2011 50th IEEE Conference on Decision and Control and
European Control Conference (CDC-ECC)
Orlando, FL, USA, December 12-15, 2011

Adaptive Control for Continuous-Time Systems in the Presence of Actuator and Sensor Hysteresis

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***Abstract*— This paper dicusses the output tracking control for a continuous-time linear plant containing uncertain hysteresis nonlinearities in actuator and sensor devices simultaneously, where the hysteresis is described by Prandtl-Ishlinskii model. A new adaptive control scheme is developed to compensate the plant, the actuator and the sensor uncertainties and to generate an adaptive estimate of the plant output. The proposed control law ensures the uniform boundedness of all signals in the closed-loop system. The tracking error between the estimated plant output and the desired output is guaranteed to converge to zero asymptotically.**

I. INTRODUCTION

The hysteresis phenomenon occurs in all the smart material-based actuators and sensors [1], [2], [4]-[11], [13]-[24], [26]. When the hysteresis nonlinearity exists in the controlled system, the system usually exhibits undesirable inaccuracies or oscillations and even instability. For the plants preceded by hysteresis which means that the system is driven by actuators with hysteresis, the control problem has received considerable attention recently. The common control approach is to construct an inverse hysteresis model to compensate the effect of the hysteresis [10], [13], [14], [24], [26]. Essentially, the inversion problem depends on hysteresis modeling methods. Some hysteretic nonlinearities are very complicated with multivalues and non-smooth features, such as those in piezo-electric actuators and magnetostrictive actuators, where the operator-based hysteresis models are generally applied. The hysteresis cancellation by the direct inversion will result in compensation errors, which may cause difficulties in stability analysis for the closed-loop system. To avoid such difficulties, some new approaches have been proposed in the literature [4], [5], [23]. Instead of directly constructing the inversion from the operator-based hysteresis model, an implicit inversion was introduced in [4] [5]for the convenience of stability analysis of the closed-loop systems.

For sensor failure detection and identification research, tremendous effort has been devoted recently. One typical design method for control of systems with sensor failures is based on the neural networks and sensor redundancy, which is rendered by the measurements from multiple sensors [16], [22]. Another typical method is reported recently in [15], where sensor characteristics are modeled as parametrizable uncertain functions and a compensator is constructed to adaptively cancel the effects of sensor uncertainties and to generate an adaptive estimate of the plant output. For the control systems measured by sensors with hysteresis uncertainties, the development of the controllers is still a challenging task [20].

In [20], the regulation problem for the MIMO plants in the presence of actuator and sensor uncertain hysteresis nonlinearities is studied, where the class of considered hysteresis can be described by ten parameters. Since the smart material-based actuators and sensors are widely used recently, the control for plants containing general hysteresis nonlinearities in both actuator and sensor devices need to be developed. In this paper, the output control for a continuous-time linear plant containing uncertain hysteresis in actuator and sensor devices is studied, where the hysterisis is expressed by Prandtl-Ishlinskii (PI) model. The adoption of PI model is based on the fact that it can describe the hysteretic nonlinearities existing in smart materials, especially the typical hysteresis behavior in piezo materials [13], [14].

In this paper, an adaptive control scheme is developed to compensate the plant, the actuator and the sensor uncertainties and to generate an adaptive estimate of the plant output, where the formulation of the inverse operator of the PI model is employed. Only the parameters directly needed in the formulation of the controller are adaptively estimated online. The proposed control law ensures the uniform boundedness of all signals in the closed-loop system. Furthermore, the tracking error between the estimated plant output and the desired output is guaranteed to converge to zero asymptotically. Generally, the zero convergence of the tracking error between the genuine plant output and the desired output can not be guaranteed except for the regulation problem (i.e. the desired plant output is zero). Finally, the proposed algorithm is illustrated by computer simulations.

II. PROBLEM STATEMENT

A. System Description

Consider the adaptive control for the continuous-time systems driven by an actuator with hysteresis and measured by a sensor with hysteresis shown in Fig. 1.

This work is partially supported by the Grants-in-Aid for Scientific Research of Japan Society for the Promotion of Science (JSPS) (No. 21560474), and the Research Foundation for the Electrotechnology of Chubu (REFEC).

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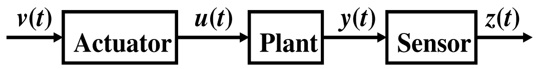

Fig. 1. Block scheme of the considered system.

The considered system is described by

$$P(s)[y](t) = k_p Z(s)[u](t), \quad (1)$$
$$u(t) = H_1[v](t), \quad (2)$$
$$z(t) = H_2[y](t), \quad (3)$$

where $y(t) \in R$ is the output of the linear plant which is also the input of the sensor, $u(t) \in R$ is the input of the linear plant which is also the output of the actuator, $v(t) \in R$ is the input to the actuator, $z(t) \in R$ is the output of the sensor which is also the measured output of the linear plant, $H_1[\cdot]$ and $H_1[\cdot]$ are the PI hysteresis operators which will be given later, k_p is the high frequency gain, $P(s), Z(s)$ are described by the following polynomials.

$$P(s) = s^{n_0} + p_{n_0-1}s^{n_0-1} + \cdots + p_1 s + p_0, \quad (4)$$
$$Z(s) = s^m + z_{m-1}s^{m-1} + \cdots + z_1 s + z_0, \;\; n_0 > m \quad (5)$$

The control purpose is to drive the output $y(t)$ of the linear plant to track the output $y_m(t)$ of the reference model described by

$$P_m(s)[y_m](t) = q(t), \quad (6)$$

where $P_m(s)$ is a monic polynomial with degree $n^* = n_0 - m$, $q(t)$ is the input of the reference model.
We make the following assumptions for the control system.

- A1: $Z(s)$ is a stable polynomial.
- A2: The upper bound for the degree n_0 of $P(s)$ is known as n.
- A3: The sign of the plant high frequency gain k_p is known.
- A4: The degree of n^* of $P_m(s)$ is known.

B. Hysteresis Model

The PI hysteresis operators will be introduced. The basic element of the PI operator is the so-called stop operator and play operator with threshold r. For arbitrary piece-wise monotone function $v(t)$, define $e_r : R \to R$ and $f_r : R \times R \to R$ as

$$e_r(v) = min(r, \; max(-r, v)), \quad (7)$$
$$f_r(v, \alpha) = max(v - r, \; min(v + r, \alpha)), \quad (8)$$

where $\alpha \in R$ can be any value. For any initial value $w_{-1} \in \bar{R}$ and $r \geq 0$, the stop operator $E_r[*; w_{-1}](t)$ and the play operator $F_r[*; w_{-1}](t)$ are respectively defined as

$$E_r[v; w_{-1}](0) = e_r(v(0) - w_{-1}), \quad (9)$$
$$E_r[v; w_{-1}](t) = e_r(v(t) - v(t_i) + E_r[v; w_{-1}](t_i)), \quad (10)$$
$$F_r[v; w_{-1}](0) = f_r(v(0) - w_{-1}), \quad (11)$$
$$F_r[v; w_{-1}](t) = f_r(v, F_r[v; w_{-1}](t_i)), \quad (12)$$

for $t_i \leq t \leq t_{i+1}$, where the function $v(t)$ is monotone for $t_i \leq t \leq t_{i+1}$ [3], [12], [19], [27]. It can be seen that

$$E_r[v; w_{-1}](t) + F_r[v; w_{-1}](t) = v(t). \quad (13)$$

The stop and play operators are rate-independent which are mainly characterized by the threshold parameter $r \geq 0$. For simplicity, denote $E_r[v; w_{-1}](t)$ and $F_r[v; w_{-1}](t)$ by respectively $E_r[v](t)$ and $F_r[v](t)$ in the following of this paper. The PI hysteresis operator $u(t) = H_1[v](t)$ is defined by

$$u(t) = r_0 v(t) + \int_0^{\bar{R}} p(r) F_r[v](t) dr. \quad (14)$$

where r_0 is a positive constant, $p(r)$ is the density function which is usually unknown, satisfying $p(r) \geq 0$ with $\int_0^\infty p(r)dr > 0$ and $\lim_{r\to\infty} p(r) = 0$. Since $p(r)$ vanishes for large r, $\bar{R}$ can be chosen as a sufficiently large positive value.

Figure 1 shows the relation between $v(t)$ and $u(t)$ for $0 \leq t \leq 10$ given by model (14) with $r_0 = 0.5$, $p(r) = e^{-0.067(r-1)^2}$, $\bar{R} = 20$, $u_{-1} = 0$ and $v(t) = 7\frac{sin(3t)}{1+t}$. It can be seen that the PI model (14) indeed generates the hysteresis curves and can be considered to be well-suited to describe the rate-independent hysteretic behavior.

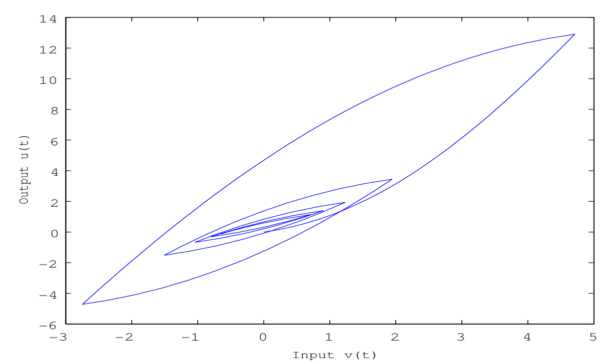

Fig. 2. Hysteresis curves given by model (14).

Similarly, for any initial value $z_{-1} \in R$, the hysteresis operator $z(t) = H_2[y](t)$ is defined by

$$z(t) = s_0 y(t) + \int_0^{\bar{R}} \bar{s}(r) F_r[y](t) dr. \quad (15)$$

where s_0 is a positive constant, $\bar{s}(r)$ is the density function satisfying $\bar{s}(r) \geq 0$ with $\int_0^\infty \bar{s}(r)dr \geq 0$ and $\lim_{r\to\infty} \bar{s}(r) = 0$.

Now, let us consider the inverse operator of $H_2[\cdot]$. The following lemma is cited (see [13] and Chapter 2 in [3]).

Lemma 1: For the operator $H_2[\cdot]$ defined in (15), there exists a density function $s(r) \geq 0$ satisfying $\int_0^\infty s(r)dr \geq 0$ such that

$$y(t) = \frac{1}{s_0} z(t) - \int_0^{\bar{R}} s(r) F_r[z](t) dr. \quad (16)$$

III. Adaptive Control

A. Some Preliminaries

In the following, without loss of generality, assume that $s_0 = r_0 = 1$. Otherwise, by observing (1), (14) and (16), $k_p r_0 s_0$ can be treated as k_p, $\frac{1}{r_0} p(r)$ can be treated as $p(r)$, $s_0 s(r)$ can be treated as $s(r)$.

To begin with, introduce an $(n-1)th$ order monic stable polynomial $\Lambda(s)$ and define $a(s) = [1, s, ..., s^{n-2}]^T$. Now, consider the polynomial equation

$$\theta_1^T a(s)P(s) + \Big(\theta_2^T a(s) + \theta_{20}\Lambda(s)\Big)k_p Z(s)$$

$$=\Lambda(s)\Big(P(s) - \theta_3 k_p Z(s)P_m(s)\Big), \tag{17}$$

where $\theta_1 \in R^{n-1}$, $\theta_2 \in R^{n-1}$, $\theta_{20} \in R$, and $\theta_3 = k_p^{-1} \in R$ are the parameters and exist uniquely (see Lemma 5.1 in [25]). Operating both sides of (17) on $y(t)$ and applying (1) yields

$$\theta_1^T a(s)Z(s)[u](t) + \Big(\theta_2^T a(s) + \theta_{20}\Lambda(s)\Big)Z(s)[y](t)$$

$$=\Lambda(s)Z(s)[u](t) - \theta_3\Lambda(s)Z(s)P_m(s)[y](t) \tag{18}$$

By observing that the polynomials $\Lambda(s)$ and $Z(s)$ are stable, the relation between the input and the output of the linear plant can thus be expressed as

$$u(t) = \theta_1^T \frac{a(s)}{\Lambda(s)}[u](t) + \theta_2^T \frac{a(s)}{\Lambda(s)}[y](t) + \theta_{20}y(t) + \theta_3 P_m(s)[y](t) \tag{19}$$

where an exponential decaying term is omitted [25]. Thus, by substituting (14) and (16) into (19), it gives

$$v(t) + \int_0^{\bar{R}} p(r)F_r[v](t)dr$$

$$=\theta_1^T \frac{a(s)}{\Lambda(s)}[v](t) + \int_0^{\bar{R}} p(r)\theta_1^T \frac{a(s)}{\Lambda(s)}[F_r[v]](t)dr$$

$$+\theta_2^T \frac{a(s)}{\Lambda(s)}[z](t) - \int_0^{\bar{R}} s(r)\theta_2^T \frac{a(s)}{\Lambda(s)}[F_r[z]](t)dr$$

$$+\theta_{20}z(t) - \int_0^{\bar{R}} \theta_{20}s(r)F_r[z](t)dr + \theta_3 P_m(s)[y](t). \tag{20}$$

Therefore, it can be easily seen that the model reference control of the linear plant can be achieved if the control input $v(t)$ to the hysteresis is chosen such that the following equation holds

$$v(t) + \int_0^{\bar{R}} p(r)F_r[v](t)dr$$

$$=\theta_1^T \frac{a(s)}{\Lambda(s)}[v](t) + \int_0^{\bar{R}} p(r)\theta_1^T \frac{a(s)}{\Lambda(s)}[F_r[v]](t)dr$$

$$+\theta_2^T \frac{a(s)}{\Lambda(s)}[z](t) - \int_0^{\bar{R}} s(r)\theta_2^T \frac{a(s)}{\Lambda(s)}[F_r[z]](t)dr$$

$$+\theta_{20}z(t) - \int_0^{\bar{R}} \theta_{20}s(r)F_r[z](t)dr + \theta_3 q(t), \tag{21}$$

where $q(t)$ is defined in (6).

B. Adaptive Control Design

Since the parameters in $P(s)$ and $Z(s)$ are unknown, the parameters θ_1, θ_2, θ_{20}, θ_3 can not be obtained. Furthermore, since the parameters and the density functions in (14) and (16) are all unkown, the control scheme in (21) can not be implemented. To overcome this difficulty, the adaptive method will be used to estimate the unknown parameters needed in the control scheme.

Suppose that the estimates of $p(r)$, θ_1, $p(r)\theta_1$, θ_2, $s(r)\theta_2$, θ_{20}, $\theta_{20}s(r)$ and θ_3 are respectively $\hat{p}(r,t)$, $\hat{\theta}_1(t)$, $\hat{p}_1(r,t)$, $\hat{\theta}_2(t)$, $\hat{s}_2(r,t)$, $\hat{\theta}_{20}(t)$, $\hat{s}_{20}(r,t)$ and $\hat{\theta}_3(t)$ at instant t, where the estimate $\hat{p}(r,t)$ should be determined such that $\hat{p}(r,t) \geq 0$ for all r and t.

With these estimates, by observing (21), the design task is to find a signal $v(t)$ as an input of the hysteresis so that the following equation holds

$$v(t) + \int_0^{\bar{R}} \hat{p}(r,t)F_r[v](t)dr = W(t), \tag{22}$$

where $W(t)$ is defined as

$$W(t) =\hat{\theta}_1^T(t)\frac{a(s)}{\Lambda(s)}[v](t) + \int_0^{\bar{R}} \hat{p}_1^T(r,t)\frac{a(s)}{\Lambda(s)}[F_r[v]](t)dr$$

$$+\hat{\theta}_2^T(t)\frac{a(s)}{\Lambda(s)}[z](t) - \int_0^{\bar{R}} \hat{s}_2^T(r,t)\frac{a(s)}{\Lambda(s)}[F_r[z]](t)dr$$

$$+\hat{\theta}_{20}^T(t)z(t) - \int_0^{\bar{R}} \hat{s}_{20}(r,t)F_r[z](t)dr + \hat{\theta}_3(t)q(t). \tag{23}$$

Without loss of generality, suppose $W(t)$ is monotonically increasing on the interval $t_i \leq t \leq t_{i+1}$. For each $t \in [t_i, t_{i+1}]$, define a new variable $\bar{V}_\mu(t)$ with $\bar{V}_0(t) = v(t_i)$ and another new variable $W_\mu(t)$, where μ is a positive parameter

$$\bar{V}_\mu(t) = \bar{V}_0(t) + \mu, \tag{24}$$

$$W_\mu(t) = \bar{V}_\mu(t) + \int_0^{\bar{R}} \hat{p}(r,t)F_r[\bar{V}_\mu(t)]dr. \tag{25}$$

The value of $v(t)$ is derived from the following algorithm.

Step 1: Let μ increase from 0.

Step 2: Calculate $\bar{V}_\mu(t)$ and $W_\mu(t)$. If $W_\mu(t) < W(t)$, then let μ increase continuously and go to Step 2; Otherwise, go to Step 3.

Step 3: Stop the increasing of μ, memorize it as μ_0 and let $v(t) = \bar{V}_{\mu_0}(t)$.

For $t = 0$, $\bar{V}_0(0)$ can be defined as $\bar{V}_0(0) = v_{min}$, where v_{min} is the admissible minimum value of $v(t)$. The calculated $v(t)$ is called the "implicit inversion" of $W(t)$.

To apply the adaptive control law derived from (22), it is necessary to develop algorithms to estimate the required parameters $\hat{p}(r,t)$, $\hat{\theta}_1(t)$, $\hat{p}_1(r,t)$, $\hat{\theta}_2(t)$, $\hat{s}_2(r,t)$, $\hat{\theta}_{20}(t)$, $\hat{s}_{20}(r,t)$ and $\hat{\theta}_3(t)$.

Now, define the estimated plant output as

$$\hat{y}(t) = z(t) - \int_0^{\bar{R}} \hat{s}(r,t) F_r[z](t)dr, \tag{26}$$

where $\hat{s}(r,t)$ is the estimate of $s(r)$ at instant t, $\hat{s}(r,t)$ should be determined such that $\hat{p}(r,t) \geq 0$ for all r and t. Define the error

$$e(t) = \hat{y}(t) - y_m(t) \tag{27}$$

Therefore, from (20) and (22), it yields

$$\begin{aligned} e(t) =& \hat{y}(t) - y(t) + y(t) - y_m(t) \\ =& -\int_0^{\bar{R}} \tilde{s}(r,t) F_r[z](t)dr \\ &+ \frac{k_p}{P_m(s)} \Big\{ -\int_0^{\bar{R}} \tilde{p}(r,t) F_r[v](t)dr \\ &+ \tilde{\theta}_1^T(t) \frac{a(s)}{\Lambda(s)}[v](t) + \int_0^{\bar{R}} \tilde{p}_1^T(r,t) \frac{a(s)}{\Lambda(s)}[F_r[v]](t)dr \\ &+ \tilde{\theta}_2^T(t) \frac{a(s)}{\Lambda(s)}[z](t) - \int_0^{\bar{R}} \tilde{s}_2^T(r,t) \frac{a(s)}{\Lambda(s)}[F_r[z]](t)dr \\ &+ \tilde{\theta}_{20}^T(t) z(t) - \int_0^{\bar{R}} \tilde{s}_{20}(r,t) F_r[z](t)dr + \tilde{\theta}_3(t) q(t) \Big\}, \end{aligned} \tag{28}$$

with

$$\begin{aligned} \tilde{s}(r,t) =& \hat{s}(r,t) - s(r) \\ \tilde{p}(r,t) =& \hat{p}(r,t) - p(r) \\ \tilde{\theta}_1(t) =& \hat{\theta}_1(t) - \theta_1 \\ \tilde{p}_1(r,t) =& \hat{p}_1(r,t) - p(r)\theta_1 \\ \tilde{\theta}_2(t) =& \hat{\theta}_2(t) - \theta_2 \\ \tilde{s}_2(r,t) =& \hat{s}_2(r,t) - s(r)\theta_2 \\ \tilde{\theta}_{20}(t) =& \hat{\theta}_{20}(t) - \theta_{20} \\ \tilde{s}_{20}(r,t) =& \hat{s}_{20}(r,t) - s(r)\theta_{20} \\ \tilde{\theta}_3(t) =& \hat{\theta}_3(t) - \theta_3 \end{aligned}$$

Let the estimate of k_p be $\hat{k}_p(t)$ at instant t. Now, define a new error $\varepsilon(t)$ as

$$\varepsilon(t) = e(t) + \hat{k}_p(t)\xi(t) \tag{29}$$

with

$$\begin{aligned} \xi(t) =& \frac{1}{P_m(s)} \int_0^{\bar{R}} \hat{p}(r,t) F_r[v](t)dr - \int_0^{\bar{R}} \hat{p}(r,t) \frac{1}{P_m(s)} F_r[v](t)dr \\ &- \frac{1}{P_m(s)} \hat{\theta}_1^T(t) \frac{a(s)}{\Lambda(s)}[v](t) + \hat{\theta}_1^T(t) \frac{1}{P_m(s)} \frac{a(s)}{\Lambda(s)}[v](t) \\ &- \frac{1}{P_m(s)} \int_0^{\bar{R}} \hat{p}_1^T(r,t) \frac{a(s)}{\Lambda(s)}[F_r[v]](t)dr \\ &+ \int_0^{\bar{R}} \hat{p}_1^T(r,t) \frac{1}{P_m(s)} \frac{a(s)}{\Lambda(s)}[F_r[v]](t)dr \\ &- \frac{1}{P_m(s)} \hat{\theta}_2^T(t) \frac{a(s)}{\Lambda(s)}[z](t) + \hat{\theta}_2^T(t) \frac{1}{P_m(s)} \frac{a(s)}{\Lambda(s)}[z](t) \\ &+ \frac{1}{P_m(s)} \int_0^{\bar{R}} \hat{s}_2^T(r,t) \frac{a(s)}{\Lambda(s)}[F_r[z]](t)dr \\ &- \int_0^{\bar{R}} \hat{s}_2^T(r,t) \frac{1}{P_m(s)} \frac{a(s)}{\Lambda(s)}[F_r[z]](t)dr \\ &- \frac{1}{P_m(s)} \hat{\theta}_{20}^T(t) z(t) + \hat{\theta}_{20}^T(t) \frac{1}{P_m(s)} z(t) \\ &+ \frac{1}{P_m(s)} \int_0^{\bar{R}} \hat{s}_{20}(r,t) F_r[z](t)dr \\ &- \int_0^{\bar{R}} \hat{s}_{20}(r,t) \frac{1}{P_m(s)} F_r[z](t)dr \\ &- \frac{1}{P_m(s)} \hat{\theta}_3(t) q(t) + \hat{\theta}_3(t) \frac{1}{P_m(s)} q(t). \end{aligned} \tag{30}$$

Thus, substituting (28) into (29) yields

$$\begin{aligned} \varepsilon(t) =& -\int_0^{\bar{R}} \tilde{s}(r,t) F_r[z](t)dr - \int_0^{\bar{R}} \tilde{p}(r,t) \frac{k_p}{P_m(s)} F_r[v](t)dr \\ &+ \tilde{\theta}_1^T(t) \frac{k_p}{P_m(s)} \frac{a(s)}{\Lambda(s)}[v](t) \\ &+ \int_0^{\bar{R}} \tilde{p}_1^T(r,t) \frac{k_p}{P_m(s)} \frac{a(s)}{\Lambda(s)}[F_r[v]](t)dr \end{aligned}$$

$$+\tilde{\theta}_2^T(t)\frac{k_p}{P_m(s)}\frac{a(s)}{\Lambda(s)}[z](t)$$

$$-\int_0^{\bar{R}}\tilde{s}_2^T(r,t)\frac{k_p}{P_m(s)}\frac{a(s)}{\Lambda(s)}[F_r[z]](t)dr$$

$$+\tilde{\theta}_{20}^T(t)\frac{k_p}{P_m(s)}z(t)-\int_0^{\bar{R}}\tilde{s}_{20}(r,t)\frac{k_p}{P_m(s)}F_r[z](t)dr$$

$$+\tilde{\theta}_3(t)\frac{k_p}{P_m(s)}q(t)+\tilde{k}_p(t)\xi(t), \tag{31}$$

where $\tilde{k}_p(t)$ is defined as $\tilde{k}_p(t)=\hat{k}_p(t)-k_p$. Define

$$\begin{aligned}m_0(t)=&1+\int_0^{\bar{R}}F_r^2[z](t)dr+\int_0^{\bar{R}}\Big(\frac{1}{P_m(s)}F_r[v](t)\Big)^2dr\\&+\Big(\frac{1}{P_m(s)}\frac{a(s)}{\Lambda(s)}[v](t)\Big)^2\\&+\int_0^{\bar{R}}\Big(\frac{1}{P_m(s)}\frac{a(s)}{\Lambda(s)}[F_r[v]](t)\Big)^2dr\\&+\Big(\frac{1}{P_m(s)}\frac{a(s)}{\Lambda(s)}[z](t)\Big)^2\\&+\int_0^{\bar{R}}\Big(\frac{1}{P_m(s)}\frac{a(s)}{\Lambda(s)}[F_r[z]](t)\Big)^2dr\\&+\Big(\frac{1}{P_m(s)}z(t)\Big)^2+\int_0^{\bar{R}}\Big(\frac{1}{P_m(s)}F_r[z](t)\Big)^2dr\\&+\Big(\frac{1}{P_m(s)}q(t)\Big)^2+\xi^2(t)\end{aligned}\tag{32}$$

By observing the expression of $\varepsilon(t)$ in (31), the parameter adaptation law is chosen as

$$\dot{\hat{s}}(r,t)=\begin{cases}\gamma_1\frac{\varepsilon(t)}{m_0(t)}F_r[v](t) & if\ \hat{p}(r,t)>0\\ \gamma_1\frac{\varepsilon(t)}{m_0(t)}F_r[v](t) & if\ \hat{p}(r,t)=0\ and\ \varepsilon(t)F_r[v](t)>0\\ 0 & otherwise\end{cases}\tag{33}$$

$$\dot{\hat{p}}(r,t)=\begin{cases}\gamma_2\frac{sign[k_p]\varepsilon(t)}{m_0(t)}\frac{1}{P_m(s)}F_r[v](t) & if\ \hat{p}(r,t)>0\\ \gamma_2\frac{sign[k_p]\varepsilon(t)}{m_0(t)}\frac{1}{P_m(s)}F_r[v](t) & if\ \hat{p}(r,t)=0\ and\\ & sign[k_p]\varepsilon(t)\frac{1}{P_m(s)}F_r[v](t)<0\\ 0 & otherwise\end{cases}\tag{34}$$

$$\dot{\hat{\theta}}_1(t)=-\Gamma_1\frac{sign[k_p]\varepsilon(t)}{m_0(t)}\frac{1}{P_m(s)}\frac{a(s)}{\Lambda(s)}[v](t), \tag{35}$$

$$\dot{\hat{p}}_1(r,t)=-\Gamma_2\frac{sign[k_p]\varepsilon(t)}{m_0(t)}\frac{1}{P_m(s)}\frac{a(s)}{\Lambda(s)}[F_r[v]](t), \tag{36}$$

$$\dot{\hat{\theta}}_2(t)=-\Gamma_3\frac{sign[k_p]\varepsilon(t)}{m_0(t)}\frac{1}{P_m(s)}\frac{a(s)}{\Lambda(s)}[z](t), \tag{37}$$

$$\dot{\hat{s}}_2(r,t)=\Gamma_4\frac{sign[k_p]\varepsilon(t)}{m_0(t)}\frac{1}{P_m(s)}\frac{a(s)}{\Lambda(s)}[F_r[z]](t), \tag{38}$$

$$\dot{\hat{\theta}}_{20}(t)=-\gamma_3\frac{sign[k_p]\varepsilon(t)}{m_0(t)}\frac{1}{P_m(s)}[z](t), \tag{39}$$

$$\dot{\hat{s}}_{20}(r,t)=\gamma_4\frac{sign[k_p]\varepsilon(t)}{m_0(t)}\frac{1}{P_m(s)}F_r[z](t), \tag{40}$$

$$\dot{\hat{\theta}}_3(t)=-\gamma_5\frac{sign[k_p]\varepsilon(t)}{m_0(t)}\frac{1}{P_m(s)}[q](t), \tag{41}$$

$$\dot{\hat{k}}_p(t)=-\gamma_6\frac{\varepsilon(t)}{m_0(t)}\xi(t), \tag{42}$$

with $\gamma_i>0$ for $i=1,\ldots,6$, $\Gamma_j=\Gamma_j^T>0$, for $j=1,\ldots,4$.

Remark 1: From (33) and (34), it can be seen that $\hat{p}(r,t)\geq 0$ and $\hat{s}(r,t)\geq 0$ for all r and t. $\hat{s}(r,0)$ and $\hat{p}(r,0)$ should be chosen such that $\hat{s}(r,0)\geq 0$, $\hat{p}(r,0)\geq 0$, $\int_0^{\bar{R}}\hat{s}(r,0)dr<\infty$ and $\int_0^{\bar{R}}\hat{p}(r,0)dr<\infty$.

In the following, the stability of the system (1)-(3) controlled by the input derived from (22) will be analyzed.

Lemma 2: The adaptive law (33)-(42) guarantees that all the estimated parameters belong to L^∞, the time derivatives of all the etimated parameters belong to $L^2\cap L^\infty$, and $\frac{\varepsilon(t)}{\sqrt{m_0(t)}}\in L^2\cap L^\infty$.

Proof: Consider the positive definite function

$$\begin{aligned}V(t)=&\gamma_1^{-1}\int_0^{\bar{R}}\tilde{s}^2(r,t)dr+\gamma_6^{-1}\tilde{k}_p^2(t)\\&+|k_p|\Big\{\gamma_2^{-1}\int_0^{\bar{R}}\tilde{p}^2(r,t)dr+\tilde{\theta}_1^T(t)\Gamma_1^{-1}\tilde{\theta}_1(t)\\&+\int_0^{\bar{R}}\tilde{p}_1^T(r,t)\Gamma_2^{-1}\tilde{p}_1(r,t)dr+\tilde{\theta}_2^T(t)\Gamma_3^{-1}\tilde{\theta}_2(t)\\&+\int_0^{\bar{R}}\tilde{s}_2^T(r,t)\Gamma_4^{-1}\tilde{s}_2(r,t)dr+\gamma_3^{-1}\tilde{\theta}_{20}^2(t)\\&+\gamma_4^{-1}\int_0^{\bar{R}}\tilde{s}_{20}^2(r,t)dr+\gamma_5^{-1}\tilde{\theta}_3^2(t)\Big\}.\end{aligned}\tag{43}$$

By taking the time derivative of $V(t)$ along the trajectory of (33)-(42), it gives

$$\dot{V}(t) \leq -\frac{2\varepsilon^2(t)}{m_0(t)} \leq 0 \tag{44}$$

The lemma can be proved based on (44).

Lemma 3: For each t, there exists a function $\hat{\hat{s}}(r,t) \geq 0$ satisfying $\int_0^{\bar{R}} \hat{\hat{s}}(r,t)dr \leq \infty$ such that

$$z(t) = \hat{y}(t) + \int_0^{\bar{R}} \hat{\hat{s}}(r,t) F_r[\hat{y}](t)dr. \tag{45}$$

Furthermore, $\hat{\hat{s}}(r,t)$ is uniformly bounded.

Proof: From (26), relation (45) is obvious by using the results in [13] and Chapter 2 in [3]. The uniform boundedness of $\hat{\hat{s}}(r,t)$ can be obtained from Lemma 2 and [13].

Theorem 1: All the signals in the closed-loop system consisting of the plant (1), reference model (6), controller derived from (22), adaptive law (33)-(42) are bounded and the tracking error $e(t) = \hat{y}(t) - y_m(t)$ belongs to $e(t) \in L^2$ and $\lim_{t\to\infty} e(t) = 0$.

Proof: Since the proof is very complicated, it is omitted.

Corollary 1: Consider the regulation problem (i.e. $\lim_{t\to\infty} y_m = 0$) of the system meeting the conditions stated in Theorem 1. Then, all the signals in the closed-loop systems remain bounded and it holds $\lim_{t\to\infty} e(t) = 0$ and $\lim_{t\to\infty} y(t) = 0$.

Proof: Theorem 1 implies $\lim_{t\to\infty} \hat{y}(t) = 0$. By (45), it can be seen that $\lim_{t\to\infty} z(t) = 0$. Then, by (16), it can be seen that $\lim_{t\to\infty} y(t) = 0$.

From (16) and (26), the genuine plant output tracking error $y(t) - y_m(t)$ is governed by

$$y(t) - y_m(t) = \int_0^{\bar{R}} \tilde{s}(r,t) F_r[z](t)dr, \tag{46}$$

which is mainly dominated by the estimation error $\tilde{s}(r,t) = \hat{s}(r,t) - s(r)$. Since the genuine plant output $y(t)$ can not be measured, this result can be considered to be reasonable.

IV. Conclusions

This paper has discussed the adaptive control for the continuous-time linear system in the presence of actuator and sensor hysteresis, where the hysteresis is described by Prandtl-Ishlinskii model. A new adaptive control scheme is developed to compensate the plant, the actuator and the sensor uncertainties and to generate an adaptive estimate of the plant output. The proposed control law ensures the uniform boundedness of all signals in the closed-loop system. The tracking error between the estimated plant output and the desired output is guaranteed to converge to zero asymptotically. The genuine output tracking error of the plant can be guaranteed to approach to zero only in the regulation problems.

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