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New Rollover Index for Detection of Tripped and Un-Tripped Rollovers

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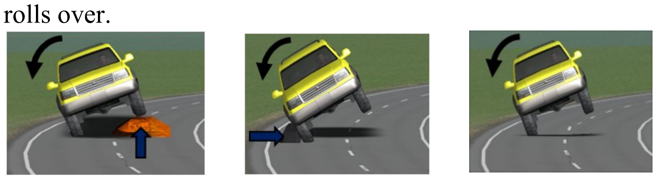
Abstract— **Accurate detection of the danger of an impending rollover is necessary for active vehicle rollover prevention. A real-time rollover index is an indicator used for this purpose. A traditional rollover index utilizes lateral acceleration measurements and can detect only un-tripped rollovers that happen due to high lateral acceleration from a sharp turn. It fails to detect tripped rollovers that happen due to tripping from external inputs such as forces when a vehicle strikes a curb or a road bump. Therefore, this paper develops a new rollover index that can detect both tripped and un-tripped rollovers. The new rollover index utilizes vertical accelerometers in addition to a lateral accelerometer and is able to predict rollover in spite of unknown external inputs acting on the system. The accuracy of the developed rollover index is evaluated with experimental tests on a 1/8th scaled vehicle. The experimental results show that the new rollover index can reliably detect both tripped and un-tripped rollovers.**

I. Introduction

Vehicles with increased dimensions and weights are known to be at higher risk of rollover. Normally, rollovers occur in one of two ways, tripped or un-tripped [1]. The two types of rollovers are shown in Figure 1. A tripped rollover happens due to tripping from external inputs. An example of this rollover happens when a vehicle leaves the roadway and slides sideways, digging its tires into soft soil or striking an object such as a curb or guardrail. An un-tripped rollover, on the other hand, happens due to high lateral acceleration from a sharp turn and not due to external inputs. An example of an un-tripped rollover is when a vehicle makes a sharp collision avoidance steering maneuver or a cornering maneuver at high speed, and consequently rolls over.



Rollover from vertical inputs — Rollover from lateral inputs

a) Tripped Rollover — b) Un-Tripped Rollover

Figure 1 Types of rollover

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Rollover accidents are dangerous. According to NHTSA's records (http://www.safercar.gov), although there were nearly 11 million crashes in 2002, only 3% involved a rollover. However, there were more than 10,000 deaths in rollover crashes in 2002. Thus, rollovers caused nearly 33% of all deaths from passenger vehicle crashes. In addition, NHTSA data also shows that 95% of single-vehicle rollovers are tripped while un-tripped rollovers occur less than 5% of the time.

Active roll prevention is a vehicle stability control system that prevents vehicles from un-tripped rollovers. It has been developed by several automotive manufacturers, e.g. Ford [2] and Volvo. To the best of the authors' knowledge, there are no assistance systems currently available that directly address tripped rollovers. Several types of actuation systems can be used in rollover prevention. The differential braking system has received the most attention from researchers [6,], [7], [8], [9] and is used for preventing rollovers by reducing the yaw rate of a vehicle and its speed. Also, steer-by-wire and active suspension systems can be potentially used to prevent rollovers.

In order to make these systems effective in their tasks, the accurate detection of the danger of a vehicle rollover is necessary [3], [4]. To detect a vehicle rollover, many researchers have developed a real-time index that provides an indication of the danger of rollover. However, they have focused on developing an indicator only for un-tripped rollovers. There are no currently published papers that have studied how to detect tripped vehicle rollovers with external inputs.

This paper focuses on developing a new rollover index that can detect both tripped and un-tripped rollovers. To begin with, let us introduce the vehicle rollover index in section 2. Then the developed new rollover indices are presented in section 3. Experimental results are shown in section 4. Finally, the conclusions are presented in section 5.

II. Vehicle Rollover Index

Accurate detection of the danger of a vehicle rollover is important. Initially, the concept of a static rollover threshold called the static stability factor (SSF) [5] was studied to quantify the propensity of a vehicle to rollover. However, the SSF by itself is not adequate for rollover prediction in dynamic situations. After that the concept of a rollover index has been introduced. A rollover index has also been known by other names such as Roll Safety Factor (RSF) and Load Transfer Ratio (LTR). A rollover index is a real-time variable used to detect wheel lift off conditions. Many researchers have tried to develop a rollover index that accurately predicts vehicle rollovers for the un-tripped case. The standard rollover index is based on a scaled lateral acceleration signal [7], [8]. References [4] and [9] have described a rollover index using a model-based roll angle estimator. Reference [9] has combined a rollover index with robustness to influential factors such as the vehicle's center of gravity. Reference [3] has developed a rollover index that utilizes predictive information from the driver's steering input, in addition to a lateral accelerometer signal. Even though there are many types of rollover indices, they are derived from the same basic model as shown in Figure 2. The basic model involves the roll angle degree-of-freedom ϕ of a vehicle body.

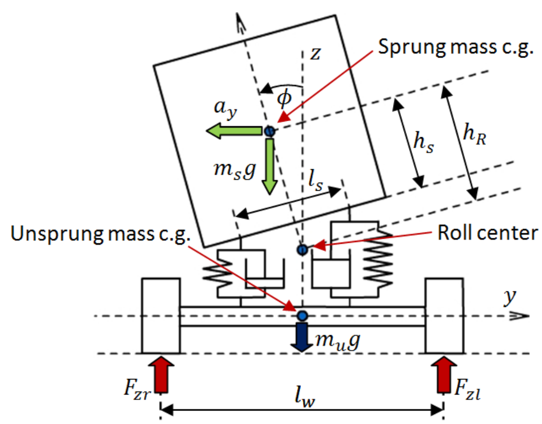


Figure 2 Un-tripped rollover model

The fundamental definition of the rollover index is described by equation (1).

$$R = \frac{F_{zr} - F_{zl}}{F_{zr} + F_{zl}}, \qquad -1 \le R \le 1 \tag{1}$$

where F_{zr} and F_{zl} are the right and left vertical tire forces of a vehicle respectively. A vehicle is considered to roll over when R is more than 1 or less than -1. Wheel lift-off occurs when R is equal to 1 or -1. It should be noted that when a vehicle is traveling straight, F_{zr} equals to F_{zl} and $R = 0$.

The definition of R in equation (1) cannot be implemented in real-time because the vertical tire forces F_{zr} and F_{zl} cannot be measured. Using the 1-degree of freedom model in Figure 2, the summation and difference of tire forces $F_{zr} + F_{zl}$ and $F_{zr} - F_{zl}$ can be calculated. An implementable version of the rollover index R can then be calculated in terms of ϕ and a_y. Such an example of a traditional rollover index calculated using a one degree of freedom is shown below in equation (2).

$$R = \frac{F_{zr} - F_{zl}}{F_{zr} + F_{zl}} = \frac{2m_s a_y h_R}{mgl_w} + \frac{2m_s h_R tan\phi}{ml_w} \tag{2}$$

where $m = m_s + m_u$, h_R is c.g. height, m_u is unsprung mass, m_s is spung mass, a_y is lateral acceleration, and ϕ is roll angle.

This type of rollover index is used for detecting un-tripped rollovers only. It is a function of lateral acceleration and roll angle. Some papers have proposed a rollover index that uses only lateral acceleration [6] since roll angle is expensive to measure. The stability control with this rollover index may arbitrarily reduce lateral acceleration capability of the vehicle. Also, as we shall show, it still fails to detect rollovers when rollovers are induced by vertical road inputs or other external inputs.

In order to detect tripped rollovers, which happens due to tripping from external inputs, a new rollover index should include the influence of road and other external inputs.

III. New Rollover Index for Tripped and Un-Tripped Rollovers

In this section, a new version of the rollover index is presented. The new rollover index can be computed without knowing any of the following variables: the road input disturbances, z_{rr} and z_{rl}, the vertical displacements of unsprung masses, z_{ur} and z_{ul}, the vertical displacement of sprung mass, z_s, and the unknown lateral force input, F_{lat}. In order to obtain the rollover index for predicting tripped rollovers, we need a model of a vehicle with 4-degrees of freedom which is shown in Figure 3.

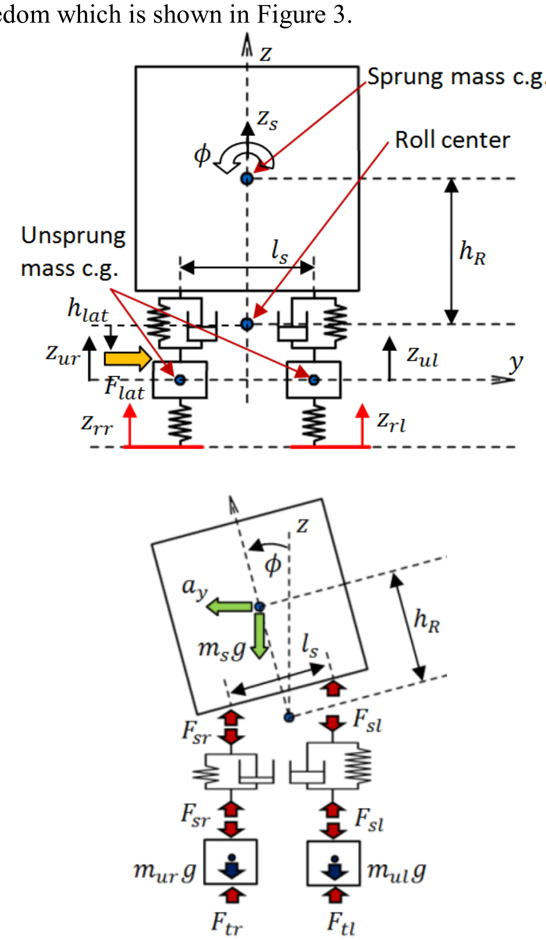


Figure 3 Four-degrees of freedom vehicle model

The vehicle body is represented by the sprung mass m_s while the mass due to the axles and tires are represented by unsprung masses m_{ul} and m_{ur}. The springs and dampers between the sprung and unsprung masses represent the vehicle suspension. The vertical tire stiffness of each side of the vehicle are represented by the springs k_{tr} and k_{tl}.

The 4-degrees of freedom of the model are the heave z_s, roll angle ϕ of the vehicle body, and the vertical motion of each side of the unsprung masses, z_{ur} and z_{ul}. The variables z_{rr} and z_{rl} are the road profile inputs that excite the system.

The external inputs z_{rl}, z_{rr}, and F_{lat} cannot be measured and are unknown. However, outputs that depend on these unknown inputs are available for measurement. For example, vertical and lateral accelerations of the vehicle can be measured using accelerometers placed on the vehicle body. These vertical and lateral accelerations can be related to the unknown inputs and to the states of the system using algebraic equations. This section develops equations for the external inputs z_{rl}, z_{rr}, and F_{lat} using measured accelerometers signals and the states of the vehicle model.

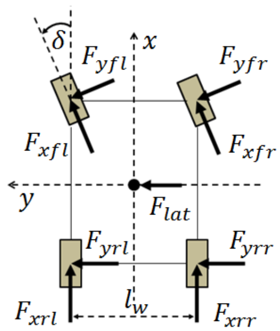


Figure 4 Lateral vehicle dynamic

Consider the vehicle lateral dynamics as shown in Figure 4. The measurement of lateral acceleration a_y involves the influence of the lateral tire forces and the unknown external lateral force F_{lat}. The measured lateral acceleration a_y is given by

$$a_y = \frac{(F_{yr} + F_{xf}\sin(\delta) + F_{yf}\cos(\delta) + F_{lat})}{m} \tag{3}$$

where $m = m_s + m_u$, F_{xf} is the longitudinal tire forces of the front wheels, F_{yf} and F_{yr} are the lateral tire forces of the front and rear wheels respectively, δ is steering angle, and F_{lat} is the unknown external lateral force.

Assume that the suspension forces always act perpendicular to the sprung mass. Then, the dynamic equations of sprung mass heave and sprung mass roll motions are given by equations (4) and (5) respectively.

$$m_s\ddot{z}_s = F_{sr} + F_{sl} - m_s g, \tag{4}$$

$$\begin{aligned}(I_{xx} + m_s h_R^2)\ddot{\phi} = \frac{l_s}{2}(F_{sl} - F_{sr}) + m_s a_y h_R cos\phi \\ + m_s g h_R sin\phi\end{aligned} \tag{5}$$

where I_{xx} is moment inertia.

The dynamic models of the unsprung mass motions are given by

$$m_{ur}\ddot{z}_{ur} = -F_{sr} + F_{tr} - m_{ur} g, \tag{6}$$

$$m_{ul}\ddot{z}_{ul} = -F_{sl} + F_{tl} - m_{ul} g \tag{7}$$

where F_{tr} and F_{tl} are left and right vertical tire forces respectively.

From unsprung mass equations, the vertical tire forces are seen to be given by

$$F_{tr} = m_{ur}\ddot{z}_{ur} + F_{sr} + m_{ur} g, \tag{8}$$

$$F_{tl} = m_{ul}\ddot{z}_{ul} + F_{sl} + m_{ul} g. \tag{9}$$

Since the vertical tire forces F_{tr} and F_{tl} equal to the normal forces F_{zr} and F_{zl}, the rollover index can be written as

$$\begin{aligned}R &= \frac{F_{zr} - F_{zl}}{F_{zr} + F_{zl}} = \frac{F_{tr} - F_{tl}}{F_{tr} + F_{tl}} \\ &= \frac{m_{ur}\ddot{z}_{ur} + F_{sr} + m_{ur} g - m_{ul}\ddot{z}_{ul} - F_{sl} - m_{ul} g}{m_{ur}\ddot{z}_{ur} + F_{sr} + m_{ur} g + m_{ul}\ddot{z}_{ul} + F_{sl} + m_{ul} g}.\end{aligned} \tag{10}$$

If $m_{ur} = m_{ul} = m_u$, then the rollover index becomes

$$R = \frac{m_u(\ddot{z}_{ur} - \ddot{z}_{ul}) + F_{sr} - F_{sl}}{m_u(\ddot{z}_{ur} + \ddot{z}_{ul}) + F_{sr} + F_{sl} + 2m_u g} \tag{11}$$

where F_{sr} and F_{sl} are suspension forces. However, F_{sr} and F_{sl} are unknown and cannot be measured. We need to replace these unknown variables.

To determine suspension forces F_{sr} and F_{sl}, we consider equations (4) and (5).

$$\begin{aligned}m_s\ddot{z}_s &= F_{sr} + F_{sl} - m_s g \\ (F_{sr} + F_{sl}) &= m_s\ddot{z}_s + m_s g\end{aligned} \tag{12}$$

$$\begin{aligned}(I_{xx} + m_s h_R^2)\ddot{\phi} &= \frac{l_s}{2}(F_{sl} - F_{sr}) + m_s a_y h_R cos\phi + m_s g h_R sin\phi \\ (F_{sl} - F_{sr}) &= \frac{2}{l_s}[(I_{xx} + m_s h_R^2)\ddot{\phi} - m_s a_y h_R cos\phi - m_s g h_R sin\phi]\end{aligned} \tag{13}$$

$\ddot{z}_s$ can be measured by using an accelerometer and ϕ can be obtained by estimation. However, $\ddot{\phi}$ is still unknown and cannot be measured.

To measure $\ddot{\phi}$, we need to place two extra accelerometers at the right and left ends on a vehicle sprung mass. The location of the extra accelerometers is shown on Figure 5.

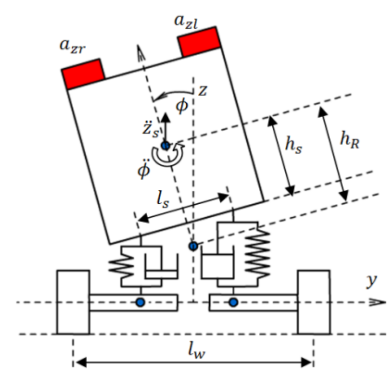


Figure 5 Extra accelerometer locations

The right accelerometer measurement a_{zr} is given by

$$a_{zr} = \ddot{z}_s cos\phi - \frac{l_s}{2}\ddot{\phi} + (\dot{y} + v_x r)sin\phi + g cos\phi. \tag{14}$$

The left accelerometer measurement a_{zl} is given by

$$a_{zl} = \ddot{z}_s cos\phi + \frac{l_s}{2}\ddot{\phi} + (\dot{y} + v_x r)sin\phi + g cos\phi. \tag{15}$$

It should be noted that the term $(\dot{y} + v_x r)$ includes the influence of the unknown lateral force F_{lat}.

Subtract equation (15) from (14).

$$a_{zl} - a_{zr} = l_s\ddot{\phi}, \qquad \ddot{\phi} = \frac{a_{zl} - a_{zr}}{l_s} \tag{16}$$

With equation (16), the equation (13) can be rewritten as

$$(F_{sl} - F_{sr}) = \frac{2}{l_s}\left[(I_{xx} + m_s h_R^2)\left(\frac{a_{zr} - a_{zl}}{l_s}\right) - m_s a_y h_R cos\phi - m_s g h_R sin\phi\right]. \tag{17}$$

Place equations (12) and (17) into (11). Then, the rollover index for this case is

$$R = \frac{m_u(\ddot{z}_{ur} - \ddot{z}_{ul}) - \frac{2}{l_s^2}(I_{xx} + m_s h_R^2)(a_{zl} - a_{zr})}{m_u(\ddot{z}_{ur} + \ddot{z}_{ul}) + m_s\ddot{z}_s + mg} + \frac{\frac{2}{l_s} m_s a_y h_R cos\phi + \frac{2}{l_s} m_s g h_R sin\phi}{m_u(\ddot{z}_{ur} + \ddot{z}_{ul}) + m_s\ddot{z}_s + mg} \tag{18}$$

where $m = m_s + 2m_u$, $(\ddot{z}_{ur} - \ddot{z}_{ul})$ is the difference between unsprung mass accelerations, $(a_{zl} - a_{zr})$ is the difference between sprung mass accelerations, a_y is lateral acceleration, and ϕ is roll angle. These variables can be measured or estimated. Since the new rollover index involves the term $(a_{zl} - a_{zr})$ or $\ddot{\phi}$, the new rollover index can handle both unknown external lateral force inputs and unknown road inputs with the same algorithm.

Note: The term involving $sin\phi$ can be ignored at small roll angles. However, it becomes important at large roll angles. In particular, if roll angle is higher for a given lateral acceleration (e.g. for higher c.g. vehicles), then the term $sin\phi$ is important and must be considered in the rollover index calculation. The roll angle may not be easily measured. However, it can be estimated from an estimation algorithm [10].

IV. Experiments with a Scaled Vehicle

The use of a full-sized vehicle for testing of a control system in this rollover application is challenging due to cost limitations and safety issues.

It is more convenient to use a scaled vehicle to test the roll control system. A scaled vehicle is inexpensive and safe for evaluation of rollover maneuvers.

Many researchers have studied scaled vehicles for testing of vehicle dynamics and control systems. For instance, references [11], and [12] developed a 1/10th scaled vehicle to study lateral vehicle dynamics. [13] developed a 1/8th scaled vehicle to study longitudinal vehicle dynamics. References [14] and [15] studied stability control algorithms with a scaled vehicle. Reference [16] studied tire characteristics with scaled tries. Reference [17] presents the Buckingham π theorem that can be used to study if a scaled vehicle has dynamics similar to a full-sized vehicle.

In order to use a scaled vehicle to describe the behavior of a full-sized vehicle, we need to show that they have dynamic similarity. This has been shown for the scaled vehicle described in the next sub-section, but is not presented here due to lack of space

Table 1 Vehicle Variables and Parameters

Variables and Parameter	Scaled Vehicle	Full-Sized vehicle
m_s (kg)	3	1600
m_u (kg)	0.2	135
I_{xx} (kg.m^2)	0.04	600
k (N/m)	900	90000
d (N.sec/m)	15	3000
l_s (m)	0.2	1.11
h_R (m)	0.18	1
k_t(N/m)	4000	400000

The variables and parameters of the scaled vehicle and of a full-sized vehicle are shown in Table 1.

Note: The parameters of the full-sized vehicle are obtained from the software CARSIM.

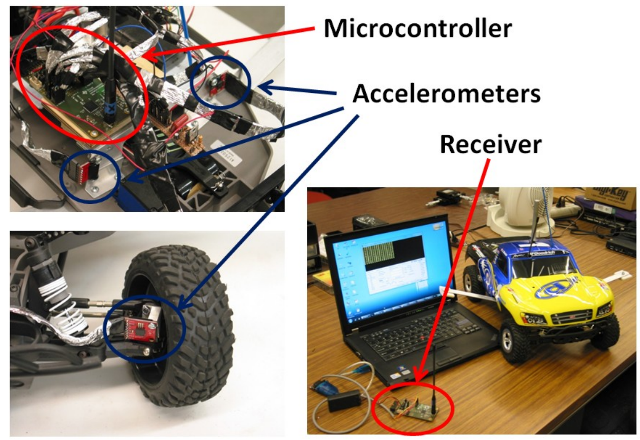


Figure 6 Microcontroller, Sensors and Scaled Vehicle 1:8

Next, we describe the experimental setup to test rollover scenarios and present the experimental results in the following subsections.

A. Experimental Set Up

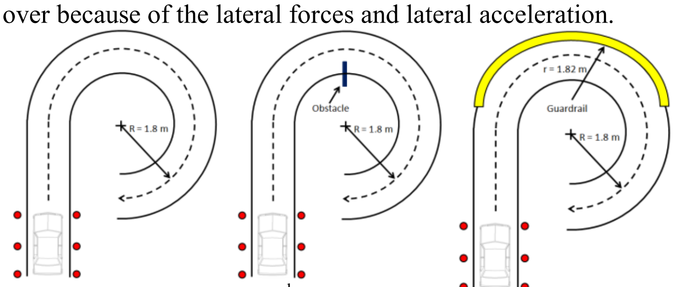
A photograph of the scaled vehicle is shown in Figure 6.

The developed rollover index will be validated with the scaled vehicle since the roll and vertical dynamics of it are similar to those of a full-size vehicle. A microcontroller (EM430F6137RF900) from Texas Instruments is used for real-time data acquisition and control of the scaled vehicle speed and steering. The microcontroller is installed on the test vehicle. It samples the data from sensors and wirelessly sends them to a receiver connected to a computer at the baud rate of 66 Hz. Three 3-axis accelerometers (MMA7260Q) from Freescale Semiconductor and a dual axis gyroscope (LPY530AL) from STMicroelectronics are used for this test to measure acceleration and angle rate, respectively. Two accelerometers installed on the left and right size of the sprung mass are used to measure the left and right vertical accelerations. Another accelerometer is placed on the front left wheel, an unsprung mass, for measuring longitudinal, lateral, and vertical accelerations. Also, the dual axis gyroscope placed near the c.g. of the vehicle is used to measure yaw rate and roll rate. The photographs of the microcontroller and the sensors are shown in Figure 6.

To evaluate the rollover index, the scaled vehicle speed and steering inputs are programmed in the microcontroller. So, the identical experiment can be repeated many times. For the first experiment, we set the vehicle to follow the path as shown in Figure 7a, at a speed of approximately 2.4 meter per second. In this case, the wheels of the scaled vehicle come close to lifting off from the ground.

For the second experiment, we put an obstacle in the path of the vehicle, as shown in Figure 7b. The size of the obstacle is 2.54 centimeters in height and 2.54 centimeters in width. Then, we set the vehicle to follow the path at a speed of approximately 2.4 meter per second. In this case, the right wheels of the scaled vehicle fully lift off.

For the third experiment, we want to evaluate the developed rollover index in the case that the vehicle is confronted with unknown lateral forces. We set a guardrail on the route of the vehicle as shown in Figure 7c. The size of the guardrail is 3.9 centimeters in height and 1.82 meters in radius. Then, we set the vehicle to follow the path at a speed of approximately 2.4 meter per second. However, it is difficult to experimentally have only unknown lateral forces applied to the vehicle. When the vehicle strikes the guardrail, the front left wheel of the vehicle confronts with both unknown lateral forces and unknown vertical forces. Therefore, in this experiment, when the vehicle strikes the guardrail, the vehicle firstly leans toward the inside of the curve because of the vertical forces. After that, the vehicle leans back toward the outside of the curve and fully rolls over because of the lateral forces and lateral acceleration.



a) 1st experiment b) 2nd experiment c) 3rd experiment

Figure 7 The scaled vehicle path

B. Experimental Results

The signals required for computing the new rollover index are the lateral acceleration, the left and right vertical accelerations of the sprung mass and of the unsprung masses, and roll angle. Since the mass of unsprung masses of the scaled vehicle is very small, it is reasonable to neglect the left and right vertical accelerations of unsprung masses. Also, the road input is a bump. The roll angle can be assumed to be small for the scaled vehicle. Then, the simplified rollover indices we examine are shown in equations (20) and (21).

$$R_1 = \frac{2m_s a_y h_R}{mgl_w} \tag{20}$$

$$R_2 = \frac{-\frac{2}{l_s^2}(I_{xx} + m_s h_R^2)(a_{zl} - a_{zr}) + \frac{2}{l_s} m_s a_y h_R}{mg} \tag{21}$$

The results of the experiments are shown in Figure 8 – Figure 10. As seen in Figure 8, the longitudinal and lateral accelerations in all three experiments are similar and of the same order, because of the same setting in all experiments. For the first and second experiments, the left vertical accelerations are close to zero. However, the right vertical acceleration from the second experiment is larger than that from the first experiment.

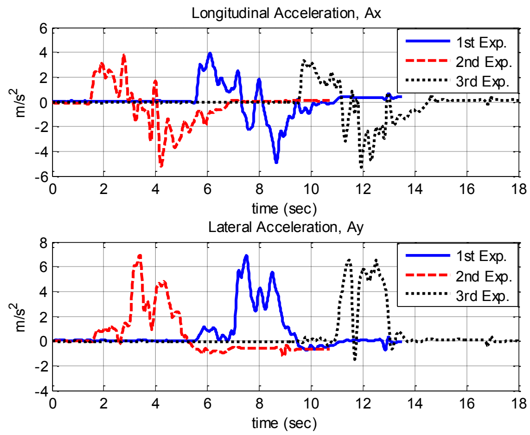


Figure 8 Longitudinal and lateral accelerations of the scaled vehicle

The left and right vertical accelerations of the third experiment can be seen in Figure 9. When the vehicle strikes the guardrail during the time, $t \approx 11.4 - 11.7$ seconds, the vertical forces apply to the left side of the vehicle. This makes the left vertical acceleration larger than the right vertical acceleration. During the time, $t \approx 11.7 - 12$ seconds, the left vertical acceleration decreases and the right acceleration increases since the vehicle leans back toward the outside of the curve and fully rolls over after the time $t > 12$ seconds. It should be noted that roll angle during the time, $t \approx 12 - 13$ seconds is very large. So the measurements during this period may not be accurate.

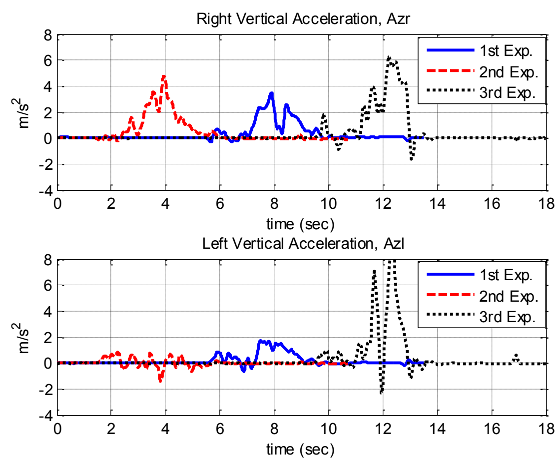


Figure 9 Right and left vertical acceleration of the scaled vehicle

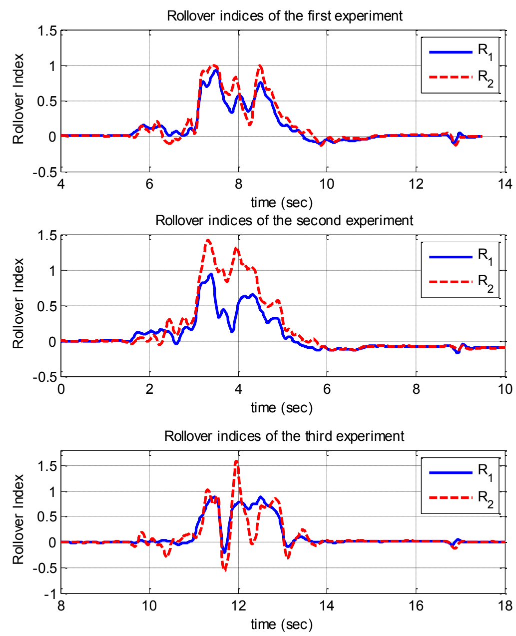


Figure 10 Comparison of rollover indices of the scaled vehicle

The first row of Figure 10 shows the rollover indices from the first experiment. In this experiment, there are no external inputs applied to the vehicle. So, the difference of vertical accelerations is small. Hence, the rollover indices from the equations (20) and (21) are almost the same. Both rollover indices show that the wheels of the vehicle are close to lift off, even though there are no external inputs acting on the system.

Likewise, the second row of Figure 10 shows the rollover indices from the second experiment. The scaled vehicle strikes the obstacle and its right wheels lift off in this case. Thus, there is a difference of vertical accelerations. The traditional rollover index R_1 in equation (20) shows that the wheels of the vehicle come close to lift off. It fails to actually detect the wheel lift off condition. However, the new rollover index R_2 of equation (21) shows that the wheels of the vehicle do lift off. Therefore, the developed rollover index is able to detect both tripped and un-tripped rollovers.

The third row of Figure 10 shows the rollover indices from the third experiment. In this case, the vehicle leans toward the inside of the curve. Then the vehicle leans back toward the outside of the curve and fully rolls over. The results show that the traditional rollover index R_1 fails to detect the wheel lift off condition for this scenario also. However, the new rollover index R_2 shows that the wheels of the vehicle do lift off. Therefore, the new rollover index can also handle unknown external lateral force inputs with the same algorithm.

V. CONCLUSIONS

This paper developed a new rollover index that can detect both tripped and un-tripped rollovers. The new rollover index utilizes vertical accelerometers in addition to a lateral accelerometer and is able to predict rollover in spite of unknown external inputs acting on the system. The accuracy of the developed rollover index is evaluated with experimental tests on a $1/8^{th}$ scaled vehicle. The experimental results show that the new rollover index can reliably detect tripped and un-tripped rollovers. This is the first ever publication on a rollover index that can handle tripped rollovers.

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