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Improved Feed-Forward Command Governor Strategies for Discrete-time Time-Invariant Linear Systems

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Abstract— **This paper presents a novel enhanced algorithm belonging to the class of so-called *Feed-Forward* Command Governor (FF-CG) strategies, characterized by the absence of any explicit on-line measure (or estimation) of the state, which have been recently proposed in [1], [2] for input/state constrained discrete-time linear systems subject to bounded disturbances. In fact, while in all traditional CG schemes the set-point manipulation is undertaken on the basis of either the actual measure of the state or its suitable estimation, feed-forward strategies are characterized by the fact that the CG design problem is explicitly solved, with limited performance degradation and with similar properties, in the absence of such a measure. This is achieved by forcing the system evolutions to stay "*not too far*" from the space of feasible steady-states. Although effective also in the case of bounded disturbances, the performance of earlier FF-CG schemes [1], [2] was mainly limited by the fact that such strategies were instructed to maintain constant their actions for a prescribed number of sampling steps between two subsequent CG action computation. Here such a restriction is completely removed and a novel class of FF-CG schemes able to update their actions at each sampling step is discussed. Finally, in order to evaluate the effectiveness of the proposed solution, numerical simulations on a physical plant have been undertaken and comparison results reported.**

I. Introduction

Command Governor or *Reference Governor* strategies are well established methods in the control literature thanks to their capability to enforce pointwise-in-time input and state-related constraints on system evolutions by suitably modifying, whenever necessary, commands and references acting on the plant. Many results and a variety of schemes now exist: see e.g. ([3])-([9]). In particular, RG and CG schemes dealing with disturbances are considered in [7], [8], with model uncertainties in [6], [8] and with partial state information in [9]. See [11] for applications to networked master/slave frameworks and [12] for recent results on hybrid piecewise-affine systems. Different perspectives on RG are reported in [10]. For recent applications refer to [13], [14].

The Command Governor (CG) is a nonlinear device which is added to a compensated plant whose primal controller is designed, typically without considering the presence of the prescribed constraints, so as to exhibit stability and good tracking performance. The CG main objective is that of modifying the reference signal supplied to such a pre-compensated system when its direct application would produce constraints violation. This modification is typically achieved by solving on-line a constrained QP optimization problem, where the prescribed constraints are enforced along the future plant predicted evolutions starting from the currently measured or estimated state, according to a receding horizon philosophy.

Because dealing with future state predictions, traditional CG schemes require the knowledge of either a measure or an estimate of the state to accurately compute them. However, in [1], [2] it has been shown that CG solutions based on a different philosophy, which do not explicitly exploit such an information, are possible at the price of some additional conservativeness. The idea behind such an approach is that, if sufficiently smooth transitions in the set-point modifications are acted by the CG unit, one can have a high confidence on the expected value of the state, even in the absence of an explicit measure of it, because of the asymptotical stability of the system at hands.

The peculiarities of FF-CG schemes may be of interest in all applications where either the measure or the estimation of the whole state may be difficult. For example, in decentralized multi-agent constrained supervisory schemes because, unlike the approaches based on distributed MPC ideas, they would not require the knowledge of the entire aggregate state (or part of it) at each time instant, the latter being unrealistic or requiring unrealistic communication infrastructures in some large scale applications. See [15] for distributed constrained supervision strategies for networked large-scale systems based on FF-CG ideas.

In [1], an early FF-CG scheme has been introduced and its main properties detailed. Although quite effective, it was shown to exhibit suboptimal tracking performance with respect to traditional state-based CG methods. The reason is that in such a scheme, unlike the traditional state-based CG approach, the FF-CG action is allowed to be computed and updated only at a certain integers multiple of the sampling time and its action has to be kept constant between two subsequent computations. Of course, this has a direct consequence on the tracking performance of the algorithm, especially when references with fast variations are considered.

The aim of this note is to present a novel class of enhanced FF-CG strategies where such a drawback is completely overcome in that the FF-CG action can now be computed and applied at each sampling time. Such a solution is achieved by observing that, under the same assumptions of

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[1], the uncertainty about the state evolution arising from the absence of measurements, involves only the dynamics related to the initial conditions. Such an uncertainty can be bounded and insulated and the resulting FF-CG strategy formulated as a standard state-feedback CG on the basis of a suitable feedback signal, under a restricted set of constraints to be fulfilled. Moreover, in the disturbance-free case it is proved that the performance of this particular FF-CG scheme asymptotically equals that of the standard state-based CG strategy.

The paper is organized as follows. In Section II, the CG problem is recalled and some preliminary aspects of the FF-CG approach are discussed. In Section III, the novel FF-CG scheme is introduced and completely analyzed. In Section IV, numerical simulations are reported in order to show the effectiveness of the proposed scheme by contrasting its tracking performance with those pertaining to earlier FF-CG and traditional state-based CG schemes. Some conclusions end the paper.

II. System description and Problem Formulation

Consider the following closed-loop system

$$\left\{\begin{array}{rcl} x(t+1) & = & \Phi x(t)+Gg(t)+G_d d(t) \\ y(t) & = & H_y x(t) \\ c(t) & = & H_c x(t)+Lg(t)+L_d d(t) \end{array}\right. \tag{1}$$

where: $t \in \mathbb{Z}$, $x \in \mathbb{R}^n$ is the state vector (which includes the controller states under dynamic regulation), $g \in \mathbb{R}^m$ the manipulable reference vector which, if no constraints (and no CG) were present, would coincide with the desired reference $r \in \mathbb{R}^m$ and $y \in \mathbb{R}^m$ the output vector which is required to track r. The vector $d \in \mathbb{R}^{n^d}$ is a disturbance signal assumed to belong to the closed ball $\mathcal{D} := \{d : \|d\|_2 \leq d_{max}\}$. Finally, $c \in \mathbb{R}^{n^c}$ represents the constrained vector which has to fulfill the set-membership constraint

$$c(t) \in \mathcal{C},\ \forall t \in \mathbb{Z}_+, \tag{2}$$

$\mathcal{C}$ being a polyhedral set. It is further assumed that:

A1. The overall system (1) is asymptotically stable.

A2. System (1) is off-set free, i.e. $H_y(I_n-\Phi)^{-1}G = I_m$.

Classical solutions to the CG design problem (see [4], [8], [11]) have been achieved by finding, at each time t, a CG action $g(t)$ as a function of the current reference $r(t)$ and measured state $x(t)$

$$g(t) := \underline{g}(r(t), x(t)) \tag{3}$$

such that $g(t)$ is the best approximation of $r(t)$ under the condition $c(t+k) \in \mathcal{C}$, $k = 0, .., \infty$ when $g(t)$ is constantly applied from t onwards at the reference terminal.

Here we will focus on a slight different approach to the CG design problem in which no measure of the state vector is assumed to determine $g(t)$. In order to better introduce the key ideas, let's consider temporarily the disturbance-free ($d(t) \equiv 0_{n_d}$) case and adopt the following notations for the steady-state solutions of (1) to a constant command $g(t) \equiv g,\ \forall t$

$$x_g := (I_n-\Phi)^{-1}Gg,\ y_g := H_y(I_n-\Phi)^{-1}Gg,\ c_g := H_c x_g + Lg. \tag{4}$$

The idea explicitly employed in this paper is that, if the manipulable reference signal $g(\cdot)$ is generated so as to be "slowly changing enough" w.r.t. system dynamics, then, because of system stability (see **A1**), the constrained vector $c(t)$ can always be maintained within a certain known (and "small") distance $\rho(t) > 0$ from the closed-loop steady-state equilibrium $c_{g(t)}$

$$c(t) - c_{g(t)} \in \mathcal{B}_{\rho(t)} \tag{5}$$

where $\mathcal{B}_{\rho(t)}$ represents the ball of radius $\rho(t)$ centered at the origin. This has been achieved in [1], [2] with strategies of the form

$$g(t) = \underline{g}(r(t), g(t-\tau), \rho(t-\tau)) \tag{6}$$

where $g(t)$ is computed every τ steps and it is constantly applied between two successive computations.

Here, a less conservative approach is presented in the next session. To this end, observe that, because $c_{g(t)}$ univocally depends on $g(t)$ and $\rho(t)$, it may be proven to be a function of its initial condition $\rho(0)$ and of the history of the commands from time 0 up to time $t-1$

$$g^{t-1} := \{g(t-1), g(t-2), ..., g(0)\} \tag{7}$$

Then, it is possible to conceive CG schemes where, instead of considering the dependence on the measured state $x(t)$, decisions can be taken on the basis of $\rho(0)$ and of the past values of $g(t)$, denoted

$$g(t) \quad = \quad \underline{g}(r(t), g^{t-1}, \rho(0)) \tag{8}$$

As it will be clear soon, we have not to memorize the entire sequence g^{t-1}. In fact, a suitable aggregate expression can be found which is equivalent to the knowledge of g^{t-1}.

A. The proposed improved FF-CG approach

In order to make precise statements consider the constrained closed-loop system (1)-(2) satisfying assumptions **A1-A2**. In order to simplify the developments, let us exploit the linearity of the system to separate the effects of initial conditions and commands from those of disturbances, i.e.

$$x(t) = \hat{x}(t)+\tilde{x}(t),\ c(t) = \hat{c}(t)+\tilde{c}(t),\ y(t) = \hat{y}(t)+\tilde{y}(t) \tag{9}$$

where $\hat{x}(t)$ (and the same for $\hat{c}(t)$ and $\hat{y}(t)$) is the disturbance-free component of the state (depending only on the initial state condition $x(0)$ and commands) whereas $\tilde{x}(t)$ depends only on the disturbances (starting from zero initial conditions). Next, consider the following set recursion

$$\begin{array}{rcl} \mathcal{C}_0 & := & \mathcal{C} \sim L_d\mathcal{D} \\ \mathcal{C}_k & := & \mathcal{C}_{k-1} \sim H_c\Phi^{k-1}G_d\mathcal{D} \\ \mathcal{C}_\infty & := & \bigcap_{k=0}^{\infty} \mathcal{C}_k \end{array} \tag{10}$$

where, for given sets $\mathcal{A}, \mathcal{E} \subset \mathbb{R}^n$, $\mathcal{A} \sim \mathcal{E}$ is the Pontryagin set difference defined as $\mathcal{A} \sim \mathcal{E} := \{a : a+e \in \mathcal{A}, \forall e \in \mathcal{E}\}$. It can been shown that the sets $\mathcal{C}_k$, if non-empty, are convex because of the convexity of $\mathcal{C}$ and nested, i.e. $\mathcal{C}_k \subset \mathcal{C}_{k-1}$.

Let us now introduce the set-valued future predictions (*virtual evolutions*) of the c-variable for all possible disturbance sequence realizations $\{d(l) \in \mathcal{D}\}_{l=0}^{k}$ along the *virtual time* k under a constant *virtual command* $g(k) \equiv g$ emanating from the initial state x (at virtual time $k=0$)

$$c(k,x,g,d(\cdot)) = \bigcup_{d(\cdot)\in\mathcal{D}} \left\{ H_c \left(\Phi^k x + \right.\right. \\ \left.\left. + \sum_{i=0}^{k-1} \Phi^{k-i-1}(Gg+G_d d(i)) \right) + Lg + L_d d(k) \right\} \tag{11}$$

The latter can be rewritten as the sum of three terms:

$$c(k,x,g,d(\cdot)) = \bar{c}(k,\hat{x},g) + \tilde{c}(k,d(\cdot)) + H_c\Phi^k\tilde{x}. \quad (12)$$

where $\overline{c}(k,\hat{x},g)$ represents the disturbance-free evolution of the c-variable along the *virtual time* k under a constant *virtual command* $g(k) \equiv g$ and disturbance-free initial state $\hat{x}$ and $\tilde{c}(k,d(\cdot))$ is the set-valued virtual evolutions of the c-variable due to all possible disturbance sequence realizations $\{d(l) \in \mathcal{D}\}_{l=0}^{k}$. It is possible to prove (see [1]) that, in spite of state unavailability,

$$\begin{array}{c} \overline{c}(k,\hat{x},g) \in \mathcal{C}_\infty,\ \forall k \in \mathbb{Z}_+ \\ \Downarrow \\ c(k,x,g,d(\cdot)) = \overline{c}(k,\hat{x},g) + \tilde{c}(k,d(\cdot)) + H_c\Phi^k\tilde{x} \subset \mathcal{C}, \forall k \in \mathbb{Z}_+ \end{array} \quad (13)$$

Thus, the constraints fulfilment can be ensured by only considering the disturbance-free evolutions of the system (1) and adopting a "worst-case" approach. To this end, let us introduce, for a given sufficiently small scalar $\delta > 0$, the sets:

$$\begin{array}{rcl} \mathcal{C}^\delta & := & \mathcal{C}_\infty \sim \mathcal{B}_\delta \\ \mathcal{W}^\delta & := & \{g \in \mathbb{R}^m :\ c_g \in \mathcal{C}^\delta\} \end{array} \quad (14)$$

where $\mathcal{B}_\delta$ is the ball of radius δ centered at the origin and $\mathcal{W}^\delta$, which we will assume non-empty, the set of all constant commands g whose corresponding disturbance-free equilibrium points $\hat{c}_g$ satisfy the constraints with margin δ. From the foregoing definitions and assumptions, it follows that $\mathcal{W}^\delta$ is closed and convex. If we manipulate the *virtual evolutions* $\bar{c}(k,\hat{x},g)$ as follows

$$\bar{c}(k,\hat{x},g) = c_g + H_c\Phi^k(\hat{x} - x_g) \quad (15)$$

it results that c_g is the steady-state component whereas $H_c\Phi^k(\hat{x} - x_g)$ the transient evolution. Like in the standard CG solution, we will restrict our attention to virtual commands g within the set $\mathcal{W}^\delta$, i.e.

$$g \in \mathcal{W}^\delta \quad (16)$$

This ensures that the steady-state component of the virtual evolutions does not violate constraints and in particular will always belong to $\mathcal{C}^\delta$. Moreover, in order to satisfy the constraints also during the transients we need to ensure

$$\bar{c}(k,\hat{x},g) = c_g + H_c\Phi^k(\hat{x} - x_g) \in \mathcal{C}_\infty \quad (17)$$

The key idea used here for the construction of an effective FF-CG algorithm is as follows. Let us assume that at time $t = 0$ a command $g(0) \in \mathcal{W}^\delta$ has been applied such that the transient components of $\bar{c}(k,\hat{x}(0),g(0)), k \geq 0$ are confined into balls of known radius $\rho(0)$ around $c_{g(0)}$ and such that constraints are not violated i.e. $c_{g(0)} \in \mathcal{C}_\infty \sim \mathcal{B}_{\rho(0)}$. The transient part of the predictions will be thus bounded as

$$\|H_c\Phi^k(\hat{x}(0) - x_{g(0)})\| \leq \rho(0), \forall k \geq 0. \quad (18)$$

We may note that, if we were waiting for a sufficient long time after the application of a new FF-CG command, the transient contribution would decrease and could be bounded within a certain percentage of its initial bound $\rho(0)$. For the forthcoming discussion, the following definitions are in order

Definition ***(Generalized Settling Time) -*** The integer $k > 0$ is said to be a *Generalized Settling Time with parameter* γ, with $0 < \gamma < 1$, for the pair (H_c, Φ), if

$$\begin{array}{c} \|H_c\Phi^k x\| \leq M(x), \quad \forall i = 0,1,...,k-1 \\ \Downarrow \\ \|H_c\Phi^{k+i}x\| \leq \gamma M(x), \forall i = 0,...,\infty \end{array} \quad (19)$$

Definition ***(Guaranteed Contraction Sequence) -*** The sequence $\gamma(k|t) \leq 1, \forall k \geq 0$ is a Guaranteed Contraction Sequence for the pair (H_c, Φ) at time t if

$$\begin{array}{ll} \|H_c\Phi^k x\| \leq M(x), & k = 0,1,...,t-1 \\ \Downarrow & \\ \|H_c\Phi^{t+k}x\| \leq \gamma(k|t)M(x), & k = 0,1,...,\infty \end{array} \quad (20)$$

holds true for each $x \in \mathbb{R}^n$, with the real $M(x) > 0$ any upper-bound to $\|H_c\Phi^k x\|, \forall k \geq 0$.

Definition ***(Maximal Guaranteed Contraction Sequence) -*** The sequence $\gamma^*(k|t) \leq 1, \forall k \geq 0$ is a Maximal Guaranteed Contraction Sequence for the pair (H_c, Φ) at time t if

i. $\gamma^*(k|t)$ is a Guaranteed Contraction Sequence for the pair (H_c, Φ) at time t.
ii. $\gamma^*(k|t) \leq \gamma(k|t), \quad \forall k$, for all Guaranteed Contraction Sequences $\gamma(k|t)$ for the pair (H_c, Φ) at t. □

In principle, one should determine any possible sequence $\gamma^*(\cdot|t)$ for every t. However, interesting enough, the following recurrent property holds true

$$\gamma^*(k|t) = \{\gamma^*(k + t|0)\}_{k=0}^{\infty} \quad (21)$$

and only $\gamma^*(k|0)$ has to be computed in practice. Moreover, by inheriting the technicalities introduced in [1] for the computation of the Generalized Settling Time, the computation of $\gamma^*(\cdot|t)$ may be performed as follows:

- $\gamma^*(0|0) = 1$;
- $\gamma^*(k|0) = 1$ if k is not a Generalized Settling Time with parameter $\gamma < 1$;
- $\gamma^*(k|0) = \gamma$ if k is a Generalized Settling Time with parameter $\gamma < 1$ and γ is the minimum amongst all parameters associate to the Generalized Settling Time k (see [1] for computational details)

Observe also that the computations should be done for any k. However, as it will be made clear later on, any approximating Guaranteed Contraction Sequence $\gamma(k|0)$ such that $lim_{k\to\infty}\gamma(k|0) = 0$ may be used in the place of $\gamma^*(k|0)$ at the price of introducing some conservativeness in the plant start-up phase of the algorithm but without affecting the feasibility properties of the FF-CG scheme. A practicable procedure is then that of computing offline and storing only the first k' samples of $\gamma^*(k|0)$ and approximating the tail with an exponentially decreasing sequence $\gamma(k|0) = M\lambda^k$, $k > k'$, with $0 < \lambda < 1$ and $M > 0$ scalar reals computed as indicated in [1].

A direct consequence of the above definitions is that if the command $g(0)$ computed at time $t = 0$ were kept constant for the subsequent t steps, i.e. $g(0) = g(1) = ... = g(t-2) = g(t-1)$, then, given a Maximal Guaranteed Contraction Sequence $\gamma^*(\cdot|t), t \in \mathcal{Z}_+$, the disturbance free c-transient would be bounded as

$$\| H_c\Phi^k(\hat{x}(t) - x_{g(0)}) \| \leq \gamma^*(k|t)\rho(0), \quad \forall k \geq 0 \quad (22)$$

because the following equalities

$$\begin{aligned} \Phi^t(\hat{x}(0) - x_{g(0)}) &= (\hat{x}(t) - x_{g(0)}) \\ &= \left(\Phi^t\hat{x}(0) + \sum_{i=0}^{t-1}\Phi^{t-i-1}Gg(0) - x_{g(0)}\right) \end{aligned} \quad (23)$$

hold true. In [1], the latter idea has been exploited to build up a FF-CG scheme where the command signal $g(t)$ is modified only every τ^* steps, being τ^* a Generalized Settling Time (see [1]).

In this work, we will overcome such a limitation as follows. Consider at time t the disturbance-free c-transient evolution along the virtual horizon k assumed that a generic sequence of inputs $g(0), g(1), ..., g(t)$ has been applied since the time $t = 0$

$$\begin{aligned}\overline{c}(k, \hat{x}(t), g(t)) &= c_g + H_c \Phi^k \left(\hat{x}(t) - x_{g(t)}\right) \\ &= c_g + H_c \Phi^k \left(\Phi^t \hat{x}(0) + \sum_{i=0}^{t-1} \left(\Phi^{t-i-1} G g(i)\right) - x_{g(t)}\right)\end{aligned} \tag{24}$$

The latter, by introducing the *translated command*

$$\Delta g(t) := g(t) - g(0) \tag{25}$$

may be rewritten as

$$\overline{c}(k, \hat{x}(t), g(t)) = c_{g(t)} + H_c \Phi^k \left(\hat{x}(t) - x_{g(t)}\right) \tag{26}$$

$$\begin{aligned}&= c_{g(t)} + H_c \Phi^k \Bigg(\Phi^t \hat{x}(0) + \sum_{i=0}^{t-1} \left(\Phi^{t-i-1} G \left(g(0) + \Delta g(i)\right)\right) \\ &\quad - x_{g(0)} - x_{\Delta g(t)}\Bigg) \\ &= c_{g(t)} + H_c \Phi^k \left(\Phi^t \hat{x}(0) + \left(\sum_{i=0}^{t-1} \Phi^{t-i-1} G g(0)\right) - x_{g(0)}\right) \\ &\quad + H_c \Phi^k \left(\sum_{i=0}^{t-1} \left(\Phi^{t-i-1} G \Delta g(i)\right) - x_{\Delta g(t)}\right)\end{aligned} \tag{27}$$

By recalling (22), the term depending from the initial conditions may be embedded as follows

$$\left(H_c \Phi^k \left(\Phi^t \hat{x}(0) + \left(\sum_{i=0}^{t-1} \Phi^{t-i-1} G g(0)\right) - x_{g(0)}\right)\right) \in \mathcal{B}_{\rho(0)\gamma^*(k|t)} \tag{28}$$

where the quantity $\gamma^*(k|t)\rho(0)$ represents an upper-bound to the effects of the initial conditions on the dynamics at time t. By definition, $\gamma^*(k|t) = 1$ for all k, t such that $k + t < \tau'$ and $\gamma^*(k|t) < 1$ for all k such that $k + t \geq \tau'$, τ' being the Minimal Generalized Settling time for the system (1). Then, it follows that $\gamma^*(k|t)\rho(0) < \rho(0), \forall k$ when $t \geq \tau'$. This inequality allows us to say that a sufficient condition for (17) to hold true is that

$$\left(c_{g(t)} + H_c \Phi^k \left(\sum_{i=0}^{t-1} \left(\Phi^{t-i-1} G \Delta g(i)\right) - x_{\Delta g(t)}\right)\right) \in \mathcal{C}^\infty \sim \mathcal{B}_{\rho(0)\gamma^*(k|t)} \tag{29}$$

By introducing now the *translated state*

$$\Delta x(t) = \left(\sum_{i=0}^{t-1} \Phi^{t-i-1} G \Delta g(i)\right) \tag{30}$$

it can be seen that it satisfies

$$\Delta x(t+1) = \Phi \Delta x(t) + G \Delta g(t) \tag{31}$$

under the assumption $\Delta x(0) = 0$. By using such a definition and remembering that $c_{g(t)} = c_{g(0)+\Delta g(t)} = c_{g(0)} + c_{\Delta g(t)}$, one may rewrite the sufficient condition (29) as

$$c_{g(0)} + \overline{c}(k, \Delta x(t), \Delta g(t)) \in \mathcal{C}^\infty \sim \mathcal{B}_{\rho(0)\gamma^*(k|t)} \forall k \geq 0. \tag{32}$$

Finally, at time t we can denoted the set of all *admissible* FF-CG commands g as the set $\mathcal{V}(\Delta x(t), \rho(\cdot|t))$, where $\rho(k|t) = \rho(0)\gamma^*(k|t)$ and

$$\begin{aligned}\mathcal{V}(\Delta x, \rho(\cdot)) := \{g \in \mathcal{W}^\delta \colon c_{g(0)} + \overline{c}(k, \Delta x, g - g(0)) \in \mathcal{C}^\infty \\ \sim \mathcal{B}_{\rho(k)}, k \geq 0\}.\end{aligned} \tag{33}$$

Because $\mathcal{C}^\infty \sim \mathcal{B}_{\rho(k)}$ is a convex set and the predictions are linear, the latter results to be a convex and compact set. Moreover, by using the same technicalities detailed in [5], it is possible to prove this set to be finitely determined. Then, by using a quadratic selection index, we may formulate the FF-CG algorithm as follows.

The FF-CG Algorithm

REPEAT AT EACH TIME t

1.1 SOLVE

$$g(t) = \arg \min_{g \in \mathcal{V}(\Delta x(t), \rho(0)\gamma^*(\cdot|t))} \| g - r(t) \|^2_\Psi, \;\; \Psi = \Psi' > 0 \tag{34}$$

1.2 APPLY $g(t)$

1.3 UPDATE (31)

The above FF-CG scheme enjoys the following properties, for the proof please refer to [16].

Theorem *1:* - Let assumptions **A1-A2** be fulfilled. Consider system (1) along with the **FF-CG** selection rule (34) and let an admissible command signal $g(0) \in \mathcal{W}^\delta$ such that $c_{g(0)} \in C_\infty \sim \mathcal{B}_{\rho(0)}$ be applied at time $t = 0$ where $\rho(0)$ is a known scalar such that

$$\| H_c \Phi^k (x(0) - x_{g(0)}) \| \leq \rho(0), \; \forall k \geq 0 \tag{35}$$

Then:

1) At each decision time t, the minimizer in (34) uniquely exists and can be obtained by solving a convex constrained optimization problem;
2) The system supervised by the FF-CG never violates the constraints, i.e. $c(t) \in \mathcal{C}$ for all $t \in \mathbb{Z}_+$ regardless of any possible admissible disturbance realization $d(\cdot) \in \mathcal{D}$;
3) The sequence of $g(t)$'s is bounded for any arbitrary bounded reference sequence $r(t) \in \mathbb{R}^m$. Moreover, whenever $r(t) \equiv r$, with r a constant set-point, the sequence of $g(t)$'s converges in finite time either to r or to its best admissible steady-state approximation $\hat{r}$:
$$\exists t' > 0 \;\; t.c. \;\; g(t) = \hat{r} := \arg \min_{g \in \mathcal{W}^\delta} \|g - r\|^2_\Psi, \forall t \geq t' \tag{36}$$
and $\lim_{t\to\infty} \hat{x}(t) = x_{\hat{r}}$, $\lim_{t\to\infty} \hat{y}(t) = y_{\hat{r}} = \hat{r}$, $\lim_{t\to\infty} \hat{c}(t) = c_{\hat{r}}$;
4) Consider the disturbance free case $d(t) \equiv 0, \forall t$, and let $g^{CG}(x, r) = \arg\min_{g \in \mathcal{V}(x)} \| g - r \|^2_\Psi$ be the standard CG solution (details in [4], [8], [11]) for the disturbance-free CG design problem where
$$\mathcal{V}(x) := \{g \in \mathcal{W}^\delta : \bar{c}(k, x, g) \in \mathcal{C}, k \geq 0\}. \tag{37}$$
is the state-dependent admissible region for the standard CG methods described in [4], [8], [11] and $\bar{c} = (k, x, g)$ defined in (12). Then, for $t \to \infty$

the time-varying regions (33) of admissible commands $g^{FF-CG}(t)$ for the FF-CG scheme asymptotically converge to the constant admissible region (37) and $lim_{t\to\infty}(g^{FF-CG}(t) - g^{CG}(t)) = 0_m$. □

Remark 1 - It is worth pointing out that the set (33) is in general time-varying because it is parameterized by sequences $\rho(\cdot|t)$ that change over time. However, this doesn't represent usually a problem from a computational viewpoint. In fact, being $\mathcal{C}_\infty$ a polyhedral set, whenever nonempty, the collection of constraint sets defined by $\mathcal{C}_\infty \sim \mathcal{B}_\delta$ is easily parameterizable in a closed form w.r.t. δ. Notice also that, as in the standard CG approach, (33) is finitely determinable with respect to k, that is only a finite number of checks for $k = 0, 1, ..., k_0$ are required for ensuring the validity of the conditions underlying (33) for all $k \geq 0$. □

III. Illustrative Example: Position Servomechanism

The proposed FF-CG scheme is applied to the position servomechanism schematically described in Figure 1. This consists of a DC-motor, a gear-box, an elastic shaft and an uncertain load. No disturbances are considered for simplicity. Technical specifications involve bounds on the shaft torsional torque T as well as on the input voltage V. System parameters are reported in Table I.

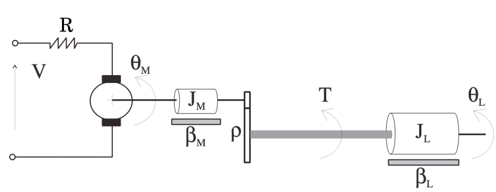

Fig. 1. Servomechanism model

TABLE I
Model parameters

Symbol	Value (MKS)	Meaning
Symbol	Value (MKS)	Meaning
L_S	1.0	shaft length
d_S	0.02	shaft diameter
J_S	negligible	shaft inertia
J_M	0.5	motor inertia
β_M	0.1	motor viscous friction coefficient
R	20	resistance of armature
K_T	10	motor constant
ρ	20	gear ratio
k_θ	1280.2	torsional rigidity
J_L	$20J_M$	load inertia
β_L	25	load viscous friction coefficient
T_s	0.1	sampling time

Let θ_M and θ_L denote respectively the motor and the load angle and let

$$x_p = \begin{bmatrix} \theta_L & \dot{\theta}_L & \theta_M & \dot{\theta}_M \end{bmatrix}'$$

be a suitable state vector, which is assumed not available for the CG unit. Then, the plant can be described by the following state-space model

$$\left\{ \begin{array}{rcl} \dot{x}_p & = & \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_\theta}{J_L} & -\frac{\beta_L}{J_L} & \frac{k_\theta}{\rho J_L} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{k_\theta}{\rho J_M} & 0 & -\frac{k_\theta}{\rho^2 J_M} & -\frac{\beta_M + k_T^2/R}{J_M} \end{bmatrix} x_p + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{k_T}{RJ_M} \end{bmatrix} V \\ \theta_L & = & \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} x_p, \; T = \begin{bmatrix} k_\theta & 0 & -\frac{k_\theta}{\rho} & 0 \end{bmatrix} x_p \end{array} \right.$$

Because the steel shaft has a finite shear strength, a maximum admissible shaft $\tau_{adm} = 50N/mm^2$ imposes the constraint $|T| \leq 78.5398\ Nm$ on the torsional torque. Moreover, the input DC voltage V has to be constrained within the range $|V| \leq 220\ V$. The model is transformed in discrete time by sampling every $T_s = 0.1s$ and using a zero-order holder on the input voltage. It is assumed that a controller acting on the motor voltage is used to guarantee assumptions A1-A2. It is also assumed that the closed-loop system state, input and output are not available for CG purposes and only the manipulation of the set-point signal is allowed.

The pre-compensated system, when not governed by a CG unit, exhibits a very fast response but inadmissible voltage inputs and torsional torques for the references of interest, as shown in Figure. 2.b for a square-wave set-point with amplitude equal to $r = 60\ deg$ (solid line) and increasing frequency. On the contrary, when a FF-CG unit is used the torque and voltage constraints happen to be fulfilled. Figure 3 shows the resulting system output (3.a) and the computed FF-CG action (3.b) for the same set-point of Figure 2. In these figures, the performance of the FF-CG and CG strategies can be compared. The comparison involves also the more conservative FF-CG technique described in [1], performed for a generalized settling time $\tau = 7$ denoted as "FF-CG(Fixed)". One can observe that the level of conservativeness introduced by the proposed FF-CG version is negligible after few istants when contrasted with the standard state-feedback CG approach. Moreover, FF-CG(Fixed) introduces a certain level of delay in the system response which is not present on the contrary in FF-CG. In Figure 4, the constrained variables are depicted: in this case, the voltage inputs and torsional torques are admissible. It is worth pointing out that the trajectories of the system controlled by the standard CG and the proposed FF-CG strategies almost coincide. On the contrary, the trajectories produced by FF-CG(Fixed) are delayed of seven sample steps.

Finally, in Table 2 the on-line computational burdens per step of all schemes are reported.

IV. Conclusions

In this paper, a novel FF-CG scheme is proposed which does not make use of any measure of the state to govern the set-point manipulations. The main idea under its development was to limit the set-point variations in order to always maintain the state trajectory "not too far" from the space of the steady-state equilibria. The properties of the proposed algorithm have been carefully analyzed and the differences with standard CG approaches pointed out. Comparisons with classical CG and previously proposed FF-CG solutions have

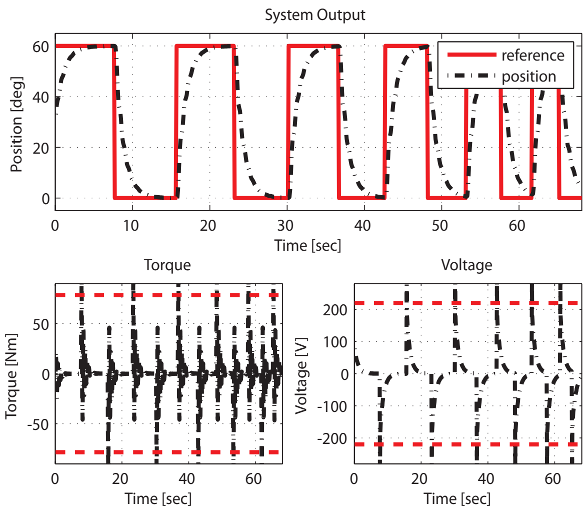

Fig. 2. System without CG unit: a) Position b) Constrained variables: Torsional Torque (left), voltage (Right)

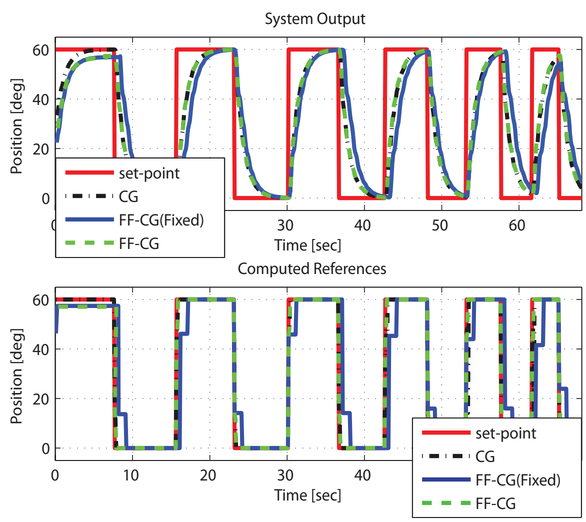

Fig. 3. System with CG unit. a) System Output: set-point (solid), CG (dash-dotted), FF-CG (dash) b) Computed Reference: set-point (solid), CG (dash-dotted), FF-CG (dash)

also been presented and discussed in the final illustrative example.

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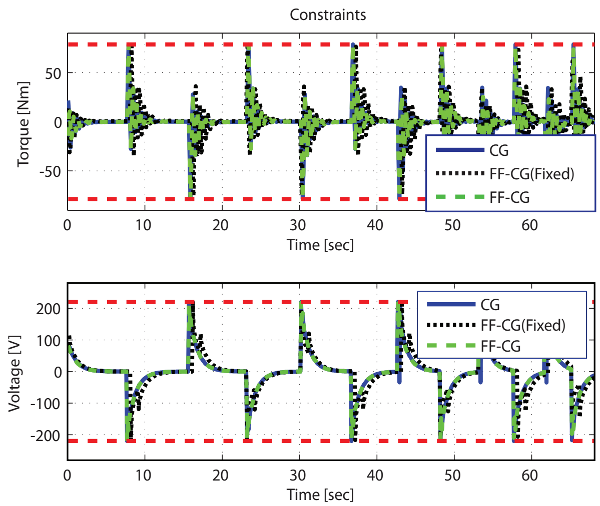

Fig. 4. Constrained variables. a) Torque: CG (dash-dotted), FF-CG (dash) b) Voltage: CG (dash-dotted), FF-CG (dash)

	CPU Time per step [ms]
CG	40
FF-CG(Fixed)	39.5
FF-CG	37.2

TABLE II
ON-LINE PHASE - CPU TIME PER STEP

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