2011 50th IEEE Conference on Decision and Control and
European Control Conference (CDC-ECC)
Orlando, FL, USA, December 12-15, 2011

A delay system method to design of event-triggered control of networked control systems

Dong Yue, Engang Tian and Qing-Long Han

Abstract— **The event-triggered H_∞ control design is investigated for networked control systems with uncertainties and transmission delays. A novel event-triggering scheme is proposed, which has some advantages over traditional ones with a continuous detector. Considering the effect of the transmission delay, a delay system model for the analysis is firstly constructed. Then, based on the model and Lyapunov functional method, criteria for the stability with an H_∞ norm bound and criteria for the co-design of both the feedback gain and the trigger parameters are derived. In order to solve the feedback gain and the trigger parameters, the linear matrix inequality technique is employed. From the simulation example, it can be concluded that the proposed event-triggering scheme is superior to some other event-triggering schemes in some existing literature.**

I. INTRODUCTION

Networked control systems (NCS) have a relatively new structure where sensors, controllers and plants are often connected over a common network medium [4], [17]. Because of the insertion of the network, the tasks in traditional systems, such as the control problem and signal estimation problem, should be re-considered. In the past decade, the control design problems for NCS have experienced an increased attention in the literature [16], [18]. In these works, most of the researchers considering NCSs use periodic triggered control method (also called time-triggered control) for system modeling and analysis due to easy implementation and analysis. In this triggering method, the fixed sampling interval should be selected to guarantee a desired performance under the worst conditions on, such as external disturbances, uncertainties, time-delays and so on. However, in practical systems, the worst cases are seldom encountered. Therefore, this kind of triggering method will lead to the sending of many "unnecessary" sampling signals through the network, which will cause a high utilization of the communication bandwidth. This kind of over-occupation may have a slight impact on the point-to-point connected system, while it will affect networked control systems greatly because the bandwidth of an NCS is often limited.

In [9], how to reduce communication requirements on networked control systems was addressed, where the maximum time allowed to elapse was obtained to guarantee the stability of the system. However, their approach leads to an inherently periodic transmission. Recently, event-triggered techniques for control design, advocating the use of actuation only when some function of the system state exceeds a threshold, have been paid an increased attention in the literature [11], [10]. Event-triggering provides a useful way of determining when the sampling action is carried out, which guarantees that only really "necessary" state signal will be sent out to the controller. Thus, the amount of the sent state signals is relatively little. Compared with periodic sampling method, the event-triggering method has the following advantages: 1) closer in nature to the way a human behaves as a controller [2], which only samples when necessary; 2) reduction in the release times of the sensor and then the burden of the network communication; 3) reduction in the computation cost of the controller and the occupation of the sensor and the actuator.

In [6], the event-triggered control with constraint $\|x(t) - x(t_k)\| \leq \bar{e}$ was proposed for linear systems with external disturbances, where the threshold $\bar{e}$ of the event generator is a constant. The criteria to guarantee the uniform boundedness of the system were derived. The methods for designs or implementations of controllers in the event-triggered form based on dissipation inequalities were proposed for both linear and nonlinear systems [11]. From [11], it can be seen that, the constructed event-triggers rely on continuous supervision of the system state in order to detect whether the current state exceeds the triggering threshold. To implement such kind of event trigger, some form of hardware event detector is needed to generate hardware interrupt to release the control task. In some applications, therefore, it may be unreasonable or impractical since the use of such kind of event detector will retrofit the existing system. In these cases, self-triggered scheme, in which the release times are determined by a software approach, was recently suggested [14], [5], [8], [7], [15]. Compared with the event-triggered scheme in [11], [6], [10], [12], [13], [14], the self-triggered scheme can provide additional energy savings for the sensor and also a less complexity in the implementation. However, as shown in [14], [8], [15], [1], the average release period based on self-triggered scheme is often smaller than that based on event-triggered scheme. Moreover, more

This work was supported in part by the Natural Science Foundation of China (Grant No. 61074025 and 60904013 and 60834002); the Australian Research Council Discovery Projects under Grant DP1096780 and Grant DP0986376, the Research Advancement Awards Scheme Program (January 2010 - December 2012) and the RDI Merit Grant Scheme Project under Gant RDIM1109 (January 2011 - December 2011) at Central Queensland University, Australia.

D. Yue is with the Department of Control Science and Engineering and the Key Laboratory of Ministry of Education for Image Processing and Intelligent Control, Huazhong University of Science and Technology, Hubei 430074, China medongy@njnu.edu.cn

E.G. Tian is with the School of Electrical and Automation Engineering, Nanjing Normal University, 78 Bancang Street, Nanjing, 210042, China teg@njnu.edu.cn

Q.-L. Han is with the Center for Intelligent and Networked Systems and the School of Information and Communication Technology, Central Queensland University, Rockhampton, Qld 4702, Australia q.han@cqu.edu.au

constraints on the system structure are needed for designs or implementations of controllers in the self-triggered form.

In this paper, we focus on the design of event-triggered H_∞ control for networked systems considering the effect of network transmission delay. The event generator is positioned between the sensor and the controller, which is used to determine whether the newly sampled state should be sent out to the controller. Our event generator is also implemented based on dissipation inequalities as in [11], [6], [10], [12], [13], [14], however, unlike the case in [11], [6], [10], [12], [13], [14], the implementation of our event generator only needs a supervision of the system state in discrete instants. Moreover, there is no need extra hardware to implement our event generator. Under the event-triggering scheme, a novel model is firstly proposed for the use of system analysis and control design. In the model, the effects of the network transmission delay and the properties of the event-triggering scheme are involved. Based on the model, criteria for the stability with an H_∞ norm bound and controller design are derived, which are expressed in the form of linear matrix inequality. The criteria also establish the relationship between the parameters of the event-trigger, the transmission delay and the feedback gain, therefore, co-design for both the controller and the parameters of triggers can be carried out by using our method. It should be noted that, the existing references [14], [15], [6], [7] can only provide an implementation of event-triggering scheme or self-triggering scheme under a given feedback gain. No method was provided in [14], [15], [6], [7] for the co-design of both the controller and the parameters of triggers. Moreover, to the best of the authors' knowledge, the event-triggered H_∞ control for networked system considering the effect of network transmission delay has not been investigated. In order to demonstrate the effectiveness of our proposed method, a simulation example is finally given. In the example, some comparisons between the existing method and our method are also provided.

II. SYSTEM DESCRIPTION AND PRELIMINARIES

Consider the following system with parameter uncertainties and external disturbance

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + B_w w(t) \quad (1)\\ z(t) &= Cx(t) + Du(t) \quad (2) \end{aligned}$$

where $x(t) \in \mathbb{R}^n$,$u(t) \in \mathbb{R}^m$, $w(t) \in \mathbb{R}^p$ are the system state vector, control input and disturbance input, respectively. A, B, B_w, C and D are the parameter matrices with only uncertainties in A and B satisfying the following assumption

$$\begin{aligned} A &= A_0 + \Delta A_0,\ B = B_0 + \Delta B_0\\ [\Delta A_0\ \ \Delta B_0] &= HF(t)\,[E_1\ \ E_2] \quad (3) \end{aligned}$$

where A_0, B_0, H, E_1 and E_2 are known matrices and $F^T(t)F(t) \leq I$. $w(t) \in L_2\,[0,\infty)$ denotes the external perturbation. Throughout this paper, we assume that system (1)-(2) is controlled through a network.

The purpose of this paper is to design a linear controller $u(t) = Kx(t)$, where K is a matrix to be determined later, such that the resulting closed-loop system satisfies the required performance. In this paper, the required performance is selected as the H_∞ performance. Different from traditional control systems, the controller considered in this paper is connected to the system (1)-(2) through a common digital communication network medium. In this case, the usual assumption in a traditional control system, that the sensor-controller-actuator communication is error-free, delay-free and without any limitation on the network bandwidth, no longer apply. Under a periodic sampling mechanism, the H_∞ control design problem was investigated for (1)-(2) in [18] considering the effect of the network transmission delay. However, how to save the network resources such as network bandwidth was not concerned in [18]. As is well known, in practical systems, periodic sampling mechanism may often lead to the sending of many "unnecessary" signals through the network, which will turn to increase the load of network transmission and waste the network bandwidth, though this method has been widely used in many control systems.

Therefore, it is significant to introduce a mechanism to determine which sampled signal should be sent out or not. In this section, a mechanism, called event generator is constructed between the sensor and the controller which is used to determine whether the newly sampled state will be sent out to the controller by using the following judgement algorithm, that is,

$$\begin{aligned} &[x\left((k+j)h\right) - x(kh)]^T\,\Omega\,[x\left((k+j)h\right) - x(kh)]\\ &\leq \sigma x^T\left((k+j)h\right)\Omega x\left((k+j)h\right) \quad (4) \end{aligned}$$

where Ω is a positive matrix, $j = 1, 2, ..., \sigma \in [0, 1)$.

Remark 1: Under (4), the sampled state $x((k+j)h)$ satisfying the inequality (4) will not be transmitted. Only the one that exceeds the threshold in (4) will be sent to the controller. This means that, in the sensor side, only some of the sampled states that violate (4) will be sent out to the controller side. Obviously, compared with the existing method in [18], [20], the burden of the network communication is reduced and the communication bandwidth in the network is saved. In the wireless network, this method will also save the transmission energy, and then increase the lifespan of the battery of the nodes. Specially, when $\sigma = 0$, the inequality (4) is not satisfied for almost all the sampled state $x\left((k+j)h\right)$, and the event-triggered scheme reduces to a periodic release scheme.

Remark 2: Different from the continuous event generator (CEG), that is, the event generator with a continuous supervision of the state [11], [6], [10], [12], [13], [14], the event generator with the algorithm (4) only supervises the difference between the states sampled in discrete instants having no interest in what happens in between updates. Moreover, unlike CEG, no extra hardware interrupt to release the control tasks is needed in our event generator.

Under the algorithm (4), assume the release times are $t_0h, t_1h, t_2h, \cdots$, where $t_0 = 0$ is the initial time. $s_ih = t_{i+1}h - t_ih$ denotes the release period which corresponds to the sampling period given by the event generator in

(4). For network uncertainties, we only consider the effect of the transmission delay on the system. Suppose that the time-varying delay in the network communication is τ_k and $\tau_k \in (0, \bar{\tau})$, where $\bar{\tau}$ is a positive real number. Therefore, the states $x(t_0 h), x(t_1 h), x(t_2 h), \cdots$ will arrive at the controller side at the instants $t_0 h + \tau_0, t_1 h + \tau_1, t_2 h + \tau_2, \cdots$, respectively.

Remark 3: The set of the release instants, i.e., $\{t_0, t_1, t_2, \cdots\}$, is a subset of $\{0, 1, 2, \cdots\}$. The amount of $\{t_0, t_1, t_2, \cdots\}$ depends on not only the value of σ, but also the variation of the system state. When $\sigma = 0$, $\{t_0, t_1, t_2, \cdots\} = \{0, 1, 2, \cdots\}$, it reduces to the case with periodic release times.

Based on above analysis, for $t \in [t_k h + \tau_k, t_{k+1} h + \tau_{k+1})$, the system model under the event generator with (4) and considering the effect of the transmission delay can be described as

$$\dot{x}(t) = Ax(t) + Bu(t_k h) + B_w w(t), \quad (5)$$

$$z(t) = Cx(t) + Du(t_k h), \quad (6)$$

Under the control $u(t) = Kx(t)$, for $t \in [t_k h + \tau_k, t_{k+1} h + \tau_{k+1})$, (5)-(6) can be rewritten as

$$\dot{x}(t) = Ax(t) + BKx(t_k h) + B_w w(t) \quad (7)$$

$$z(t) = Cx(t) + DKx(t_k h) \quad (8)$$

Case a): if $t_k h + h + \bar{\tau} \geq t_{k+1} h + \tau_{k+1}$, where $\bar{\tau} = \max\{\tau_k\}$, define a function $\tau(t)$ as

$$\tau(t) = t - t_k h, for\ t \in [t_k h + \tau_k, t_{k+1} h + \tau_{k+1}) \quad (9)$$

Obviously

$$\tau_k \leq \tau(t) \leq (t_{k+1} - t_k) h + \tau_{k+1} \leq h + \bar{\tau} \quad (10)$$

Case b): if $t_k h + h + \bar{\tau} < t_{k+1} h + \tau_{k+1}$, consider the following intervals

$$[t_k h + \tau_k, t_k h + h + \bar{\tau}), \quad [t_k h + ih + \bar{\tau}, t_k h + ih + h + \bar{\tau})$$

Since $\tau_k \leq \bar{\tau}$, it can be easily shown that d_M exists such that

$$t_k h + d_M h + \bar{\tau} < t_{k+1} h + \tau_{k+1} \leq t_k h + d_M h + h + \bar{\tau}$$

and $x(t_k h)$ and $x(t_k h + ih)$ with $i = 1, 2, \cdots, d_M$ satisfy (4). It can also be seen that

$$[t_k h + \tau_k, t_{k+1} h + \tau_{k+1}) = I_1 \cup I_2 \cup I_3 \quad (11)$$

where $I_1 = [t_k h + \tau_k, t_k h + h + \bar{\tau})$, $I_2 = \cup_{i=1}^{d_M - 1} \{I_2^i\} = \cup_{i=1}^{d_M - 1} [t_k h + ih + \bar{\tau}, t_k h + ih + h + \bar{\tau})$, $I_3 = [t_k h + d_M h + \bar{\tau}, t_{k+1} h + \tau_{k+1})$. Define

$$\tau(t) = \begin{cases} t - t_k h, & for\ t \in I_1 \\ t - t_k h - ih, & for\ t \in I_2^i \\ t - t_k h - d_M h, & for\ t \in I_3 \end{cases} \quad (12)$$

We can easily show that

$$\begin{cases} \tau_k \leq \tau(t) \leq h + \bar{\tau}, & t \in I_1 \\ \tau_k \leq \bar{\tau} \leq \tau(t) \leq h + \bar{\tau}, & t \in I_2^i \\ \tau_k \leq \bar{\tau} \leq \tau(t) \leq h + \bar{\tau}, & t \in I_3 \end{cases} \quad (13)$$

where the third row in (13) holds because $t_{k+1} h + \tau_{k+1} \leq t_k h + (d_M + 1) h + \bar{\tau}$. Therefore, for $t \in [t_k h + \tau_k, t_{k+1} h + \tau_{k+1})$, $0 \leq \tau_k \leq h + \bar{\tau} \triangleq \tau_M$, that is, $\tau(t) \in [0, \tau_M]$.

In Case a), for $t \in [t_k h + \tau_k, t_{k+1} h + \tau_{k+1})$, define $e_k(t) = 0$. In Case b), define

$$e_k(t) = \begin{cases} 0, & for\ t \in I_1 \\ x(t_k h) - x(t_k h + ih), & for\ t \in I_2^i \\ x(t_k h) - x(t_k h + d_M h), & for\ t \in I_3 \end{cases} \quad (14)$$

Remark 4: From the definition of $e_k(t)$ and the triggering algorithm (4), it can be seen that, for $t \in [t_k h + \tau_k, t_{k+1} h + \tau_{k+1})$,

$$e_k^T(t) \Omega e_k(t) \leq \sigma x^T(t - \tau(t)) \Omega x(t - \tau(t)) \quad (15)$$

Combining the definitions of $\tau(t)$ and $e_k(t)$ in (12) and (14), for $t \in [t_k h + \tau_k, t_{k+1} h + \tau_{k+1})$, (7)-(8) can be rewritten as

$$\dot{x}(t) = Ax(t) + BKx(t - \tau(t)) + BKe_k(t) + B_w w(t) \quad (16)$$

$$z(t) = Cx(t) + DKx(t - \tau(t)) + DKe_k(t), \quad (17)$$

$$x(t) = \phi(t), t \in [-\tau_M, 0] \quad (18)$$

where $\phi(t)$ is the initial function of $x(t)$.

Before giving the main results, we need some definition and lemma.

Definition 1: The closed-loop system (16)-(18) is said to be robustly exponentially stable if, in case of $w(t) \equiv 0$, for any initial condition, there exist constant $\epsilon > 0$ and $\lambda > 0$ such that

$$\|x(t)\| \leq \epsilon \sup_{-\tau_M \leq s \leq 0} \|\phi(s)\| e^{-\lambda t}$$

Lemma 1: For matrices $R > 0$ and $X^T = X$, we have

$$-XR^{-1}X \leq \rho^2 R - 2\rho X \quad (19)$$

where ρ is any chosen constant.

III. MAIN RESULTS

Our aim in this paper is to develop techniques to deal with the robust H_∞ event-triggered control problem for networked systems with parameter uncertainties and transmission delay. More specifically, given a disturbance attenuation level γ, we design a state feedback controller such that the system (16)-(18) under the event generator with (4) satisfies the following two requirements:

(1) The closed-loop system (16)-(18) with $w(t) \equiv 0$ is robustly exponentially stable in the sense of Definition 1.

(2) under zero initial condition, the controlled output $z(t)$ satisfies $\|z(t)\|_2 \leq \gamma \|w(t)\|_2$ for any nonzero $w(t) \in L_2[0, \infty)$.

Based on the Lyapunov functional method, we first conclude the following result.

Theorem 1: For given parameters γ, σ and matrix K, the system (16)-(18) is exponentially stable with an H_∞ norm bound γ if there exist matrices $P > 0, Q > 0, R > 0, \Omega >$

0 and N, M with appropriate dimensions such that for $l = 1, 2$

$$\begin{bmatrix} W & * & * & * & * \\ \Phi_{21}(l) & -R & * & * & * \\ \Phi_{31} & 0 & -\gamma^2 I & * & * \\ \Phi_{41} & 0 & \sqrt{\tau_M} RB_\omega & -R & * \\ \Phi_{51} & 0 & 0 & 0 & -I \end{bmatrix} < 0, \qquad (20)$$

where

$$\begin{aligned} W &= \Phi_{11} + \Gamma + \Gamma^T \\ \Phi_{11} &= \begin{bmatrix} PA + A^T P + Q & * & * & * \\ K^T B^T P & \sigma\Omega & * & * \\ 0 & 0 & -Q & * \\ K^T B^T P & 0 & 0 & -\Omega \end{bmatrix} \\ \Gamma &= \begin{bmatrix} -N & N - M & M & 0 \end{bmatrix} \\ \Phi_{21}(1) &= \sqrt{\tau_M} N^T, \Phi_{21}(2) = \sqrt{\tau_M} M^T, \\ \Phi_{41} &= \begin{bmatrix} \sqrt{\tau_M} RA & \sqrt{\tau_M} RBK & 0 & \sqrt{\tau_M} RBK \end{bmatrix}, \\ \Phi_{31} &= \begin{bmatrix} B_\omega^T P & 0 & 0 & 0 \end{bmatrix}, \\ \Phi_{51} &= \begin{bmatrix} C & DK & 0 & DK \end{bmatrix} \end{aligned}$$

Proof: Construct the following Lyapunov functional as

$$\begin{aligned} V(t) &= x^T(t)Px(t) + \int_{t-\tau_M}^{t} x^T(s)Qx(s)ds \\ &\quad + \int_{t-\tau_M}^{t} \int_{s}^{t} \dot{x}^T(v)R\dot{x}(v)dvds \end{aligned} \qquad (21)$$

where P, Q, and R are positive matrix with appropriate dimensions. Similar to the proof in [19] and recalling (15) and Lemma 1, the proof can be completed. ■

The obtained result can be easily extended to the system with uncertain parameters. Considering the parameter uncertainties defined in (3), the following theorem can be obtained.

Theorem 2: For given parameters σ, γ and matrix K, the system (16)-(18) is robustly exponentially stable with an H_∞ norm bound γ if there exist matrices $P > 0, Q > 0, R > 0, \Omega > 0, N, M$ and scalar $\varepsilon > 0$ such that for $l = 1, 2$

$$\begin{bmatrix} W' & * & * & * & * & * \\ \Phi_{21}(l) & -R & * & * & * & * \\ \Phi_{31} & 0 & -\gamma^2 I & * & * & * \\ \Phi'_{41} & 0 & \sqrt{\tau_M} RB_\omega & -R & * & * \\ \Phi_{51} & 0 & 0 & 0 & -I & * \\ \Phi_{61} & 0 & 0 & \Phi_{64} & 0 & \Phi_{66} \end{bmatrix} < 0 \qquad (22)$$

where $W' = \Phi'_{11} + \Gamma + \Gamma^T$, Φ'_{11} and Φ'_{41} are obtained from Φ_{11} and Φ_{41} by replacing A and B with A_0 and B_0,respectively, and

$$\begin{aligned} \Phi_{61} &= \begin{bmatrix} H^T P & 0 & 0 & 0 \\ \varepsilon E_1 & \varepsilon E_2 K & 0 & \varepsilon E_2 K \end{bmatrix}, \\ \Phi_{64} &= \begin{bmatrix} \sqrt{\tau_M} H^T R \\ 0 \end{bmatrix}, \Phi_{66} = diag\{-\varepsilon I, -\varepsilon I\} \end{aligned}$$

In the following, a criterion is proposed to design the feedback gain in case of network-induced delay and parameter uncertainties and under the trigger (4).

Theorem 3: For given parameters σ, γ and ρ, the system (16)-(18) under the trigger condition (4) and feedback gain $K = YX^{-1}$ is robustly exponentially stable with an H_∞ norm bound γ, if there exist matrices $X > 0, \tilde{Q} > 0, \tilde{R} > 0, \tilde{\Omega} > 0, \tilde{N}, \tilde{M}, Y$ of appropriate dimensions and a scalar $\mu > 0$ such that for $l = 1, 2$

$$\begin{bmatrix} \hat{W} & * & * & * & * & * \\ \Sigma_{21}(l) & -\tilde{R} & * & * & * & * \\ \Sigma_{31} & 0 & -\gamma^2 I & * & * & * \\ \Sigma_{41} & 0 & \sqrt{\tau_M} B_\omega & \hat{W_{44}} & * & * \\ \Sigma_{51} & 0 & 0 & 0 & -I & * \\ \Sigma_{61} & 0 & 0 & \Sigma_{64} & 0 & \Sigma_{66} \end{bmatrix} < 0 \qquad (23)$$

$$\begin{aligned} \hat{W} &= \Sigma_{11} + \tilde{\Gamma} + \tilde{\Gamma}^T, \hat{W_{44}} = \rho^2 \tilde{R} - 2\rho X \\ \Sigma_{11} &= \begin{bmatrix} A_0 X + X A_0^T + \tilde{Q} & * & * & * \\ Y^T B_0^T & \sigma\tilde{\Omega} & * & * \\ 0 & 0 & -\tilde{Q} & * \\ Y^T B_0^T & 0 & 0 & -\tilde{\Omega} \end{bmatrix} \\ \tilde{\Gamma} &= \begin{bmatrix} -\tilde{N} & \tilde{N} - \tilde{M} & \tilde{M} & 0 \end{bmatrix} \\ \Sigma_{21}(1) &= \sqrt{\tau_M} \tilde{N}^T, \Phi_{21}(2) = \sqrt{\tau_M} \tilde{M}^T, \\ \Sigma_{41} &= \begin{bmatrix} \sqrt{\tau_M} A_0 X & \sqrt{\tau_M} B_0 Y & 0 & \sqrt{\tau_M} B_0 Y \end{bmatrix}, \\ \Sigma_{51} &= \begin{bmatrix} CX & DY & 0 & DY \end{bmatrix}, \\ \Sigma_{31} &= \begin{bmatrix} B_\omega^T & 0 & 0 & 0 \end{bmatrix} \\ \Sigma_{61} &= \begin{bmatrix} \mu H^T & 0 & 0 & 0 \\ E_1 X & E_2 Y & 0 & E_2 Y \end{bmatrix}, \\ \Sigma_{64} &= \begin{bmatrix} \sqrt{\tau_M} \mu H^T \\ 0 \end{bmatrix}, \Sigma_{66} = \begin{bmatrix} -\mu I & 0 \\ 0 & -\mu I \end{bmatrix} \end{aligned}$$

Proof: Defining $X = P^{-1}$, pre and post multiplying (22) with $diag\{X, X, X, X, X, I, R^{-1}, I, I, I\}$, defining new matrix variables $\tilde{Q} = XQX$, $\tilde{R} = XRX$, $\tilde{\Omega} = X\Omega X$, $\tilde{N} = diag(X, X, X, X)NX$, $\tilde{M} = diag(X, X, X, X)MX$, $Y = KX$, $\mu = \varepsilon^{-1}$ and using Lemma 1 with the inequality

$$-X\tilde{R}^{-1}X \leq \rho^2 \tilde{R} - 2\rho X \qquad (24)$$

(23) can be obtained from (22). ■

Remark 5: In [8], the authors investigated self-triggered control for linear systems, the proposed technique can guarantee exponential input-to-state (ISS) stability of the system. However, the considered system is free of delay and the method can not be applied into networked control systems. Furthermore, the method in [8] can not be used for the co-design of both feedback gain and the trigger parameters.

Remark 6: Theorem 3 provides a useful way of co-design for both the feedback gain and the trigger parameters σ and Ω by solving a set of LMIs in (23). Moreover, the information of the transmission delay is also involved in the condition (23). Therefore, our method can be used to deal with the case with network transmission delay. For given condition on the transmission delay, by solving (23), the corresponding feedback gain and trigger parameters can be obtained, which can be used to guarantee the required performance even though the transmission delay exists in the network communication.

Remark 7: When σ, ρ and γ are fixed, the upper bound for τ_M can be solved in terms of (23). Appropriately selecting the value of ρ will lead to a relatively larger value of the maximum τ_M. However, how to find an optimal value of ρ is still open. On the other hand, from the simulation example in the following section, it can be seen that larger σ will lead to smaller upper bound of τ_M. It can also be found from the simulation example that, larger σ produces a larger average release period, which turns to reduce the load of network communication and thus decrease the transmission delay.

Remark 8: Note that $\tau_M = h + \bar{\tau}$. When τ_M is solved, selecting a sampling period $h < \tau_M$, the allowable maximum transmission delay is $\bar{\tau} = \tau_M - h$. If $\tau_k \equiv 0$, that is, no transmission delay exists or the effect of the transmission delay can be omitted, the solved τ_M denotes the maximum sampling period.

IV. SIMULATION EXAMPLE

Consider the system (1)-(2) with the parameter matrices satisfying the following two cases:

Case 1 :

$$A_0 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{-mg}{M} & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & g/l & 0 \end{bmatrix}, B_0 = \begin{bmatrix} 0 \\ 1/M \\ 0 \\ -1/Ml \end{bmatrix} \quad (25)$$
$$C = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}, D = 0, B_w^T = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}$$

where $M = 10$ is the cart mass and $m = 1$ is the mass of the pendulum bob, $l = 3$ is the length of the pendulum arm and $g = 10$ is gravitational acceleration. The eigenvalues of A are $\{0, 0, 1.8257, -1.8257\}$, the system is unstable without a controller. The state $\begin{bmatrix} x_1 & x_2 & x_3 & x_4 \end{bmatrix} = \begin{bmatrix} y & \dot{y} & \theta & \dot{\theta} \end{bmatrix}$, where $x_i (i = 1, 2, 3, 4)$ are the cart's position, the cart's velocity, the pendulum bob's angle and the pendulum bob's angular velocity respectively. The initial state is $x_0 = \begin{bmatrix} 0.98 & 0 & 0.2 & 0 \end{bmatrix}$.

Case 2 : A_0 and B_0 are the same as in (25), the other parameter matrices are given as follows:

$$C = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}, D = 0.1, B_w^T = C, \quad (26)$$
$$E_1 = \begin{bmatrix} 0 & 0.1 & 0 & 0 \\ 0 & 0 & -0.1 & 0 \\ 0 & 0 & 0 & 0.1 \\ 0 & 0 & 0.1 & 0.01 \end{bmatrix}, \quad (27)$$
$$H = diag\{1, 1, 1, 1\}, \quad (28)$$
$$\omega(t) = \begin{cases} sgn(\sin t), & if\ t \in [0, 10] \\ 0, & otherwise \end{cases} \quad (29)$$

In the following, under Case 1 or Case 2, we will demonstrate the design process of the feedback gain and the proposed event-triggering scheme and also provide some comparison results showing the advantages of our own method over the existing ones.

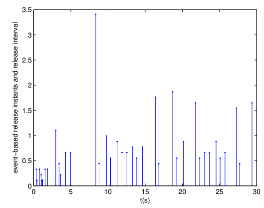
Under Case 1. To compare with the existing methods in the design of event-triggering scheme, we use the following feedback gain

$$K = \begin{bmatrix} 2 & 12 & 378 & 210 \end{bmatrix} \quad (30)$$

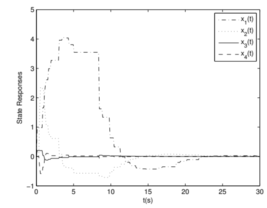
TABLE I

AVERAGE SAMPLING PERIOD BY DIFFERENT TRIGGERING SCHEMES

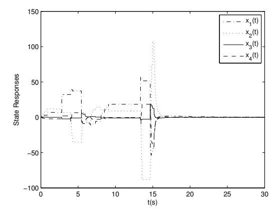
Event-triggering Schemes	Average Periods
Theorem 1 with $h = 0.11$	0.7729
Event-triggering scheme in [13]	0.4816
Event-triggering scheme in [14]	0.3375
The bound on the MATI in [3]	0.0168
Event-triggering scheme in [12]	$< 10^{-5}$



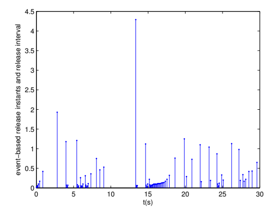
(a) The release instants and release interval with feedback gain (30) and trigger matrix (31)



(b) The state responses of system with feedback gain (30) and trigger matrix (31)



(c) The state responses of system with feedback gain and Ω in (35)



(d) The release instants and release interval with feedback gain K and trigger matrix Ω in (35)

which is the same as in [14]. Applying Theorem 1 with $\rho = 0.53$ and $\sigma = 0.3$, we can solve the maximum value of τ_M as 0.11 and the corresponding Ω as

$$\Omega = \begin{bmatrix} 0.0047 & 0.0190 & 0.6555 & 0.3391 \\ 0.0190 & 0.1325 & 3.7101 & 2.1602 \\ 0.6555 & 3.7101 & 119.0910 & 65.0458 \\ 0.3391 & 2.1602 & 65.0458 & 37.2885 \end{bmatrix} \quad (31)$$

Suppose $\tau_k \equiv 0$ as in [13]. Since $\tau_M = h + \bar{\tau}$, it can be known that the maximum sampling period is 0.11. Taking $h = 0.11$ and using the algorithm (4), the simulation results for $t \in [0, 30]$ show that, only 37 sampled signals need to be sent out to the controller, which takes 13.6% of the sampled signals. Moreover, it can be computed that our event generator leads to a maximum release interval of 3.41. The comparison results on the average interval under our scheme and other schemes in [13], [14], [12], [3] are shown in Table I. It can be found that our event-triggering scheme can provide a larger average release interval than those obtained by the existing ones in [13], [14], [12], [3]. The release instants and release intervals are shown in Fig. a.

The state responses of system (16)-(18) with feedback gain (30) and trigger matrix (31) are shown in Fig. b.

For the given feedback gain (30), applying Theorem 1 with $\sigma = 0$, the upper bound of τ_M is solved as 0.1989. Suppose $\tau_k \equiv 0$, letting $\bar{\tau} = 0$ and from $\tau_M = h + \bar{\tau}$, the maximum allowed sampling period can be obtained as

0.1989. From Remark 1, when $\sigma = 0$, the event-triggered scheme reduces to the periodic release scheme. Compared 0.1989 with the result 0.7729 obtained above, the proposed event-triggered scheme can reduce much signal transmission and thus reduce the network burden than the periodic release scheme.

Using Theorem 3 with $\rho = 0.53$ and $\sigma = 0.3$, the upper bound for τ_M is computed as 0.14 and the corresponding feedback gain and the trigger matrix are

$$K = \begin{bmatrix} 0.0051 & 0.4089 & 247.1533 & 135.4933 \end{bmatrix} \quad (32)$$

$$\Omega = \begin{bmatrix} 1.1376 & -1.8096 & -3.3038 & 6.0318 \\ -1.8096 & 3.3143 & 6.0510 & -11.0477 \\ -3.3038 & 6.0510 & 11.0477 & -20.1702 \\ 6.0318 & -11.0477 & -20.1702 & 36.8255 \end{bmatrix} \quad (33)$$

If h is selected as 0.01, under the controller with feedback gain (32) and the trigger (4) with Ω in (33), the closed-loop system can tolerate the transmission delay bounded by 0.13.

Under Case 2. Suppose $\tau_k \equiv 0$ and $\Delta A_0 = 0$ and $\Delta B_0 = 0$ as in [14]. Using the feedback gain (30), applying Theorem 1 with $\rho = 0.53$, $\gamma = 200$ and $\sigma = 0.1$, the upper bound of τ_M is solved as 0.144 and the event-triggering matrix Ω as

$$\Omega = 10^6 \times \begin{bmatrix} 0.0001 & 0.0004 & 0.0113 & 0.0062 \\ 0.0004 & 0.0023 & 0.0664 & 0.0378 \\ 0.0113 & 0.0664 & 2.1190 & 1.1628 \\ 0.0062 & 0.0378 & 1.1628 & 0.6520 \end{bmatrix} \quad (34)$$

Setting $h = 0.144$ and under the controller with the feedback gain (30) and the algorithm (4) with Ω in (34), the average release period is shown to be 0.4608, which is larger than 0.2830 obtained in [14] using the same feedback gain (30).

For the case with parameter uncertainties, applying Theorem 3 with $\rho = 0.53$, $\gamma = 200$ and $\sigma = 0.1$, the upper bound of $\tau_M = 0.06$, the corresponding K and Ω are

$$K = 10^3 \times \begin{bmatrix} 0.0134 & 0.0919 & 1.0023 & 0.5687 \end{bmatrix}$$

$$\Omega = \begin{bmatrix} 0.0449 & 0.2177 & -0.0028 & -0.0644 \\ 0.2177 & 1.3902 & -0.0273 & -0.3900 \\ -0.0028 & -0.0273 & 0.0411 & -0.0630 \\ -0.0644 & -0.3900 & -0.0630 & 0.2325 \end{bmatrix} \quad (35)$$

Select the sampling period $h = 0.01$. Since $\tau_M = h + \bar{\tau}$, the allowable maximum transmission delay can be 0.05. Considering the effect of the transmission delay, the average release period is obtained as 0.2278, the state response of the system (1)-(2) with K and Ω in (35) are shown in Fig c. Fig d plots the release instants and release interval.

V. CONCLUSION

To reduce the communication load in the network, a novel event-triggering scheme has been proposed, which can be used to determine when the sampled signals by sensors will be transmitted. Under the event trigger, an event-triggered H_∞ control design method has been proposed for networked control systems with uncertainties and network-induced delay. A delay system model has been used to describe the prosperities of the event trigger and effects of the transmission delay on the system. Based on Lyapunov functional method, criteria for stability with an H_∞ norm bound and H_∞ control design have been obtained, which are expressed in the form of linear matrix inequalities. A simulation example has shown that, our event-triggering scheme can lead to a larger average release period than those by some existing methods. As the special case of the proposed event-triggering scheme, a periodic release scheme can be obtained by solving (20), (22) or (23) with $\sigma = 0$.

REFERENCES

[1] A. Anta and P. Tabuada, "To sample or not to sample: Self-triggered control for nonlinear systems," *IEEE Transactions on Automatic Control*, vol. 55, no. 9, pp. 2030–2042, 2010.

[2] K. Årzén, "A simple event-based PID controller," in *Proc. 14th IFAC World Congress*, vol. 18, 1999, pp. 423–428.

[3] D. Carnevale, A. Teel, and D. Nesic, "Further results on stability of networked control systems: a lyapunov approach," in *American Control Conference, 2007. ACC'07*, 2007, pp. 1741–1746.

[4] J. P. Hespanha, P. Naghshtabrizi, and Y. G. Xu, "A survey of recent results in networked control systems," *Proceedings of the IEEE*, vol. 95, no. 1, pp. 138–162, 2007.

[5] M. Lemmon, T. Chantem, X. Hu, and M. Zyskowski, "On self-triggered full-information H-infinity controllers," *Hybrid Systems: Computation and Control*, pp. 371–384, 2007.

[6] J. Lunze and D. Lehmann, "A state-feedback approach to event-based control," *Automatica*, vol. 46, no. 1, pp. 211–215, 2010.

[7] M. Mazo and P. Tabuada, "Input-to-state stability of self-triggered control systems," in *Decision and Control, Proceedings of the 48th IEEE Conference on*, 2010, pp. 928–933.

[8] M. Mazo Jr, A. Anta, and P. Tabuada, "An ISS self-triggered implementation of linear controllers," *Automatica*, vol. 52, pp. 1310–1314, 2010.

[9] D. Seto, J. Lehoczky, L. Sha, and K. Shin, "On task schedulability in real-time control systems," in *Real-Time Systems Symposium, 1996., 17th IEEE*, 1996, pp. 13–21.

[10] J. Sijs and M. Lazar, "On event based state estimation," *Hybrid Systems: Computation and Control*, pp. 336–350, 2009.

[11] P. Tabuada, "Event-triggered real-time scheduling of stabilizing control tasks," *IEEE Transactions on Automatic Control*, vol. 52, no. 9, pp. 1680–1685, 2007.

[12] P. Tabuada and X. Wang, "Preliminary results on state-trigered scheduling of stabilizing control tasks," in *Proc. of the 45th IEEE Conference on Decision and Control*, 2006, pp. 282–287.

[13] X. Wang, "Event-Triggering in Cyber-Physical Systems," *PhD thesis, 2009, Department of Electrical Engineering, University of Notre Dame, Notre Dame, Indiana, USA.*, 2009.

[14] X. Wang and M. Lemmon, "Self-triggered feedback control systems with finite-gain L2 stability," *IEEE Transactions on Automatica Control*, vol. 45, no. 3, pp. 452–467, 2009.

[15] X. Wang and M. Lemmon, "Self-triggering under state-independent disturbances," *IEEE Transactions on Automatic Control*, vol. 55, no. 6, pp. 1494–1500, 2010.

[16] Z. Wang, F. Yang, D. W. C. Ho, and X. Liu, "Robust H_∞ control for networked systems with random packet losses," *IEEE Transactions on Systems, Man and Cybernetics, (Part B)*, vol. 37, pp. 916–924, 2007.

[17] T. Yang, "Networked control system: a brief survey," *IEE Proc.-Control Theory Appl*, vol. 153, no. 4, pp. 403–412, 2006.

[18] D. Yue, Q.-L. Han, and J. Lam, "Network-based robust H_∞ control with systems with uncertainty," *Automatica*, vol. 41, pp. 999–1007, 2005.

[19] D. Yue, E. Tian, Z. Wang, and J. Lam, "Stabilization of systems with probabilistic interval input delays and its applications to networked control systems," *IEEE Transactions on Systems, Man and Cybernetics, (Part A)*, vol. 39, pp. 939–945, 2009.

[20] P. V. Zhivoglyadov and R. H. Middleton, "Networked control design for linear systems," *Automatica*, vol. 39, pp. 743–750, 2003.