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Online Distributed Interdependency Estimation for Critical Infrastructures.

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***Abstract*— The paper deals with the problem to provide a global estimation of a scenario composed by several and interdependent infrastructures. Due to the increasing presence of interdependencies, to correctly manage any infrastructure it is more and more relevant to have information about the "state" of the others, especially in critical situations. However, it is unfeasible both any centralized solution and to exchange detailed data about the state of each infrastructure, due to their huge volumes and in order to avoid the disclosure of critical information. To overcome such problems in this paper a fully distributed approach is proposed, attesting a copy of the whole model inside each one of the infrastructures' control room; each copy directly receives information coming from its own field and shares with the others only aggregated and non-sensible data. Moreover, in order to provide consistent outputs, the different copies have to synchronize. To this end a specific procedure has been developed, and some conditions for the convergence have been identified. In this way the proposed system is able to operate as an Online Distributed Interdependency Estimation (ODIE) System. The approach is under experimental tests on a national wide electric and telecommunication networks inside the European Project MICIE.**

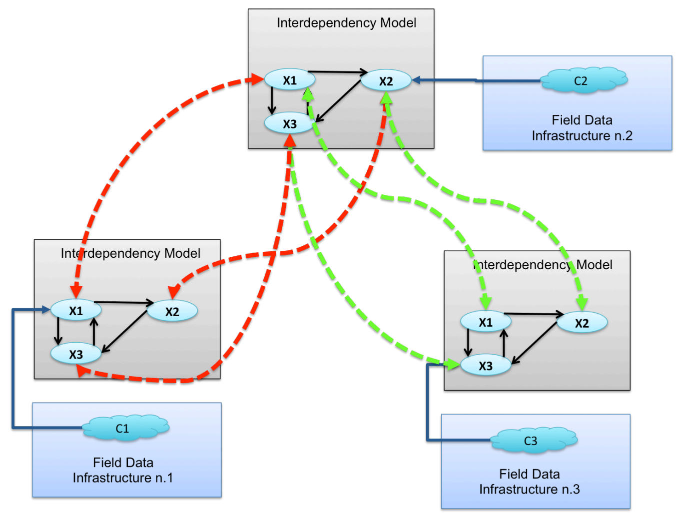

Fig. 1. Example of ODIE framework for 3 interconnected Critical Infrastructures. Each estimator has the same, although simplified, model of the system of systems, but directly receives only data generated within its own field. The different evolutions are then synchronized in a distributed way (i.e, exchanging data only with the neighbors).

I. Introduction

Nowadays the protection of the critical national *infrastructures* (e.g. energy grids, transportation networks, telecommunications systems, etc.) is one of the main issues for national and international security. Even if such a topic always gathered large attention, it has become more prominent due to the increased relevance of critical infrastructures and because of the increased presence of functional relationships, i.e. dependencies and interdependencies, among the different infrastructures, which amplify the negative effect of any natural or man-made accident [14].

In the literature, many approaches have been introduced, in order to address the challenging complexity of interdependency; some of them, like the *Input-Output Inoperability Model* (IIM) proposed in [11], [12], considers infrastructures as unique and abstract entities, while others decompose the overall system into a set of interconnected elements, where the level of abstraction depends on the specific reference scenario (see, for example, the approach proposed in [13]).

Hence the stakeholders of each infrastructure, in order to manage their assets, need to have information about the actual and near future state of the other infrastructures. However this is nearly impossible, due to the huge quantity of data that has to be shared and especially because such data is often considered sensible information, which the stakeholders are not willing to share. To overcome such limits, in the EU project MICIE [20] a distributed online risk predictor has been developed, which is able to notice the actual effects of negative phenomena, to estimate (or even predict) the evolution of the state of the overall system and then support the decisions of human operators and actors. Specifically, the tool operates as an alerting system providing the operators with an estimation of the global state of the other infrastructures, and of the consequential potential impact.

To this end it is assumed that each control room is equipped with an although abstract global model of the overall System of Systems, while only data originated within its own infrastructure is directly available to the corresponding tool (see Figure 1). The tools, attested in the different control rooms, in order to provide a consistent system, exchange their own state that, due to its abstract and aggregated nature, does not represent sensible data. To reduce the communication effort, moreover, the information exchange among tools has been restricted according to a given network topology, hence leading to a distributed synchronization. More in detail each control center is equipped with a Input-Output Inoperability Model (IIM) [11] representing the whole scenario, and the different copies are then synchronized.

An initial synchronization framework has been proposed in [15] in the case of distributed and interconnected continuous-time IIM models; in this case the synchronization was reached by letting each tool reach consensus on the actual degradation phenomena occurring over each infrastructure.

A slightly different approach has been proposed in [18], where a linear, discrete-time interdependency model was considered, and each tool received only the data originated within its field, while the status of each tool were composed with a *distributed consensus protocol*; the resulting shared values were used to influence the further evolution of each tool.

In this paper, exploiting the peculiarities of the IIM models, a simplified procedure to evaluate a synchronization gain matrix is provided. Moreover, a formal, analytical method to grant the synchronization of identical, continuous-time, linear models is given, where each model receives exogenous disturbances, that model the variation of the working condition of the infrastructure (e.g., the consequences of an electrical fault).

Extending traditional synchronization approaches, the above disturbances are seen as additional states for each system, and a characterization of the evolution is given, exploiting the conditions that assure that the synchronized systems converge. To this end a synchronization algorithm is used, and when a variation in the disturbances is noticed, the algorithm is launched again, assuming new initial conditions.

The paper is organized as follows: after some preliminary notations and definitions, the IIM interdependency model is reviewed and commented in Section II; Section III is devoted to introduce the problem of Synchronization of identical, continuous-time, linear systems, as well as the synchronization with constant disturbances; Section IV details the proposed framework and describes its application to a real case study, providing also a small simulative example; finally some conclusive remarks are collected in Section V.

A. Preliminaries

In the following vectors will be represented by boldface letters. Let $\mathbb{R}$ denote the set of real numbers. Let's define $I_n \in \mathbb{R}^{n\times n}$ as the $n \times n$ identity matrix and $\mathbf{1}_p$ as a vector with p components, each equal to one.

Let $A \otimes B$ denote the Kronecker product of two matrices A and B.

An *interconnection* is a matrix $\Gamma = \{\gamma_{ij}\} \in \mathbb{R}^{p\times p}$ such that $\gamma_{ij} \geq 0$ for $i \neq j$ and $\gamma_{ii} = -\sum_{j\neq i} \gamma_{ij}$.

The (directed) graph of Γ is the pair (N, E) where $N = \{n_1, n_2, \ldots, n_p\}$ and $(n_i, n_j) \in E \iff \gamma_{ij} > 0$. Interconnection Γ is *connected* if its graph is connected (i.e., if each node can be reached from each other node by means of the arcs). For connected Γ , it follows from definition that $\lambda = 0$ is an eigenvalue with eigenvector $\mathbf{1}_p$ (i.e. $\Gamma\mathbf{1}_p = 0$). Moreover, all the other eigenvalues have strictly negative real parts [24].

A graph is *balanced* if, for each node $\{n_1, n_2, \ldots, n_p\}$ the *incoming* degree (i.e., the number of incoming links) equals the *outgoing* degree (i.e., the number of outgoing links). If a graph is balanced and $\Gamma\mathbf{1}_p = 0$ it follows that $\mathbf{1}_p^T\Gamma = 0$.

II. IIM Interdependency Modeling

The main objective of the IIM model, introduced in [11] and refined in [12], is to represent within a simple framework the global effects of negative events in scenarios composed by highly interdependent infrastructures. The approach analyzes how the effects of natural outages or terroristic attacks in one economic sector or infrastructure may affect the others, highlighting cascading effects and intrinsic vulnerabilities. Based on the economic equilibrium theory of [17], the static *input-output inoperability model* is defined as:

$$\mathbf{x} = A^*\mathbf{x} + \mathbf{u} \tag{1}$$

where $\mathbf{x}$ is a vector of n components each of one representing the degree of *inoperability* of the corresponding infrastructure, i.e., its inability (in percentage) to correctly operate. A^* is a square $n \times n$ matrix whose elements a^*_{ij} represent the fraction of inoperability transmitted from the j-th infrastructure the the i-th one.

Finally, $\mathbf{u}$ assumes the role of external, induced inoperability; from this point of view it can be seen as an actual *perturbation* generated by an adverse event.

In [19] model (1) is further extended, considering a *dynamic* term:

$$\mathbf{x}(t) = A^*\mathbf{x}(t) + \mathbf{u}(t) + W\dot{\mathbf{x}}(t) \tag{2}$$

Matrix W is a square $n \times n$ matrix that represents the willingness of the economy to invest in capital resources. Many choices are possible for the W matrix; however, as exposed in [19], the elements of W must be either zero or negative for an economic system to be stable. In [12] the authors adopt a diagonal B matrix in the form:

$$W = -H^{-1}; \quad k_{ii} \geq 0; \quad \forall i = 1, \ldots, n \tag{3}$$

Substituting inside Eq. (2):

$$\dot{\mathbf{x}}(t) = H[(A^* - I_n)\mathbf{x}(t) + \mathbf{u}(t)] \tag{4}$$

Matrix H is generally referred as the *industry resilience* coefficient matrix because each element k_{ii} can be seen as the recovery rate with respect to adverse or malicious events.

The IIM model, in its dynamic fashion, can be adopted to represent the response of interdependent infrastructures to an induced perturbation, until the equilibrium is reached.

In order to evaluate the level of dependencies of an infrastructure, in [13] the *dependency index* is introduced, as the sum of the IIM coefficients along a single row:

$$\delta_i = \sum_{j=1}^{n} a^*_{ij} \tag{5}$$

This index represents a measurement of the robustness of the corresponding infrastructure with respect to the inoperability of the others; in fact, it represents the maximum inoperability that i-th infrastructure may receive when each other element is completely inoperable. Consequently, $\delta_i \le 1$ means that the i-th infrastructure maintains some operative capability also in the case in which all the other infrastructures in the scenario have completely collapsed.

In [16] it is proved that a sufficient condition to guarantee the stability of system (2) is that the maximum of the dependency indexes (5) of matrix A^* is less than one. This represents the condition that the infrastructures are provided with buffers or batteries, thus increasing their resistance to the inoperability of the other infrastructures.

III. Linear System Synchronization

In the literature the *synchronization* of identical linear distributed systems has been widely investigated [1], [2], [3], [6], [7], [8], [4], [5]. By synchronization it is intended the convergence of the solutions of the systems to a common trajectory; the synchronization approach is said to be *distributed* if each system receives data only by a subset of the other systems (i.e., only by its *neighborhood*). When the trajectory is a stationary point, the problem reduces to a *consensus* problem [9], [10].

In the following the theory of linear system synchronization will be reviewed and a condition for the synchronization under rather general hypotheses will be provided.

Let p identical linear systems, where i-th system is in the form:

$$\dot{\mathbf{x}}_i(t) = A\mathbf{x}_i(t) + B\mathbf{u}_i(t); \tag{6}$$

where $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $\mathbf{x}_i \in \mathbb{R}^n$ and $\mathbf{u}_i \in \mathbb{R}^m$. The input $\mathbf{u}_i$ only depends on the state of the neighbors of system i, according to the topology Γ; in other terms:

$$\mathbf{u}_i(t) = P\textstyle\sum_{j=1}^{p} \gamma_{ij}(\mathbf{x}_j(t) - \mathbf{x}_i(t)); \tag{7}$$

where P is an $m \times n$ matrix.

The above systems in the form of Eq. (6) are said to *synchronize* if

$$\lim_{t\to+\infty} ||\mathbf{x}_i(t) - \mathbf{x}_j(t)|| = 0\ ; \quad \forall i, j = 1, \ldots, p; \quad i \ne j \tag{8}$$

Let L be the *graph Laplacian* induced by Γ, whose elements $\{l_{ij}\}$ are in the form:

$$l_{ij} = \begin{cases} \sum_{k=1,k\ne i}^{p} \gamma_{ik}, & j = i \\ -\gamma_{ij}, & j \ne i \end{cases} \tag{9}$$

Let $\mathbf{x}(t) = [\mathbf{x}_1(t), \ldots, \mathbf{x}_p(t)]^T$, the overall dynamic for the p stacked systems is given by:

$$\dot{\mathbf{x}}(t) = [I_p \otimes A - L \otimes BP]\mathbf{x}(t) \tag{10}$$

In [2] a Lyapunov-based approach is adopted to grant the stability and then the synchronization; conversely in [5], under the hypothesis of a stable matrix A, a sophisticated algorithm is used for the choice of matrix P.

In the following theorem we will show that, under some additional hypotheses, the complexity of choosing P can be considerably reduced.

Theorem 3.1: Let $K = BP$, and chose a matrix P such that K is diagonal and has nonnegative elements. If matrix A is such that: for all $i = 1, \ldots, n$ $a_{ii} \le 0$; for all $j = 1, \ldots, n$, $j \ne i$ $a_{ij} \ge 0$ and $\sum_{j=1}^{n} a_{ij} \le 0$, then System (10) is stable.

Proof: The dynamic matrix of System (10) has the following structure:

$$\begin{bmatrix} A - l_{11}K & l_{12}K & \cdots & l_{1p}K \\ l_{21}K & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & l_{p-1,p}K \\ l_{p1}Kn & \cdots & l_{p,p-1}K & A - l_{pp}K \end{bmatrix} \tag{11}$$

From Gershgorin Circle Theorem [23] the eigenvalues of an $n \times n$ matrix M lie, in the complex plane, in the union of circles centered in $C(i) = m_{ii}$ with radius equal to $R(i) = \sum_{j=1;j\ne i}^{n} |m_{ij}|$. Consider the i-th row of the q-th block row; the center is given by

$$C(q, i) = a_{ii} - \sum_{g=1}^{p} \gamma_{qg} k_{ii} \tag{12}$$

Since γ_{qg}, k_{ii} are all nonnegative by hypothesis, it follows that $C(q, i) \le 0$. Conversely off diagonal elements for row i of block row q are all positive, therefore the $|\cdot|$ is not required for the radius, which is given by:

$$\begin{aligned} R(q,i) &= \textstyle\sum_{j=1,j\ne i}^{n} a_{ij} + \sum_{g=1}^{p} \gamma_{qg} \sum_{j=1,j\ne i}^{n} k_{ij} + \\ &\quad + \textstyle\sum_{g=1,g\ne q}^{p} \gamma_{qg} \sum_{j=1}^{n} k_{ij} = \\ &= \textstyle\sum_{j=1,j\ne i}^{n} a_{ij} + 2\sum_{g=1}^{p} \gamma_{qg} \sum_{j=1}^{n} k_{ij} + \\ &\quad - \gamma_{qq} \textstyle\sum_{j=1}^{n} k_{ij} - \sum_{g=1}^{p} \gamma_{qg} k_{ii} \end{aligned} \tag{13}$$

The system is stable if $C(q, i) + R(q, i) \le 0$ for each $q = 1, \ldots p$ and $i = 1, \ldots, n$, or in other terms if

$$\begin{aligned} &a_{ii} - \textstyle\sum_{g=1}^{p} \gamma_{qg} k_{ii} + \sum_{j=1,j\ne i}^{n} a_{ij} + \\ &+ 2\textstyle\sum_{g=1}^{p} \gamma_{qg} \sum_{j=1}^{n} k_{ij} - \gamma_{qq} \sum_{j=1}^{n} k_{ij} + \\ &- \textstyle\sum_{g=1}^{p} \gamma_{qg} k_{ii} \le 0 \end{aligned} \tag{14}$$

Since for hypothesis $\sum_{j=1} a_{ij} \le 0$, Inequality (14) is true if

$$\left(\textstyle\sum_{g=1}^{p} 2\gamma_{qg} - \gamma_{qq}\right) \sum_{j=1}^{n} k_{ij} \le \sum_{g=1}^{p} 2\gamma_{qg} k_{ii} \tag{15}$$

where some terms have been rearranged. Clearly, Inequality (15) is true for all (q, i) if K is diagonal and the k_{ii} are positive.

■

In [5] it is proved that, for a connected Γ, the p systems converge to

$$\bar{\mathbf{x}}(t) = (e^{At} \otimes \mathbf{r}^T) \begin{bmatrix} \mathbf{x}_{10} \\ \vdots \\ \mathbf{x}_{p0} \end{bmatrix} \tag{16}$$

where $\mathbf{r} \in \mathbb{R}^p$ is a vector such that $r^T\Gamma = 0$ and $\sum_{h=1}^{p} r_h = 1$.

It is possible to specify some conditions, in order to further characterize the synchronization reached:

Corollary 3.2: If Γ is a balanced and connected graph, then the p systems synchronize to the *average* evolution.

Proof: Since systems synchronize, it follows that

$$\mathbf{r}^T\Gamma = 0 \quad \text{ and } \mathbf{r}^T\mathbf{1}_p = 1 \tag{17}$$

Moreover, a balanced Γ ensures that $\Gamma\mathbf{1}_p = 0$ and $\mathbf{1}_p^T\Gamma = 0$. Therefore the only $\mathbf{r}$ that satisfies (17) is such that $r_j = \frac{1}{p}$ for each $j = 1, \ldots, p$, proving the statement. ■

Note that it is also possible to obtain the synchronization to a weighted average or to the sum of the evolutions. It is sufficient to use the synchronization algorithm with modified initial conditions $\mathbf{x}_{i0}^*$ obtained from the real initial condition detected x_{i0}. For example if $\mathbf{x}_{i0}^* = p\mathbf{x}_{i0}$, where p is the number of distributed systems, for each system i, the sum of the evolutions is obtained. Note that, to achieve this result, each system needs to know the number of systems involved in the synchronization.

Although the above approach is very powerful, it is not able to consider systems which depend also on exogenous signals, as in our case when each copy is "perturbated" by inputs coming from its own field.

In the next subsection an approach for the synchronization of distributed linear systems with constant disturbance will be introduced.

A. Synchronization with Constant Disturbance

Consider for each of the p systems in the form (6) an additional constant disturbance $\mathbf{w}_i(t) = \mathbf{w}_i \in \mathbb{R}^n$ whose effect on the system is determined by the $n \times n$ matrix D; hence each copy has the form

$$\dot{\mathbf{x}}_i(t) = A\mathbf{x}_i(t) + B\mathbf{u}_i(t) + D\mathbf{w}_i(t); \tag{18}$$

In order to address the synchronization problem, the state of each system has to be extended in the following way. Let $\mathbf{z}_i \in \mathbb{R}^{2n}$ be defined as:

$$\mathbf{z}_i(t) = [\mathbf{x}_i(t)^T, \mathbf{w}_i(t)^T]^T; \quad \forall i = 1, 2, \ldots, p \tag{19}$$

Transforming systems (6) and introducing a new input $\mathbf{v}_i(t) \in \mathbb{R}^m$ in the form

$$\mathbf{v}_i = Q \sum_{j=i}^{p} \gamma_{ij}[\mathbf{w}_j(t) - \mathbf{w}_i(t)] \tag{20}$$

let $\mathbf{e}_i(t) = [\mathbf{u}_i(t)^T, \mathbf{v}_i(t)^T]^T$; the resulting *extended* systems are in the form

$$\dot{\mathbf{z}}_i(t) = \tilde{A}\mathbf{z}_i(t) + \tilde{B}\mathbf{e}_i(t); \quad \forall i = 1, \ldots, p \tag{21}$$

where matrices $\tilde{A} \in \mathbb{R}^{2n\times 2n}$ and $\tilde{B} \in \mathbb{R}^{2n\times 2m}$ are in the form

$$\tilde{A} = \begin{bmatrix} A & D \\ 0 & 0 \end{bmatrix}; \quad \tilde{B} = \begin{bmatrix} B & 0 \\ 0 & I_n \end{bmatrix} \tag{22}$$

Note that $\tilde{A}$ is block triangular and the evolution of $\mathbf{w}_i$ is independent on $\mathbf{x}_i$; therefore the system is stable if A is stable. Hence it is possible to apply the approach in [5], which only requires the stability of A. In order to simplify the choice of matrices P and Q the results of Theorem 3.1 are extended in the following way

Theorem 3.3: Let p linear systems in the form of Eq. (21), where D is such that $d_{ii} \leq 0$ for all $i = 1, \ldots, n$, $d_{ij} \geq 0$ for all $j = 1, \ldots, n$ and $\sum_{j=1}^{n} d_{ij} \leq 0$. Then, under the hypotheses on the matrix A, required by Theorem 3.1, a state feedback in the form

$$\mathbf{e}_i(t) = \begin{bmatrix} P \\ Q \end{bmatrix} \sum_{j=1}^{p} \gamma_{ij}(\mathbf{z}_j(t) - \mathbf{z}_i(t)) \tag{23}$$

solves the synchronization problem if P and Q are such that

$$\tilde{K} = \begin{bmatrix} BP & 0 \\ 0 & Q \end{bmatrix} \tag{24}$$

is diagonal and the entires of $\tilde{K}$ are nonnegative.

Proof: It is sufficient to show that the elements $\tilde{a}_{ij}$ and $\tilde{k}_{ij}$ respect the conditions required by Theorem 3.1. To this end note that, due to the hypotheses on D, the conditions on $\tilde{A}$ are satisfied. Moreover Inequality (15) becomes

$$\left(\sum_{g=1}^{p} 2\gamma_{qg} - \gamma_{qq}\right) \sum_{j=1}^{n} \tilde{k}_{ij} + \sum_{j=1}^{n} d_{ij} \leq \sum_{g=1}^{p} 2\gamma_{qg}\tilde{k}_{ii} \tag{25}$$

and, for a diagonal $\tilde{K}$ it is sufficient that $\sum_{j=1}^{n} d_{ij} \leq 0$. ■

Note that the choice of D is not limitative since in typical situations D is assumed as diagonal and the presence of negative d_{ii} does not preclude to represent any perturbation, i.e., it is sufficient to suitably change the sign of w_i.

Note further that, since the evolution of the w_i is decoupled from the evolution of x_i, the above theorem addresses, at the same time, the synchronization of linear systems and the *consensus* for the disturbances, in a unified approach.

Note that it is possible to represent, at the same time, the case of non-overlapping disturbances, i.e. the j-th disturbance variable for each system is different from zero only for the i-th system ($w_{ij} \neq 0$, $w_{hj} = 0$ for each $h \neq i$), and the case of inputs with different beliefs, i.e. each copy of the model receives a different value for the j-th disturbance variable.

IV. Online Distributed Interdependency Estimation

In the field of Critical Infrastructure Protection a crucial aspect is the capability to identify possible risks induced by cascading failures. Unfortunately critical infrastructures operators are very reluctant to share detailed information (i.e., field data) about their infrastructures, because this data is considered sensible information.

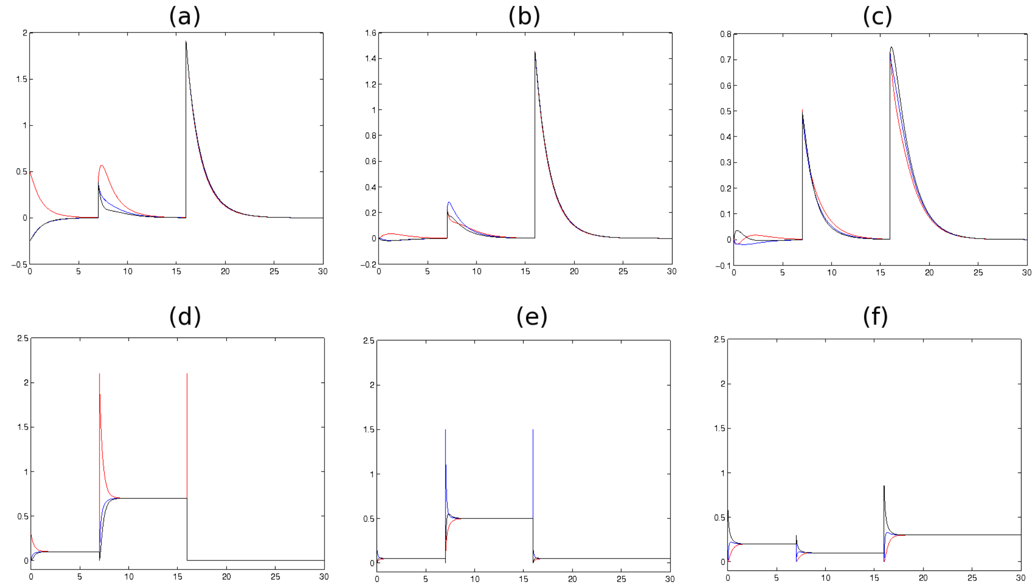

Fig. 2. Simulation Results: Figures (a), (b) and (c) show the convergence of the error between the $i-th$ state variable for the three systems and the corresponding state variable for a system which directly receives the complete disturbance vector, i.e., in each plot the error for the tree systems is plotted. Since the evolutions converge to zero, the trajectories of the three systems converge to the sum of their isolated evolutions. Figures (d), (e) and (f) show the synchronization for first second and third disturbance variable. The red, blue and black curves characterize the first, second and third system, respectively. Note that the algorithm is executed with updated initial conditions after a change in the disturbance is noticed, i.e. in $t_1 = 6.99s$ and in $t_2 = 15.99s$.

To overcome such a difficulty and provide the operators with a useful tool, in the EU project MICIE [20], an approach based on a distributed architecture that implements an *Online Distributed Interdependency Estimator* (ODIE) has been proposed. The control room of each infrastructure is equipped with an identical copy of an abstract and high-level dynamic model that represents the interdependencies among the different infrastructures, hence able to capture the most relevant domino effects.

Each instance of the model, attested in a given infrastructures' control room, acquires as inputs information coming from its own field; in other terms the copy in the control room of the first infrastructure receives as inputs the severity of failure affecting the first infrastructure, the control room of second infrastructure receives as inputs those related to the second infrastructure, and so on.

Allowing to synchronize the different copies, the system is able to provide to the different operators a coherent picture of the global situation, without exchanging sensible information.

The result is that the trajectory of each system converges to the trajectory of an "hypothetical" centralized estimator, i.e., an estimator which receives all the inputs (or a more reliable input in the case of contradictory information).

Note further that the disturbances are assumed to vary impulsively and their value remains constant between variations. In order to cope with such disturbances, their variation is monitored and, if a variation is noticed at time $t_c + \epsilon$, the synchronization protocol is executed again considering the updated initial condition $[x(t_c)^T, w(t_c)^T]^T$.

In the following subsection a simple example will be given to show the potentialities of the proposed framework.

A. *Simulation Results*

Consider a scenario composed of three interdependent infrastructures depicted in Figure 1, and an IIM model, defined in Equation (1), where H is assumed as the identity matrix. The A^* matrix and the interconnection Γ are:

$$A^* = \begin{bmatrix} 0 & 0.4 & 0.5 \\ 0.5 & 0 & 0.5 \\ 0.5 & 0.6 & 0 \end{bmatrix}; \quad \Gamma = \begin{bmatrix} -1 & 1 & 0 \\ 1 & -3 & 2 \\ 0 & 2 & -2 \end{bmatrix} \tag{26}$$

Note that $A^* - I_n$ is stable and that the conditions required by Theorem 3.1 are verified; we chose a matrix $D = -I_n$ and P, Q both equal to the identity matrix, in order to satisfy conditions required by Theorem 3.3. Note further that Γ is balanced, and therefore it is possible to achieve the sum synchronization.

Consider the disturbance vector $\mathbf{w}_i = [w_{i1}, w_{i2}, w_{i3}]^T$; the $i-th$ copy receives an input where $i-th$ component is equal to w_{ii} and other entries are equal to zero (i.e. non-overlapping inputs). Moreover, the initial conditions of systems are equal to zero, except for system 1, whose initial conditions are $\mathbf{x}_1(0) = [0.25, 0, 0]^T$. The disturbances are assumed to vary twice during the simulation: at time $t_0 = 0s$ they assume the following values:

$$w_{11}(0) = 0.1, \quad w_{22}(0) = 0.05, \quad w_{33}(0) = 0.2$$

then at time $t_1 = 6.99s$ they abruptly change assuming the values:

$$w_{11}(6.99) = 0.7, \quad w_{22}(6.99) = 0.5, \quad w_{33}(6.99) = 0.1$$

finally in $t_2 = 15.99s$ they become:

$$w_{11}(15.99) = 0, \quad w_{22}(15.99) = 0.05, \quad w_{33}(15.99) = 0.3$$

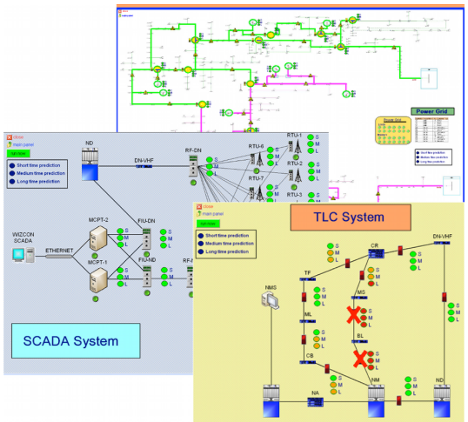

Fig. 3. Scenario considered within the MICIE European Project [20], composed of a portion of a national-wide medium voltage power grid, a telecommunication network and a SCADA system.

Figure (2) shows the results for synchronization of the three systems; more specifically the error of the three systems with respect to a system which directly receives the complete disturbance vector $[w_{11}, w_{22}, w_{33}]^T$ is plotted in Figures 2.(a), 2.(b), 2.(c), i.e., each figure represents the error on a given state variable. Conversely, Figures 2.(d), 2.(e), 2.(f) show the consensus reached on the disturbances, i.e., in each figure the value of the i-th disturbance is plotted for the three systems. Note that, after any perturbation, the synchronization procedure is executed considering the updated initial conditions. Note further that to achieve sum synchronization the initial condition inside the algorithm is multiplied by $p = 3$; moreover since $D = -I$ the value of the disturbance is changed in sign inside the algorithm: therefore $w^*_{i0} = 3[x^T_{i0}, -w^T_{i0}]^T$.

Such an approach has been adopted in FP7 MICIE European Project, considering a scenario composed of a portion of a real national-wide medium voltage power grid, with the connected telecommunication network and the corresponding SCADA systems operated by one of the project's partner (see Figure 3). A copy of the model is installed inside the control room of each infrastructure and directly receives the data available from its own field, while the different tools exchange abstract and non-sensitive data to reach synchronization(for further details on the results of MICIE Project see [21], [22]).

V. Conclusions

In this paper the synchronization problem for linear systems with disturbances has been addressed and some conditions for the convergence have been provided. The results are applied to the design of an online distributed estimator for critical infrastructures. Specifically a copy of the Input-Output Inoperability model isattested in each control room and is able to collect information from its own field and share aggregated data with the other copies. The approach has been exploited within the MICIE European Project, considering a real case study in Israel. Future works will be devoted to introduce time-delays and switching interconnection topologies, as well as more complex and nonlinear interdependency models, considering also the discrete time perspective.

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