\hat{\chi}_t^{TCA,i} = \Pi)\mu_t^{\cancel{\chi},i}]$$

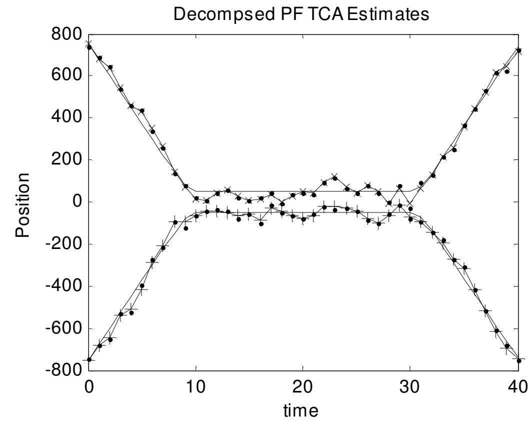
VI. MONTE CARLO SIMULATION

We consider a two target scenario from [3]. Two initially separated targets move towards each other, each with constant initial velocity 75m/s. At some moment in time both objects start decelerating at -50m/s^2 until they both have zero velocity. The deceleration starts at a moment such that the minimum distance between the two targets equals d = 100m. After spending a significant number of scans with zero velocity, both objects start accelerating at 50m/s^2 away from each other until their velocity equals the opposite of their initial velocity. From that moment on the velocity of both objects remains constant again.

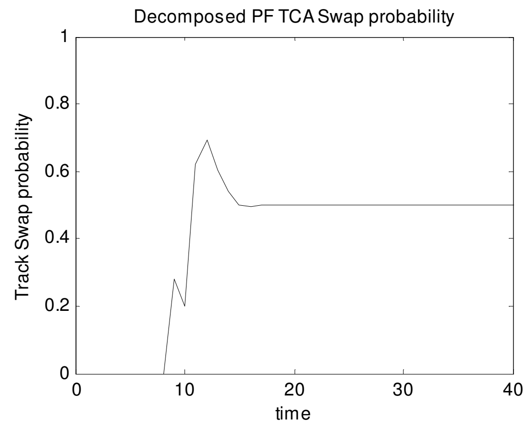
For this scenario, Monte Carlo simulations containing 100 runs are performed for the novel decomposed particle filter using $N_p = 100$ joint particles. The initial track estimates are accurate. The target and observation model used by this particle filter evolves according to discretized continuous white-noise acceleration [2], and target position is observed in noise, i.e. for the coefficients in (1)-(2) holds:

$$a_i = \begin{bmatrix} 1 & T_s \\ 0 & 1 \end{bmatrix},\ W_i = \sigma_a \begin{bmatrix} \frac{1}{3}T_s^3 & \frac{1}{2}T_s^2 \\ \frac{1}{2}T_s^2 & T_s \end{bmatrix},\ h_i = \begin{bmatrix} 1 & 0 \end{bmatrix},\ g_i = \sigma_m$$

with σ_a the standard deviation of acceleration noise and σ_m the standard deviation of measurement error. The scenario parameter values are V_{initial} = 75m/s, T_s = 1s, σ_m = 30m and σ_a = 50m/s^2.



1a. Measurements (•) and TCA estimated positions (x and +)



1b. Track swap probability estimated by decomposed PF with TCA

Figure 1. Track output of decomposed PF with TCA (Table 3).

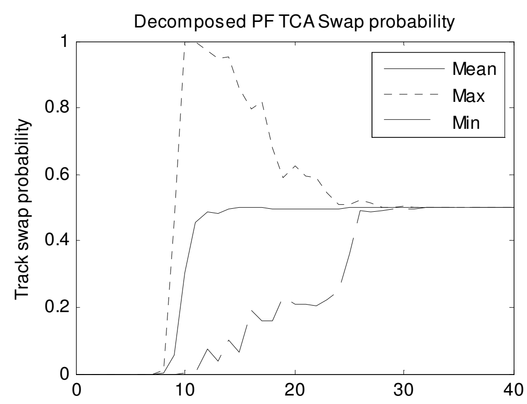


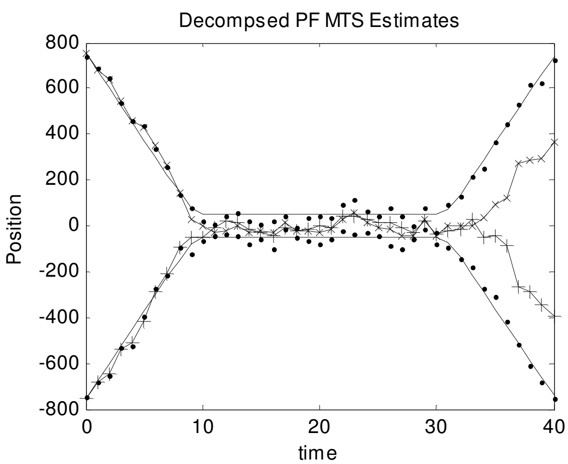
Figure 2 Mean, maximum and minimum track swap probability estimated over 100 MC runs for decomposed PF with TCA (Table 3).

Monte Carlo simulation results are presented in Figures 1-6. Figure 1 shows typical results for one run by the novel decomposed particle filter and TCA track output (Table 3). The optimal TCA track position output stays remarkably close to the true targets (Figure 1a), in spite of significant track swap probability (Figure 1b). The peak in TCA Swap probability in Figure 1b indicates that $\hat{\chi}_t^{TCA,i}$ often favours permutated contributions to the track output as soon as the two tracks start to move close to each other.

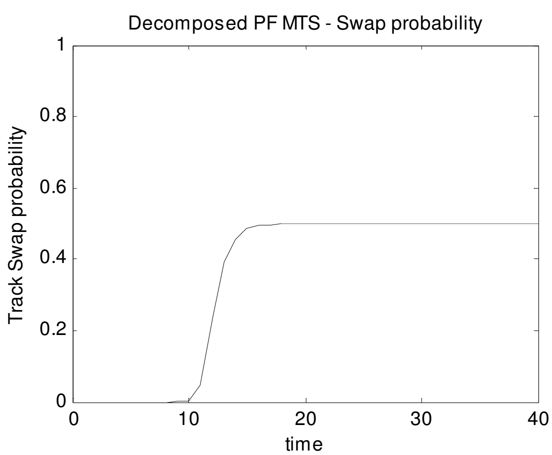
Figure 2 shows that the optimal TCA track state output approach leads to a very dynamic kind of behaviour by the novel decomposed particle filter in the estimated Track Swap probabilities.

Figure 3 shows typical results for one run by the novel decomposed particle filter (Table 1) and MTS track output (Table 2). The track swap probability behaves much smoother (Figure 3b) than in Figure 1b, though at the cost of less good position output (Figure 3a versus Figure 1a).

Figure 4 shows that MMSE track output from the decomposed particle filter ends with 100% track coalescence. Figure 5 shows that for a SIR PF this track coalescence partially dissolves, which phenomenon is in line with the theory of [8].



3a. Measurements (●) and Decomposed PF with MTS estimated positions (x and +). This shows some improved performance over IPS with MMSE output.



3b. Track swap probability estimated by decomposed PF with MTS

Figure 3. Track output of decomposed PF with MTS (Table 2).

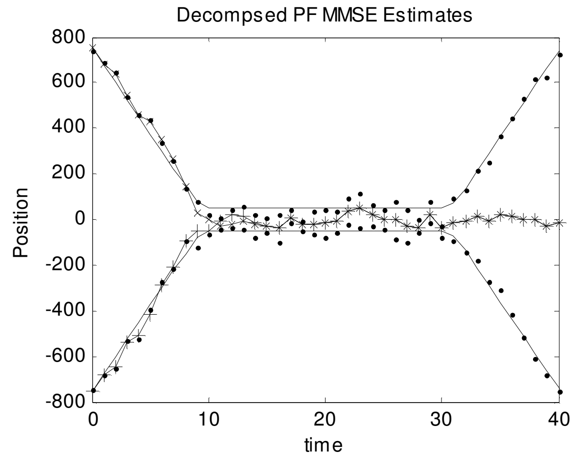


Figure 4. Measurements (●) and Decomposed PF with MMSE estimated positions (x and +). This shows the full track coalescence announced by equation (8).

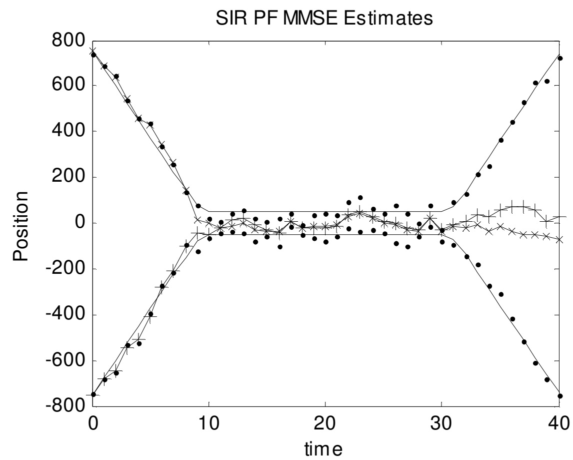


Figure 5. Measurements (●) and SIR PF MMSE estimated positions (x and +). This shows slowly dissolving track coalescence, in line with the theory [8].

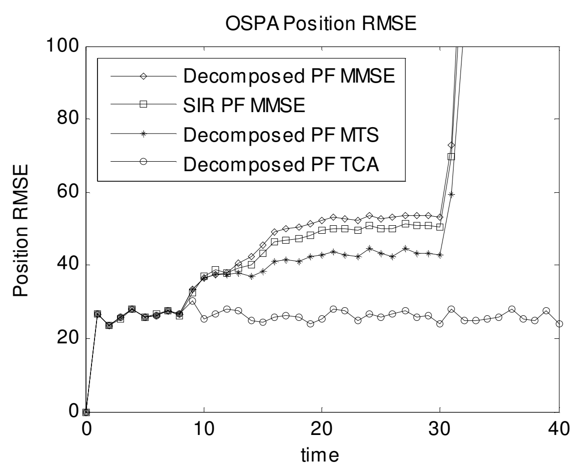


Figure 6 RMS of OSPA position errors for the decomposed particle filter with TCA, MTS and MMSE outputs, and for a SIR particle filter with MMSE output.

Figure 6 shows the RMS of OSPA errors [21] in position for the novel decomposed particle filter with TCA output, with Minimum Track Swap (MTS) output, with MMSE output, and a normal SIR Particle Filter (using ten thousand particles) with MMSE output. This shows that the remarkable smooth behaviour of optimal TCA track output in Fig. 1a is typical behaviour. The decomposed particle filter with TCA track output has superior performance over the three other track outputs. Moreover, the position RMS of the OSPA errors in the TCA track output is almost constant over the simulated period.

VII. CONCLUDING REMARKS

For the problem of maintaining tracks of two Gaussian targets from unassociated Gaussian observations, this paper studied minimization of track swapping and track switching. Explicit use has been made of the unique decomposition by [6] and the decomposed particle filter of [7]. For this decomposed particle filter, two specific track outputs have been studied:

- Minimizing track swapping
- Minimizing track switching

Thanks to the decomposed particle filter framework, both track outputs come with an estimation of the conditional probability that the track outputs are swapped relative to the true target locations.

The decomposed particle filter with different track outputs have been compared to each other and to a standard SIR particle filter with MMSE track output. This comparison is accomplished through running MC simulations for an example of two Gaussian targets that start well separated, then fly towards each other, and then separate again. The simulation results obtained show that the decomposed particle filter with minimal track switching performs much better than the one with minimal track swapping, and also much better than a standard SIR particle filter with MMSE track output. The decomposed particle filter with minimal track switching output has two unique capabilities:

- It provides track output that behaves continuous and stays remarkably close to the true target locations;
- It provides an estimate of the conditional probability that the presented tracks are swapped regarding the true target locations.

Directions for follow up research are to extend the novel decomposed particle filter for covering missed detections and false measurements, to extend it to hybrid and non-linear targets, to cover limited sensor resolution, and to cover more than two targets. A start of the latter is in [10].

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