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Control Design for LPV Systems with Input Saturation and State Constraints: an Application to a Semi-Active Suspension

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Abstract— **This paper proposes a control design strategy for LPV systems subject to additive disturbances in the presence of actuator saturation and state constraints. LMI conditions are derived in order to simultaneously compute an LPV controller and an anti-windup gain that ensures the boundedness of the trajectories, considering that the disturbances belong to a given admissible set. The disturbance attenuation is addressed via an H_∞ constraint. Besides, state constraints (corresponding to the local validity of the LPV model and system structural limits) are always assured. The theoretical results are applied to a quarter-car model rewritten in the LPV framework where the passivity constraint is recast to the saturation one. The interest of the provided methodology is emphasized by simulations.**

I. Introduction

In the last years, many studies have focused on the control of saturated (in states, control inputs...) systems which are present in almost real applications. For a system with input saturation, there is usually an inconsistency between the states of the plant and those of the controller because of the saturated actuator between the system control input and the controller output. This effect, usually called windup, dramatically degrades the closed-loop performances or even worse causes the system instability. To preserve the consistency, the input to controller needs to be changed by an appropriate signal, which is provided by a called anti-windup compensator. Usually, when a system is subject to actuator saturation, two main issues arise: the guarantee of stability (global or local) and the minimization of the performance degradation. There are two methods to solve these problems: two-step and one-step design. The traditional two-step method first designs a linear controller without considering the input saturation effect and then add an anti-windup compensator to minimize the adverse effects of control input saturation on closed-loop performance [1], [2]. For the one step approach, the controller and an anti-windup compensator (static in general) are simultaneously computed [3], [4]. It can be noticed that the control design with input saturation is a nonlinear problem. However, many solutions have been proposed to model the saturation effect in such a way that the problem can be treated within a linear framework, for example: the polytopic differential inclusion model [5], [6], [7] and the use of sector conditions [8], [9], [4]. Up to now, numerous results have been obtained for LTI systems. On the other hand, very few papers dealing with switched or LPV systems can be found in the literature, see for instance [10], for switching systems, and [6], [11], [12] for LPV systems.

In this paper, we aim at using the one-step anti-windup design for semi-active suspension control to achieve the best compromise among conflicting objectives: passenger comfort, road holding and suspension deflection. Indeed semi-active suspensions have recently received a lot of attention since they provide the best compromise between cost (energy-consumption and actuators/sensors hardware) and performance (see e.g. [13], [14]). For such suspensions, numerous control approaches have been developed. An overview of some recent methodologies in terms of performances is found in [15].

In our previous works [16] and [17], the LPV framework is used to model the nonlinear damper characteristics, and, also to consider the actuator saturation as a scheduling parameter (this approach can be referred to [6]). The performance on suspension deflection, along with comfort and road holding, is managed by using some frequency-based weighting functions. An LPV controller is then synthesized using a global analysis (global stability and performance). In this work, instead of considering the suspension deflection as a performance objective, we will treat it as a constraint. Besides, we are only interested in a certain working range of the damper because, in real applications, its deflection velocity is limited. Since the states are physically bounded, due to the limit in the suspension deflection, and the LPV polytopic model is not globally valid in the state space, a regional stabilization approach is considered. First, a general design method for LPV system with input saturation and state constraints is proposed. Precisely, a sufficient condition to guarantee the regional asymptotic stability of the origin for arbitrary scheduling parameters and to guarantee bounded trajectories in the presence of disturbances (which are assumed to be limited in amplitude) is derived based on the modified sector condition [8] and on the use of a quadratic Lyapunov function. The condition ensures also an upper bound on the induced-L_2 gain between the disturbance input and the controlled output when there is no saturation. Moreover, the state constraints on the system are always assured for the considered class of disturbances. Then we apply the result to enhance the performance of a semi-active suspension system rewritten in the LPV framework where the passivity constraint is recast in an input saturation one.

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The rest of the paper is organized as follows. In Section II, we introduce the control problem of LPV system subject to input saturation. In Section III, some useful preliminaries are presented. The main result is stated in Section IV. In Section V, the proposed method is applied to semi-active suspension control. Finally, some conclusions and perspectives are drawn in Section VI.

II. Problem Formulation

In the following, X_i denotes the i^{th} row of matrix X. (*) stands for symmetric blocks and $sym(X) = X + X^T$. ($\bullet$) stands for an element that has no influence on the development.

A. System description

Consider a quasi-LPV plant

$$\begin{array}{rcl} \dot{x} & = & A(\theta)x + B_w(\theta)w + B_u u \\ z & = & C_z(\theta)x + D_{zw}(\theta)w + D_{zu}u \\ y & = & C_y x + D_{yw}w \end{array} \quad (1)$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, $w \in \mathbb{R}^q$, $z \in \mathbb{R}^r$ and $y \in \mathbb{R}^p$ are the state, the input, the disturbance vectors, the control output and the measured output, respectively. θ is a vector of scheduling parameters which are supposed to depend on states and assumed to be known (measured or estimated).
consider also an LPV controller

$$\begin{array}{rcl} \dot{x}_c & = & A_c(\theta)x_c + B_c(\theta)u_c + v \\ y_c & = & C_c(\theta)x_c + D_c(\theta)u_c \end{array} \quad (2)$$

where $x_c \in \mathbb{R}^{n_c}$, $u_c \in \mathbb{R}^p$, $y_c \in \mathbb{R}^m$, v is an additional input used for anti-windup compensation.

The unconstrained closed-loop system composed by the plant and the controller is defined by the following interconnections

$$u = y_c, \quad u_c = y, \quad v = 0 \quad (3)$$

The following assumptions are considered:

- *Assumption 1:* The matrices B_u, D_{zu}, C_y and D_{yz} are supposed to be parameter-independent (to satisfy the hypotheses of polytopic design for LPV systems [18]).
- *Assumption 2:* The input disturbance is limited in amplitude, that is $\forall t > 0, w(t) \in \mathscr{W}$ with
$$\mathscr{W} = \{w \in \mathbb{R}^q : w^T w < \delta\} \quad (4)$$
- *Assumption 3:* The scheduling parameters depend on the system's states $\theta = \theta(x,t)$ and are bounded in
$$\Theta = \{\theta : \underline{\theta}_i \leqslant \theta_i \leqslant \overline{\theta}_i, i = 1, ..., k\} \quad (5)$$
- *Assumption 4:* The control inputs are bounded in amplitude:
$$-\overline{u}_i \leqslant u_i(t) \leqslant \overline{u}_i, \quad i = 1, ..., m \quad (6)$$

B. LPV controller

We consider a dynamic LPV controller with a static anti-windup action

$$\begin{array}{rcl} \dot{x}_c & = & A_c(\theta)x_c + B_c(\theta)u_c + E_c(\theta)(sat(y_c) - y_c) \\ y_c & = & C_c(\theta)x_c + D_c(\theta)u_c \end{array} \quad (7)$$

where $x_c \in \mathbb{R}^{n_c}$, $u_c \in \mathbb{R}^p$, $y_c \in \mathbb{R}^m$ and $E_c(\theta)$ is a static anti-windup term [8], [10]. In the presence of the control bounds, the interconnections between the plant and the controller are given (according to (2)) by

$$u = sat(y_c), \ u_c = y, \ v = E_c(\theta)(sat(y_c) - y_c) \quad (8)$$

where the saturated function $sat(.)$ is defined by

$$sat(y_{c_i}) = sign(y_{c_i}) \min(|y_{c_i}|, \overline{u}_i) \quad (9)$$

From (1) and (2), the closed-loop system is given by

$$\begin{array}{rcl} \dot{\xi} & = & \mathscr{A}(\theta)\xi + \mathscr{B}(\theta)w - (\mathscr{B}_u + \mathscr{R}E_c(\theta))\psi(y_c) \\ z & = & \mathscr{C}(\theta)\xi + \mathscr{D}(\theta)w + \mathscr{D}_\psi \psi(y_c) \end{array} \quad (10)$$

where

$$\begin{array}{l} \xi = [x^T x_c^T]^T, \psi(y_c) = y_c - sat(y_c) \\ \mathscr{A}(\theta) = \begin{bmatrix} A(\theta) + B_u D_c(\theta) C_y & B_u C_c(\theta) \\ B_c(\theta) C_y & A_c(\theta) \end{bmatrix} \\ \mathscr{B}(\theta) = \begin{bmatrix} B_w(\theta) + B_u D_c(\theta) D_{yw} \\ B_c(\theta) D_{yw} \end{bmatrix} \\ \mathscr{B}_u = \begin{bmatrix} B_u \\ 0 \end{bmatrix}, \mathscr{R} = \begin{bmatrix} 0 \\ I_{n_c} \end{bmatrix} \\ \mathscr{C}(\theta) = \begin{bmatrix} C_z(\theta) + D_{zu} D_c(\theta) C_y & D_{zu} C_c(\theta) \end{bmatrix} \\ \mathscr{D}(\theta) = D_{zw}(\theta) + D_{zu} D_c(\theta) D_{yw} \\ \mathscr{D}_\psi = -D_{zu} \end{array} \quad (11)$$

The controller output is rewritten as

$$y_c = \mathscr{K}(\theta)\xi + \mathscr{K}_w(\theta)w \quad (12)$$

where
$$\mathscr{K}(\theta) = \begin{bmatrix} D_c(\theta) C_y & C_c(\theta) \end{bmatrix}, \ \mathscr{K}_w(\theta) = \ D_c(\theta) D_{yw}$$

C. Problem Definition

In this paper, we look for an LPV controller (7) for the LPV system (1) such that the following conditions are satisfied:
(i) in the absence of disturbances, or if the disturbances are vanishing, the controller guarantees the regional asymptotic stability of the origin for an arbitrary scheduling parameter θ provided that the initial states belong to a specific set in the state space. In the presence of disturbances satisfying Assumption 2, the controller guarantees that the trajectories of (10) are bounded.
(ii) the controller guarantees the respect of some constraints on the states of the closed-loop system.
(iii) for the unconstrained closed-loop system, i.e. when the saturation is not active, the controller guarantees an upper bound γ on the L_2-gain between the disturbance input w and the controlled output z.

Remark: Considering the same L_2 performance when the system operates linearly and under control saturation can lead to very conservative results. Hence, we consider that the L_2 performance should be satisfied only by the unconstrained system, which corresponds to a classic H_∞ problem. On the other hand, if the control saturates, we should ensure that the trajectories are bounded and do not violate the state constraints.

III. PRELIMINARIES

A. Practical validity region

In practice, besides the constraint on the control input, the system states are usually bounded because of structural limits. Furthermore, the local validity of the LPV model can be also translated in state constraints. We assume the state constraint can be represented by a polyhedron $\mathscr{X}$ defined by

$$\mathscr{X} = \{\xi \in \mathbb{R}^{2n} : H_i\xi \leq h_{0i},\ i = 1 : s\} \tag{13}$$

Note that only the state of the plant is constrained, so we have $H = \begin{bmatrix} H_1 & 0 \end{bmatrix}$.

B. Saturation model validity region

Due to the boundness of w and to the fact that the states of the real system are constrained to belong to $\mathscr{X}$, a regional stabilization approach is adopted in this paper. In order to take into account the saturation effects, an "LPV" version of the modified sector condition proposed in [8] is applied. With this aim, define the matrix $\mathscr{G}(\theta) = \begin{bmatrix} G_1(\theta) & G_2(\theta) \end{bmatrix}$ and the following polyhedral set

$$S_\theta = \left\{\xi \in \mathbb{R}^{2n}, |(\mathscr{K}_i(\theta) - \mathscr{G}_i(\theta))\xi| \leqslant \bar{u}_i, i = 1, ..., m\right\}, \tag{14}$$

$\forall\theta \in \Theta$. Hence, the following Lemma can be stated.

Lemma 1: If $\xi(t) \in S_\theta$, then the following inequality

$$\psi(y_c)^T T\left(\psi(y_c) - \begin{bmatrix} \mathscr{G}(\theta) & 0 & \mathscr{K}_w(\theta) \end{bmatrix}\begin{bmatrix} \xi \\ \psi(y_c) \\ w \end{bmatrix}\right) \leqslant 0 \tag{15}$$

holds for any diagonal and positive definite matrix $T \in \mathbb{R}^{m\times m}$.

Proof: The result can be inferred directly from [8].

C. W-invariance

Because the disturbance input is bounded in amplitude, we use the W-invariance concept to ensure the boundedness of the trajectories (see [19]).

Definition 1: A set $\mathscr{E} \subset \mathbb{R}^{2n}$ is W-invariant with respect to system (10) if $\forall\xi(0) \in \mathscr{E}$, $w(t) \in \mathscr{W}$ and for any scheduling parameter signal $\theta(t)$, it follows that the state trajectory remains in $\mathscr{E}$, i.e $\xi(t) \in \mathscr{E}$, $\forall t > 0$.

In the approach, $\mathscr{E}$ is considered as an ellipsoidal set associated to a quadratic function $V(t) = \xi^T P\xi$, $P = P^T \succ 0$

$$\mathscr{E} = \{\xi \in \mathbb{R}^{2n} : \xi^T P\xi < 1\} \tag{16}$$

To ensure that $\mathscr{E}$ is a W-invariant set, it suffices to ensure that

$$\dot{V}(t) < 0, \begin{cases} \forall\xi(t) : \xi^T P\xi > 1 \\ \forall w(t) : w^T w < \delta \end{cases} \tag{17}$$

along the trajectories of (10). By using the S-procedure, this condition can be satisfied if there exist scalars $\beta_1 > 0$ and $\beta_2 > 0$, such that

$$\dot{V} + \beta_1(\xi^T P\xi - 1) + \beta_2(\delta - w^T w) < 0 \tag{18}$$

IV. MAIN RESULTS

In this section, an LMI-based constructive condition to solve the problem stated in II-C is stated.

Theorem 1: If, for given $\beta_1 > 0$ and $\gamma > 0$, there exist symmetric positive definite matrices $X, Y \in \mathbb{R}^{n\times n}$, a positive scalar β_2, positive diagonal matrices $S \in \mathbb{R}^{m\times m}$, matrices $\hat{A}(\theta) \in R^{n\times n}$, $\hat{B}(\theta) \in R^{n\times p}$, $\hat{C}(\theta)$, $\hat{Z}_1(\theta)$, $\hat{Z}_2(\theta) \in R^{m\times n}$, $\hat{D}(\theta) \in R^{m\times p}$, $\hat{Q}(\theta) \in R^{n\times m}$ such that the matrix inequalities (20)-(24) are verified, then the LPV controller (2) with matrices

$$\begin{cases} E_c(\theta) = N^{-1}\hat{Q}(\theta)S^{-1} - N^{-1}YB_u \\ D_c(\theta) = \hat{D}(\theta) \\ C_c(\theta) = [\hat{C}(\theta) - D_c(\theta)C_yX]M^{-T} \\ B_c(\theta) = N^{-1}[\hat{B}(\theta) - YB_uD_c(\theta)] \\ A_c(\theta) = N^{-1}[\hat{A}(\theta) - NB_c(\theta)C_yX - YB_uC_c(\theta)M^T \\ \quad -Y(A(\theta) + B_uD_c(\theta)C_y)X]M^{-T} \end{cases} \tag{19}$$

where M and N verify $MN^T = I - XY$, solves the problem defined in Section II-C.

$$\begin{bmatrix} \mathscr{L}_{11}(\theta) & \mathscr{L}_{12}(\theta) & \mathscr{L}_{13}(\theta) & \mathscr{L}_{14}(\theta) \\ * & \mathscr{L}_{22}(\theta) & \mathscr{L}_{23}(\theta) & \mathscr{L}_{24}(\theta) \\ * & * & \mathscr{L}_{33}(\theta) & \mathscr{L}_{34}(\theta) \\ * & * & * & \mathscr{L}_{44}(\theta) \end{bmatrix} \prec 0 \tag{20}$$

$$\begin{bmatrix} \mathscr{O}_{11}(\theta) & \mathscr{O}_{12}(\theta) & \mathscr{O}_{13}(\theta) & \mathscr{O}_{14}(\theta) \\ * & \mathscr{O}_{22}(\theta) & \mathscr{O}_{23}(\theta) & \mathscr{O}_{24}(\theta) \\ * & * & \mathscr{O}_{33}(\theta) & \mathscr{O}_{34}(\theta) \\ * & * & * & \mathscr{O}_{44}(\theta) \end{bmatrix} \prec 0 \tag{21}$$

$$\begin{bmatrix} X & * & * \\ I & Y & * \\ \hat{C}_i(\theta) - \hat{Z}_{1i}(\theta) & (\hat{D}(\theta)C_y)_i - \hat{Z}_{2i}(\theta) & \bar{u}_i^2 \end{bmatrix} \succeq 0 \quad \text{for } i = 1 : m \tag{22}$$

$$\begin{bmatrix} X & * & * \\ I & Y & * \\ H_{1i}X & H_{1i} & h_{0i}^2 \end{bmatrix} \succeq 0 \quad \text{for } i = 1 : s \tag{23}$$

$$\beta_2\delta - \beta_1 \prec 0 \tag{24}$$

where

$$
\begin{aligned}
&\mathscr{L}_{11}(\theta) = A(\theta)X + XA(\theta)^T + B_u\hat{C}(\theta) + \hat{C}(\theta)^T B_u^T + \beta_1 X\\
&\mathscr{L}_{12}(\theta) = A(\theta) + \hat{A}(\theta)^T + B_u\hat{D}(\theta)C_y + \beta_1 I_n\\
&\mathscr{L}_{13}(\theta) = -B_u S + \hat{Z}_1(\theta)^T, \mathscr{L}_{14}(\theta) = B_u\hat{D}(\theta)D_{yw} + B_w(\theta)\\
&\mathscr{L}_{22}(\theta) = YA(\theta) + A(\theta)^T Y + \hat{B}(\theta)C_y + C_y^T\hat{B}(\theta)^T + \beta_1 Y\\
&\mathscr{L}_{23}(\theta) = -\hat{Q}(\theta) + \hat{Z}_2(\theta)^T, \mathscr{L}_{24}(\theta) = \hat{B}(\theta)D_{yw} + YB_w(\theta)\\
&\mathscr{L}_{33}(\theta) = -2S, \mathscr{L}_{34}(\theta) = \hat{D}(\theta)D_{yw}, \mathscr{L}_{44}(\theta) = -\beta_2 I\\
&\mathscr{O}_{11}(\theta) = A(\theta)X + XA(\theta)^T + B_u\hat{C}(\theta) + \hat{C}(\theta)^T B_u^T\\
&\mathscr{O}_{12}(\theta) = \hat{A}(\theta)^T + A(\theta) + B_u\hat{D}(\theta)C_y\\
&\mathscr{O}_{13}(\theta) = B_w(\theta) + B_u\hat{D}(\theta)D_{yw}\\
&\mathscr{O}_{14}(\theta) = XC_z(\theta)^T + \hat{C}(\theta)^T D_{zu}^T\\
&\mathscr{O}_{22}(\theta) = YA(\theta) + A(\theta)^T Y + \hat{B}(\theta)C_y + C_y^T\hat{B}(\theta)^T\\
&\mathscr{O}_{23}(\theta) = YB_w(\theta) + \hat{B}(\theta)D_{yw}\\
&\mathscr{O}_{24}(\theta) = C_z(\theta)^T + C_y^T\hat{D}(\theta)^T D_{zu}^T, \mathscr{O}_{33}(\theta) = -\gamma I_m\\
&\mathscr{O}_{34}(\theta) = D_{zw}(\theta)^T + D_{yw}^T\hat{D}(\theta)^T D_{zu}^T, \mathscr{O}_{44}(\theta) = -\gamma I_p
\end{aligned}
\tag{25}
$$

Proof of theorem 1

Sufficient condition for stability - related to problem (i)

First, we look for the stability condition for the closed-loop system with controller (10). From (15) and (18), by employing the *S*-procedure, if there exist a positive definite matrix T and positive scalars β_1 and β_2 such that

$$
\frac{dV}{dt} + \beta_1(\xi^T P\xi - 1) + \beta_2(\delta - w^T w) - 2\psi(y_c)^T T \times \left(\psi(y_c) - \begin{bmatrix} \mathscr{G}(\theta) & 0 & \mathscr{K}_w(\theta) \end{bmatrix} \begin{bmatrix} \xi \\ \psi(y_c) \\ w \end{bmatrix}\right) < 0
\tag{26}
$$

then it follows that $\dot{V} < 0$, for all ξ in the boundary of $\mathscr{E}$ that belongs to the region S_θ, and for all $w \in W$. Hence, in order to ensure that $\mathscr{E}$ is a W-invariant set, we must also satisfy:

$$
\mathscr{E} \subset S_\theta \tag{27}
$$

The condition (26) is in fact guaranteed if both following inequalities hold [20]

$$
\frac{dV}{dt} + \beta_1\xi^T P\xi - \beta_2 w^T w - 2\psi(y_c)^T T_1 \times \left(\psi(y_c) - \begin{bmatrix} \mathscr{G}(\theta) & 0 & \mathscr{K}_w(\theta) \end{bmatrix} \begin{bmatrix} \xi \\ \psi(y_c) \\ w \end{bmatrix}\right) < 0
\tag{28}
$$

$$
\beta_2\delta - \beta_1 < 0 \tag{29}
$$

The condition (28) is equivalent to the matrix inequality (33). Note that (33) is not an LMI in terms of β_1, β_2, P, T and the controller matrices A_c, B_c, C_c, D_c, E_c. By first assuming that β_1 is known and applying some congruence transformations, similar to the ones proposed in [18], we show in the sequel that (33) is equivalent to (20). With this aim, let P and P^{-1} be partitioned as follows

$$
P = \begin{bmatrix} Y & N \\ N^T & \bullet \end{bmatrix} \text{ and } P^{-1} = \begin{bmatrix} X & M \\ M^T & \bullet \end{bmatrix} \tag{30}
$$

and define the matrices

$$
\Pi = \begin{bmatrix} X & I \\ M^T & 0 \end{bmatrix}, S = T^{-1} \tag{31}
$$

$$
\begin{cases}
\hat{A}(\theta) = NA_c(\theta)M^T + NB_c(\theta)C_y X + YB_u C_c(\theta)M^T \\
\qquad\qquad + Y(A(\theta) + B_u D_c(\theta)C_y)X \\
\hat{B}(\theta) = NB_c(\theta) + YB_u D_c(\theta) \\
\hat{C}(\theta) = C_c(\theta)M^T + D_c(\theta)C_y X \\
\hat{D}(\theta) = D_c(\theta) \\
\hat{Z}_1(\theta) = G_1(\theta)X + G_2(\theta)M^T \\
\hat{Z}_2(\theta) = G_1(\theta) \\
\hat{Q}(\theta) = YB_u S + NE_c(\theta)S
\end{cases}
\tag{32}
$$

Pre and post-multiplying (33) by $diag(\Pi^T, S, I)$ and its transpose, we obtain the LMI (34) (which corresponds exactly to the LMI (20)).

On the other hand, it can be seen that the following inequality implies $\mathscr{E} \subset S_\theta$:

$$
\begin{bmatrix} P & * \\ \mathscr{K}_i(\theta) - \mathscr{G}_i(\theta) & \bar{u}_i^2 \end{bmatrix} \succeq 0, \quad i = 1,..,m \tag{35}
$$

Pre and post-multiplying (35) by $diag(\Pi^T, 1)$, we obtain LMI (22).

State constraint - related to problem (ii) To ensure the state constraint (13) is not violated, it suffices to guarantee the inclusion of W-invariant set $\mathscr{E}$ in the practical validity region $\mathscr{X}$. Similarly to the previous manipulation, the inequality

$$
\begin{bmatrix} P & * \\ H_i & h_{0i}^2 \end{bmatrix} \succeq 0 \tag{36}
$$

implies that $\mathscr{E} \subset \mathscr{X}$. Pre and post-multiplying (36) by $diag(\Pi^T, 1)$ and its transpose, one obtains the LMI (23).

Sufficient condition of L_2 gain performance in linear mode (without saturation) - related to problem (iii)

Consider now $\dot{V}$ computed with the unconstrained system, i.e. satisfying (3), and the following inequality

$$
\frac{dV}{dt} + \frac{1}{\gamma}z^T z - \gamma w^T w < 0 \tag{37}
$$

Following the same steps as in [18], we can show that (21) ensures that (37) is verified. Hence, we can conclude that the L_2 gain of the unconstrained system is smaller than γ.

□

V. Application to semi-active suspension control

A. Quarter car model

Consider a simple quarter vehicle model made up of a sprung mass (m_s) and an unsprung mass (m_{us}). A spring with the stiffness coefficient k_s and a semi-active damper connect these two masses. The wheel tire is modeled by a spring with the stiffness coefficient k_t. In this model, z_s (respectively z_{us}) is the vertical position of m_s (respectively m_{us}) and z_r is the road profile. It is assumed that the wheel-road contact is ensured. The dynamical equations of a quarter vehicle are given by

$$
\begin{aligned}
m_s\ddot{z}_s &= -k_s z_{def} - F_{damper} \\
m_{us}\ddot{z}_{us} &= k_s z_{def} + F_{damper} - k_t(z_{us} - z_r) - c_t(\dot{z}_{us} - \dot{z}_r)
\end{aligned}
\tag{38}
$$

$$\begin{bmatrix} sym(P\mathscr{A}(\theta))+\beta_1 P & -P(\mathscr{B}_u+\mathscr{R}E_c(\theta))+\mathscr{G}^T(\theta)T & P\mathscr{B}(\theta) \\ * & -2T & T\mathscr{K}_w(\theta) \\ * & * & -\beta_2 I \end{bmatrix} < 0 \tag{33}$$

$$\begin{bmatrix} sym(A(\theta)X+B_u\hat{C}(\theta))+\beta_1 X & A(\theta)+\hat{A}(\theta)^T+B_u\hat{D}(\theta)C_y+\beta_1 I_n & -B_uS+\hat{Z}_1(\theta)^T & B_u\hat{D}(\theta)D_{yw}+B_w(\theta) \\ * & sym(YA(\theta)+\hat{B}(\theta)C_y)+\beta_1 Y & -\hat{Q}(\theta)+\hat{Z}_2(\theta)^T & \hat{B}(\theta)D_{yw}+YB_w(\theta) \\ * & * & -2S & \hat{D}(\theta)D_{yw} \\ * & * & * & -\beta_2 I \end{bmatrix} \prec 0 \tag{34}$$

where $z_{def} = z_s - z_{us}$ is the damper deflection (m) (assumed to be measured or estimated), $\dot{z}_{def} = \dot{z}_s - \dot{z}_{us}$ is the deflection velocity (m/s) (can be directly computed from z_{def}) and F_{damper}, the damper force, is given as follows:

$$F_{damper} = c\dot{z}_{def} \tag{39}$$

The passivity constraint of a semi-active damper is

$$0 \leqslant c_{min} \leqslant c \leqslant c_{max} \tag{40}$$

Rewrite $F_{damper} = c_{nom}\dot{z}_{def} + u\dot{z}_{def} = c_{nom}\dot{z}_{def} + u\theta$, where $c_{nom} = (c_{max}+c_{min})/2$ and $\theta = \dot{z}_{def}$ considered as a scheduling parameter. We suppose that the absolute deflection velocity $|\dot{z}_{def}|$ is smaller than 1.2 m/s, hence θ is within the range $[-1.2, 1.2]$. Note that the knowledge of this bound is necessary, when using the polytopic approach.

It can be seen that u is the control input and the passivity constraint is now recast into the *saturation constraint*

$$|u| < (c_{max} - c_{min})/2 \tag{41}$$

The quarter vehicle used in this paper is the "Renault Mégane Coupé" model whose specific parameters are: m_s = 315 kg, m_{us} = 37.5 kg, k_s = 29500 N/m, k_t = 210000 N/m, c_t=100 Ns/m. The damping coefficient varies between c_{min} = 700 Ns/m and c_{max} = 5000 Ns/m. The maximum suspension deflection is 0.125 m (which corresponds to the *state constraint*).

B. State-space representation and control objective

The state-space representation of the quarter car model is given by

$$\begin{aligned} \dot{x_s} &= A_s x_s + B_{s1} w + B_{s2} u \\ z &= C_z x_s + D_z u \\ y &= C_s x_s \end{aligned} \tag{42}$$

where $x_s = (z_s - z_{us}, \dot{z}_s, z_{us} - z_r, \dot{z}_{us})^T$, $w = \dot{z}_r$, $z = \ddot{z}_s$, $y = (z_s - z_{us}, \dot{z}_s - \dot{z}_{us})^T$.

$$A_s = \begin{bmatrix} 0 & 1 & 0 & -1 \\ \frac{-k_s}{m_s} & \frac{-c_{nom}}{m_s} & 0 & \frac{c_{nom}}{m_s} \\ 0 & 0 & 0 & 1 \\ \frac{k_s}{m_{us}} & \frac{c_{nom}}{m_{us}} & \frac{-k_t}{m_{us}} & \frac{-c_{nom}-c_t}{m_{us}} \end{bmatrix}, \mathrm{D}_z = \begin{bmatrix} \frac{-\theta}{m_s} \end{bmatrix}$$

$$B_{s1} = \begin{bmatrix} 0 & 0 & -1 & \frac{c_t}{m_{us}} \end{bmatrix}, B_{s2} = \begin{bmatrix} 0 & \frac{-\theta}{m_s} & 0 & \frac{\theta}{m_{us}} \end{bmatrix}$$

$$C_z = \begin{bmatrix} \frac{-k_s}{m_s} & \frac{-c_{nom}}{m_s} & 0 & \frac{c_{nom}}{m_s} \end{bmatrix}, C_s = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix}.$$

Note that the input matrices B_{s2} and D_z are parameter dependent so the *Assumption 1* is not guaranteed. Adding a strict low-pass filter on the control input as in [16], the system can be represented in such a way that *Assumption 1* is satisfied.

In this preliminary study, the only state constraint is the suspension deflection constraint $|z_{def}| < 0.125$ m (because we suppose that the bound of θ is guaranteed during the work of the damper). Hence, in (13), $H = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}$ and $h_0 = 0.125$.

We aim at improving the passenger comfort by minimizing the disturbance attenuation level γ of the closed-loop transfer function from w (the road disturbance) to z (the car acceleration $\ddot{z}_s$) (while taking into account the constraints on the system input and states). To enhance the performance, the weighting function on z is chosen (using the optimization procedure in [21])

$$W_z(s) = \frac{0.4901s^2 + 1563s + 360.9}{s^2 + 217.7s + 788.9} \tag{43}$$

The augmented system (42)-(43) is written in the form of (1) and is used for the controller synthesis.

C. Simulation Results

A common example of road disturbance is described by

$$z_r = \begin{cases} \pm\frac{A}{2}\left(1-\cos\left(\frac{2\pi V}{L}t\right)\right), & 0 \le t \le \frac{L}{V} \\ 0, & t > \frac{L}{V} \end{cases}$$

where A and L the height and length, V the vehicle velocity, "+" a bump, "-" a pothole. As in figure 1, we consider a road profile with a bump where $A = 0.15$ m, $L = 5$ m, $V = 27$ km/h at 0 s (corresponding to a low frequency disturbance) and a pothole where $A = 0.055$ m, $L = 5$ m, $V = 72$ km/h at 2.5 s (corresponding to a high frequency disturbance). The chosen road profile corresponds to a disturbance satisfying Assumption 2, with $\delta = 0.5$ m^2/s^2.

As seen in Fig. 2-3, we can improve the passenger comfort (by minimizing the peak value of the car body acceleration) of the closed-loop system with the proposed method w.r.t the passive open-loop cases (Soft Damper ($c = c_{min}$), Hard Damper ($c = c_{max}$) and Nominal Damper ($c = c_{nom}$)). Observe that between 2.5 s and 3 s, the control effectively saturates, but the stability is kept. Indeed, during the saturation, the anti-windup acts and the performance does not degrades. Furthermore, it should be noticed that the limits of the suspension travel and the validity for the LPV system are not violated by the trajectory.

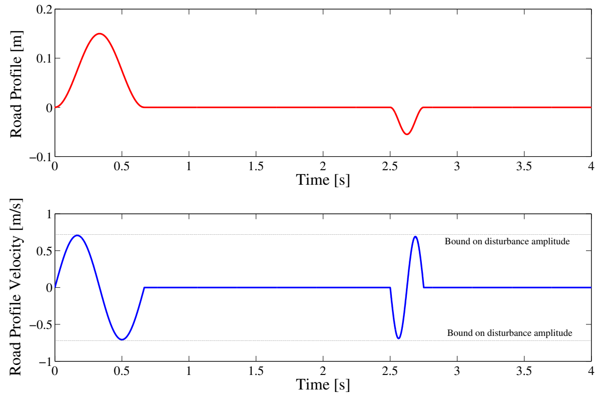

Fig. 1. Road Profile.

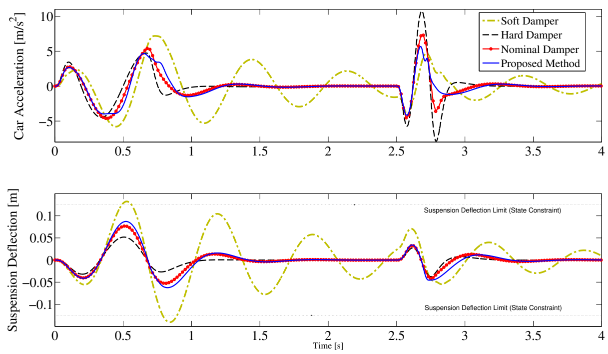

Fig. 2. Performances Comparison.

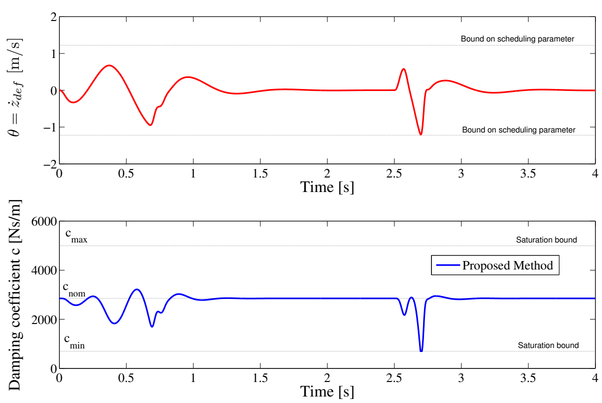

Fig. 3. Scheduling parameter and saturation constraint.

VI. Conclusions

The contribution of the paper is twofold: the proposition of an LMI method to synthesize LPV controllers taking into account input saturation and state constraints; and the application of the method in a semi-active suspension control problem. The simulation results have shown the efficiency of the proposed methodology w.r.t several passive cases. For future work, the application for semi-active suspension control will also be extended further i.e the optimization of comfort, road holding and suspension deflection.

References

[1] M. V. Kothare, P. J. Campo, M. Morari, and C. N. Nett, "A unified framework for the study of anti-windup designs," *Automatica*, vol. 30, pp. 1869–1883, 1994.

[2] G. Grimm, J. Hatfield, I. Postlethwaite, A. Teel, M. Turner, and L. Zaccarian, "Antiwindup for stable linear systems with input saturation: An LMI-based synthesis," *IEEE Transaction on Automatic Control*, vol. 48, no. 9, pp. 1509–1525, september 2003.

[3] J. M. Gomes da Silva Jr., D. Limon, T. Alamo, and E. F. Camacho, "Dynamic output feedback for discrete-time systems under amplitude and rate actuator constraints," *IEEE Transaction on Automatic Control*, vol. 53, no. 10, pp. 2367–2372, 2008.

[4] E. F. Mulder, P. Y. Tiwari, and M. V. Kothare, "Simultaneous linear and anti-windup controller synthesis using multiobjective convex optimization," *Automatica*, vol. 45, pp. 805–811, 2009.

[5] J. M. Gomes da Silva Jr. and S. Tarbouriech, "Local stabilization of discrete-time linear systems with saturating controls: An lmi-based approach," *IEEE Transaction on Automatic Control*, vol. 46, no. 1, pp. 119–125, 2001.

[6] F. Wu, K. M. Grigoriadis, and A. Packard, "Anti-windup controller design using linear parameter-varying control methods," *International Journal of Control*, vol. 73, no. 12, pp. 1104–1114, 2000.

[7] T. Hu, Z. Lin, and B. M. Chen, "Analysis and design for discrete-time linear systems subject to actuator saturation," *System & Control Letters*, vol. 45, pp. 97–112, 2002.

[8] J. M. Gomes da Silva Jr. and S. Tarbouriech, "Antiwindup design with guaranteed regions of stability: An lmi-based approach," *IEEE Transaction on Automatic Control*, vol. 50, no. 1, pp. 106–111, 2005.

[9] F. Wu and B. Lu, "Anti-windup control design for exponentially unstable LTI systems with actuator saturation," *System & Control Letters*, vol. 52, pp. 305–322, 2004.

[10] V. F. Montagner, J. M. Gomes da Silva Jr., and P. L. D. Peres, "Regional stabilization of switched systems subject to input saturation," in *Proceedings of the 3rd IFAC Symposium on System Structure and Control (SSSC)*, 2007.

[11] V. F. Montagner, R. C. L. F. Oliveira, P. L. D. Peres, S. Tarbouriech, and I. Queinnec, "Gain-scheduled controllers for linear parameter-varying systems with saturating actuators: Lmi-based design," in *Proceedings of the IEEE American Control Conference (ACC)*, New York City, USA, July 2007, pp. 6067–6072.

[12] Y.-Y. Cao, Z. Lin, and Y. Shamash, "Set invariance analysis and gain-scheduling control for lpv systems subject to actuator saturation," in *Proceedings of the IEEE American Control Conference (ACC)*, Anchorage, AK, May 2002, pp. 668–673.

[13] T. D. Gillespie, *Fundamentals of Vehicle Dynamics*. Society of Automotive Engineers Inc, 1992.

[14] U. Kiencke and L. Nielsen, *Automotive Control Systems for Engine, Driveline, and Vehicle*. Springer Verlag, 2000.

[15] S. Savaresi, C. Poussot-Vassal, C. Spelta, O. Sename, and L. Dugard, *Semi-Active Suspension Control for Vehicles*. Elsevier, 2010.

[16] A. L. Do, O. Sename, and L. Dugard, "An LPV control approach for semi-active suspension control with actuator constraints," in *Proceedings of the IEEE American Control Conference (ACC)*, Baltimore, Maryland, USA, June 2010.

[17] A. L. Do, C. Spelta, S. Savaresi, O. Sename, L. Dugard, and D. Delvecchio, "An LPV control approach for comfort and suspension travel improvements of semi-active suspension systems," in *Proceedings of the 49th IEEE Conference on Decision and Control (CDC)*, Atlanta, GA, December 2010, pp. 5660–5665.

[18] C. Scherer, P. Gahinet, and M. Chilali, "Multiobjective output-feedback control via LMI optimization," *IEEE Transaction on Automatic Control*, vol. 42, no. 7, pp. 896–911, july 1997.

[19] F. Blanchini, "Set invariance in control," *Automatica*, vol. 35, no. 11, pp. 1747–1767, 1999.

[20] S. Boyd, L. E. Ghaoui, E. Feron, and V. Balakrishnan, *Linear Matrix Inequalities in System and Control Theory*. SIAM Studies in Applied Mathematics, 1994.

[21] A. L. Do, O. Sename, L. Dugard, and B. Soualmi, "Multi-objective optimization by genetic algorithms in $\mathscr{H}_\infty$/LPV control of semi-active suspension," in *Proceedings of the 18th IFAC World Congress (WC)*, Milan, Italy, September 2011.