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A Robust Adaptive Fuzzy Sliding mode Controller for Trajectory Tracking of ROVs

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Abstract— **This study deals with dynamic modeling and tracking control of a remotely underwater vehicle (ROV) with six degrees of freedom (DOF). The sliding mode scheme for tracking control of an ROV is a powerful approach to compensate structured and unstructured uncertainties. In this study, performance of sliding mode approach modified by robust adaptive fuzzy control algorithm for an ROV is presented. Fuzzy algorithm is used for on-line estimation of external disturbances as well as unknown nonlinear terms of dynamic model of the ROV. A robust control rule is employed to compensate for estimation errors. The boundedness and asymptotic convergence properties of the control algorithm and its semi-global stability are analytically proven using Lyapunov stability theory and Barbalat's lemma. Moreover, adaptation laws and robust control terms are derived from Lyapunov stability synthises. The adopted control scheme is implemented in numerical simulations, based on the dynamic parameters of Shiraz University Remotely Operated Vehicle (Ariana I ROV). Simulations show the effectiveness of the adopted controller for trajectory tracking.**

I. INTRODUCTION

THE significance of utilizing Remotely Underwater Vehicles in marine applications is a forgone conclusion. Industrial applications of ROV include inspection and maintenance of offshore oil and gas subsea structures, ships underwater bodies, damps structures and equipments, and data gathering for seabed studies and marine archeology, etc. [1], [2]. Designing an ROV system, especially its control system is a major challenge in which engineers and researchers are faced with a number of complexities such as considering inherently nonlinear dynamics, time-varying and undeterministic hydrodynamic parameters, disturbances caused by underwater currents and waves, etc [1].

In order to meet the requirements of control systems for underwater robots, various types of control schemes are implemented in the literatures. Autopilot deign for an unmanned underwater vehicle based on the technique of $H\infty$ is reported [3]. The tracking problem for low-speed maneuvering of (JHUROV) ROV using a linear proportional-derivative (PD) control and a family of fixed and adaptive model-based controllers is also reported [4]. Several adaptive control methodologies presented in other studies. [5], [6].

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An adaptive PD controller for the dynamic positioning of ROVs working in close proximity of off-sure structures is introduced by [1]. Yoerger and Slotine applied sliding mode scheme for trajectory control of underwater vehicle quite successfully [7]. Walchko, Novick and Nechyba applied sliding mode control to Subjugator ROV [8] and Cristi, Papoulias and Healy employed an adaptive sliding mode control for their underwater vehicle [9]. In both of these studies uncertainties were assumed to be bounded. Hoang, Kreuzer proposed a robust adaptive sliding mode control for dynamic of an ROV in which prior knowledge of bounds for uncertainties in parameters was not required [10].

Artificial intelligence approaches have been widely used in the field of underwater robots. Autopilots formulated using fuzzy logic [11]. Some studies applying fuzzy control to underwater robots can be found in [2], [12] and [13]. Implementing Artificial Neural Network (ANN) methods to underwater robots control are also reported [14], [15].

Ordinary fuzzy control algorithm needs a large number of training to achieve the desired performance. Training of neural network may be time consuming and it may be not proper for real-time control [16]. Moreover, in many cases stability and stronger mathematical approach should be considered. Other stable schemes have been combined with fuzzy logic control to achieve more stability [17]. Labiod, Boucherit and Guerra presented adaptive fuzzy control scheme for a class of MIMO nonlinear systems [18].

Some types of combinations of fuzzy logic and sliding mode control have been reported [19]-[24]. In [21], this method was applied to an underwater vehicle robot. Bessa and Barreto chose the switching variable "s" instead of the state variables in the premise of fuzzy rules in order to avoid forming incredibly large fuzzy sets and fuzzy rules in higher-order systems [23] and applied it to depth regulation of underwater vehicles [24]. Adaptive fuzzy sliding mode control is applied in identification of external disturbances to control the dynamic positioning of underwater vehicles with four controllable degrees of freedom [22]. They experienced applying a fuzzy inference system to approximate random external disturbances, using sliding mode switching variable "s" as premise variable. They also assumed the estimations of mass, centripetal and hydrodynamic matrices can be achieved with some small bounds on the parameters.

In this paper, adaptive fuzzy sliding mode algorithm with robustifying control term is employed for trajectory tracking of underwater vehicle with six controllable degrees of freedom. In this approach weight, centripetal, hydrodynamic and disturbance matrices are assumed to be unknown. In order to guarantee the stability of the close loop system a robustifying term is added to control rule. Semi-global stability, asymptotic convergence to minimize the tracking error and boundedness of the close-loop signals are assured by Lyapunov stability theory and Barbalat's lemma. Results show that the proposed control law can provide fine performance in trajectory tracking problem in spite of the system unknown parameters and external disturbances.

In the next section the architecture of fully actuated Ariana-I ROV is described. In the third section, motion dynamics equations of the underwater vehicles is explained. In section 4 the proposed control scheme is introduced, simulation results are presented in section 5, conclusions can be found in section 6.

II. SYSTEM ARCHITECTURE

The ROV constructed for this study is laboratory underwater vision based robot called Ariana-I. The Center of gravity and the center of buoyancy of the vehicle are positioned such that Ariana-I ROV is self-stabilized; however, double actuators in lateral direction are provided in order to provide stronger reaction to some particular situations in which the system is forced to disturbances. The distribution of thrusters is designed in such a way that the vehicle is able to maneuver with high accuracy both in the horizontal and vertical plane and compensate for the disturbances and noises during its operation.

The net weight of the vehicle out of water is about 130kg and it is almost neutrally buoyant in the water. The dimensions of the frame are $130 \times 100 \times 65\ cm$. The frame is made from ABS whose density is $1040 \frac{kg}{m^3}$. In the bottom plate center plate of the frame, a box with a volume of $370 \times 370 \times 150\ mm^3$ is installed, which contains six 24VDC sealed acid batteries and electronic board. The board includes drivers of thruster dc motors, Analog Devices micro processor for data acquisition and interfacing, heading magnetic compass, MEMS sensor for roll and pitch angles measurements, acceleration sensor and pressure sensor.

III. MODELING AND DYNAMIC EQUATIONS OF MOTION

ROVs in the water would have at most six degree of freedom including three rotations and three translational motions. Dynamic equations of motion of ROVs include six non-linear differential equations which in general cannot be decoupled. However, many ROVs are designed such that the metacentric height would be sufficiently large and provide self-stabilization of roll and pitch angles. In this particular condition the order of dynamic model would be reduced to four DOF and the vertical motion could be decoupled from the horizontal motion plane [2], [7], [22].

In this paper we consider all six degrees of freedom whose equations are written in two coordinate frames, body-fixed frame and earth-fixed frame. Both coordinate systems are shown in Figure2.

Figure1. Ariana-I ROV

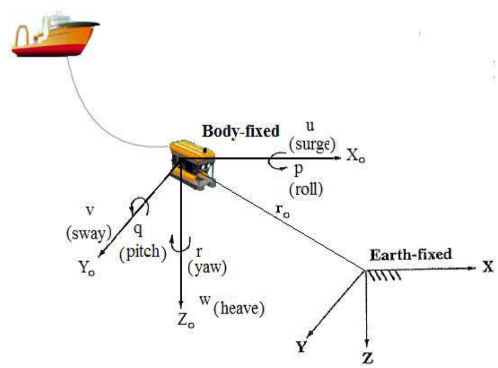

Figure2. Body-fixed frame and coordinate-fixed frame

A. Kinematic Transformation

By using Jacobian matrix $J(\eta)$, kinematic transformation can be performed to transform linear and angular velocities between the two body and fixed coordinate frames.

$$\begin{bmatrix} \dot{\eta}_1 \\ \dot{\eta}_2 \end{bmatrix} = \begin{bmatrix} J_1(\eta_2) & 0_{3\times3} \\ 0_{3\times3} & J_2(\eta_2) \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \Leftrightarrow \dot{\eta} = J(\eta) v \qquad (1)$$

Where v_1 and v_2 denote the linear and angular velocities in body-fixed frame, $\eta = [\eta_1^T, \eta_2^T]^T$; $\eta_1 = [x, y, z]^T$; $\eta_2 = [\phi, \theta, \psi]^T$; ($\phi, \theta, \psi$ are Euler angles).

B. Dynamic Model

The general vectorial representation of 6-DOFs rigid-body equation of motion in the space is written in the compact form as:

$$M_{RB}\dot{v} + C_{RB}(v)v = \tau_{RB} \qquad (2)$$

where M_{RB}, $C_{RB}(v)$ and τ_{RB} are the inertia matrix, coriolis and centripetal matrix and the vector of external forces and moments acting on the vehicle. For underwater applications external forces and moments acting on vehicle can be classified to hydrodynamic forces and moments τ_H which include added mass $M_A\dot{v} + C_A(v)v$, hydrodynamic damping $D(v)$ and restoring forces $g(\eta)$, environmental forces and moments τ_E which include; propulsion forces and moments τ_T which include actuators forces and moments.

$$\tau_{RB} = \tau_H + \tau_E + \tau_T \quad (3)$$
$$\tau_H = -M_A\dot{v} - C_A(v)v - D(v)v - g(\eta) \quad (4)$$

C. Hydrodynamic Terms

By a rough approximation it is possible to assume that if the vehicle has three planes of symmetry in performing a non-coupled motion with low speed and it assumed that the terms higher than second order are negligible, then diagonal structure of $D(v)$ with only linear and quadratic damping can be adopted in the modeling [25]. Moreover:

$$\begin{aligned} D(v) = &-diag\left\{X_u, Y_v, Z_w, K_p, M_q, N_r\right\} \\ &-diag\left\{X_{u|u|}|u|, Y_{v|v|}|v|, Z_{w|w|}|w|,\right. \\ &\left. K_{p|p|}|p|, M_{q|q|}|q|, N_{r|r|}|r|\right\} \end{aligned} \quad (5)$$

Coefficients in (5) ($X_u, \ldots, X_{u|u|}, \ldots$) should be determined analytically (for example by use of strip theory) or identified experimentally.

D. Added inertia terms

In general, if the vehicle is geometrically symmetric and it moves at low speeds, M_A obtained as:

$$M_A = -diag\left\{X_{\dot{u}}, Y_{\dot{v}}, Z_{\dot{w}}, K_{\dot{p}}, M_{\dot{q}}, N_{\dot{r}}\right\} \quad (6)$$

and C_A is derived from the matrix, M_A. More details can be found in [25].

E. Hydrostatic Forces

The Ariana-I ROV is nearly neutrally buoyant.
We consider restoring forces and moments can be passively compensated [22].

F. Thruster Force (τ_T)

In general, thruster force and moment vector has a complicated function form. An extensive study on thrusters and their effects on the underwater vehicles is reported in [29]. The simplified nonlinear equation for steady-state axial thrust T produced by fixed pitch propeller marine thrusters is presented in the literature as below [30]:

$$T = C_T n|n| \quad (7)$$

The parameter C_T should be identified experimentally. n is the propeller rate of rotation.

G. Forces and Moments on the ROV due to the Umbilical Cable

Disturbance forces and moments caused by umbilical cable can be modeled in different ways. Some modeling for umbilical cable of ROV can be found in [26], [27]. Adoption of any of these schemes is not proper for on-line identification and control action, which is mainly because of complexity and large amount of computation required in such approaches. It is common to address this problem by considering the forces and moments due to umbilical cable as random and add them to other disturbance terms in the disturbance vector P.

H. Representation of Equation of Motion of ROV in the Body-Fixed Frame

Replacement of (4) into (3) and together with (2) yields the following representation of 6-DOFs dynamic equation of motion of underwater vehicles:

$$\begin{aligned} &M\dot{v} + C(v)v + D(v)v + g(\eta) + P = \tau \\ &\dot{\eta} = J(\eta)v \\ &M \triangleq M_{RB} + M_A; C(v) = C_{RB}(v) + C_A(v); \tau = \tau_T + \tau_E \end{aligned} \quad (8)$$

I. Earth-fixed coordinate frame equation of motion of ROV

The following representation in earth-fixed coordinate frame can be obtained by applying kinematic transformation [25] to eliminate v and $\dot{v}$ from (8):

$$M_\eta(\eta)\ddot{\eta} + C_\eta(v,\eta)\dot{\eta} + D_\eta(v,\eta)\dot{\eta} + g_\eta(\eta) + P_\eta = \tau_\eta \quad (9)$$

Based on III.E, we eliminate the term $g_\eta(\eta)$ in the following sections.

IV. CONTROL

Generally ROVs are underwater robots which are moving at low speeds (less than 2 knots) and the Ariana_I ROV almost has three planes of symmetry. As referred in III.C under these conditions vehicle is performing a non-couple motion. In the following, sliding mode scheme is combined with adaptive fuzzy algorithm and robust adaptive control. The nonlinear dynamic system can be rewritten in the following form:

$$\ddot{\eta} = \underbrace{\left(\bar{C} + \bar{D} + \bar{P}\right)}_{F(\eta)} + \underbrace{\bar{M}^{-1}\bar{\tau}}_{G(\eta)U} \quad (10)$$

In order to simplify control analysis, in this section nonlinear equation (10) system in the earth-reference is adopted.

Where $\bar{M} = J^{-T}MJ^{-1}, \bar{C} = -\bar{M}^{-1}\left(J^{-1}C(v)v + J^{-T}M\dot{J}^{-1}\dot{\eta}\right)$, $\bar{D} = -\bar{M}^{-1}\left(J^{-1}(\eta)D(v)v\right), \bar{P} = -\bar{M}^{-1}\left(J^{-1}(\eta)P\right)$ and $\bar{\tau} = J^{-1}\tau$. For further simplicity the term $\bar{M}^{-1}$ is multiplied to the centripetal, hydrodynamics and disturbance terms. It should be noted that the off-diagonal terms of matrix $\bar{M}$ are incorporated in the vector $\bar{P}$. In addition, the following assumptions have been made for control analysis:

Assumption1.

the matrix $\bar{M}^{-1}$ is positive definite. Indeed, a strictly positive constant μ_0 exists such that:

$\bar{M}^{-1} > \mu_0 I_6$.

Assumption 2.

The desired trajectories η_{d_i} and $\dot{\eta}_{d_i}$ are at least once differentiable and η_{d_i}, $\dot{\eta}_{d_i}$ and $\ddot{\eta}_{d_i}$ are bounded and known. Also, states η_i and $\dot{\eta}_i$ are available through measurements.

Let tracking error $\tilde{\eta}_i$ due to each DOF be defined as

$$\tilde{\eta}_i = \eta_{d_i} - \eta_i \tag{11}$$

where η_{d_i} is the desired path for each DOF. Assume that $S_i(t)$ is the sliding surface and it is defined as

$$s_i\left(\tilde{\eta}_i, \dot{\tilde{\eta}}_i\right) = 0 \text{ where} \tag{12}$$

$$s_i(t) = \dot{\tilde{\eta}}_i + \lambda_i \tilde{\eta}_i, \quad i = 1, 2, \ldots, 6 \tag{13}$$

λ_i is strictly positive constant. Obviously, $s_i(t) \longrightarrow 0$ derive $\tilde{\eta}_i(t) \longrightarrow 0$ asymptotically for $i = 1, \ldots 6$. Therefore, designing a controller which contributes to $s_i(t) \longrightarrow 0$ for each DOF is the control objective. The time derivative of (13) can be written as

$$\dot{s}_i = \ddot{\eta}_{d_i} + \lambda_i \dot{\tilde{\eta}}_i - \left(\bar{c}_i + \bar{d}_i + \bar{p}_i\right) - \bar{m}_i^{-1} \bar{\tau}_i \tag{14}$$

If nonlinear functions of the system $(\bar{C} + \bar{D} + \bar{P})$ and $\bar{M}^{-1}$ are known, the following control law can be employed to satisfy the control purpose

$$\bar{\tau}_i = \bar{m}_i \left[-\left(\bar{c}_i + \bar{d}_i + \bar{p}_i\right) + \ddot{\eta}_{d_i} + \lambda_i \dot{\tilde{\eta}}_i + k_i \operatorname{sgn}(s_i) \right] \tag{15}$$

Where sgn(.) is the Sign function. Substituting (15) into (14) results in

$$\dot{s}_i(t) = -k_i \operatorname{sgn}(s_i), \quad i = 1, 2, \ldots, 6 \tag{16}$$

Equation (16) implies the finite time convergence to the sliding surface $S_i(t)$, contributing to convergence of the tracking error $\tilde{\eta}_i$ to 0. In this study, we assumed that nonlinear functions are unknown. In order to design a proper control law to cope with this problem we employ fuzzy inference system for online estimation of unknown functions.

The fuzzy inference system used in this study is zero-order Sugeno. It can be characterized by a set of if-then rules in the form:

$$R^k: \ if \ s_{i_k} = S_{i_k} \ then \ \hat{\hat{h}}_{i_r} = \hat{\theta}_{h_{i_r}} \tag{17}$$

Where S_{i_k} are fuzzy sets whose membership functions should be properly selected. The parameter $\hat{\hat{h}}_i$ will be replaced by approximation of the nonlinear functions in (10) and are shown by $\left(\hat{\hat{c}}_i + \hat{\hat{d}}_i + \hat{\hat{p}}_i\right)$ and $\hat{\hat{m}}_i^{-1}$. $\hat{\theta}_{h_{i_r}}$ is output value of each fuzzy rule k, with $k = 1, \ldots, R$. The final output of the fuzzy system is calculated by a weighted average formula:

$$\hat{\hat{h}}_i(s_i) = \frac{\sum_{r=1}^{R} \omega_{i_r} \hat{\theta}_{h_{i_r}}}{\sum_{r=1}^{R} \omega_{i_r}} \tag{18}$$

Equation (18) can be written in compact form as:

$$\hat{\hat{h}}_i(s_i) = \Omega_i^T(s_i) \hat{\Theta}_{h_i} \tag{19}$$

where $\hat{\Theta}_{\mathrm{h_i}} = [\hat{\theta}_{h_{i_1}}, \hat{\theta}_{h_{i_2}}, \ldots, \hat{\theta}_{h_{i_R}}]^{\mathrm{T}}$, $\Omega_{\mathrm{i}}(s_i) = [\Omega_{\mathrm{i_1}}, \Omega_{\mathrm{i_2}}, \ldots, \Omega_{\mathrm{i_R}}]^T$ whose components are $\Omega_{\mathrm{i_k}} = \frac{\omega_{\mathrm{i_k}}}{\sum_{k=0}^{R} \omega_{i_k}}$ and $\omega_{\mathrm{i_k}}$ is the firing strength of each rule. Let us consider the approximation of the nonlinear functions as follows:

$$\hat{\hat{c}}_i + \hat{\hat{d}}_i + \hat{\hat{p}}_i = \Omega^T_{c_i+d_i+p_i} \hat{\Theta}_{c_i+d_i+p_i}$$
$$\hat{\hat{m}}_i = \Omega^T_{m_i} \hat{\Theta}_{m_i}, \quad i = 1, 2, \ldots, 6 \tag{20}$$

We define optimal parameters $\hat{\theta}_{c_i}{}^* + \hat{\theta}_{d_i}{}^* + \hat{\theta}_{p_i}{}^*$ and $\hat{\theta}_{m_i}{}^*$ based upon optimal estimation [18], [22]. Minimum parameter estimation errors are defined as:

$$\tilde{\theta}_{c_i+d_i+p_i} = \hat{\theta}^*_{c_i+d_i+p_i} - \hat{\theta}_{c_i+d_i+p_i}$$
$$\tilde{\theta}_{m_i} = \hat{\theta}^*_{m_i} - \hat{\theta}_{m_i}, \quad i = 1, 2, \ldots, 6 \tag{21}$$

Minimum fuzzy approximation errors are defined as:

$$\varepsilon_{c_i+d_i+p_i} = \left(\bar{c}_i + \bar{d}_i + \bar{p}_i\right) - \left(\hat{\hat{c}}_i^* + \hat{\hat{d}}_i^* + \hat{\hat{p}}_i^*\right)$$
$$\varepsilon_{m_i} = \bar{m}_i^{-1} - \hat{\hat{m}}_i^{-1*}, \ i = 1, 2, \ldots, 6 \tag{22}$$

Here, the employed fuzzy sets should not be larger than universal approximation property. As a consequence, minimum approximation error supposed to be bounded as:

$$\left|\varepsilon_{c_i+d_i+p_i}\right| \le \bar{\varepsilon}_{c_i+d_i+p_i}, \left|\varepsilon_{m_i}\right| \le \bar{\varepsilon}_{m_i}; \ i = 1, 2, \ldots, 6 \tag{23}$$

where $\bar{\varepsilon}_{c_i+d_i+p_i}$ and $\bar{\varepsilon}_{m_i}$ are given constants. Now, let us define certainty control term $\bar{\tau}_{c_i}$ as follows:

$$\bar{\tau}_{AFSMC_i} = \hat{\hat{m}}_i \left[-\left(\hat{\hat{c}}_i + \hat{\hat{d}}_i + \hat{\hat{p}}_i\right) + \ddot{\eta}_{d_i} + \lambda_i \dot{\tilde{\eta}}_i + k_i \operatorname{sgn}(s_i) \right] \tag{24}$$

It should be noted that the term $\hat{\hat{m}}_i^{-1}$ is estimated on-line and it may contribute to a singular estimation for this term. In order to address this problem, in (25) regularized inverse of $\hat{\hat{m}}_i^{-1}$ will be used. This regularized inverse is well-defined when $\hat{\hat{m}}_i^{-1}$ is singular. Modified control law is:

$$\bar{\tau}_{AFSMC_i} = \left(\frac{\hat{\hat{m}}_i^{-1}}{\varepsilon_0 + \hat{\hat{m}}_i^{-2}} \right) \left[-\left(\hat{\hat{c}}_i + \hat{\hat{d}}_i + \hat{\hat{p}}_i\right) + \ddot{\eta}_{d_i} + \lambda_i \dot{\tilde{\eta}}_i + k_i \operatorname{sgn}(s_i) \right] \tag{25}$$

Control law (25) is well-defined, but the stability of the close-loop system would not be guaranteed by this term alone. Hence, a robustifying control term $\bar{\tau}_{r_i}$ will be added to the control law:

$$\bar{\tau}_i = \bar{\tau}_{AFSMC_i} + \bar{\tau}_{r_i} \tag{26}$$

Incorporating $\bar{\tau}_{r_i}$ to control term can guarantee the stability of close loop system and it define as

$$\bar{\tau}_{r_i} = \frac{s_i \left|s_i\right| \left[\bar{\varepsilon}_{c_i+d_i+p_i} + \bar{\varepsilon}_{m_i} \left|\bar{\tau}_{AFSMC_i}\right| + \left|\bar{\tau}_{0_i}\right| \right]}{\mu_0 \left\|s_i\right\|^2 + \sigma_i} \tag{27}$$

where $\bar{\tau}_{0_i}$ is

$$\bar{\tau}_{0_i} = \left(\frac{\varepsilon_0}{\varepsilon_0 + \hat{\hat{m}}_i^{-2}} \right) \left[-\left(\hat{\hat{c}}_i + \hat{\hat{d}}_i + \hat{\hat{p}}_i\right) + \ddot{\eta}_{d_i} + \lambda_i \dot{\tilde{\eta}}_i + k_i \operatorname{sgn}(s_i) \right] \tag{28}$$

and σ is a time-varying design parameter. The adaptation laws (29), (30) are used to achieve the most appropriate approximation for unknown nonlinear functions and design parameter σ is updated (31) as follows:

$$\dot{\hat{\Theta}}_{c_i+d_i+p_i} = -\gamma_{c_i+d_i+p_i} \Omega_{c_i+d_i+p_i} s_i \tag{29}$$

$$\dot{\hat{\Theta}}_{m_i} = -\gamma_{m_i}\Omega_{m_i} s_i \overline{\tau}_{AFSMC_i} \tag{30}$$

$$\dot{\sigma}_i = -\gamma_{i_0} \frac{|s_i|\left[\overline{\varepsilon}_{c_i+d_i+p_i} + \overline{\varepsilon}_{m_i}\left|\overline{\tau}_{AFSMC_i}\right| + \left|\overline{\tau}_{0_i}\right|\right]}{\mu_0\|s_i\|^2 + \sigma_i} \tag{31}$$

Here, we should prove that the adaptation laws (29) -(31) give suitable approximations for nonlinear functions of the system and also guarantee the convergence of the tracking error to zero. First, by substituting (26) into (14), $\dot{s}_i$ can be rewritten as

$$\begin{aligned} \dot{s}_i &= \ddot{\eta}_{d_i} + \lambda_i\dot{\tilde{\eta}}_i - \left(\overline{c}_i + \overline{d}_i + \overline{p}_i\right) - \left(\overline{m}_i^{-1} - \hat{\overline{m}}_i^{-1}\right)\overline{\tau}_{AFSMC_i} \\ &-\hat{\overline{m}}_i^{-1}\overline{\tau}_{AFSMC_i} - \overline{m}_i^{-1}\overline{\tau}_{r_i} = -k_i \operatorname{sgn}(s_i) \\ &-\left[\left(\hat{\overline{c}}_i^* + \hat{\overline{d}}_i^* + \hat{\overline{p}}_i^*\right) - \left(\hat{\overline{c}}_i + \hat{\overline{d}}_i + \hat{\overline{p}}_i\right)\right] - \left(\hat{\overline{m}}_i^{*-1} - \hat{\overline{m}}_i^{-1}\right)\overline{\tau}_A \\ &+\overline{\tau}_{0_i} - \overline{m}_i^{-1}\overline{\tau}_{r_i} - \varepsilon_{c_i+d_i+p_i} - \varepsilon_{m_i}\overline{\tau}_{AFSMC_i} \end{aligned} \tag{32}$$

Multiplying (32) by s_i from right, gives:

$$\begin{aligned} s_i\dot{s}_i &= -k_i|s_i| - \left(\Omega^T_{c_i+d_i+p_i}\tilde{\Theta}_{c_i+d_i+p_i} s_i\right) \\ &-\left(\Omega^T_{m_i}\tilde{\Theta}_{m_i}\overline{\tau}_{AFSMC_i}\right) - \overline{m}_i^{-1}\overline{\tau}_{r_i} s_i + \overline{\tau}_{0_i} s_i \\ &-\varepsilon_{c_i+d_i+p_i} s_i - \varepsilon_{m_i} s_i \overline{\tau}_{AFSMC_i} \end{aligned} \tag{33}$$

Now, let's consider the following positive definite Lyapunov function candidate:

$$\begin{aligned} V_i(t) &= \frac{1}{2}s_i^2 + \frac{1}{2}\left(\frac{1}{\gamma_{c_i+d_i+p_i}}\tilde{\Theta}^T_{c_i+d_i+p_i}\tilde{\Theta}_{c_i+d_i+p_i}\right. \\ &\left.+\frac{1}{\gamma_{m_i}}\tilde{\Theta}^T_{m_i}\tilde{\Theta}_{m_i}\right) + \frac{1}{2\gamma_{i_0}}\sigma_i^2 \end{aligned} \tag{34}$$

By use of (34) and (20) and applying (29), (30), the time derivative of Lyapunov function become:

$$\dot{V}_i(t) = -k_i|s_i| + \dot{V}_i'(t) \tag{35}$$

$$\dot{V}_i'(t) = -\overline{m}_i^{-1}\overline{\tau}_{r_i}s_i + \overline{\tau}_{0_i}s_i - \varepsilon_{c_i+d_i+p_i}s_i - \varepsilon_{m_i}s_i\overline{\tau}_{AFSMC_i} + \frac{1}{\gamma_{i_0}}\sigma_i\dot{\sigma}_i \tag{36}$$

Equation (36) can be bounded by using (23)

$$\begin{aligned} \dot{V}_i'(t) &\le -\overline{m}_i^{-1}\overline{\tau}_{r_i}s_i + |s_i|\left[\overline{\varepsilon}_{c_i+d_i+p_i}\right. \\ &\left.+\overline{\varepsilon}_{m_i}\left|\overline{\tau}_{AFSMC_i}\right| + \left|\overline{\tau}_{0_i}\right|\right] + \frac{1}{\gamma_{i_0}}\sigma_i\dot{\sigma}_i \end{aligned} \tag{37}$$

By considering assumption 1 and (27), one can easily verify that

$$\begin{aligned} \overline{m}_i^{-1}\overline{\tau}_{r_i}s_i &\ge |s_i|\left[\overline{\varepsilon}_{c_i+d_i+p_i} + \overline{\varepsilon}_{m_i}\left|\overline{\tau}_{AFSMC_i}\right| + \left|\overline{\tau}_{0_i}\right|\right] \\ &-\frac{\sigma_i|s_i|\left[\overline{\varepsilon}_{c_i+d_i+p_i} + \overline{\varepsilon}_{m_i}\left|\overline{\tau}_{AFSMC_i}\right| + \left|\overline{\tau}_{0_i}\right|\right]}{\mu_0\|s_i\|^2 + \sigma_i} \end{aligned} \tag{38}$$

Therefore, (37) becomes:

$$\dot{V}_i'(t) \le \frac{\sigma_i|s_i|\left[\overline{\varepsilon}_{c_i+d_i+p_i} + \overline{\varepsilon}_{m_i}\left|\overline{\tau}_{AFSMC_i}\right| + \left|\overline{\tau}_{0_i}\right|\right]}{\mu_0\|s_i\|^2 + \sigma_i} + \frac{1}{\gamma_{i_0}}\sigma_i\dot{\sigma}_i \tag{39}$$

By applying adaptation law (31) to (39) yields

$$\dot{V}_i'(t) \le 0 \Rightarrow \dot{V}_i(t) \le -k_i|s_i| \tag{40}$$

As a result, $\dot{V}_i(t)$ is negative semi-definite and $V_i \epsilon L_\infty$. It implies the boundedness of the signals $s_i(t)$, $\tilde{\Theta}_{c_i+d_i+p_i}$, and $\tilde{\Theta}_{m_i}$, which in turn contribute to boundedness of $\hat{\Theta}_{c_i+d_i+p_i}$,$\hat{\Theta}_{m_i}$, $\overline{\tau}_i$ and $\dot{s}_i(t)$. Since $V_i(t) \le V_i(0)$, by integrating both sides of (40):

$$\lim_{t\to\infty}\int_0^t k_i|s_i|d\theta \le \lim_{t\to\infty}\left[V_i(0) - V_i(t)\right] \le \infty \tag{41}$$

It implies that $s_i \epsilon L_2$. Since $s_i \epsilon L_2 \cap L_\infty$ and $\dot{s}_i(t) \epsilon L_\infty$, using Barbalat's lemma [28] yields $s_i(t) \to 0$ as $t \to \infty$. Hence, the asymptotically convergence of trajectory tracking error to zero is guaranteed.

Although this scheme demonstrates good stability and tracking properties, discontinuous term in the control law contributes to a chattering phenomenon. To surmount chattering, a thin boundary layer can be adopted to smooth out the control discontinuity [28].

The term $K_i sgn(s_i)$ would be substituted by $k_i sat(s_i/\phi_i)$ in the control law (25). It will be guaranteed that the boundary layer is an attractive set; hence, it would be an invariant set. Let us define a parameter s_{ϕ_i}

$$s_{\phi_i} = s_i - \phi_i sat\left(\frac{s_i}{\phi_i}\right) \tag{42}$$

In order to show the stability and boundedness of the closed loop signals, in the new proposded control law, instead of s_i in Lyapunov function (34), s_{ϕ_i} should be used. Inside the boundary layer $s_{\phi_i} = 0$, and outside the boundary the first term in the time derivative of adopted Lyapunov function would be

$$\dot{W}_i(t) = s_{\phi_i}\dot{s}_{\phi_i} = s_{\phi_i}\dot{s}_i = \left(\ddot{\eta}_{d_i} - \ddot{\eta}_i + \lambda_i\dot{\tilde{\eta}}_i\right)s_{\phi_i} \tag{43}$$

According to definition of saturation function outside the boundary layer $\overline{\tau}_{AFSMC_i}$ would become (25). Consequently, it is easy to show that the time derivative of new Lyapunov function would also be negative semi-definite.

Remark.

One may argue that the terms $k_i sgn(s_i)$ can be substituted with the term $k_i s_i$ in the control term (25). That would be exactly the same as control scheme in [18]. It should be noted that this approach may face the problem of saturation for control term $\overline{\tau}_{AFSMC_i}$ in the simulations. In this approach (40) becomes :

$$\dot{V}_i(t) \le -k_i s_i^2 \tag{44}$$

which also satisfies the control objective. It should be noted that when s_i is in small orders comparing (40) and

(44) shows that the rate of the convergency when using $sgn(.)$ function will be faster than the other one in close vicinity of s_i and it implies that the rate of convergency of tracking error by using $sgn(.)$ is faster, although both approaches satisfy the objectives of control problem. In [18] state variables are used as the premise variables of fuzzy rules, contributing to the problem of forming large fuzzy sets. In this study, in order to avoid this difficulty "s" is adopted as the premise variable in the fuzzy rules.

V. SIMULATION RESULTS

The usefulness and strength of the adopted controller has been checked on numerical simulations. Forth-order Rung-Kutta method is employed in order to solve differential equations in the numerical simulations. We performed an adaptive fuzzy control with robustifying term which described in equation (26) to obtain trajectory tracking goal. The parameters of the model are

$$M = diag\{141.78, 178.51, 159.39, 49.25, 44, 57.82\}$$

$$D(v)v = diag\{5.8\mathrm{u} + 36.76\mathrm{u}|\mathrm{u}|, 47.1\mathrm{v} + 246.43\mathrm{v}|\mathrm{v}|,$$
$$33.5\mathrm{w} + 238.5\mathrm{w}|\mathrm{w}|, 40\mathrm{p}|\mathrm{p}|, 80\mathrm{q}|\mathrm{q}|, 20.73\mathrm{r} + 40.5\mathrm{r}|\mathrm{r}|\}$$

For the thruster DC motors which we used the forces are varied in the range of $(-29.4N, 49N)$. The results are demonstrated for the tracking of the trajectory $z_d = [1 - \cos(.1\pi t)]$. There are 13 triangular fuzzy sets take into account for fuzzy system. Their ranges are followed in proportion with the variation of the switching variable s in the related state. The initial value for j$^{\text{th}}$ rule-base for the unknown parameter $\left(\hat{c}_i + \hat{\bar{d}}_i + \hat{p}_i\right)$ were set to 0 and the j$^{\text{th}}$ rule-base were set to 0.1 for $\hat{\bar{m}}_i^{-1}$ where $j = 1, \ldots, 13$. Rule-bases are updated at each time step due to the adaptation laws, Eq. (30), (31). The disturbance signal which is acted on the heave motion is produced by a random source. Other disturbance forces are in the range of ± 5N and moments in the range of ± 1N.m, respectively. Firstly, in the simulation the nonlinear functions $\left(\hat{c}_i + \hat{\bar{d}}_i + \hat{p}_i\right)$ and $\hat{\bar{m}}_i^{-1}$ are assumed to be completely unknown. Fuzzy systems in each degree of freedom are used to estimate the nonlinear functions. The design parameters in the first case are chosen as follows:

$$K = \left[20, 2, 10^2, 2, 2, 2\right]^T, \varepsilon_0 = 10^{-4}, \Lambda = 2I_6$$

$$\Phi = 10^{-3} \times \left[10, 1, 50, 1, 1, 1\right]^T, \bar{\varepsilon}_{m_i} = 0.01$$

$$\bar{\varepsilon}_{c_i + d_i + p_i} = \left[5, 5, 1, 5, 5, 5\right]^T, \gamma_{c_i + d_i + p_i} = \left[10^{-4}, 1, 10^{-2}, 1, 0.1, 5\right]^T$$

$$\gamma_m = \left[10^{-4}, 1, 4 \times 10^{-6}, 1, 1, 1\right]^T, \mu_0 = 0.1$$

$$\gamma_0 = \left[10^{-3}, 10^{-3}, 10^{-6}, 10^{-2}, 10^{-2}, 10^{-2}\right]^T$$

The trajectory tracking of position and velocity in the heave motion are presented in Figs. 3(a), (b). In spite of the fact that dynamics of underwater systems are highly nonlinear, as observed the desired and actual trajectories are closely overlap each other and the control signal which is shown in Fig. 3(c) is smooth. It is capable to supply high accuracy trajectory tracking with no chattering.

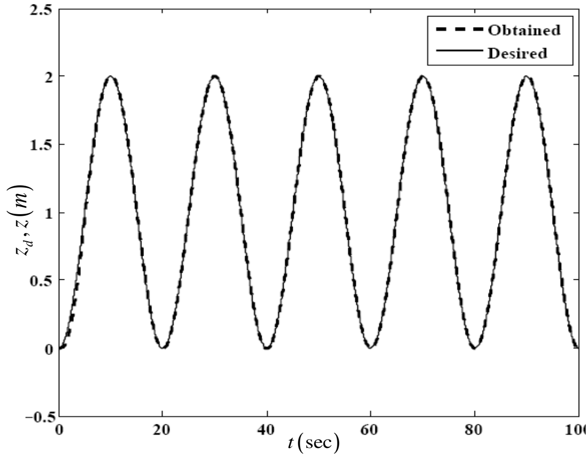

Figure3 (a). Desired and actual displacement trajectories (RAFSMC)

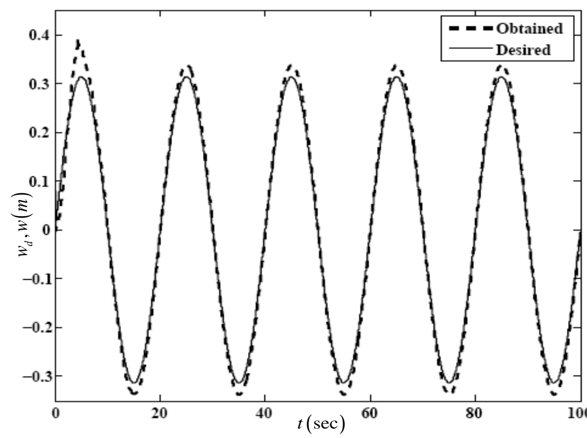

Figure3 (b). Desired and actual velocity trajectories (RAFSMC)

The second simulation case is similar to the previous one except it was assumed that the parameter $\hat{\bar{m}}_i^{-1}$ is not approximated online, so the robustifying term is not needed in this case.

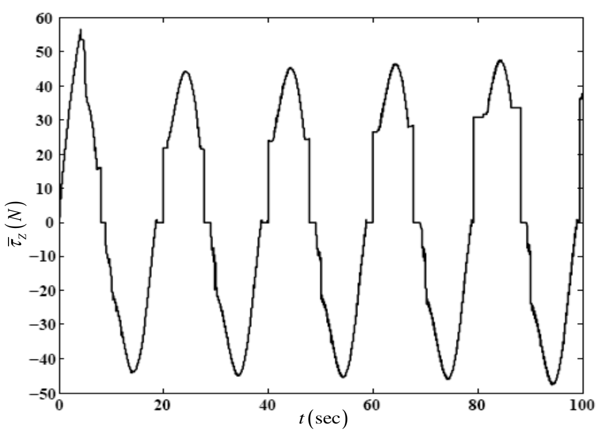

Figure3 (c). Control variable $\bar{\tau}_z$ (RAFSMC)

Also $\hat{\bar{m}}_i^{-1}$ is not exactly known and it has been chosen with a maximal uncertainty of $\pm 40\%$ of the exact values which described before. Online estimations were carried out for $\left(\hat{\hat{c}}_i + \hat{\hat{d}}_i + \hat{p}_i\right)$ and the adaptive fuzzy sliding mode controller (AFSMC) has a well performance for trajectory tracking of position and velocity. Figs. 4(a), (b) display the desired and actual trajectories of position and velocity of this case. Control signal of this case is demonstrated in Fig. 4(c).

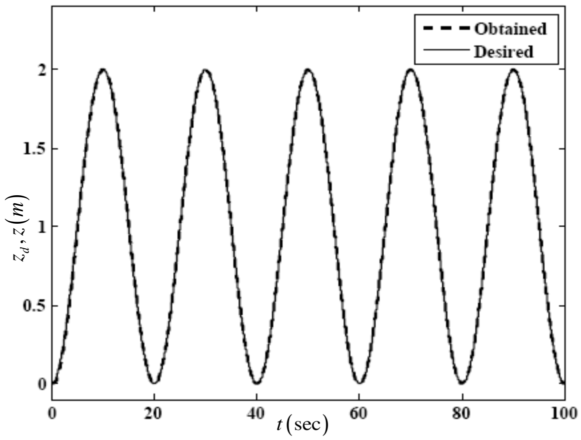

Figure4 (a). Desired and actual displacement trajectories (AFSMC2)

Fig. 3 presented the tracking error of the first and second cases and traditional sliding mode control. In the simulation that sliding mode control (SMC) used, K is similar to previous cases and $\phi_3 = 0.15$, another terms of Φ were chosen as before. In the SMC simulation, disturbance forces and moments vector P were not estimated but the tracking error still remain in the satisfactory bounds. In the SMC simulation, disturbance forces and moments vector P were not estimated but the tracking error still remain in the satisfactory bounds.

According to the Fig. 5 the second simulation controller (AFMC2) has the minimum tracking error on the basis of knowing the parameter $\widehat{m}_i^{-1}$ almost. Whilst all nonlinear functions in the first simulation study are unknown, tracking error of this simulation controller (AFMC1) is smaller than SMC and close to the first study with no chattering.

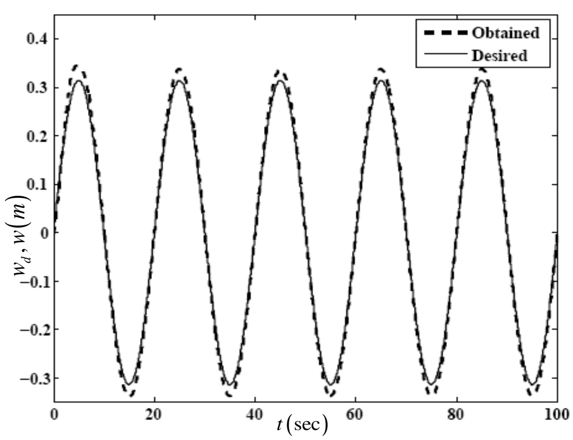

Figure4 (b). Desired and actual velocity trajectories (AFSMC2)

Indeed, the capability and effectiveness of the adopted controller is clearly shown and it can be verified that an adaptive fuzzy sliding mode controller with robustifying term has a satisfactory performance in trajectory tracking despite disturbances and unknown functions of nonlinear systems.

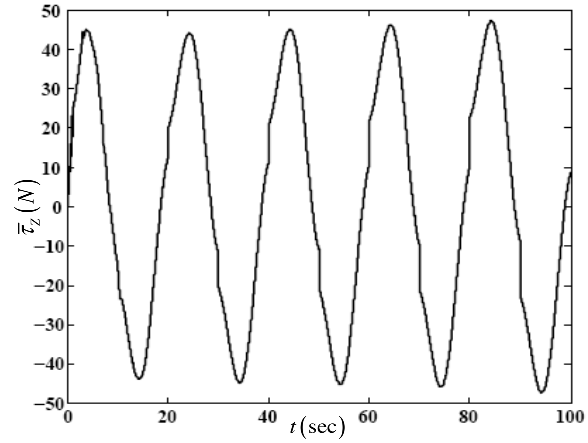

Figure4 (c). Control variable $\bar{\tau}_z$ (AFSMC2)

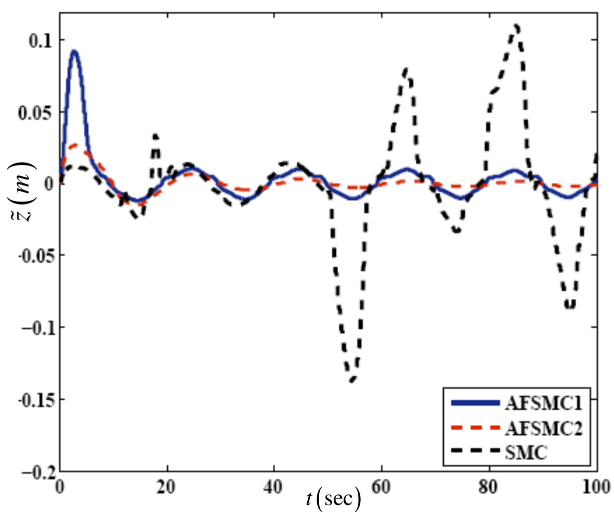

Figure5. Tracking error $\tilde{z}$

VI. CONCLUSION

This paper described the dynamic modeling and designing a strong controller for the trajectory tracking problem of Remotely Operated underwater Vehicles (ROVs). The main objective of this study was to design a controller which is able to provide on-line estimation of nonlinear functions of dynamic equation of the vehicle and also external disturbances acting on the vehicle when it moves in the water. The adopted control law was an adaptive fuzzy sliding mode controller with regularized inverse matrix to avoid singularity problem incorporated with a robustifying term to deal with the approximation error and guarantee the boundedness and stability of the closed-loop control signals. This approach ensured the convergence of the trajectory tracking error to zero, as well. The boundedness and stability of the closed-loop signals are clearly shown by Lyapunov stability theory and Barbalat's lemma. Simulation results show that the proposed controller can compensate the time-varying unknown uncertainties and lead to good performance. The results were compared to the cases in which we have some knowledge from the mathematical dynamic modeling of the plant. Dynamic features of the plant were taken from the Ariana-I ROV. Whilst the control algorithm which we used is a complicated approach; we are working to implement it on the experimental set up.

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