2011 50th IEEE Conference on Decision and Control and
European Control Conference (CDC-ECC)
Orlando, FL, USA, December 12-15, 2011

Improving parameter estimation using minimal analytically redundant subsystems

D. Garcia-Alvarez, A. Bregon, M.J. Fuente and B. Pulido

***Abstract*— This work presents a novel parameter estimation approach for system modelling based on model decomposition. This approach uses Possible Conflicts to decompose the system model into minimal *submodels* that are used to obtain minimal parameter estimators for non-faulty situations. A laboratory plant was used to test the approach. The results obtained were compared against two classical parameter estimation techniques, the SQP optimization method and a curve-fitting approach using non-linear least squares. Both classical approaches use the global simulation model of the plant to carry out the optimization. The properties of the three techniques are presented and discussed. The developed parameter estimation approach improves the results obtained with the cited classical approaches.**

I. INTRODUCTION

One of the most important stages in the process simulation field is model validation. Model validation can be defined as the procedure to determine whether a simulation model is an accurate representation of the real system fulfilling some specific objectives. The simulation model has to be able to replace the real system in order to satisfy a set of pre-established requirements. A notable task into this phase consists of comparing the real system outputs against the simulation model.

The process to design simulation models, similar to other development processes, is an iterative procedure [1]. When a simulation model is obtained and its conceptual correctness has been verified, the validation phase begins. In the validation phase, the response of the real system and the simulation model are compared under equivalent working conditions. Then, it can be decided if the model has to be changed or the tuning needs to be improved.

Typically, even though the model is empirically correct and it is able to capture the process trends for the operating points studied, the behaviour is not accurate with respect to the real plant response. In these situations, it is necessary to fit the simulation model based on the real process data. This task is known as parameter estimation.

The parameter estimation tries to solve problems related to discrepancies between the real system, affected by uncertainty sources, and the simulation model, based on theoretical principles. The most common cost function used by parameter estimation schemes is the minimization of the squared error between the real plant outputs and the simulated model outputs. The decision variables are the estimated parameters. An important task that needs to be performed by the model developers is to decide which parameters have to be estimated and which parameters have to be set to a fixed value. Usually, estimated parameters are those parameters subject to uncertainty.

The parameter estimation procedure is a very intense computational problem, especially for nonlinear systems. Moreover, computational complexity increases exponentially with the size of the system. In real industrial plants, the number of parameters to be estimated is usually very big. Consequently, the computational effort for parameter estimation in real plants can be very high.

For system modelling, the parameter estimation process is performed only once in a modelling and simulation approach, but in those cases the parameter estimation techniques can be applied to highly detailed large scale modelled plants (for example in training operators simulators), where the solution can be unreachable.

In Fault Tolerant Control [2], the system identification procedure is performed *on-line* several times. Hence, a quick and robust system identification approach is needed, and classical techniques can be unable to fulfill these requirements. Also, data reconciliation [3] approaches require several runs of the optimization algorithms using simulation models in a similar way to the parameter estimation techniques. The problem of parameter estimation must be simplified in order to reduce the computational effort.

In [4], a model decomposition approach to reduce the computational burden of the faulty parameter identification was proposed. The main idea was to use Possible Conflicts [5] to decompose the global estimation problem into smaller estimation tasks, called minimal faulty parameter estimators. Empirical studies showed that the minimal faulty parameter estimators provide faster and more accurate fault identification for nonlinear systems. The present work uses similar ideas, and presents a novel approach for system identification using actual parameter estimation in non-faulty situations. The main goal is to reduce the estimation time while being accurate in those systems where a timely online system identification process is required.

In this work, real data from a two-tank laboratory plant are used. Using such data, online parameter estimation for system identification is carried out. First, two well-known parameter estimation techniques, SQP optimization and non-

D. Garcia-Alvarez and M.J. Fuente's work has been partially supported by Spanish government grant DPI2009-14410-C02-02. Anibal Bregon and Belarmino Pulido's work has been partially supported by the Spanish MCI TIN2009-11326 and DPI2008-01996 grants

D. Garcia-Alvarez and M.J. Fuente are with the Department of Systems Engineering and Automatic Control, University of Valladolid, Valladolid, Spain dieggar@cta.uva.es, maria@autom.uva.es

Anibal Bregon and Belarmino Pulido are with the Department of Computer Science, University of Valladolid, Valladolid, Spain anibal@infor.uva.es, belar@infor.uva.es

linear least squares [6], are used to estimate the parameters of the plant. Then, model decomposition using Possible Conflicts is applied together with the non-linear least squares approach. Efficiency and accuracy for parameter estimation with each one of these techniques is studied. Results showed that using Possible Conflicts, efficiency and accuracy of the parameter estimation task is highly improved.

The rest of the paper is organized as follows: Section II presents the laboratory plant that were used as the case study, and the experiments that were carried out. Section III describes the different parameter estimation configurations considering the global system model. Section IV describes our parameter estimation in a non-faulty situation approach using minimal parameter estimators. Section V shows the results obtained for each one of the parameter estimation techniques. Finally, Section VI presents the main conclusions and future work.

II. CASE STUDY

A. Physical Configuration

Theoretical concepts will be tested on a real laboratory plant, made up of two cylindrical tanks T_1 and T_2, connected through a narrow cylindrical pipe placed 5 cm over the tanks bases. There are two drain pipes at the same height than the connection pipes. Fig. 1 shows a scheme of the laboratory plant.

The main goal of the control strategy in this plant is to maintain the level of the tanks close to the level selected by the operator. The level of both tanks is measured by two level sensors. The water is drained into the tanks by two variable velocity pumps.

The control strategy is performed by two PI controllers. The PC and the sensors and actuators are connected using an I/O card (CIO-DA16). The communication between the card and Simulink©, that supports the control system, is based on the OPC communication protocol. The I/O card works as an OPC server using a VC++ application developed specifically for this card. Simulink© works as a client using the OPC Simulink© blocks provided by the Matlab© OPC library. Further information about the physical configuration of the plant can be found in [7].

B. Mathematical model

The mathematical model of the laboratory plant is based on first principles, i.e. based on mass balances. The model has been implemented using Simulink©.

Flows drained through drain pipes in both tanks (q_{10} and q_{20}) are defined as follows:

$$q_{10} = C_{10} S_n \sqrt{2gh_1} \tag{1}$$

$$q_{20} = C_{20} S_n \sqrt{2gh_2} \tag{2}$$

where g is the force of gravity; S_n is the section of the drain pipes; and, h_1 and h_2 represent the level in tanks.

Both tanks are connected through a pipe whose flow q_{12} is expressed using the *Torricelli* theorem:

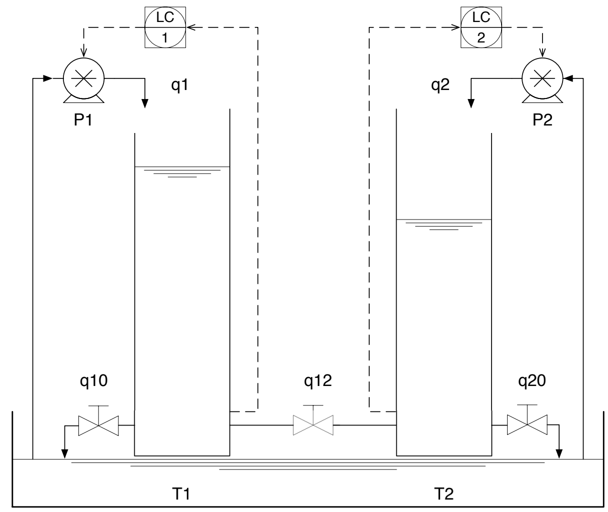

Fig. 1. Schema of the laboratory plant

$$q_{12} = C_{12} S_n \text{sign}(h_1 - h_2)\sqrt{2g|h_1 - h_2|} \tag{3}$$

The function $sign$ in equation 3 represents the direction of the flow inside the pipe given the heights of tanks T_1 and T_2: h_1 and h_2.

The variation in the level of the tanks is given by the following mathematical expressions:

$$A\frac{dh_1}{dt} = q_1 - q_{12} - q_{10} \tag{4}$$

$$A\frac{dh_2}{dt} = q_2 + q_{12} - q_{20} \tag{5}$$

where A is the area of the tanks bases.

Table I shows the physical parameters that appear in all the previous equations.

TABLE I

PHYSICAL PARAMETERS OF THE LABORATORY PLANT.

Parameter	Value
S_n	0.5 cm
A	314 cm^2

Parameters C_{10}, C_{20} and C_{12} in equations 1, 2 and 3 are constant, and they model the flow inside the three pipes. These will be the three estimated parameters. The presence of manual valves are the source of uncertainty for parameters C_{10}, C_{20} and C_{12}.

C. Experiments design

A set of close-loop experiments were designed to perform the tuning of parameters C_{10}, C_{20} and C_{12}. The set of experiments explores the different operating points of the plant through random changes in the reference in both tanks.

The experiments consider the different stationary states in the process with long time periods without changes in the references. Also the transient states were explored with changes in the references with continuous high frequencies. Figure 2 shows an example of these experiments.

Typical experiments for fault detection and diagnosis tasks were used in order to validate the parameters calculated.

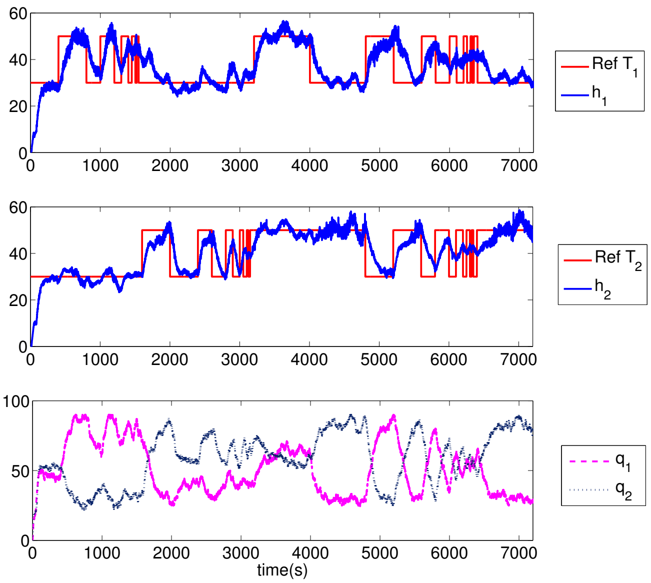

Fig. 2. Closed loop experiments

III. PARAMETER ESTIMATION

This section describes two classical approaches that have been applied to the case study to carry out the parameter estimation: the sequential quadratic programming (SQP) optimization approach, and a curve-fitting approach using non-linear least squares. Both configurations can be seen as equivalent but the cost function of each method is adapted to use the suitable Matlab© command.

$\mathbf{X}$ is the input vector for the system. In our case study $\mathbf{X} = [\mathbf{q_1}\ \mathbf{q_2}]$ represents the input flows obtained for pumps p_1 and p_2. The outputs from the model are the measured level in the tanks $\mathbf{Y} = [\mathbf{h_1}\ \mathbf{h_2}]$. The parameter vector to be optimized is $\theta = [C_{12}\ C_{10}\ C_{20}]$.

A. SQP

The first configuration used to estimate the parameters is a dynamic optimization approach. Fig. 3 shows a flow chart with the basic idea of the approach.

The cost function of this solution consists of a function whose arguments are the value of the parameters. The cost function runs the simulation experiment by means of a callback to the model using the parameters received. The model uses the designed experiments and the parameters in order to generate an output. When the simulation ends, the model returns the output with the current parameters values, and the cost function computes the sum of the squared error between the real plant output and the simulated plant output:

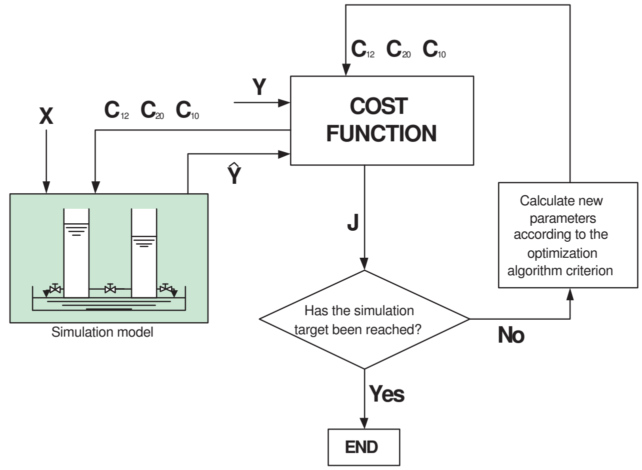

Fig. 3. Dynamic optimization solution.

$$\min_{\theta} \sum_{i=1}^{T_{max}} \sum_{j=1}^{2} (y_{i,j} - \hat{y}_{i,j})^2 \qquad (6)$$

The optimization problem posed in this configuration has been solved using an implementation of sequential quadratic programming (SQP) method. This method is suitable for non-linear optimization problems. The Matlab© command that encapsulate this optimization method is the *fmincon* command [8].

Table II shows the selected optimization options, the maximum number of iterations, and maximum number of function evaluations. The table also shows the initial value of the parameters (θ_0). The termination criteria for the optimization algorithm depends on either the number of iterations or the function evaluations. The optimization algorithm terminates if if the tolerances shown in the table reach the minimum value selected.

TABLE II

PARAMETERS FOR SQP SOLUTION.

Parameter	**Value**
Algorithm	SQP (*active set*)
θ_0 $[C_{12}\ C_{10}\ C_{20}]$	$[0.5\ 0.5\ 0.5]$
θ_{min} $[C_{12}\ C_{10}\ C_{20}]$	$[10^{-2}\ 10^{-2}\ 10^{-2}]$
θ_{max} $[C_{12}\ C_{10}\ C_{20}]$	$[10\ 10\ 10]$
Termination criteria	
Maximum number of iterations allowed	400
Maximum number of function evaluations	600
Termination tolerance on the function value	10^{-6}
Termination tolerance on the parameters	10^{-6}
Termination tolerance on the constraint violation	10^{-6}

B. Non-linear least squares

The second method proposed for the parameter estimation problem considering the whole model is based on a curve-fitting by means of non-linear least-squares. This function is implemented by the Matlab© function *lsqcurvefit* [8].

This solution is equivalent to the previous one but some changes must be performed in order to use correctly the command. Function F that appears in equation 7 receives as arguments the parameters and the input data of the designed experiments $\mathbf{X} = [\mathbf{q_1}\ \mathbf{q_2}]$. This function returns the estimated value of the tank levels $\hat{\mathbf{Y}} = [\hat{\mathbf{h_1}}\hat{\mathbf{h_2}}]$. The quadratic difference between the $\hat{\mathbf{Y}}$ and $\mathbf{Y}$ is the target to minimize in this configuration.

$$\min_{\theta} \sum_{i} (F(\theta, \mathbf{X}) - \mathbf{Y})^2 \tag{7}$$

The command *lsqcurvefit* solves optimization problems using non-linear least-squares. This method is based on the inner reflexive Newton method [8]. In this case an approximation to a big lineal system solution is calculated using the Preconditioned Conjugate Gradient (PCG) method.

TABLE III

PARAMETERS FOR NON-LINEAR LEAST SQUARES SOLUTION.

Parameter	Value
Algorithm	*Non-linear least squares*
θ_0 $[C_{12}\ C_{10}\ C_{20}]$	[0.1 0.1 0.1]
θ_{min} $[C_{12}\ C_{10}\ C_{20}]$	$[10^{-2}\ 10^{-2}\ 10^{-2}]$
θ_{max} $[C_{12}\ C_{10}\ C_{20}]$	[10 10 10]
Termination criteria	
Maximum number of iterations allowed	400
Maximum number of function evaluations	600
Termination tolerance on the function value	10^{-6}
Termination tolerance on the parameters	10^{-6}
Termination tolerance on the constraint violation	10^{-6}

Table III shows the parameters selected for the optimization method for this configuration. Non linear least squares algorithm was not able to converge to the true solution using same initial conditions and number of iterations that the SQP approach. Even using values as shown in table III the performance of this method was worse than SQP as shown in table IV. Both the accuracy in the estimation and the computation time is slightly better with the SQP approach than with the non-linear least squares. The main problem regarding this second solution is related with the maximum number of function evaluations, that is reached without being able to find the best solution.

Even testing in a small plant the non-linear least squares method shows problems in convergence as the results show. So, this method is used in order to prove the improvement in the parameter estimations using model decomposition.

IV. MODEL DECOMPOSITION FOR PARAMETER ESTIMATION

The solution presented in this work for parameter estimation in the modelled laboratory plant is based on the minimal parameter estimators technique [4], [9]. The main contribution of the minimal parameter estimators deals with the size of the cost function rather than the optimization technique itself. The basic idea is to decompose the global estimation problem into smaller optimization tasks, later, each one of these optimization tasks can be carried out using dynamic optimization using SQP, least-squares curve-fitting, or any other optimization technique.

TABLE IV

ESTIMATION RESULTS FOR THE SQP AND THE NON-LINEAR LEAST SQUARES APPROACHES.

Parameter	SQP	Non-linear least squares
CPU mean time	$375.05\ s$	$509.99\ s$
Number of iterations	73	150
Number of function evaluations	444	600
Total error	99033.94	102375.81
Parameter estimated	[0.1076 0.1992 0.1976]	[0.1029 0.1984 0.1969]
Termination criterion	Tolerance on the parameters	Maximum number of function evaluations

The minimal parameter estimators are computed from the set of Possible Conflicts, PCs [5], of a system. The PCs approach is a dependency-compilation technique from the Artificial Intelligence, DX, community equivalent to the Analytical Redundancy Relations (ARR) [5], [10], [11]. PCs are computed *off-line* through variable elimination. Each PC identifies a minimal over-determined set of equations that can be solved using only observed, i.e. measured variables. The PCs are those subsystems from the global system model that can lead to a conflict when a fault occurs within the Consistency-based Diagnosis framework [12]. Because PCs are over-determined sets of equations, thus providing the analytical redundancy necessary to perform fault detection and diagnosis.

At the same time, these subsystem can be used for a more efficient parameter estimation task due to the decoupling of the system model, allowing to define cost functions with a smaller number of parameters than the global system model [9]. The minimal parameter estimators method adapts a similar approach developed within the Moriarty system (developed by NASA [13], [14]) that used the concept of Model-based Decomposition for parameter learning within an *on-line* reconfiguration process. PCs are conceptually equivalent to Dissents [5], used by Williams and Millar [13], [14] for model-based parameter learning in Moriarty.

The main advantage regarding the use of minimal parameter estimators relies on the use of smaller fragments of the system, what entails having faster and more simple estimation tasks (this is specially significant when highly non-linear systems are considered). Moreover, since minimal parameter estimators contain only a subset of the system model and measurements, the estimations will be less influenced by noisy measurements.

PCs are made up of a subset of equations, one output variable, that is the variable estimated by the PC, and a subset of input variables, that are the input and measured variables of the system. A PC, PC_k, is described in state space form as follows:

$$\dot{\hat{x}}_{pc_k} = f_{pc_k}(x_{pc_k}, u_{pc_k}, \theta_{pc_k})$$

$$\hat{y}_{pc_k} = g_{pc_k}(x_{pc_k}, u_{pc_k}, \theta_{pc_k})$$

where f_{pc_k} and g_{pc_k} are nonlinear functions; $\hat{x}_{pc_k}$ and u_{pc_k} are the state and input vectors, respectively; $\hat{y}_{pc_k}$ is the variable estimated by the PC; and θ_{pc_k} is the subset of parameters for PC_k.

Based on the model description shown in section II a set of two Possible Conflicts have been found (shown in table V). These Possible Conflicts are minimal w.r.t. the set of constraints in the models. In the table, the first column shows the PCs, the second column illustrate the set of components or sensors used by the PC, the third column describes the set of equations or support of the PC, and finally the fourth column presents the variable estimated by the PC.

TABLE V

PCS FOUND FOR THE PLANT: COMPONENTS AND ESTIMATED VARIABLE FOR EACH POSSIBLE CONFLICT

PC_x	Components or Sensors	Eq. Support	$Estimate_x$
PC_1	T_1, h_1, h_2	Eq. 1, Eq. 3, Eq. 4	h_1
PC_2	T_2, h_1, h_2	Eq. 2, Eq. 3, Eq. 5	h_2

Once the equations of the Possible Conflicts are computed from the global system model, these equations are used to compute a parameterized function, F_{pc_k}, where the parameter to be estimated, θ_i, is set as an input variable. Then, F_{pc_k} can be used to solve non-linear optimization problems for parameter θ_i as established by the following proposition, [9]:

Proposition 1: A Possible Conflict, PC_k, and a set of input variables for PC_k, X_{pc_k}, can be used as a parameter estimator, $\hat{y}_{pc_k} = F_{pc_k}(X_{pc_k}, \theta_i)$, by using the measured variable estimated by the Possible Conflict, $\hat{y}_{pc_k}$, and solving $\hat{y}_{pc_k}$ based on the rest of the measured variables.

Figure 4 shows a flow diagram describing this proposed solution. This algorithm has to be run twice because there are two PCs. As shown in Figure 4, main difference with Figure 3 is that PC models need to be computed, but is done only once, and off-line. Once computed, the minimal parameter estimators define the cost functions F_{pc_k} used in Figure 4.

The non-linear least squares approach (described in the previous section) has been used in order to solve the curve-fitting problem for each one of the minimal estimators. In this case, the optimization algorithm parameters are those described in Table III, but in this case it was not necessary to modify the initial value of the parameters, that was initially set to 0.5 for all the cases. The cost function used to minimize the error in the estimation is described as follows:

$$\min_{\theta_i} \sum (F_{pc_k}(X_{pc_k}, \theta_i) - Y_{pc_k})^2 \tag{8}$$

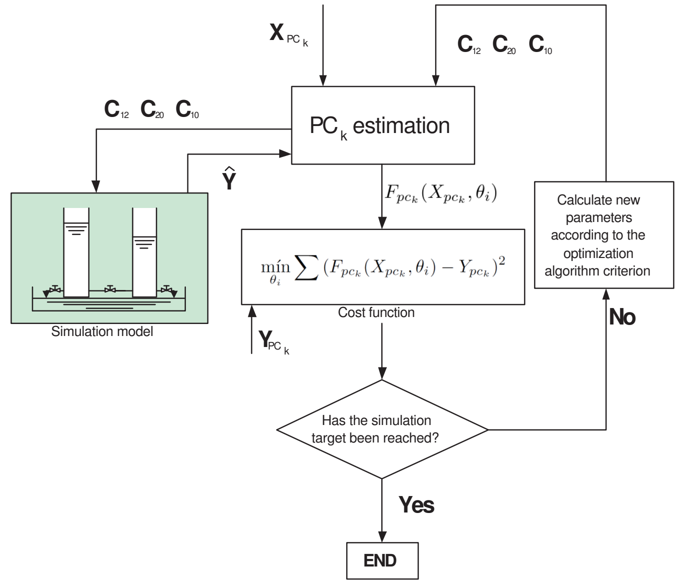

Fig. 4. Minimal parameter estimators solution

V. RESULTS

Table VI compares the performance of each one of the three considered solutions when applied to different scenarios of the laboratory plant. Table VI contains the following information:

- The CPU mean time employed by each solution. This value was computed by measuring the CPU time of 10 experiments run using a PC Intel© core duo ($2,53$ GHz) and 3 Gb of memory.
- The number of iterations.
- The total number of cost function evaluations employed by each optimization approach to solve the optimization problem.
- The global quadratic error in the estimation committed by each one of the three solutions for the best case solution.
- The value of the parameters vector estimated.
- The termination criterion for each method.

Regarding the minimal parameter estimators, since two Possible Conflicts were found for this system (one for each measurement), two minimal parameter estimators were implemented. Results related to minimal parameter estimators (shown in last column) show the result for both minimal parameter estimators, i.e., the number of iterations employed by the first minimal estimator was 15, and the number of iterations employed by the second minimal estimator was 14. Then, the total amount of iterations employed by this approach was 15+14, as described in the table.

Columns 2 and 3 in Table VI show the results for SQP and Curve-fitting approaches, respectively. Both techniques used the global model of the system to perform the parameter estimation. Results show that both approaches are very similar in terms of accuracy, since the estimation error obtained with each approach is very similar. However, the computational effort to obtain the same accuracy results is smaller for the

TABLE VI
RESULTS

Parameter	Dynamic optimization	Curve-fitting	Minimal parameter estimators
CPU mean time	375.05 s	509.99 s	28.54 s for PC_1 + 25.82 s for PC_2
Number of iterations	73	150	15 for PC_1 + 14 for PC_2
Number of function evaluations	444	600	48 for PC_1 + 45 for PC_2
Total error	99033.94	102375.81	60120.5
Parameter estimated	[0.1076 0.1992 0.1976]	[0.1029 0.1984 0.1969]	[0.0998 0.1940 0.1933]
Termination criterion	Tolerance on the parameters	Maximum number of function evaluations	Tolerance on the parameters

SQP than the Curve-fitting approach. On the other hand, the results for the minimal parameter estimators show that the CPU mean computation time was highly decreased, whereas the total error in the estimation was also decreased. Moreover, looking at the number of iterations and the number of function evaluations, it is possible to see that the approach converged much faster that the other two techniques.

The termination criterion was the tolerance on the parameters in the approaches of SQP solution and minimal parameter estimators (meaning that the parameter value between one iteration and the previous one was lower than the tolerance fixed for this case). The termination criterion in the curve-fitting solution was reached because the maximum number of cost function evaluations was raised. The minimal parameter estimators used the curve-fitting optimization algorithm to perform the parameter estimation. Hence, the results obtained with this real data experiments are very significative from a practical point of view. It is possible to see the great advantage that the approach of system decomposition to compute minimal parameter estimators is able to obtain: faster convergence to the true solutions, smaller computation time, and more accurate results.

VI. CONCLUSIONS

In this work a parameter estimation approach based on model decomposition has been presented, and the estimation results for a laboratory plant using real data have been obtained and compared against two classical estimation techniques: SQP and non-linear least squares for the entire system.

The main conclusion of this work is that applying Possible Conflicts to decompose the system model generates smaller estimation tasks that can be used for fast convergence of the parameter estimation task for on-line estimation. Moreover, results with real data also showed that the estimations obtained with the minimal estimators approach are more accurate than the estimations obtained with the other two approaches.

As future work, we are planning to run the SQP approach together with the minimal estimators. Our guess (based on the comparison between the SQP and the non-linear least squares approaches) is that the minimal parameter estimators implemented using dynamic optimization, will improve even more the current estimation results. Moreover, the comparative study with real data from a more complex case study with high nonlinearities will be carried out to see how the decomposition approach behaves when nonlinearities arise.

REFERENCES

[1] W.D. Kelton A.M. Law. *Simulation and Modeling Analisys*. Mc Graw Hill, 2000.

[2] M. Blanke, M. Kinnaert, J. Schröder, and J. Lunze. *Diagnosis and fault-tolerant control*. Springer, 2003.

[3] C. Crowe. Data reconciliation Progress and challenges. *Journal of Process Control*, 6(2-3):89–98, June 1996.

[4] A. Bregon, G. Biswas, and B. Pulido. A Decomposition Method for Nonlinear Parameter Estimation in TRANSCEND. *IEEE Transactions on Systems, Man, and Cybernetics. Part A: Systems and Humans*, to appear.

[5] B. Pulido and C. Alonso-González. Possible Conflicts: a compilation technique for consistency-based diagnosis. *IEEE Trans. on Systems, Man, and Cybernetics. Part B: Cybernetics*, 34(5):2192–2206, Octubre 2004.

[6] H.W. Sorenson. *Parameter Estimation. Principles and Problems*. Mc Graw Hill, 1980.

[7] M.J. Fuente, G. Garcia, and G.I. Sainz. Fault diagnosis in a plant using Fisher discriminant analysis. *Proceding of the 16th Mediterranean Conference on Control and Automation Congress Centre, Ajaccio, France*, pages 53–58, 2008.

[8] *MATLAB & Simulink Release Notes for R2009b*.

[9] A. Bregon. *Integration of FDI and DX Techniques within Consistency-based Diagnosis with Possible Conflicts*. PhD thesis, Departamento de Informática. Universidad de Valladolid., 2010.

[10] M.O. Cordier, P. Dague, F. Lévy, J. Montmain, and M. Staroswieckiand L. Travé-Massuyès. Conflicts versus Analytical Redundancy Relations: a comparativeanalysis of the Model-based Diagnosis approach from the Artificial Intelligence and Automatic Control perspectives. *IEEE Trans. on Systems, Man, and Cybernetics. Part B: Cybernetics*, 34(5):2163–2177, 2004.

[11] J. Armengol, A. Bregon, T. Escobet, E. Gelso, M. Krysander, M. Nyberg, X. Olive, B. Pulido, and L. Travé-Massuyès. Minimal Structurally Overdetermined sets for residual generation: A comparison of alternative approaches. In *Proceedings of the 7th IFAC Symposium on Fault Detection, Supervision and Safety of Technical Processes, SAFEPROCESS09*, pages 1480–1485, Barcelona, Spain, 2009.

[12] R. Reiter. A Theory of Diagnosis from First Principles. *Artificial Intelligence*, 32:57–95, 1987.

[13] B.C. Williams and B. Millar. Automated decomposition of Model-based Learning Problems. In *Proceedings of the 7th International Workshop on Principles of Diagnosis, DX96*, pages 258–266, Val Morin, Quebec, Canada, 1996.

[14] B.C. Williams and B. Millar. Decompositional Model-based Learning and its analogy to diagnosis. In *Proceedings of the 15th National Conference on Artificial Intelligence, AAAI98*, Madison, Wisconsin, USA, 1998.