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Reconfigurable Control of Networked Nonlinear Euler-Lagrange Systems Subject to Fault Diagnostic Imperfections

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Abstract— **The main objective of this paper is to design a distributed reconfigurable controller for networked nonlinear Euler-Lagrange (EL) systems in presence of actuator faults and imperfections in the fault detection and identification (FDI) algorithm. Specifically, we propose an adaptive distributed control algorithm which has the capability of estimating the faults (*both* intermittent and permanent). We incorporate the information provided from the FDI algorithm (which is assumed to be running in parallel with the controller) in the design of the adaptive controller. We consider three main types of imperfections in the FDI algorithm, namely, (1) *fault detection imperfection*, that is when fault is not detected by the FDI algorithm, (2) *fault isolation imperfection*, that is when the fault is detected in the wrong channel or in the wrong agent, and (3) *fault identification imperfection*, that is when the fault estimation is not exact. We show that our proposed distributed reconfigurable controller can maintain the closed-loop networked EL systems stability under these scenarios and can improve the performance of the closed-loop system in the third scenario. Simulation results for the attitude control of a network of spacecraft demonstrate the effectiveness and capabilities of our proposed distributed reconfigurable control algorithm.**

I. INTRODUCTION

Input imperfections and actuator faults increase the associated difficulties in the design of control algorithms for dynamical systems. This problem becomes more challenging and also more important when one is dealing with a network of multiple nonlinear systems. Actuator faults can suddenly occur during the operation of networked systems. If the network is operating in hazardous or unstructured environments, where any undesired motion can be critical, achieving position synchronization control and tracking after actuator faults become of utmost importance. Formation control and consensus seeking for networked nonlinear Euler-Lagrange (EL) systems have been considered in the literature recently. Specifically, in [1] consensus seeking of a class of networked EL systems, namely robot manipulators is studied under a time-invariant communication network topology. Formation control of robot manipulators is also considered in [2]. Furthermore, in [3] state/output synchronization of networked passive systems has been studied. Formation control of EL systems under switching communication network topologies has been studied in [4]. Distributed optimal synchronization and formation control of networked EL systems has been considered in [5]. In addition, in our recent studies we have considered synchronization control of networked EL systems in presence of actuator (*both* intermittent and permanent) faults with switchings in the communication network topologies [6], [7].

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The controller recovery algorithm that is proposed in [6], [7] requires the knowledge of fault bounds for controller reconfiguration. This information has to be provided by the fault detection, isolation and identification (FDI) algorithm that is working in parallel with the controller. However, in the present work, we propose an adaptive distributed reconfigurable control algorithm, which has the capability of estimating the faults (*both* intermittent and permanent). We incorporate the information provided by the FDI module in the design of the adaptive controller. We consider three main types of imperfections in the FDI algorithm, namely, (1) *fault detection imperfection*, that is when fault is not detected by the FDI algorithm, (2) *fault isolation imperfection*, that is when the fault is detected in the wrong channel or in the wrong agent, and (3) *fault identification imperfection*, that is when the fault estimation is not exact. We show that our proposed distributed reconfigurable controller can maintain the closed-loop networked EL systems stability under these scenarios and can improve the performance of the closed-loop networked EL systems in the third case. Simulation results for the attitude control of a network of spacecraft demonstrate the effectiveness and capabilities of our proposed distributed reconfigurable control algorithms.

II. BACKGROUND AND PRELIMINARIES

A. Euler-Lagrange (EL) Systems

In this work, we consider $m > 1$ Euler-Lagrange (EL) systems, where the j-th system is governed by the following *nonlinear* dynamic equations, namely,

$$\mathbf{D}_j(\boldsymbol{q}_j)\ddot{\boldsymbol{q}}_j + \mathbf{C}_j(\boldsymbol{q}_j,\dot{\boldsymbol{q}}_j)\dot{\boldsymbol{q}}_j + g_j(\boldsymbol{q}_j) = \mathrm{DI}(\mathfrak{u}_j) + d(t) \quad (1)$$

where $j \in \{1,\ldots,m\}$, $\boldsymbol{q}_j = \{q_{1,j},\ldots,q_{k,j}\} \in \Re^k$ is the generalized coordinates vector, $\mathbf{D}_j(\boldsymbol{q}_j) \in \Re^{k\times k}$ is a symmetric positive definite matrix known as the general inertia matrix, $\mathbf{C}_j(\boldsymbol{q}_j,\dot{\boldsymbol{q}}_j) \in \Re^{k\times k}$ is the matrix of Coriolis and centrifugal forces, and $g_j(\boldsymbol{q}_j) = \frac{\partial}{\partial \boldsymbol{q}_j}\mathscr{P}_j(\boldsymbol{q}_j)$, is the gravitational force vector (GFV). Furthermore, $\mathfrak{u}_j \in \Re^k$ is the input vector, DI(.) is a nonlinear function of inputs, and $d(t)$ represents the external bounded time-varying disturbance on the system.

The dynamic model (1) has the following properties [8], [9], namely, **P1:** The general inertia matrix is bounded, specifically, there exists bounded positive scalars $\underline{k}_j$, and $\overline{k}_j$ such that: $\underline{k}_j\ \mathfrak{I}_k < \mathbf{D}_j(\boldsymbol{q}_j) < \overline{k}_j\ \mathfrak{I}_k$, $\forall \boldsymbol{q}_j$, where $\mathfrak{I}_k$ is an $k \times k$

identity matrix, **P2:** GFV is assumed to be upper bounded, that is, $0 \leq \sup_{\boldsymbol{q}_j \in \Re^k}\{|g_{i,j}(\boldsymbol{q}_j)|\} \leq \bar{g}_{i,j},\ \forall i \in \{1,\ldots,k\}$, where $g_{i,j}(\boldsymbol{q}_j)$ denotes the elements of $g_j(\boldsymbol{q}_j)$, and **P3:** $\dot{\mathbf{D}}_j(\boldsymbol{q}_j) - 2\mathbf{C}_j(\boldsymbol{q}_j,\dot{\boldsymbol{q}}_j)$ is a skew-symmetric matrix.

The nonlinear function of the input to the system $\mathrm{DI}(\mathfrak{u}_j)$ has the following form, namely,

$$\mathrm{DI}(\mathfrak{u}_j) \triangleq \bar{\mathfrak{u}}_j(t) + \mathfrak{u}_j(t) \tag{2}$$

where $\mathfrak{u}_j(t) \in \Re^k$ is the input to the system and $\bar{\mathfrak{u}}_j(t) = [\bar{\mathfrak{u}}_{j,1}(t),\ldots,\bar{\mathfrak{u}}_{j,k}(t)]^T \in \Re^k$. The vector $\bar{\mathfrak{u}}_j(t)$ represents additive actuator faults and FDI imperfections.

We make the following assumptions explicit.

Assumption 1: The function $\bar{\mathfrak{u}}_{j,1}(t)$ is defined as $\bar{\mathfrak{u}}_{j,1}(t) = \bar{\mathfrak{u}}_{j,1,l}(t)$, for $t_{l-1} \leq t < t_l,\ l = 1,2,\ldots$, where $\bar{\mathfrak{u}}_{j,1,l}(t) \in \mathfrak{C}^1$ (class of continuously differentiable functions). This implies that $\bar{\mathfrak{u}}_j(t)$ is a vector of piecewise bounded continuous functions of time. The time derivative of $\bar{\mathfrak{u}}_j(t)$ is well-defined everywhere except at time t_l where $\frac{d}{dt}\bar{\mathfrak{u}}_j(t)$ consists of a dirac delta function.

The input imperfection $\bar{\mathfrak{u}}_j(t)$ considered in Assumption 1 is a function of time. Therefore, through this formulation one can represent *both* intermittent and permanent actuator faults.

Assumption 2: The disturbance signal $d(t) \in \Re^k$ is a vector of uniformly bounded and piecewise continuous functions of time, i.e. $\sup_{t>0} d(t) < \infty$.

We provide the following definition which will be used subsequently in this work.

Definition 1: There is a *non-vanishing dwell-time* for a switched system if there exists a sequence $\{\tau_l\}$ of switching times such that $\inf_l(\tau_{l+1} - \tau_l) \geq \bar{\tau}$. Any value of $\bar{\tau} > 0$ for which this inequality holds will be denoted as a *non-vanishing dwell-time*.

Assumption 3: There exists a *non-vanishing dwell-time* between two sequential switchings in $\bar{\mathfrak{u}}_{j,1,l}(t)$. This essentially implies that we have a finite number of switchings in a finite time.

Assumption 4: It is assumed that an FDI algorithm is operating in parallel with the controller.

Assumption 5: We also assume that only the generalized coordinates vector $\boldsymbol{q}_j$ and its time derivative $\dot{\boldsymbol{q}}_j$ are available for feedback and exchange among the agents.

B. Graph Theory and Communication Topology

In this work, it is assumed that information exchanges among the m EL systems can be represented by a graph. Graph $\mathscr{G}$ consists of a node set $\mathscr{V} = \{1,\ldots,m\}$, an edge set $\mathscr{E} \subseteq \mathscr{V} \times \mathscr{V}$, and a weighted adjacency matrix $\Lambda = [\lambda_{jn}] \in \Re^{m\times m}$. The m agents in the network are considered as nodes of a graph. The communication links among the agents are considered as the graph edge set.

The weighted adjacency matrix Λ is defined such that $\lambda_{jn} = \lambda_{nj}$ is a positive weight if $(j,n) \in \mathscr{E}$, while $\lambda_{jn} = \lambda_{nj} = 0$, otherwise. Associated with Λ we introduce a symmetric positive semi-definite matrix known as the Laplacian matrix $\mathscr{L} = [l_{jn}] \in \Re^{m\times m}$ such that $l_{jj} = \sum_{n=1,n\neq j}^{m} \lambda_{jn}$ and $l_{jn} = -\lambda_{jn}$, where $k \neq j$. Furthermore, if the graph is connected, $\mathscr{L}$ has a simple eigenvalue 0 with an associated eigenvector of $\mathbf{1}_m$, where $\mathbf{1}_m$ is an $m\times 1$ column vector of ones. All the other eigenvalues of $\mathscr{L}$ are positive if and only if the graph $\mathscr{G}$ is connected. For a given node j in the communication network the set of agents from which it can receive information is called a neighboring set $\mathscr{N}_j$, that is $\forall j = 1,\ldots,n : \mathscr{N}_j = \{n = 1,\ldots,m | (j,n) \in \mathscr{E}\}$. In addition, the number of neighbors of the j-th agent is denoted by $|\mathscr{N}_j|$.

It should be noted that in this work all the communication graphs are assumed to be *connected*, therefore, $\mathscr{L}$ is a positive semi-definite matrix.

C. Input-to-State Stability of General Networked Nonlinear Systems

In this subsection, we extend the standard definition (Definition 4.7 in [10]) of an input-to-state stability (ISS) of general nonlinear systems to general *networked* nonlinear systems.

Definition 2: Consider a network of 'm' multiple heterogeneous nonlinear systems where the dynamics of the j-th agent can be expressed by

$$\begin{aligned} \dot{x_j} &= f_j(x_j) + g_j(x_j)u_j + \bar{g}_j(x_j)w_j \\ y_j &= h_j(x_j) \end{aligned} \tag{3}$$

where $x_j \in \Re^{\bar{n}}$, $u_j \in \Re^{\bar{m}}$, $y_j \in \Re^p$, $w_j \in \Re^l$, $g_j(x_j) \in \Re^{\bar{n}\times\bar{m}}$, and $\bar{g}_j(x_j) \in \Re^{\bar{n}\times l}$. A nonlinear state-feedback control law $u_j = K_j(x_j) + K_{jn}(x_{jn})$ for the j-th nonlinear system, where $x_{jn} = x_j - x_n,\ j \in \mathscr{V}, n \in \mathscr{N}_j$, with $K_j(0) = 0$ and $K_{jn}(0) = 0$, is said to be ISS if for the closed-loop system there exists a class $\mathscr{KL}$ function $\bar{\beta}_j$ and a class $\mathscr{K}$ function [1] $\bar{\gamma}_j$ such that for any initial condition $x_j(0)$ and $x_{jn}(0)$ and any bounded input $w_j(t)$, the solutions $x_j(t)$ and $x_{jn}(t)$ exist for all $t \geq 0$ and satisfy,

$$\begin{aligned} \|x_j(t)\| + \|x_{jn}(t)\| \leq \bar{\beta}_j\Big(\|x_j(0)\| + \|x_{jn}(0)\|, t\Big) \\ + \bar{\gamma}_j\Big(\sup_{0\leq\xi\leq t}\|w_j(\xi)\|\Big) \end{aligned} \tag{4}$$

The above inequality guarantees that for any bounded disturbance $w_j(t)$, the states $x_j(t)$ and $x_{jn}(t)$ will remain bounded. In addition, as time evolves (t increases) the states $x_j(t)$ and $x_{jn}(t)$ will remain ultimately bounded by a class $\mathscr{K}$ function of $\sup_{0\leq\xi\leq t}\|w_j(\xi)\|$. One can further show that if $w_j(t) \to 0$ as $t \to \infty$, then, $x_j(t) \to 0$ and $x_{jn}(t) \to 0$ as $t \to \infty$.

The ISS can be shown by using a Lyapunov-like theorem as discussed below.

Lemma 1: Consider a network of 'm' multiple heterogeneous nonlinear systems where the dynamics of the j-th agent can be expressed by (3). Suppose there exists a nonlinear state-feedback control law $u_j = K_j(x_j) + K_{jn}(x_{jn})$ for the j-th nonlinear system, with $K_j(0) = 0$ and $K_{jn}(0) = 0$, and a continuously differentiable positive definite radially

[1] See e.g. page 144 in [10] for the definitions of class $\mathscr{KL}$, $\mathscr{K}$ and $\mathscr{K}_\infty$ functions.

unbounded Lyapunov function $\mathscr{W}$ for the networked heterogeneous nonlinear system such that for the closed-loop system we have,

$$\begin{aligned}\dot{\mathscr{W}} &\leq -\bar{\bar{\gamma}}(\|x_j(t)\| + \|x_{jn}(t)\|) + \underline{\gamma}\|w_j(t)\|, \text{ and}\\ \dot{\mathscr{W}} &\leq -\underline{\underline{\gamma}}(\|x_j(t)\| + \|x_{jn}(t)\|)\\ &\quad \Leftrightarrow \|x_j(t)\| + \|x_{jn}(t)\| \geq \rho(\|w_j(t)\|)\end{aligned} \tag{5}$$

for all $x_j(t)$, $x_{jn}(t)$, and $w_j(t)$, where $\bar{\bar{\gamma}}$, $\underline{\gamma}$, and $\underline{\underline{\gamma}}$ are class $\mathscr{K}_\infty$ functions and ρ is a class $\mathscr{K}$ function. Then the closed-loop system is ISS.

Proof: The proof is similar to that of Theorem 4.19 in [10] and is therefore omitted due to space limitations. ■

Definition 3: Any positive definite radially unbounded Lyapunov function $\mathscr{W}$, which satisfies (5) is defined as the *ISS-Lyapunov function*.

III. Distributed H_∞-Optimal State Synchronization and Formation Control of Euler-Lagrange Systems

Let us denote the desired position, velocity and acceleration coordinates vector for the j-th EL system by $\boldsymbol{q}^\star(t)$, $\dot{\boldsymbol{q}}^\star(t)$, and $\ddot{\boldsymbol{q}}^\star(t)$, respectively, where all are smooth functions of time. Let the biased desired position for the j-th EL system be denoted as $\mathfrak{c}_j(t) = \boldsymbol{q}^\star(t) + \boldsymbol{q}_j^\flat$, , where $\boldsymbol{q}_j^\flat$ is added to guarantee the EL systems do not collide at the steady state. Also, let $\tilde{\boldsymbol{q}}_j(t) = \boldsymbol{q}_j(t) - \mathfrak{c}_j(t)$, and $\boldsymbol{q}_{jn} = \tilde{\boldsymbol{q}}_j - \tilde{\boldsymbol{q}}_n$. Our goal, in this section is to introduce a distributed control law which guarantees synchronization and trajectory tracking of the EL system (1) coordinates.

Let us decompose the control input $\mathfrak{u}_j$ as follows (which is also known as the modified computed-torque control input), namely,

$$\mathfrak{u}_j = \mathbf{D}_j(\boldsymbol{q}_j)\dot{\mathfrak{r}}_j + \mathbf{C}_j(\boldsymbol{q}_j, \dot{\boldsymbol{q}}_j)\boldsymbol{r}_j + g_j(\boldsymbol{q}_j) + \tau_j \tag{6}$$

where $\tau_j = \bar{\tau}_j + \bar{\bar{\tau}}_j + \Gamma_j$ is to be designed with $\bar{\tau}_j$, $\bar{\bar{\tau}}_j$, and Γ_j to be specified subsequently. Note that the modified computed-torque control input only requires measurements from the generalized coordinates vector $\boldsymbol{q}_j$ and its time derivative $\dot{\boldsymbol{q}}_j$ (refer to Assumption 5). Therefore, the dynamics of system (1) becomes

$$\dot{x}_j = \mathbf{A}_j(x_j)x_j + \mathbf{B}_j(x_j)\tau_j + \mathbf{B}_j(x_j)\bar{\mathfrak{u}}_j(t) + \mathbf{B}_j(x_j)d_j \tag{7}$$

where $x_j = [\int_0^t \tilde{\boldsymbol{q}}_j^T d\xi, \tilde{\boldsymbol{q}}_j^T, \dot{\tilde{\boldsymbol{q}}}_j^T]^T \in \mathfrak{R}^{3k}$, and

$$\mathbf{A}_j(x_j) = \begin{bmatrix} 0 & \mathfrak{I}_k & 0\\ 0 & 0 & \mathfrak{I}_k\\ -\mathbf{D}_j^{-1}\mathbf{C}_j\bar{\bar{\mathbf{K}}}_j & -\mathbf{D}_j^{-1}\mathbf{C}_j\bar{\mathbf{K}}_j - \bar{\bar{\mathbf{K}}}_j & -\mathbf{D}_j^{-1}\mathbf{C}_j - \bar{\mathbf{K}}_j \end{bmatrix}$$

$$\mathbf{B}_j(x_j) = \begin{bmatrix} 0\\ 0\\ \mathbf{D}_j^{-1}\end{bmatrix}$$

In the following we define the main three objectives that we pursue in this work, namely, (a) synchronization control, (b) tracking control, and (c) control recovery.

Definition 4: Our first objective (a) is to design $\bar{\tau}_j$ such that it guarantees trajectory tracking, i.e. $\tilde{\boldsymbol{q}}_j \to 0$ and $\dot{\tilde{\boldsymbol{q}}}_j \to 0$ as $t \to \infty$. Our second objective (b) is to design $\bar{\bar{\tau}}_j$ so that it guarantees state synchronization among the agents, i.e. $\boldsymbol{q}_{jn} \to 0$ and $\dot{\boldsymbol{q}}_{jn} \to 0$ as $t \to \infty$. Our third objective (c) is to design Γ_j such that it compensates for the effects of actuator faults and FDI imperfections and is denoted as the *control recovery*.

We first present the following result from [11].

Lemma 2: Consider a network of 'm' multiple heterogeneous *nonlinear* EL systems with the state-space dynamics (7). Let us define the manifold $\boldsymbol{r}_j = \dot{\boldsymbol{q}}^\star - \bar{\mathbf{K}}_j\, \tilde{\boldsymbol{q}}_j - \bar{\bar{\mathbf{K}}}_j \int_0^t \tilde{\boldsymbol{q}}(\xi)_j d\xi$ and $\mathfrak{s}_j = \boldsymbol{q}_j(t) - \boldsymbol{r}_j \triangleq [\bar{\bar{\mathbf{K}}}_j \quad \bar{\mathbf{K}}_j \quad \mathfrak{I}_k]x_j$. In absence of actuator faults and FDI imperfections, i.e. $\bar{\mathfrak{u}}_j(t) = 0$, and for a given $\bar{\gamma}_j > 0$ and $0 < \alpha_j < 1$, let

$$\begin{aligned}\tau_j &= \underbrace{-\frac{1}{2}\left(\mathbf{K}_j - \frac{1}{\bar{\gamma}_j^2}\mathfrak{I}_3\right)\mathfrak{s}_j}_{\bar{\tau}_j} + \underbrace{\frac{\alpha_j}{2}\mathbf{K}_j \sum_{n\in\mathscr{N}_j}\frac{1}{|\mathscr{N}_j|}\mathfrak{s}_n}_{\bar{\bar{\tau}}_j}\\ &\triangleq -\frac{1}{2}\mathfrak{s}_j^T\left(\mathbf{K}_j - \frac{\alpha_j}{\bar{\gamma}_j^2}\mathfrak{I}_3\right)\mathfrak{s}_j\\ &\quad -\frac{\alpha_j}{4}\mathfrak{s}_{jn}^T\mathbf{K}_j\sum_{n\in\mathscr{N}_j}\frac{1}{|\mathscr{N}_j|}\mathfrak{s}_{jn}\end{aligned} \tag{8}$$

where $\mathfrak{s}_{jn} = \mathfrak{s}_j - \mathfrak{s}_n$, subject to the following conditions

$$\bar{\mathbf{K}}_j \succ 0,\ \bar{\bar{\mathbf{K}}}_j \succ 0,\ \mathbf{K}_j - \frac{1}{\bar{\gamma}_j^2}\mathfrak{I}_3 \succ 0, \tag{9}$$

$$1 > \alpha_j > 0 \tag{10}$$

$$(1-\alpha_j)\bar{\mathbf{K}}_j^2 - 2\bar{\bar{\mathbf{K}}}_j \succ 0 \tag{11}$$

Then by employing the control law (8) the following inequality [2] is always obtained for the j-th agent, namely,

$$\begin{aligned}\sum_{j=1}^m \int_0^\infty \left[\frac{1}{2}x_j^T\mathbf{Q}_j x_j + \bar{\tau}_j^T\mathbf{R}_j\bar{\tau}_j + \frac{1}{4}\sum_{n\in\mathscr{N}_j}x_{jn}^T\mathbf{Q}_{jn}x_{jn}\right]dt\\ \leq \frac{1}{2}\sum_{j=1}^m\bar{\gamma}_j^2\int_0^\infty d_j^T(t)d_j(t)dt\end{aligned} \tag{12}$$

where we have

$$\mathbf{R}_j = \left(\mathbf{K}_j - \frac{1}{\bar{\gamma}_j^2}\mathfrak{I}_3\right)^{-1} \tag{13}$$

and,

$$\sum_{n\in\mathscr{N}_j}\mathbf{Q}_{jn} = \alpha_j\begin{bmatrix}\bar{\bar{\mathbf{K}}}_j^2\mathbf{K}_j & 0 & 0\\ 0 & \bar{\mathbf{K}}_j^2\mathbf{K}_j & 0\\ 0 & 0 & \mathbf{K}_j\end{bmatrix} \tag{14}$$

as well as,

$$\begin{aligned}\mathbf{Q}_j =&(1-\alpha_j)\begin{bmatrix}\bar{\bar{\mathbf{K}}}_j^2\mathbf{K}_j & 0 & 0\\ 0 & \bar{\mathbf{K}}_j^2\mathbf{K}_j & 0\\ 0 & 0 & \mathbf{K}_j\end{bmatrix}\\ &-\begin{bmatrix}0 & 0 & 0\\ 0 & 2\bar{\bar{\mathbf{K}}}_j\mathbf{K}_j & 0\\ 0 & 0 & 0\end{bmatrix}\end{aligned} \tag{15}$$

[2]This inequality is known as the $\mathfrak{L}_2$-gain from the input $d_j(t)$ to the states $x_j(t)$ and $x_{jn}(t)$, and the control command $\bar{\tau}_j(t)$. For more details see [11].

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Proof: The proof can be found in [11]. ■

IV. Control Recovery of the Networked Euler-Lagrange Systems

According to Assumption 4, an FDI unit is operating in parallel with the distributed controller (6) and (8). As discussed in Definition 4, in presence of a fault the term Γ_j is added to the system to compensate for the effects of a fault and recover, as much as possible, the performance of the closed-loop system.

In reality, however, no FDI algorithm is 100% perfect and reliable. Consequently, the controller must be robust to imperfections in the FDI algorithm. Let the fault in the j-th system satisfy Assumption 1. We now consider the following three cases.

Case 1: The FDI algorithm is not capable of detecting the fault. Consequently, the controller for the j-th system will not reconfigure itself in presence of the fault. This is designated as *imperfection in the fault detection*.

Case 2: The FDI algorithm has detected the fault in an incorrect input channel or an incorrect agent. Consequently, the controller is reconfigured in an inappropriate channel or agent. This is designated as *imperfection in the fault isolation*.

Case 3: The FDI algorithm has detected the fault in the correct input channel or agent. However, the magnitude or the severity of the fault is not correctly identified. In other words, the FDI algorithm provides a piecewise continuous estimation of the vector $\bar{\mathrm{u}}_j(t)$, which is denoted by $\bar{\mathrm{u}}_j^*(t)$ such that $\left\|\bar{\mathrm{u}}_j^* - \bar{\mathrm{u}}_j\right\|$ is always bounded, i.e. $\sup_{t>0}\left\|\bar{\mathrm{u}}_j^* - \bar{\mathrm{u}}_j\right\| < \infty$. This is designated as *imperfection in the fault identification*.

A. Control of Networked Euler-Lagrange Systems Subject to Imperfection in the Fault Detection

In presence of imperfection in the fault detection the controller is not reconfigured. However, one can still guarantee boundedness of the synchronization and the trajectory tracking errors in presence of the fault. Our next result is provided in the following lemma.

Lemma 3: Consider a network of 'm' multiple heterogeneous *nonlinear* EL systems (1) under the distributed control law (6) and (8). Suppose for a given $\gamma_j > 0$ the controller gains, $\bar{\bar{\mathbf{K}}}_j$, $\bar{\mathbf{K}}_j$, $\mathbf{K}_j$, and α_j are selected such that the conditions (9), (10), and (11) are satisfied. Then under Assumptions 1-5 the closed-loop networked system remains globally bounded under Case 1 for $t \geq 0$.

Proof: When no fault recovery is invoked one can combine the actuator fault signal $\bar{\mathrm{u}}_j(t)$ as part of the disturbance $d(t)$. Consequently, let $\bar{d}(t) = \bar{\mathrm{u}}_j(t) + d(t)$. Now, consider the following positive definite, radially unbounded, ISS-Lyapunov function candidate for the j-th system, namely,

$$\mathscr{Y}_j(x_j) = \frac{1}{2}x_j^T\mathbf{P}_j(x_j)x_j \tag{16}$$

Let $\mathscr{Y} \triangleq \sum_{j=1}^m \mathscr{Y}_j$ be the ISS-Lyapunov function candidate for the networked EL systems. One can show that the time derivative of the ISS-Lyapunov function candidate $\mathscr{Y}$ along the trajectories of the closed-loop system (1), (6), (8), (13), (14), and (15) can be written as

$$\dot{\mathscr{Y}} \leq -\frac{1}{4}\sum_{j=1}^m x_j^T\left[\mathbf{P}_j(x_j)\mathbf{B}_j(x_j)\mathbf{R}_j\mathbf{B}_j^T(x_j)\mathbf{P}_j(x_j)\right]x_j - \frac{1}{2}\sum_{j=1}^m x_j^T\mathbf{Q}_jx_j - \frac{1}{4}\sum_{j=1}^m\sum_{n\in\mathscr{N}_j} x_{jn}^T\mathbf{Q}_{jn}x_{jn} + \frac{1}{2}\sum_{j=1}^m \gamma_j^2\left\|\bar{d}_j\right\|^2 \tag{17}$$

The positive definite matrices $\mathbf{Q}_j$, $\mathbf{R}_j$, and $\mathbf{Q}_{jn}$ $\forall j \in \mathscr{V}$, $n \in \mathscr{N}_j$ imply that the first two terms in the right hand side of the inequality (17) are $\mathscr{K}_\infty$ function of x_j and x_{jn}, respectively. Define the bounded region $\mathfrak{B}_r$ that includes the origin, that is $\mathfrak{B}_r = \Big\{x_j, (x_j - x_n) \;\Big|\; \frac{1}{2}x_j^T\left[\mathbf{Q}_j + \frac{1}{2}\mathbf{P}_j(x_j)\mathbf{B}_j(x_j)\mathbf{R}_j\mathbf{B}_j^T(x_j)\mathbf{P}_j(x_j)\right]x_j + \frac{1}{4}\sum_{n\in\mathscr{N}_j} x_{jn}^T\mathbf{Q}_{jn}x_{jn} \leq \frac{1}{2}\gamma_j^2\left\|\bar{d}_j\right\|^2\Big\}$. For all x_j and x_{jn} outside this region we have $\frac{d}{dt}\mathscr{Y} < 0$. Consequently, by invoking Lemma 1, one can conclude that the closed-loop networked EL system under the distributed control law (6) and (8) is ISS and the synchronization and the tracking trajectory errors remain globally ultimately bounded. ■

Lemma 3 guarantees boundedness of the synchronization and the trajectory tracking errors in presence of a fault. However, in presence of actuator faults and without invoking a controller reconfiguration, this bound may be too large and may exceed the mission specifications (as shown in the simulations in Section V). Therefore, one may need to appropriately adjust the controller to recover the performance of the networked EL system as described in the next subsection.

B. Adaptive Control of Networked Euler-Lagrange Systems Subject to Imperfection in the Fault Identification

Consider Case 3 holds. The purpose of this subsection is to design $\Gamma_j = \mathrm{diag}(\gamma_{1,j}, \ldots, \gamma_{k,j}) \in \Re^k$ in order to compensate for the effects of FDI imperfections and actuator faults. Our result is now presented below.

Theorem 1: Consider a network of 'm' multiple heterogeneous EL systems that are governed by the dynamics (1) and subject to the distributed control law (6) where $\bar{\tau}_j$ and $\bar{\bar{\tau}}_j$ are specified from (8) for the the j-th system. Given that the conditions in Case 3 hold, let us set $\gamma_{p,j}$ according to,

$$\gamma_{p,j} = -\,\mathrm{sgn}(\mathfrak{s}_{p,j})\hat{\mathrm{u}}_{p,j}(t), p \in \{1,\ldots,k\}, j \in \{1,\ldots,m\} \tag{18}$$

where $\hat{\mathrm{u}}_{p,j}(t)$ is an estimate of $\bar{\mathrm{u}}_{p,j}(t)$ and is governed by

$$\dot{\hat{\mathrm{u}}}_{p,j}(t) = \bar{\sigma}_{p,j}\mathfrak{s}_{p,j} - \bar{e}_{p,j}[\hat{\mathrm{u}}_{p,j}(t) - \bar{\mathrm{u}}_{p,j}^*(t)] \tag{19}$$

where $\bar{\sigma}_{p,j} > 0$, $\bar{\mathrm{u}}_{p,j}^*$ denoted the estimate of the fault severity that is provided by the FDI algorithm, and $\bar{e}_{p,j} > 0$, $p \in \{1,\ldots,k\}$ are diagonal elements of the positive definite matrix $\bar{\mathbf{E}}_j \succ 0$. Then under Assumptions 1-5 by application of the above-mentioned distributed adaptive control law the closed-loop states of the j-th nonlinear EL system, i.e. $\bar{x}_j =$

$[x_j^T \quad \tilde{u}_j^T]^T$, with $\tilde{u}_j = \hat{u}_j - \bar{u}_j$ remain globally bounded under Case 3 for $t \geq 0$.

Proof: The time derivative of the estimation error $\tilde{u}_{p,j}(t)$ defined as $\tilde{u}_{p,j}(t) = \hat{u}_{p,j}(t) - \bar{u}_{p,j}(t)$, along the trajectories of (19) is given by

$$\begin{aligned}\dot{\tilde{u}}_{p,j}(t) =& \dot{\hat{u}}_{p,j}(t) - \dot{\bar{u}}_{p,j}(t)\\ =& \bar{\sigma}_{p,j}\mathfrak{s}_{p,j} - \bar{e}_j[\hat{u}_{p,j}(t) - \bar{u}^*_{p,j}(t)] - \dot{\bar{u}}_{p,j}(t)\\ =& \bar{\sigma}_{p,j}\mathfrak{s}_{p,j} - \bar{e}_j\tilde{u}_{p,j}(t) + \bar{e}_j[\bar{u}^*_{p,j}(t) - \bar{u}_{p,j}(t)] - \dot{\bar{u}}_{p,j}(t)\end{aligned}$$

Consider the following positive definite radially unbounded function as the ISS-Lyapunov function candidate for the closed-loop networked EL system

$$\mathscr{W}(\bar{x}_j) = \sum_{j=1}^{m}\left[\mathscr{Y}_j(x_j) + \sum_{p=1}^{k}\frac{1}{2\bar{\sigma}_{p,j}}\tilde{u}^2_{p,j}\right] \tag{20}$$

where $\bar{x}_j = [x_j^T, \tilde{u}_j^T]^T$, $\mathscr{Y}_j(x_j)$ is given by (16) and

$$\mathbf{P}_j(x_j) = \begin{bmatrix} \bar{\bar{\mathbf{K}}}_j\mathbf{D}_j\bar{\bar{\mathbf{K}}}_j + \bar{\bar{\mathbf{K}}}_j\bar{\mathbf{K}}_j\mathbf{K}_j & \bar{\bar{\mathbf{K}}}_j\mathbf{D}_j\bar{\mathbf{K}}_j + \bar{\bar{\mathbf{K}}}_j\mathbf{K}_j & \bar{\bar{\mathbf{K}}}_j\mathbf{D}_j\\ \bar{\mathbf{K}}_j\mathbf{D}_j\bar{\bar{\mathbf{K}}}_j + \bar{\bar{\mathbf{K}}}_j\mathbf{K}_j & \bar{\mathbf{K}}_j\mathbf{D}_j\bar{\mathbf{K}}_j + \bar{\mathbf{K}}_j\mathbf{K}_j & \bar{\mathbf{K}}_j\mathbf{D}_j\\ \mathbf{D}_j\bar{\bar{\mathbf{K}}}_j & \mathbf{D}_j\bar{\mathbf{K}}_j & \mathbf{D}_j\end{bmatrix} \tag{21}$$

where $\mathbf{P}_j(x_j)$ is a positive definite matrix provided that the conditions (9), (10), and (11) are satisfied. This essentially implies that there exists positive scalars k_0 and $\bar{k}_0$ such that $k_0\left\|\bar{x}_j\right\|^2 \leq \mathscr{W}(\bar{x}_j) \leq \bar{k}_0\left\|\bar{x}_j\right\|^2$. Therefore, $\mathscr{W}(\bar{x}_j)$ is a positive definite radially unbounded function.

The time derivative of the ISS-Lyapunov function candidate along the trajectories of the closed-loop system is obtained as,

$$\begin{aligned}\frac{d}{dt}\mathscr{W} =& \sum_{j=1}^{m}\Bigg[\frac{\partial\mathscr{Y}_j}{\partial x_j}\mathbf{A}_j(x_j)x_j + \frac{\partial\mathscr{Y}_j}{\partial x_j}\mathbf{B}_j(x_j)\tau_j\\ &+ \frac{\partial\mathscr{Y}_j}{\partial x_j}\mathbf{B}_j(x_j)d_j + \tilde{u}_j^T\bar{\Sigma}_j^{-1}\dot{\tilde{u}}_j\Bigg]\\ =& \sum_{j=1}^{m}\Bigg[\frac{1}{2}x_j^T[\dot{\mathbf{P}}_j(x_j) + \mathbf{P}_j(x_j)\mathbf{A}_j(x_j)\\ &+ \mathbf{A}_j^T(x_j)\mathbf{P}_j(x_j)]x_j + x_j^T\mathbf{P}_j(x_j)\mathbf{B}_j(x_j)\tau_j\\ &+ x_j^T\mathbf{P}_j(x_j)\mathbf{B}_j(x_j)d_j + \tilde{u}_j^T\bar{\Sigma}_j^{-1}\dot{\tilde{u}}_j\Bigg]\\ \leq& -\frac{1}{4}\sum_{j=1}^{m}x_j^T\mathbf{P}_j\mathbf{B}_j\mathbf{R}_j^{-1}\mathbf{B}_j^T\mathbf{P}_jx_j - \frac{1}{2}\sum_{j=1}^{m}x_j^T\mathbf{Q}_jx_j\\ &- \frac{1}{4}\sum_{j=1}^{m}\sum_{n\in\mathscr{N}_j}x_{jn}^T\mathbf{Q}_{jn}x_{jn} - \sum_{j=1}^{m}\tilde{u}_j^T\bar{\Sigma}_j^{-\frac{1}{2}}\bar{\mathbf{E}}_j\bar{\Sigma}_j^{-\frac{1}{2}}\tilde{u}_j\\ &+ \sum_{j=1}^{m}\tilde{u}_j^T\bar{\Sigma}_j^{-\frac{1}{2}}\bar{\mathbf{E}}_j\bar{\Sigma}_j^{-\frac{1}{2}}(\bar{u}^*_j - \bar{u}_j)\\ &- \sum_{j=1}^{m}\tilde{u}_j^T\bar{\Sigma}_j^{-1}\dot{\bar{u}}_{p,j} + \sum_{j=1}^{m}\left\|\mathfrak{s}_j\right\|\left\|d_j\right\|\end{aligned} \tag{22}$$

where $\bar{\Sigma}_j = \mathrm{diag}(\bar{\sigma}_{1,j},\ldots,\bar{\sigma}_{k,j}) \in \Re^k$. Consequently, we obtain,

$$\begin{aligned}\frac{d}{dt}\mathscr{W} \leq& -\frac{1}{4}\sum_{j=1}^{m}x_j^T\mathbf{P}_j\mathbf{B}_j\mathbf{R}_j\mathbf{B}_j^T\mathbf{P}_jx_j - \frac{1}{2}\sum_{j=1}^{m}x_j^T\mathbf{Q}_jx_j\\ &- \frac{1}{4}\sum_{j=1}^{m}\sum_{n\in\mathscr{N}_j}x_{jn}^T\mathbf{Q}_{jn}x_{jn} - \sum_{j=1}^{m}\tilde{u}_j^T\bar{\Sigma}_j^{-\frac{1}{2}}\bar{\mathbf{E}}_j\bar{\Sigma}_j^{-\frac{1}{2}}\tilde{u}_j\\ &+ \sum_{j=1}^{m}\tilde{u}_j^T\bar{\Sigma}_j^{-\frac{1}{2}}\bar{\mathbf{E}}_j\bar{\Sigma}_j^{-\frac{1}{2}}(\bar{u}^*_j - \bar{u}_j)\\ &- \sum_{j=1}^{m}\tilde{u}_j^T\bar{\Sigma}_j^{-1}\dot{\bar{u}}_{p,j} + \sum_{j=1}^{m}\left\|\mathfrak{s}_j\right\|\left\|d_j\right\|\end{aligned} \tag{23}$$

which implies that there exists constant positive scalars $k_i,\ i \in \{1,\ldots,8\}$, such that

$$\begin{aligned}\frac{d}{dt}\mathscr{W} \leq& \sum_{j=1}^{m}\Bigg[-k_1\left\|x_j\right\|^2 - k_2\left\|\tilde{u}_j\right\|^2 - k_3\left\|x_{jn}\right\|^2\\ &+ k_4\left\|\tilde{u}_j\right\|\left\|\bar{u}^*_j - \bar{u}_j\right\| + k_5\left\|\tilde{u}_j\right\|\left\|\dot{\bar{u}}_j\right\| + k_6\left\|d_j\right\|^2\Bigg]\\ \leq& \sum_{j=1}^{m}\left[-k_7\left\|\bar{x}_j\right\|^2 + k_5\left\|\bar{x}_j\right\|\left\|\dot{\bar{u}}_j\right\| + k_8\left\|\bar{x}_j\right\|\left\|\bar{v}_j\right\|\right]\end{aligned}$$

where $\left\|\bar{v}_j\right\| = \left\|\bar{u}^*_j - \bar{u}_j\right\| + \left\|d_j\right\|^2$. Consequently,

$$\begin{aligned}k_0\sum_{j=1}^{m}\frac{d}{d\xi}\left\|\bar{x}_j\right\|^2 \leq& -k_7\sum_{j=1}^{m}\left\|\bar{x}_j\right\|^2 + k_5\sum_{j=1}^{m}\left\|\bar{x}_j\right\|\left\|\dot{\bar{u}}_j\right\|\\ &+ k_8\sum_{j=1}^{m}\left\|\bar{x}_j\right\|\left\|\bar{v}_j\right\|\end{aligned}$$

Therefore, when $\left\|\bar{x}_j\right\| \neq 0$ one obtains,

$$\sum_{j=1}^{m}\frac{d}{d\xi}\left\|\bar{x}_j\right\| \leq -\frac{k_7}{k_0}\sum_{j=1}^{m}\left\|\bar{x}_j\right\| + \frac{k_5}{k_0}\sum_{j=1}^{m}\left\|\dot{\bar{u}}_j\right\| + \frac{k_8}{k_0}\sum_{j=1}^{m}\left\|\bar{v}_j\right\|$$

By integrating the above inequality we obtain

$$\begin{aligned}\sum_{j=1}^{m}\left\|\bar{x}_j(t)\right\| \leq& \sum_{j=1}^{m}\left\|\bar{x}_j(0)\right\|e^{-\frac{k_7}{k_0}(t)} + \sum_{j=1}^{m}\int_0^t\frac{k_5}{k_0}e^{\frac{k_7}{k_0}\xi}\left\|\dot{\bar{u}}_j(\xi)\right\|d\xi\\ &+ \sum_{j=1}^{m}\int_0^t\frac{k_8}{k_0}e^{\frac{k_7}{k_0}\xi}\left\|\bar{v}_j\right\|d\xi\\ \leq& \sum_{j=1}^{m}\left\|\bar{x}_j(0)\right\|e^{-\frac{k_7}{k_0}(t)} + k_9 + \frac{k_8}{k_7}\sum_{j=1}^{m}\bar{v}_{j,0}\end{aligned} \tag{24}$$

where $\bar{v}_{j,0} = \sup_{t\geq 0}\left\|\bar{v}_j\right\|$ and $k_9 = \sum_{j=1}^{m}\int_0^t\frac{k_5}{k_0}e^{\frac{k_7}{k_0}\xi}\left\|\dot{\bar{u}}_j(\xi)\right\|d\xi$. Assumptions 2 and 4 imply that $\bar{v}_{j,0} < \infty$. Consequently, in view if Assumptions 1 and 3 one can conclude that the states of the j-th closed-loop system, $\bar{x}_j(t)$, are globally uniformly bounded. This completes the proof of the theorem. ■

The over-all distributed adaptive control law can be written in the following form, namely,

$$\mathfrak{u}_j = \mathbf{D}_j(\boldsymbol{q}_j)\dot{\mathfrak{r}}_j + \mathbf{C}_j(\boldsymbol{q}_j, \dot{\boldsymbol{q}}_j)\boldsymbol{r}_j + g_j(\boldsymbol{q}_j) - \frac{1}{2}\mathfrak{s}_j^T\left(\mathbf{K}_j - \frac{\alpha_j}{\bar{\gamma}_j^2}\mathfrak{I}_3\right)\mathfrak{s}_j - \frac{\alpha_j}{4}\mathfrak{s}_{jn}^T\mathbf{K}_j\sum_{n\in\mathcal{N}_j}\frac{1}{|\mathcal{N}_j|}\mathfrak{s}_{jn} - \text{sgn}(\mathfrak{s}_j)^T\hat{\mathfrak{u}}_j(t) \tag{25}$$

with

$$\dot{\hat{\mathfrak{u}}}_j(t) = \bar{\Sigma}_j\mathfrak{s}_j - \bar{\mathbf{E}}_j[\hat{\mathfrak{u}}_j(t) - \bar{\mathfrak{u}}_j^*(t)] \tag{26}$$

where $\bar{\mathfrak{u}}_j^*(t)$ is an estimate of $\bar{\mathfrak{u}}_j(t)$ provided by the FDI module.

C. Control of Networked Euler-Lagrange Systems Subject to Imperfection in the Fault Isolation

In presence of an imperfection in the fault isolation, the controller is reconfigured according to (25) and (26) but for an incorrect agent or incorrect input channel. The stability of the networked EL system, however, can still be guaranteed. Our specific result is now given in the following lemma.

Lemma 4: Consider a network of 'm' multiple heterogeneous EL systems that are governed by the dynamics (1) and subject to (6) where $\bar{\tau}_j$ and $\bar{\bar{\tau}}_j$ are defined in (8) for the j-th system. Let conditions of Case 2 hold. For the pth input channel of the j-th agent, which is fault-free, let us set $\gamma_{p,j}$ according to (18) and let the time derivative of $\hat{\mathfrak{u}}_{p,j}(t)$ be chosen according to (19), where $\bar{\sigma}_{p,j} > 0$, and $\bar{e}_{p,j} > 0$, $p \in \{1,\ldots,k\}$ are diagonal elements of the positive definite matrix $\bar{\mathbf{E}}_j \succ 0$. Then under Assumptions 1-5 by application of the above distributed adaptive control law the closed-loop system states of the j-th nonlinear EL system, i.e. $\bar{x}_j = [x_j^T \quad \tilde{\mathfrak{u}}_j^T]^T$, remain globally bounded under Case 2 for $t \geq 0$.

Proof: The proof is straightforward and can be carried out similar to the proofs of Lemma 3 and Theorem 1, and is therefore omitted due to space limitations. ■

Remark 1: The discontinuity of the distributed adaptive control law (25) can cause complications for the numerical solvers in performing simulations. It also can lead to chattering phenomenon of the system (high-frequency actuation and vibration) in practice. This chattering is due to the fact that the variables $\mathfrak{s}_j$ are never exactly zero in control calculations. Therefore, the discontinuous term keeps switching from a small positive $\mathfrak{s}_j$ to a small negative $\mathfrak{s}_j$. To avoid chattering, a saturation function can replace the sign function in the control law (25). The saturation function is continuous around the surface $\mathfrak{s}_j = 0$, which allows $\mathfrak{s}_j$ to smoothly converge to a neighborhood of origin.

V. Simulation Studies: Distributed Control of Spacecraft Formation

In this section, our proposed distributed reconfigurable control strategy is applied to spacecraft formation flying problem, which is an application area of significant strategic interest. The 3-degree of freedom (DOF) attitude dynamics of a spacecraft can be written in the form of (1) with $g_j(\boldsymbol{q}_j) = \frac{\partial \mathscr{F}_j(\dot{\boldsymbol{q}}_j)}{\partial \dot{\boldsymbol{q}}_j} = 0$, where we specifically have [5], [12],

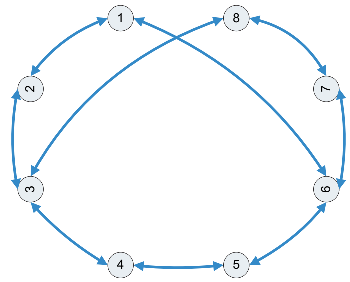

Fig. 1. The communication network graph of the eight spacecraft considered in our simulation studies.

$$\mathbf{D}_j(\boldsymbol{q}_j) = \bar{\mathbf{R}}_j^{-T}\mathbf{J}_j\bar{\mathbf{R}}_j^{-1} \tag{27}$$

and

$$\mathbf{C}_j(\boldsymbol{q}_j, \dot{\boldsymbol{q}}_j) = -\bar{\mathbf{R}}_j^{-T}\mathbf{S}(\mathbf{J}_j\bar{\mathbf{R}}_j^{-1}\dot{\boldsymbol{q}}_j)\bar{\mathbf{R}}_j^{-1} + \bar{\mathbf{R}}_j^{-T}\mathbf{J}_j\frac{d}{dt}\bar{\mathbf{R}}_j^{-1} \tag{28}$$

where $\boldsymbol{q}_j = [\theta_j, \phi_j, \psi_j]^T$ is the vector of the Euler angles (pitch, roll and yaw), $\mathbf{J}_j = \mathbf{J}_j^T$ is the j-th spacecraft positive definite moment of inertia matrix, and $\bar{\mathbf{R}}_j$ is defined as

$$\bar{\mathbf{R}}_j = \frac{1}{c_{\theta_j}}\begin{bmatrix} c_{\theta_j} & s_{\phi_j}s_{\theta_j} & c_{\phi_j}s_{\theta_j} \\ 0 & c_{\phi_j}c_{\theta_j} & -s_{\phi_j}c_{\theta_j} \\ 0 & s_{\phi_j} & c_{\phi_j} \end{bmatrix}$$

where c_{θ_j} stands for $\cos(\theta_j)$, s_{θ_j} stands for $\sin(\theta_j)$, s_{ϕ_j} stands for $\sin(\phi_j)$, and c_{ϕ_j} stands for $\cos(\phi_j)$. In addition, $\mathbf{S}(x)$ is the skew-symmetric operator, namely,

$$\mathbf{S}(x) = \begin{bmatrix} 0 & -x_3 & x_2 \\ x_3 & 0 & -x_1 \\ -x_2 & x_1 & 0 \end{bmatrix}$$

In simulations we consider a network of 8 spacecraft. The communication network graph is depicted in Fig. 1. One can observe from this figure that the network is strongly connected and the connections are bi-directional.

For simulations, we set $\gamma_j = 0.6$, $\alpha_j = 0.86$, $\forall j \in \{1,\ldots,8\}$. In addition, in view of (9), (10), and (11) the distributed controller (25) and (26) gains are selected as: $\mathbf{K}_j = 20\mathfrak{I}_3$, $\bar{\mathbf{K}}_j = 0.16\mathfrak{I}_3$, and $\bar{\mathbf{K}}_j = 0.001\mathfrak{I}_3$. This results in the following parameters for the j-th EL system, namely, $\mathbf{R}_j = 17.22\mathfrak{I}_3$, $\sum_{n\in\mathcal{N}_j}\mathbf{Q}_{jn} = \text{diag}([0.17e-4,\ 0.17e-4,\ 0.17e-4,\ 0.44,\ 0.44,\ 0.44,\ 17.22,\ 17.22,\ 17.22])$, and $\mathbf{Q}_j = \text{diag}([0.27e-5,\ 0.27e-5,\ 0.27e-5,\ 0.031e-3,\ 0.031e-3,\ 0.031e-3,\ 2.78,\ 2.78,\ 2.78])$. The above selection of the controller gains imposes more emphasis on the synchronization of the spacecraft attitudes and their attitude rates and considerably less emphasis on the state regulation. Our desired objective is to keep the spacecraft attitude states in the neighborhood of origin.

The inertia matrix of a deployed spacecraft without a propulsion system does not change during its mission. For spacecraft with a propulsion system, the fuel tanks are placed usually close to the center of mass of the spacecraft so that as

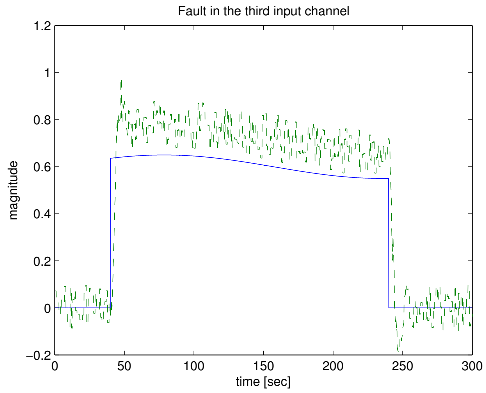

Fig. 2. Fault magnitude in the third input channel of the spacecraft # 1 (blue line) versus its estimate obtained from the FDI algorithm (dashed green line).

the fuel is consumed, the center of mass and inertia do not change significantly. Therefore, we assume that the inertia matrix of the spacecraft in the network is known within a $\pm 10\%$ accuracy, i.e. $\mathbf{J}_j = \hat{\mathbf{J}}_j \pm 0.10\hat{\mathbf{J}}_j$, where $\mathbf{J}_j$ is the actual spacecraft inertia matrix and $\hat{\mathbf{J}}_j$ is it's nominal value. The disturbance $d(t)$ is considered to be a Gaussian distributed noise with the mean value of zero and variance of 0.001. The initial attitudes of the spacecraft are selected randomly between zero to 60 degrees.

An additive actuator fault occurs in the third input channel of the first spacecraft, i.e. $\bar{u}_{1,3}(t) = 0.05\sin(0.02t) + 0.6$ for $40 \leq t \leq 240$. The fault and its estimate, which is provided by a typical FDI algorithm (which is beyond the scope of this work) are depicted in Fig. 2. One can observe from this figure that there exists an error in the fault identification by the FDI algorithm.

A. Distributed Control of Spacecraft Formation Subject to Imperfection in the Fault Detection and without Controller Reconfiguration

In the first part of our simulation results we assume imperfection in the fault detection where the controller is *not* reconfigured in presence of the actuator fault, i.e. $\Gamma_j(t) = 0,\ \forall j \in 1,\ldots,8,\ \ t \geq 0$. Attitudes of the spacecraft in this scenario are shown in Fig. 3 for the first 300 seconds.

Fig. 3 shows that without controller reconfiguration attitude synchronization is not achieved. However, in view of the fact that the proposed controller is robust to input disturbances and actuator faults (refer to Lemma 3), the states remain bounded and stable.

B. Distributed Control of Spacecraft Formation with Controller Reconfiguration Subject to Imperfection in the Fault Identification

In this part of simulations we assume that the adaptive reconfiguration part of the controller is present. The parameters of the controller (25) and (26) are selected as: $\bar{\Sigma}_j = 0.5\mathfrak{I}_3$ and $\bar{\mathbf{E}}_j = \mathfrak{I}_3$. Furthermore, it is assumed that it takes 5 seconds for the FDI algorithm to detect the fault and activate the controller reconfiguration.

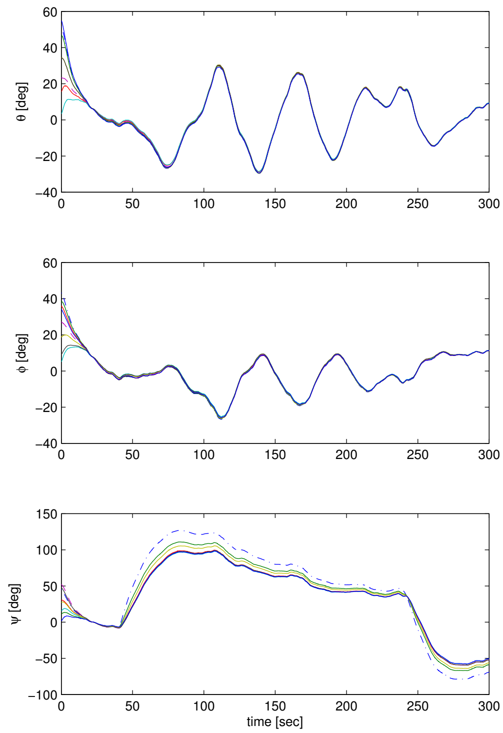

Fig. 3. Spacecraft attitudes without controller reconfiguration for the first 300 seconds. The dash-dotted line represents the spacecraft # 1.

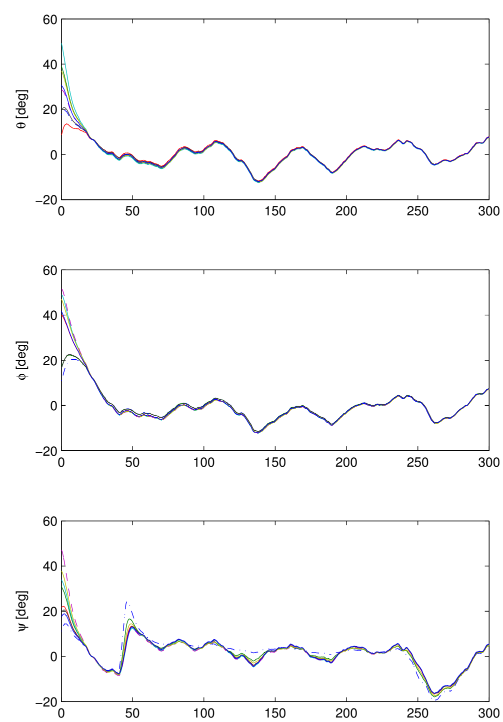

Fig. 4. Spacecraft attitudes with the the controller reconfiguration for the first 300 seconds subject to imperfection in the fault identification. The dash-dotted line represents the spacecraft # 1.

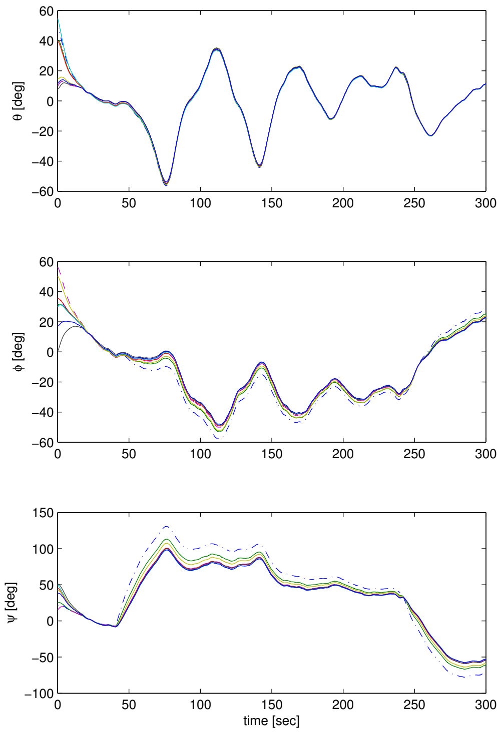

Fig. 5. Spacecraft attitudes with the controller reconfiguration for the first 300 seconds subject to imperfection in the fault isolation. The dash-dotted line represents the spacecraft # 1.

Fig. 4 depicts the attitudes of the eight spacecraft in the formation under the distributed adaptive control law (25) and (26) for the first 300 seconds. By comparing Fig. 4 with Fig. 3 one notices a great improvement in the synchronization and tracking performance of the closed-loop networked EL systems. Specifically, the synchronization error is considerably decreased by employing the adaptive controller. In addition, the attitudes are closer to zero when compared with those obtained in Fig. 3.

C. Distributed Control of Spacecraft Formation with Controller Reconfiguration Subject to Imperfection in the Fault Isolation

In the last part of our simulation results we consider imperfection in the fault isolation. Specifically, we assume that the FDI algorithm detects the fault, however, it incorrectly reconfigures the second input channel of the first spacecraft.

Attitudes of the networked eight spacecraft are shown in Fig. 5. By comparing the results presented in Fig. 4 with those of Fig. 5 the degradations in the synchronization of the roll angle, $\phi(t)$ can be observed. This is in addition to the performance degradation in the third channel, ψ. This, however, confirms our analysis (refer to Lemma 4) which guarantees boundedness of the closed-loop signals in presence of imperfection in the fault isolation.

In other words, despite the incorrect application of the controller reconfiguration to a healthy actuator and no controller reconfiguration to a faulty actuator, the overall closed-loop system still remains stable.

VI. Conclusion

The main contribution of this paper is a formal development of distributed adaptive state synchronization and tracking control law for nonlinear Euler-Lagrange (EL) systems subject to FDI imperfections in actuator faults. Specifically, in presence of actuator faults, our proposed distributed control algorithm has the capability of compensating for the fault and taking proper controller reconfiguration actions. We consider three main types of imperfections in the FDI algorithm, namely, (1) *fault detection imperfection* that arises when fault is not detected by the FDI algorithm, (2) *fault isolation imperfection* that arises when the fault is detected in the wrong channel or in the wrong agent, and (3) *fault identification imperfection* that arises when the fault estimation is not accurate. We show that our proposed distributed controller can maintain the closed-loop networked EL systems stability under all these scenarios, and can moreover improve the performance of the resulting closed-loop networked EL systems corresponding to the last scenario. Simulation results for the attitude control of a network of eight spacecraft demonstrate the effectiveness and capabilities of our proposed distributed control algorithm.

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